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# THE PRELIMINARY CHOICE OF DESIGN PARAMETERS FOR NEUTRON CHOPPERS

by

George S. Stanford

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George S. Stanford

Reactor Engineering Division

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# TABLE OF CONTENTS

## LIST OF FIGURES



## TABLE

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## ABSTRACT

Some of the factors affecting the counting rate achieved by a neutron chopper spectrometer are investigated. It is shown that in a chopper of optimum design the counting rate per channel at any given energy is proportional both to the slit width and to the square of the resolution in $\mu$sec/m, as well as to the cutoff velocity $v_c$ of the rotor; this last is because a larger number of bursts per second become possible as $v_c$ is increased. For an idealized cutoff function, it is shown that the total running time for measuring a spectrum is minimum when the ratio of cutoff velocity to peripheral velocity of the rotor is such that approximately 1.67 runs per energy decade are required; this means that the chopper speed and the time-channel width are changed by a factor of two from one run to the next.

## I. INTRODUCTION

In designing a neutron chopper for a specific purpose, one of the predetermined quantities is usually the maximum (i.e., worst) permissible energy resolution $\Delta E/E$. Also, it is usually important that the data be obtainable in as short a time as possible, since rarely is a chopper spectrometer plagued by too many neutrons. Therefore, it is appropriate to relate the counting rate achieved by a chopper system to the resolution, in order to see the effect on the ultimate counting rate of the various design parameters.

The burst shapes and cutoff functions of various kinds of neutron chopper have been discussed in detail by several authors.(1-7) The purpose of the present paper is to provide some criteria to use in planning the gross characteristics of a chopper, so that the range of choice can be narrowed considerably before detailed calculations are made. Accordingly, several approximations are made which would not be used in determining the final designs: the fact is neglected that the mean time of arrival of neutrons of given velocity is to some extent a function of the lateral portion of the detector, the approximation for the aperture of the chopper is better in some cases than in others, the effect of leakage of neutrons

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In designing a neutron chopper for a specific purpose, one of the predetermined quantities is usually the maximum (i.e., worst) permissible energy resolution $\Delta E/E$. Also, it is usually important that the data be obtainable in as short a time as possible, since rarely is a chopper spectrometer plagued by too many neutrons. Therefore, it is appropriate to relate the counting rate achieved by a chopper system to the resolution, in order to see the effect on the ultimate counting rate of the various design parameters.

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A basic assumption of this treatment is that the maximum rate of accumulation of data is desired.

## List of Symbols

| | |
|---|---|
| $a_e$ | Area of spectrometer entrance aperture, if only one slit is used at a time. If n slits are open at once, then $a_e$ is the area the entrance aperture would have if all slits but one were blocked off. |
| $a_x$ | Area of spectrometer exit aperture, if only one slit is used at a time. If n slits are open at once, then $a_x$ is the area the exit aperture would have if all slits but one were blocked off. |
| B | Fraction of running time during which no chopped neutrons reach the detector. |
| $c(\tau/\tau_c)$ | Cutoff function. |
| D | Diameter of rotor. |
| E | Kinetic energy of neutron. |
| $\Delta E/E$ | Energy resolution, after adjustment for effect of detector thickness. Defined by Eq. (3.9). |
| $(\Delta E/E)_t$ | True energy resolution. |
| h | Height of slit entrance. |
| H | Half-width of resolution function. |
| j | The fraction of $\tau_c$ which is defined by the point at which, due to the cutoff function, the data cease to be sufficiently precise, i.e., data for $\tau > j\tau_c$ are not useful. |
| $J_\omega(\tau)$; $J_\omega(E)$ | Instantaneous number of neutrons/($cm^2$-steradian-sec-ev) with reciprocal speed $\tau$ or energy E at the spectrometer source, travelling along the axis of the spectrometer. |

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H — Half-width of resolution function.

J — The fraction of [illegible] which is defined by the point at which, due to the [illegible], the data cease to be sufficiently precise, i.e., data for $\tau/\tau_c$ [illegible] not useful.

$I_e(\gamma)$; $I_e(E)$ — Instantaneous number of neutrons/(cm²-second-sec-ev) with reciprocal speed γ or energy E at the spectrometer [illegible], travelling along the axis of the spectrometer.

| | |
|---|---|
| $K$; $K(\tau)$ | Spectrometer constant, defined by Eqs. (4.1) and (4.2). |
| $\ell$ | Distance from entrance aperture to exit aperture. In the special case of Fig. 2, $\ell$ and $\ell_f$ are the same. |
| $\ell_d$ | Thickness of detector. |
| $\ell_f$ | Length of flight path. |
| $m$ | Mass of neutron. |
| $n$ | Number of slits used per neutron burst. |
| $N$ | Number of chopper runs (at different speeds) required to cover a specified energy interval. |
| $p$ | Peripheral velocity of rotor. |
| $r$ | Number of neutron bursts per unit time. |
| $R(E)$ | Counting rate per unit energy. |
| $R(\tau)$ | Counting rate per unit reciprocal speed. |
| $R_c$; $R_c(\tau)$ | Counting rate in the time channel corresponding to reciprocal speed $\tau$. |
| $R_E \equiv (\Delta E/E)_{max}$ | Maximum permissible energy resolution. |
| $R_{es}$ | Spectrometer resolution in microseconds per meter. $R_{es}$ is a function of the speed of the rotor. |
| $s$ | Width of slit entrance. (Effective width, if neutron leakage through the slit edges is appreciable.) |
| $S$ | Specific area of chopper, defined by Eq. (1.2). |
| $T$; $T'$ | Quantities proportional to the total running time required to cover a specified energy region. $T$ and $T'$ are defined by Eqs. (4.8) and (4.10). |
| $v_c$ | Cutoff velocity; i.e., velocity of slowest neutron accepted by spectrometer. |
| $w$ | Duration (full width at half maximum) of burst of neutrons passed by chopper. |

| | |
|---|---|
| $\alpha \equiv \mu j \tau_{c1}/\tau_{01}$ | A measure of the time-of-flight interval which is to be covered in a single run, after subtraction of the overlap region. |
| $\beta$ | A constant for a given spectrometer which relates burst duration, slit width, and peripheral velocity according to Eq. (3.6). |
| $\gamma$ | A constant relating the resolution and the burst width, defined by Eq. (3.5). |
| $\delta$ | Duration of a channel in the time analyzer. |
| $\epsilon(\tau)$ | Efficiency of the detector. |
| $\mu$ | The extent of the overlap of adjacent runs, defined by Eq. (4.5). |
| $\nu$ | The number of neutron bursts per revolution of the chopper. |
| $\tau$ | Reciprocal speed of neutron. |
| $\tau_0$ | The minimum value of $\tau$, for a given chopper speed, consistent with the resolution requirements. |
| $\tau_c \equiv 1/v_c$ | The reciprocal speed of a cutoff neutron. |
| $\psi \equiv \delta/w$ | The ratio of time-channel duration to burst half-width. |
| $\Omega$ | The solid angle subtended at the entrance aperture by the exit aperture. |

Figure 1 shows a generalized schematic of the spatial aspects of a spectrometer, and Fig. 2 depicts some features of a diverging-slit chopper.

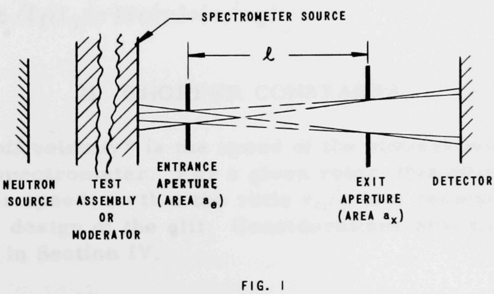


FIG. 1
GENERALIZED SPATIAL ASPECTS OF A NEUTRON SPECTROMETER

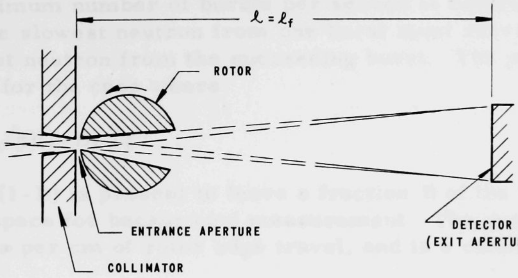


FIG. 2
DIVERGING-SLIT CHOPPER

It is shown in Ref. (8) that for a neutron chopper the counting rate per unit energy at any energy E is given approximately by

$$R(E) = S\epsilon(E)c(E/E_c)J_\omega(E) \quad , \tag{1.1a}$$

where the "specific area" S is given by

$$S \equiv na_e a_x \ell^{-2} rw \tag{1.2}$$

In terms of the variable $\tau$,

$$R(\tau) = S\epsilon(\tau)c(\tau/\tau_c)J_\omega(\tau) \quad . \tag{1.1b}$$

Equation (1.1b) gives the counting rate per unit reciprocal speed as a function of the reciprocal speed $\tau$. Under the assumption that the channel width in the time analyzer is the optimum fraction of the burst width (see below), it is the counting rate per channel $R_c(\tau)$ which determines the running time required for adequate statistics in the energy region around $\tau$. Since the reciprocal-speed interval represented by a time channel of duration $\delta$ is $\delta/\ell_f$, then $R_c(\tau) = R(\tau) \ \delta/\ell_f$, or

$$R_c(\tau) = (\delta/\ell_f)J_\omega(\tau)S\epsilon(\tau)c(\tau/\tau_c) \quad . \tag{1.3}$$

## II. CHOPPER CONSTANTS

The "cutoff velocity" is the speed of the slowest neutron which is accepted by the spectrometer. For a given rotor, this quantity is proportional to the rotor speed, so that the ratio $v_c/p$ is a constant whose value depends upon the design of the slit. Considerations affecting the choice of $v_c/p$ are treated in Section IV.

It is shown in Ref. (8) that for a neutron chopper the counting rate per unit energy at any energy E is given approximately by

$$R(E) = S\epsilon(E)c(E/E_c)J_0(E) \qquad (1.1a)$$

where the "specific area" S is given by

$$S = \eta a_s a_x \ell^{-2} r w \qquad (1.2)$$

In terms of the variable $\tau$,

$$R(\tau) = S\epsilon(\tau)c(\tau/\tau_c)J_0(\tau) \qquad (1.1b)$$

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The maximum number of bursts per second is determined by the condition that the slowest neutron from one burst must reach the detector before the fastest neutron from the succeeding burst. The practical maximum will occur for the case where

$$r/p = (v_c/p)(1-B)/\ell_f \quad , \tag{2.1}$$

where the term (1-B) is present to leave a fraction B of the time between bursts as dead space for background measurement. The quantity $r/p$ is the number of bursts per cm of rotor edge travel, and is a constant for the rotor.

The areas $a_e$ and $a_x$ of the entrance and exit apertures for each slit are to be determined for a rotor position such that one set of slits is parallel to the axis of the spectrometer. [See Fig. 2. Determination of the apertures is discussed more fully in Ref. (8). The effect of the rotation of the rotor is taken into account by means of the cutoff function.] We will restrict ourselves to the case where

$$a_e = hs \quad ; \tag{2.2}$$

that is, the slit defines the entrance aperture, which is usually true. (It is probably also usually true that the detector constitutes the exit aperture, as in Fig. 2, but the relationships to be derived do not require this restriction. The treatment that follows is equally applicable to the case where the slit defines the <u>exit</u> aperture, and the source constitutes the entrance aperture: it is necessary only to interchange $a_e$ and $a_x$.) It will be seen below that, subject to certain requirements, the slit width s is to be made as large as possible.

The solid angle $\Omega$ of the spectrometer is the solid angles subtended at the entrance aperture by the exit aperture. It is assumed that this is determined by experimental or practical considerations and will be the same regardless of the distance between the apertures; therefore we will use the relationship

$$a_x/\ell^2 = \Omega \quad . \tag{2.3}$$

Within reason, it is possible to construct a rotor to give any integral number of bursts per revolution $\nu$. For a given rotor diameter, it is desirable to use the maximum $\nu$ consistent with the cutoff velocity and the flight path. We can write

$$\nu = \pi Dr/p \quad , \tag{2.4}$$

where $p/\pi D$ is the number of revolutions per second. It will be kept in mind that $\nu$ must be integral. In view of Eq. (2.1),

$$\nu = (\pi D/\ell_f)(\nu_c/p)(1-B) \quad . \tag{2.5}$$

The maximum number of bursts p[illegible] is determined by the condition that the slowest neutron from one [illegible] reach the detector before the fastest neutron from the succeeding [illegible]. The [illegible] maximum will occur for the case where

$$r/p = (v_c/p)(1 - B)/\ell \qquad (2.1)$$

where the term (1 - B) is provided to leave a fraction B of the time between bursts as dead space for [illegible] measurement. The quantity [illegible] is the number of bursts per [illegible] edge travel, and is a constant for the rotor.

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$$a_e \leq a_x \qquad (2.2)$$

that is, the slit defines the entrance aperture, which is usually true. (It is probably also usually true that the detector constitutes the exit aperture, as in Fig. 2, but the relationships to be derived do not require this restriction. The development that follows is equally applicable to the case where the slit defines the exit aperture, and the source constitutes the entrance aperture. It is necessary only to interchange $a_e$ and $a_x$.) It will be seen below that, subject to certain requirements, the slit width s is to be made as large as possible.

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## III. RESOLUTION

We went to find the physical conditions which will maximize the counting rate per channel $R_c(\tau)$ at a given time-of-flight resolution $R_{es}$. It is necessary therefore to express $R_{es}$ in terms of the physical parameters of the system. A definition of the word "resolution" is required. One can consider a "resolution function" with components due to the burst width, time-channel width, detector thickness, etc. Frequently, the word "resolution" is used to mean the half-width H of this function. It should be realized, however, that two monoenergetic neutron lines separated by a reciprocal-speed interval $\Delta\tau$ equal to H would not be resolved: if the resolution function were Gaussian, an approximation which has been used,[5] the conventional Rayleigh criterion in optics would require $\Delta\tau \approx 1.14$ H. This question will be discussed further a little later.

The problem arises of determining the optimum relationship between the burst half-width w and the channel width $\delta$. Let

$$\delta = \psi w \quad , \tag{3.1}$$

where $\psi$ is a constant of proportionality. From Eqs. (1.2), (1.3), (2.1), and (3.1), one gets

$$R_c(\tau) = J_\omega(\tau) n a_e a_x \ell^{-2} v_c \epsilon(\tau) c(\tau/\tau_c)(1-B)\cdot\psi(w/\ell_f)^2 \quad . \tag{3.2}$$

To determine the optimum value of $\psi$, it is necessary to express the half-width H in terms of $\psi$ and w. If we approximate the burst and the time channel by Gaussian distributions of half-width w and $\psi$w, respectively, then

$$H^2 = w^2(\psi^2 + 1) \quad . \tag{3.3}$$

When this equation is solved for w and substituted in Eq. (3.2), there results an expression for $R_c(\tau)$ which is maximum for $\psi = 1$.

The burst and time channel, however, are not Gaussian. The latter is usually rectangular, and the former is roughly triangular. Neglecting the effect of the detector thickness, Mostovoi et al.,[2] present without derivation what is perhaps a better representation. Converted to the present notation, it is as follows:

$$H = w\left(1 + \frac{1}{4}\psi\right) \quad , \qquad \psi < 4/5 \tag{3.4a}$$

$$H = w\left(2 + \psi\left\{1 - [(4/\psi) - 1]^{1/2}\right\}\right) \quad , \qquad \psi > 4/5 \tag{3.4b}$$

where, by letting 2w be the full width of the burst, we have assumed a triangular burst shape. In this case after solving for w and substituting

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We want to find the physical conditions which will maximize the counting rate per channel $R_c(\tau)$ at a given time-of-flight resolution. It is necessary therefore to express $R_c$ in terms of the physical parameters of the system. A definition of the word "resolution" is required. One can consider a "resolution function" with components due to the burst width, time-channel width, detector thickness, etc. Frequently, the word "resolution" is used to mean the half-width H of this function. It should be realized, however, that two monoenergetic neutron lines separated by a reciprocal-speed interval $\Delta\tau$ equal to H would not be resolved. If the resolution function were Gaussian, an approximation which has been used,[5] the conventional Rayleigh criterion in optics would require $\Delta\tau \approx 1.14$ H. This question will be discussed further a little later.

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$$R_c(\tau) = J_\omega(\tau) n a_e a_x \ell^{-2} v \text{ [illegible] } (1-B) \text{ [illegible] } (w\,\tau_1)^2 \quad . \tag{3.2}$$

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in Eq. (3.2), one gets an expression for $R_c(\tau)$ which is maximum when $\psi \approx 1.3$. It is a broad maximum, with the counting rate for $\psi = 1$ being down by only ~4%.

Thus we can conclude that the ratio $\psi$ of channel width to burst half-width should be unity or somewhat greater, but that its value is not critical. The ratio must be known, however, if the resolution is to be calculated.

Now to return briefly to the meaning of the word "resolution": if one does some graphical experimenting with a triangular burst shape of half-width w and a rectangular time channel whose duration $\delta$ is equal to w (i.e., $\psi = 1$), it appears that two monoenergetic lines of equal intensity would not always be resolved unless $\Delta\tau > 2w$. Since in this case $H \equiv \sqrt{2}w$ (Eq. 3.3) or $H \equiv 1.27w$ (Eq. 3.4b), the two lines are reliably resolved only for $\Delta\tau > 1.4$ H or $\Delta\tau > 1.6$ H, depending on the definition of H. The word "resolved" is used here to imply that there would be between the two peaks at least one time channel containing fewer counts than the ones at the peaks.

We will avoid further controversy by letting

$$R_{es} = \gamma w / \ell_f \quad , \tag{3.5}$$

where $\gamma$ is an unspecified constant which leaves the reader complete freedom in selecting the value of $\psi$ and the functional dependence of H on w and $\psi$, as well as the definition of "resolution." Equation (3.5) contains the implicit condition that the channel duration is always adjusted to be the same fraction $\psi$ of the burst duration.

For any chopper, the duration of the burst will be proportional to the slit width and inversely proportional to the peripheral velocity. If w is the full width at half-maximum of the neutron burst, then

$$w = \beta s / p \quad . \tag{3.6}$$

The constant $\beta$ depends upon the type of spectrometer: for the single-ended type, $\beta = 1$; for the double-ended type, $\beta = \frac{1}{2}$, if the detector is narrow with respect to the neutron beam.[1] For the latter type of slit, however, it is implicit in a suggestion by Larsson et al.,[5] that the detector (or source) width should be adjusted so that $\beta \approx 2^{-1/2}$.

From Eqs. (3.5) and (3.6) it follows that

$$R_{es} = \beta\gamma s / p\ell_f \quad . \tag{3.7}$$

in Eq. (3.2), one gets an expression for $R_c(\psi)$ which is maximum when $\psi = 1.3$. It is a broad maximum, with the counting rate for $\psi = 1$ being down by only ~4%.

Thus we can conclude that the ratio $\psi$ of channel width to burst half-width should be unity or somewhat greater, but that its value is not critical. The ratio must be known, however, if the resolution is to be calculated.

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From Eqs. (3.5) and (3.6) it follows that

$$R_{es} = \beta\gamma s/p\ell_f \quad . \qquad (3.7)$$

The time-of-flight resolution $R_{es}$ is easily related to the energy resolution $\Delta E/E$: since $E = \frac{1}{2} m/\tau^2$, if $\Delta E/E$ is small we can write $\Delta E/E = 2\Delta\tau/\tau \equiv 2R_{es}/\tau$, so that

$$R_{es} = \frac{1}{2} \tau \Delta E/E \quad . \tag{3.8}$$

Until now we have assumed that the detector thickness $\ell_d$ is negligible with respect to $\ell_f$. If it is not, a convenient way to take it into account is to let $\Delta E/E$ be an adjusted energy resolution for use in the calculations. If $(\Delta E/E)_t$ is the actual resolution required, and if we use the Gaussian approximation, then $(\Delta E/E)_t^2 = (\Delta E/E)^2 + (2\ell_d/\ell_f)^2$, where $\ell_d/\ell_f$ is the fractional uncertainty in $\tau$ due to the detector length. Thus,

$$\Delta E/E = [(\Delta E/E)_t^2 - (2\ell_d/\ell_f)^2]^{1/2} \quad . \tag{3.9}$$

## IV. OPTIMUM CUTOFF POINT

Starting with Eq. (1.3) and substituting Eqs. (1.2), (2.1), (2.2), (2.3), (3.1), and (3.5), one can obtain

$$R_c(\tau) = (1-B)\psi\epsilon(\tau)c(\tau/\tau_c)h\gamma^{-2}J_\omega(\tau)\cdot\Omega nsv_c R_{es}^2 \quad . \tag{4.1}$$

The quantities preceding the dot are all predetermined, so we will write

$$R_c(\tau) = K(\tau)\cdot\Omega nsv_c R_{es}^2 \quad . \tag{4.2}$$

The reason for the presence of the cutoff velocity in Eqs. (4.1) and (4.2) is our assumption that the burst rate is proportional to it [see Eq. (2.1)]. Although the counting rate at any $\tau$ can be increased by designing for increased cutoff velocity (and burst rate), this is done at the expense of reducing the size of the energy region that can be covered in a single run. The problem thus arises of determining the optimum cutoff velocity for a given chopper speed - that is, of determining the optimum size of the energy region that is to be covered at any one chopper speed. For purposes of calculation we will make some simplifying assumptions:

(a) In recognition of the fact that the cutoff function $c(\tau/\tau_c)$ for any real chopper approaches zero gradually rather than abruptly as $\tau \rightarrow \tau_c$, it will be assumed that there is a fraction j such that, at any particular chopper speed, data for $\tau > j\tau_c$ are not useful. Specifically, we will let

$$\begin{aligned} c(\tau/\tau_c) &\equiv 1 \quad , \qquad \tau < j\tau_c \\ c(\tau/\tau_c) &\equiv 0 \quad , \qquad \tau > j\tau_c \quad , \end{aligned} \tag{4.3}$$

The time-of-flight resolution $R_{es}$ is easily related to the energy resolution $\Delta E/E$: since $E = \frac{1}{2}mv^2$. If $\Delta E/E$ is small we can write $\Delta E/E = 2\Delta v/v = 2R_{es}/\tau$, so that

$$R_{es} = \frac{1}{2}\tau\Delta E/E \qquad (3.8)$$

Until now we have assumed that the detector thickness $\ell_d$ is negligible with respect to $\ell_f$. If it is not, a convenient way to take it into account is to let $\Delta E/E$ be an adjusted energy resolution for use in the calculations. If $(\Delta E/E)_t$ is the actual resolution required, and if we use the Gaussian approximation, then $(\Delta E/E)_t^2 = (\Delta E/E)^2 + (2\ell_d/\ell_f)^2$, where $\ell_d/\ell_f$ is the fractional uncertainty in $\tau$ due to the detector length. Thus,

$$\Delta E/E = [(\Delta E/E)_t^2 - (2\ell_d/\ell_f)^2]^{\frac{1}{2}} \qquad (3.9)$$

## IV. OPTIMUM CUTOFF POINT

Starting with Eq. (1.3) and substituting Eqs. (1.2), (2.1), (2.2), (2.3), (3.1), and (3.5), one can obtain

$$R_c(\tau) = (1-B)\,[illegible]\,c(\tau/\tau_c)\,[illegible]\,\sin v_c R_{es}^2 \qquad (4.1)$$

The quantities preceding the [illegible] are all predetermined, so we will write

$$R_c(\tau) = K(\tau)\,\sin v_c R_{es} \qquad (4.2)$$

The reason for the presence of the cutoff velocity in Eqs. (4.1) and (4.2) is our assumption that the burst rate is proportional to it [see Eq. (2.1)]. Although the counting rate at any $\tau$ can be increased by designing for increased cutoff velocity (and burst rate), this is done at the expense of reducing the size of the energy region that can be covered in a single run. The problem thus arises of determining the optimum cutoff velocity for a given chopper speed - that is, of determining the optimum size of the energy region that is to be covered at any one chopper speed. For purposes of calculation we will make some simplifying assumptions:

(a) In recognition of the fact that the cutoff function $c(\tau/\tau_c)$ for any real chopper approaches zero gradually rather than abruptly as $\tau \rightarrow \tau_c$, it will be assumed that there is a fraction $j$ such that, at any particular chopper speed, data for $\tau > j\tau_c$ are not useful. Specifically, we will let

$$c(\tau/\tau_c) = 1 \;, \qquad \tau < j\tau_c$$

$$c(\tau/\tau_c) = 0 \;, \qquad \tau > j\tau_c$$

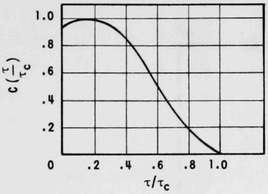


FIG. 3
TYPICAL CUTOFF FUNCTION (REF.9)

where the fraction j is a constant for a given rotor. (It is still necessary to design the spectrometer so as to prevent overlap involving neutrons with $\tau = \tau_C$.) An actual cutoff function for a slit of the type in Fig. 2 is shown in Fig. 3 (adapted from Reference 9).

(b) It will be assumed for the moment that $K(\tau)$ is independent of $\tau$, for $\tau \leq j\tau_C$. (This condition actually would exist for a $1/E$ spectrum with a $1/v$ detector if it were true that $c(\tau/\tau_C) = 1$ for $\tau \leq j\,\tau_C$.)

(c) It will be assumed that the required energy resolution $R_E \equiv (\Delta E/E)_{max}$ is independent of E. (It is permissible for the actual resolution to be smaller than $R_E$ at parts of the spectrum, but never greater than $R_E$.) It follows from Eq. (3.8) that the maximum permissible value of $R_{es}$ is a function of $\tau$, and is given by $\frac{1}{2}\tau R_E$. If $\tau_0$ is the minimum value of $\tau$ for a given chopper speed, consistent with the resolution requirements, and since $R_E$ is the maximum permissible value of $\Delta E/E$, then, from (3.8), $\tau_0 = 2R_{es}/R_E$, or

$$R_{es} = \frac{1}{2}\tau_0 R_E \quad . \tag{4.4}$$

(d) When two or more runs are needed to cover an energy interval, some overlap of adjacent runs is desirable for a normalization check. Consider two adjacent runs, denoted by subscripts 1 and 2. Let $\tau_{01}$ be, for a particular chopper speed, the minimum $\tau$ consistent with the resolution requirements. Then, if $j\tau_{C1}$ is the effective cutoff, we will assume that the region covered by the adjacent (slower) run will overlap the region of the first run, starting at some fraction $\mu$ of $j\tau_{C1}$, so that $\tau_0$ for the second run will be given by

$$\tau_{02} = \mu j \tau_{C1} \quad . \tag{4.5}$$

A quantity $\alpha$ will be used, defined by

$$\alpha \equiv \mu j \tau_{C1}/\tau_{01} \quad . \tag{4.6}$$

Thus $\alpha$ is a measure of the time-of-flight interval which is to be covered in a single run, after subtracting the overlap region. The various quantities are illustrated in Fig. 4. We want to find the optimum value of $\alpha$ - that is, the time-of-flight interval which results in the shortest total running time when an extended region is to be measured.

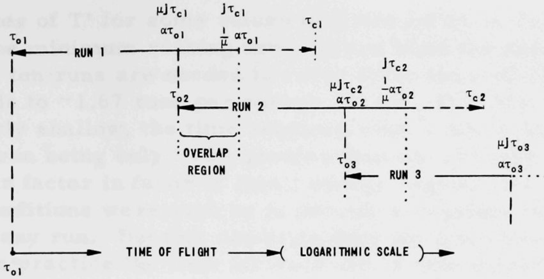


FIG. 4
COVERAGE OF EXTENDED TIME-OF-FLIGHT
REGION BY A SUCCESSION OF CHOPPER RUNS

Under our assumptions, for any run the counting rate is the same in all time channels, and is given (from Eqs. 4.2 and 4.4) by

$$R_c = K\Omega n s v_c \left(\frac{1}{2}\,\tau_0 R_E\right)^2 = \frac{1}{4}\,K\Omega n s R_E^2 \quad . \tag{4.7}$$

For each run, the running time is inversely proportional to $R_c$, and if T is a number proportional to the total running time, then for a series of N runs,

$$T = \sum_{i=1}^{N} \frac{1}{R_{ci}} = \frac{4}{K\Omega n s R_E^2} \sum_{i=1}^{N} \frac{\tau_{ci}}{\tau_{0i}^2} \quad . \tag{4.8}$$

From Eq. (4.6), $\tau_{ci} = \alpha\tau_{0i}/\mu j$; moreover, it can be seen from Fig. 4 that $\tau_{0i} = \alpha^{i-1}\tau_{01}$, so that (4.8) becomes

$$T = \frac{4}{\mu j K\Omega n s R_E^2 \tau_{01}} \sum_{i=1}^{N} \alpha^{-(i-2)} \quad . \tag{4.9}$$

To make the calculation, we will assume that three $\tau$-decades (six energy decades) are to be covered, and let N be the number of runs used. Then $N = 3/\log\alpha$, or $\alpha = 10^{3/N}$. Therefore, in this case,

$$T' = \sum_{i=1}^{N} 10^{-3(i-2)/N} \quad , \tag{4.10}$$

where the constant before the summation in Eq. (4.9) has been included in T'.

Under our assumptions, for any run the counting rate is the same in all time channels, and is given (from Eqs. 4.2 and 4.4) by

$$R_E = [illegible] \left(\frac{1}{2} \tau_c R_E\right)^2 [illegible]$$ (4.7)

For each run, the running time is inversely proportional to $R_E$, and if T is a number proportional to the total running time, then for a series of N runs,

$$T = \sum_{i=1}^{N} \frac{1}{R_{Ei}} = [illegible] \sum_{i=1}^{N} \frac{1}{\tau_{ci}^2}$$ (4.8)

From Eq. (4.6), $\tau_{ci} = \alpha \tau_{c,i+1}$ [illegible] seen from Fig. 4 that $\tau_{ci} = \alpha^{i-1} \tau_{c1}$, so that (4.8) becomes

$$T = \frac{4}{K \sigma n R_F^2 \tau_{c1}^2} \sum_{i=1}^{N} [illegible]$$ (4.9)

To make the calculation, we [illegible] decades (six energy decades) are to be covered [illegible] number of runs used. Then, $N = 3/\log \alpha$, or [illegible]

$$T = \sum_{i=1}^{N} 10^{-3(i-2)/N}$$ (4.10)

where the constant before the summation in Eq. (4.9) has been included in T.

The values of T' for some values of N are listed in Table I, where it is seen that the minimum running time occurs when the spectrometer is designed so that ten runs are needed to cover three time-of-flight decades. This corresponds to ~1.67 runs per energy decade. It is also seen that the minimum is fairly shallow, the time required when a whole energy decade is covered in a run being only ~15% greater than the optimum. There is, however, another factor in favor of small energy regions per run: we have assumed that conditions were such as to provide a constant counting rate per channel for any run. But this condition does not often prevail, with the result that in practice the time for each run is determined by the channel with the smallest counting rate, resulting in accumulation of better-than-required statistics in the rest of the channels. The smaller the energy region per run, the more efficiently are the time channels used. Thus a value of 2 for $\alpha$ is perhaps still larger than the true optimum in practice.

Table I

RELATIVE RUNNING TIMES FOR DIFFERENT VALUES OF $\alpha$

| Number of Runs, N | $\alpha$ | Runs per Energy Decade | Relative Total Running Time, T' |
|---|---|---|---|
| 3 | 10.000 | 0.50 | 11.100 |
| 5 | 3.981 | 0.83 | 5.311 |
| 6 | 3.162 | 1.00 | 4.620 |
| 7 | 2.684 | 1.17 | 4.273 |
| 9 | 2.154 | 1.50 | 4.016 |
| 10 | 1.995 | 1.67 | 3.996 |
| 11 | 1.874 | 1.83 | 4.015 |

## V. DETERMINATION OF THE PARAMETERS

Solving Eq. (4.6) for $\tau_c$ and substituting in (4.7), one gets

$$R_c(\tau) \equiv \frac{1}{4} K(\tau) \Omega ns(\mu j/\alpha) \tau_0 R_E^2 \quad , \tag{4.11}$$

where the dependence upon $\tau$ has been reinserted, and where

$$K(\tau) \equiv \psi h \gamma^{-2} J_\omega(\tau) c(\tau/\tau_c) \epsilon(\tau)(1-B) \quad . \tag{4.12}$$

In designing a chopper, the counting rate $R_c(\tau_0)$ is to be maximized. The various design parameters can be chosen as follows:

(a) The maximum permissible energy resolution $(\Delta E/E)_{max} \equiv R_E$ is determined by the nature of the experiment.

(b) The upper limit of the energy region of interest is determined by the experiment. Thus the minimum reciprocal speed $\tau_{01}$ is specified, which determines the minimum chopper resolution $R_{es}$ by means of Eq. (4.4):

$$(R_{es})_{min} = \frac{1}{2} \tau_{01} R_E \quad . \tag{4.13}$$

(c) In view of Table I, we let $\alpha = 2.00$. (It is fortunate that it is usually easy to change the width of a time-analyzer channel by a factor of 2, thereby maintaining the same ratio of channel width to burst width from run to run.)

(d) Taking the cutoff function of Fig. 3 as representative, some reasonable value of the fraction $\mu j$ can be selected, say $\mu j = 0.5$, which should allow room for sufficient overlap.

(e) The cutoff point is now determined by Eq. (4.6): $\tau_c/\tau_0 = \alpha/\mu j$. Combined with the value of the slit width and the rotor diameter (see below), this will permit design of the slit.

(f) The product $\Omega ns$ is to be maximized; the result will depend upon the geometry of the experiment and perhaps on the resolution requirements (see the following item). The quantities $\Omega$, n, and s are not in general independent: it can happen, for example, that increasing the number of slits per burst decreases the possible $\Omega$ for each slit, and also reduces the slit width that can be used. In high-resolution work, it can also happen that, with p and $\ell_f$ as large as possible, Eq. (3.7) still requires that s be limited, in which case $\Omega n$ must be maximized. In such a case it is advantageous to use the two-ended slit, provided that undue reduction in $\Omega$ is not required, because the smaller value of $\beta$ permits a larger s.

(g) The peripheral velocity and the length of the flight path must be chosen so that $p_{max} \ell_f = \beta\gamma s/(R_{es})_{min}$ (see Eq. 3.7). The requirement that s be large usually means that the chopper should be capable of high-speed operation, with a flight path as long as is convenient. The upper limit to possible peripheral velocity is set by the strength of the rotor material, and is more or less independent of rotor diameter. Speeds in excess of 400 m/sec have been achieved.[10]

(h) The rotor diameter must be large enough for the required attenuation of unwanted neutrons. The exact diameter can be found from Eq. (2.5):

$$(1-B)D = \ell_f \nu p \tau_c/\pi \quad , \tag{4.14}$$

where $p\tau_c$ is a constant for the rotor (see the first paragraph of Part II), and where the number of bursts per revolution $\nu$ is selected so that D is sufficiently large.

(b) The upper limit of the energy region of interest is determined by the experiment. Thus the minimum reciprocal speed is specified, which determines the minimum chopper resolution by means of Eq. (4.6):

$$(R_{es})_{min} = \tau_M R_E \qquad (4.13)$$

(c) In view of Table I, we let $n = 2.56$. (It is fortunate that it is usually easy to change the width of a time analyzer channel by a factor of 2, thereby maintaining the same ratio of channel width to burst width from run to run.)

(d) Taking the cutoff function of Fig. 3 as representative, some reasonable value of the fraction can be selected, say 0.5, which should allow room for sufficient overlap.

(e) The cutoff point is now determined by Eq. (4.5): $r_c/r_a = \alpha/\mu_j$. Combined with the value of the slit width and the rotor diameter (see below), this will permit design of the slit.

(f) The product $\Omega s$ is to be maximized; the result will depend upon the geometry of the experiment and perhaps on the resolution requirements (see the following item). The quantities $\Omega$, $n$, and $s$ are not in general independent: it can happen, for example, that increasing the number of slits per burst decreases the possible $\Omega$ for each slit, and also reduces the slit width that can be used. In high-resolution work, it can also happen that, with $p$ and $f_j$ as large as possible, Eq. (3.7) still requires that $s$ be limited, in which case $\Omega$ must be maximized. In such a case it is advantageous to use the two-ended slit, provided that undue reduction in $\Omega$ is not required, because the smaller value of $\beta$ permits a larger $\Omega$.

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where $p r_c$ is a constant for the rotor (see the first paragraph of Part II), and where the number of bursts per revolution $\nu$ is determined so that $D$ is

## VI. CONCLUSIONS

We have investigated some of the factors affecting the counting rate achieved by a mechanical neutron chopper system. Formulas have been derived which interrelate the counting rate, the resolution, and various geometrical factors such as slit dimensions, rotor speed, and flight-path length. In particular, we have observed (see Eq. 4.1) that the counting rate, for a fixed chopper speed and neutron flux, is proportional to:

(1) the slit height (no surprise);

(2) the square of the resolution, in microseconds per meter;

(3) the product $\Omega ns$, where $\Omega$ is the solid angle of the detector, n is the number of slits per burst, and s is the width of a slit. It was remarked that these three quantities are not always independent. (In view of the preceding item, the truth of this one is not evident *a priori*. In fact, if other things are constant, the resolution is proportional to the slit width (Eq. 3.7), so that if (3.7) is substituted in (4.1) and s is the only variable, the counting rate is seen to be proportional to the cube of the slit width.);

(4) the cutoff velocity $v_c$, since the possible number of bursts per second is proportional to it.

For the maximum counting rate, the first three quantities are to be made as large as possible. The fourth is to be adjusted (by means of slit design) so that, for any chopper run, the lowest energy for which data are useful, plus allowance for overlap, is one quarter of the highest energy for which data are useful, i.e., the chopper speed is changed by a factor of two in going from one run to the next, and five runs are needed to cover three decades in energy.

The product $p\ell_f$ of the rotor speed and the flight path is determined by the slit width and the resolution (see Eq. 3.7). With this condition fulfilled, the individual values of p and $\ell_f$ have no further effect on the counting rate. However, the condition that the slit be as wide as possible usually means that the rotor should be capable of high-speed operation.

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The product $\rho \ell_f$ of the rotor speed and the flight path is determined by the slit width and the resolution (see Eq. 3.7). With this condition fulfilled, the individual values of $\rho$ and $\ell_f$ have no further effect on the counting rate. However, the condition that the slit be as wide as possible usually means that the rotor should be capable of high-speed operation.

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