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## STABILITY ANALYSIS OF EBR-II

by

H. H. Hummel and L. T. Bryant

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## ABSTRACT

Calculations have been made for predicting the resonance of EBR-II to oscillator measurements to be made during startup of the reactor. Because of assumptions made in the calculations, which are believed to be justified by the design of the reactor, the only feedbacks are prompt negative ones. No instability in the calculated behavior is therefore possible.

The radially lumped pin model gives good results for EBR-II pins for frequencies up to 2 to 3 radians per second because of the short pin time constant, of the order of 0.1 sec.

## 1. ASSUMPTIONS AND CALCULATIONS

Calculations have been made for predicting the behavior of EBR-II during oscillator measurements to be made during startup of the reactor. Because of assumptions made in the calculations, which are believed to be justified by the design of the reactor, no instability in its operation appears possible.

The design of EBR-II has been discussed in detail elsewhere.(1) A vertical section of the reactor is shown in Fig. 1. A drawing of the core subassemblies is shown in Fig. 2. The core region of a normal subassembly contains 91 pins each 14.22 in. long. The fuel is uranium metal approximately 50% enriched in $U^{235}$ and containing 5 wt-% simulated fission products (this alloy is referred to as fissium). The fuel pins are 0.144 in. in diameter and are surrounded by a sodium bond, 6 mils thick, and a Type 304 stainless steel clad, 9 mils thick. The upper and lower blankets contain similar but larger pins, 19 to a subassembly, the uranium diameter being 0.317 in. in this case. Above and below the core are coolant header regions in which sodium is redistributed between core and blanket pins.

The following description of the subassemblies and their method of support has been reproduced from Ref. 3.

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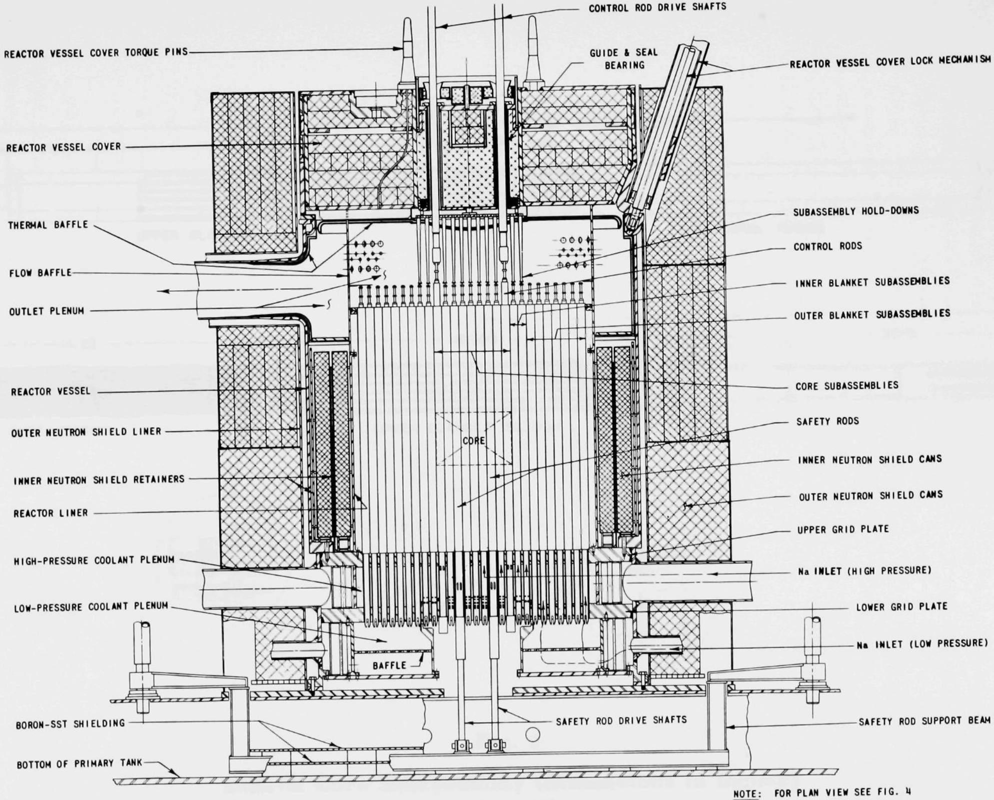


Fig. 1

EBR-II Reactor (Vertical Section)

112-496-A

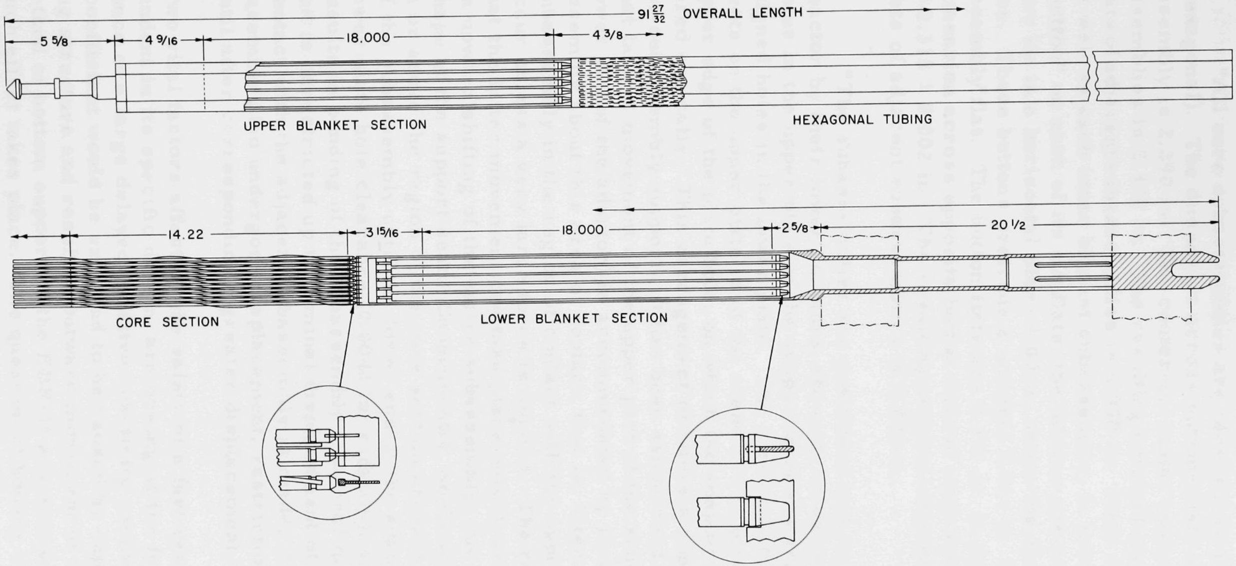


Fig. 2

EBR-II Core Subassembly (Dimensions in Inches)
111-5157

"All core subassemblies are identical in size and shape (hexagonal). The dimension across outside flats of each subassembly is 2.290 in. The center-to-center spacing of the subassemblies is 2.320 in. The resulting nominal clearance between flats of adjacent subassemblies is 0.030 in. Each core subassembly, as well as each inner blanket subassembly, is provided with a "button" on each of its six flats; the buttons are positioned so that they lie in a horizontal plane 1.00 in. above the core (fuel) center line. These buttons protrude a nominal 0.014 in. from the subassembly flat. The button flats are 0.375 in. in diameter. The dimension across opposite button flats of each subassembly is held to 2.318 ± 0.002 in. The resulting nominal clearance between button flats of adjacent subassemblies is 0.002 in.

"The subassemblies are positioned and supported in the reactor by their lower adaptors, the ends of which pass through holes in the upper plate of the support grid and engage in the axially aligned holes in the lower plate. The portion of the adaptor which rests on the upper plate is of the shape of a truncated sphere; the upper edge of the plate hole, on which the adaptor rests, is chamfered conically. This arrangement provides a continuous line contact for subassembly support. It has been established experimentally that lateral movement of the upper part of the subassembly (or of the lower end of the adaptor) is accommodated by pivoting of the subassembly about this area of contact; that is, lateral movement of the subassembly in the region of contact with the upper plate does not occur unless a very large force is applied. The reason for this is that the latter movement can take place only in accompaniment with an upward shifting of the entire subassembly, due to the conical shape of the support seat. Consequently, application of lateral force in or above the region of the core section produces only a pivoting of the subassembly until the lower end of the adaptor closes the lower plate hole clearance (0.0042 in. radially), and, thereafter, results in bending of the subassembly. Lateral movement of the top end is unrestricted up to nominal displacement of 0.030 in., when contact with the adjacent subassembly is made; if the adjacent subassembly also undergoes displacement, restriction is not effected until after correspondingly greater displacement."

Two vital factors affecting the safety of a fast reactor, which are both dependent in its specific design, are bowing of the fuel elements and the presence of a large delayed negative reactivity coefficient.(2) Such a delayed coefficient would be expected to be caused by expansion of an upper supporting structure and result in outward movement of the fuel. Because of the method of bottom support of the EBR-II subassemblies, it is assumed that no such effect takes place. The question of bowing in EBR-II has been discussed in Ref. 3. This matter is currently under reconsideration, but

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There are adequate experimental and theoretical results to indicate that the Doppler effect will be insignificant in EBR-II, and it has therefore been ignored. The sodium void coefficient, which has been found to be positive in certain large reactors,[4,5] is strongly negative in EBR-II.

Because of the large sodium inventory in the primary coolant tank, the temperature of the sodium entering the reactor has been assumed to be constant.

Because of the above assumptions, no prompt positive or delayed negative reactivity coefficients are present in the feedback model, and the predicted behavior is therefore quite stable. The assumed feedback is a prompt negative one due to unrestrained thermal expansion of fuel and steel and to coolant expansion.

No non-linearities are present in the feedback model used so far. Limited bowing would be one possible source of non-linearity. Another possible source of non-linearities is phase transformation in the fuel. It is believed, however, that this will be too sluggish in the case of fissium to affect the results of oscillator experiments.

## 2. TEMPERATURE CALCULATIONS

Oscillating temperatures in the EBR-II fuel pins have been calculated both with an IBM-704 code prepared for Long,[6] by means of the exact integration technique of Storrer,[7] and with an analog computer. These calculations were made for two-region pins, the inner region being the metallic fuel and the outer, the homogenized sodium bond and steel clad. Although the digital computer technique is more accurate, its application is limited to linear problems in which time dependence can be separated out. Use of the analog is desirable because it permits study of the effect of non-linear feedbacks (although none has so far been assumed in EBR-II) and of non-linearity in the kinetics equation.

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Because of limited capacity of the analog, a simplified temperature calculation must be made when it is used. It has been found that acceptable accuracy in temperature calculation for the EBR-II core is obtained for frequencies up to 5 radians/sec with the use of steady-state radial temperature distributions; that is, the oscillating component of the temperature in a fuel pin at a given axial position is equal to the oscillating coolant temperature at that point plus a source-produced term which has the same ratio to the oscillating source as it would have in the steady state. The solution in this approximation for the oscillating temperature of the coolant $T_c(z)$ as a function of axial variable $z$ for a heat source $a(z)$ per unit length of pin has been given by Storrer[7] as

$$T_c(z) = T_c(0)\, e^{-\lambda z/v} + \frac{e^{-\lambda z/v} \int_0^z a(z')\, e^{\lambda z'/v}\, dz'}{\lambda h \tau_c (1 + i\omega\tau_f)} , \qquad (1)$$

where $h$ is an overall heat transfer coefficient defined so that $h(\overline{T}_1 - T_c)$ is the total heat transferred per unit length of pin, $\overline{T}_1$ being the "lumped" fuel temperature; $v$ is the coolant velocity; $\omega$ is the frequency, radians/sec; $\tau_c = C_c/h$, where $C_c$ is heat capacity of coolant per unit length; and $\tau_f$ is a fuel pin time constant, which represents the ratio of heat stored in the pin per unit temperature rise of the fuel to heat lost per unit temperature difference between fuel temperature and coolant temperature. This is related to the ratio of the average fuel temperature (relative to $T_c$) to the temperature at the outside of the pin (relative to $T_c$), and this ratio is contained in the constant $h$, which can be defined as

$$h = 2\pi R h_F \frac{T(R) - T_c}{\overline{T}_1 - T_c} \qquad (2)$$

where $R$ is the outer radius of the pin and $h_F$ is the coolant film heat transfer coefficient.

Storrer's development was for an unclad fuel pin, and for this case

$$\frac{1}{h} = \frac{1}{8\pi k} + \frac{1}{2\pi R h_F} , \qquad (3)$$

where $k$ is the thermal conductivity of the pin.

Now $\tau_f$ is given by $C_1/h$, where $C_1$ is the heat capacity of the pin per unit length. Then $\lambda$ is given by

$$\lambda = \frac{\omega}{1 + \omega^2 \tau_f^2} \left[ \frac{\tau_f^2}{\tau_c} \omega + i \left( 1 + \frac{\tau_f}{\tau_c} + \tau_f^2 \omega^2 \right) \right] \quad . \tag{4}$$

In order to obtain improved accuracy in phase-shift calculations over that for the bare pin model, the EBR-II pin is treated as having two radial regions, a fuel region 1, and a homogenized bond and clad region 2. In this case Eq. (1) still applies, but it is necessary to modify the definition of $\lambda$ to the following:

$$\lambda = \frac{\omega}{1 + \omega^2 \tau_f^2} \left[ \frac{\omega \tau_f \tau_f'}{\tau_c} + i \left( 1 + \frac{\tau_f'}{\tau_c} + \omega^2 \tau_f^2 \right) \right] \quad , \tag{5}$$

in which

$$\tau_f = \frac{C_1 + C_2[(\overline{T}_2 - T_c)/\overline{T}_1 - T_c)]}{h} \quad , \tag{6}$$

$$\tau_f' = \frac{C_1 + C_2}{h} \quad , \tag{7}$$

and h is found from Eq. (2) by using the two-region steady-state temperature solution with a constant heat source in region 1, rather than by using Eq. (3).

The phase shifts in EBR-II core temperatures relative to the power are very nearly linear in $\omega$ up to 2-3 radians/sec. This is to be expected when typical values of constants for EBR-II are examined for a coolant velocity of 537 cm/sec, the average full-flow value. In this case,

$$\tau_f = 0.104 \text{ sec}$$

$$\tau_f' = 0.136 \text{ sec}$$

$$\tau_c = 0.0410 \text{ sec}$$

$$\tau_f'/\tau_c = 3.32$$

and

$$z/v = 36.12/537 = 0.0672 \text{ sec (for full height of core).}$$

For low frequency $\lambda$ is approximately

$$\lambda = i\omega \left( 1 + \frac{\tau_f'}{\tau_c} \right) , \tag{8}$$

the terms in $\omega^2$ being small. The coolant transport lag in the core is then of the order of $[1 + (\tau_f'/\tau_c)]\, z/v$, which is 0.29 sec. A complete discussion of the validity of lumped parameter models and of low-frequency approximations for a bare fuel pin is given in Refs. 7 and 8.

The application of the bare pin approximation, using Eq. (3) for h and the definition of $\tau$ according to Eq. (7), leads to a value of $\tau_f$ of 0.087 sec. This would not make $\lambda$ significantly different from the value given by Eq. (5) at low frequency, since h divides out of the term linear in $\omega$. An error of 20% would be produced in the part of the phase shift independent of coolant velocity because of the $i\omega\tau_f$ term in Eq. (1). It has been found that Eq. (1) with the use of Eqs. (5), (6), and (7) gives phase shifts very accurately in the low-frequency range.

Solutions for coolant temperature in the core by means of the radially lumped model have been obtained with the analog computer, using four axial segments in the core. Equation (1) is the continuous solution for this case. This is compared with the rigorous solution from the IBM-704 in Figs. 3 to 6. Although agreement is not perfect, the analog computation is seen to be a reasonable approximation.

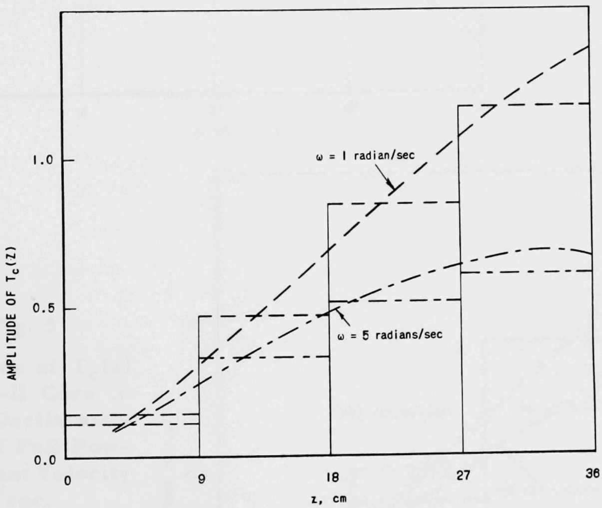


Fig. 3

Amplitude of $T_c(z)$ in EBR-II Core in °C for Oscillations of 1% of Full Power Multiplied by 270/537. Coolant Velocity 270 cm/sec
112-812

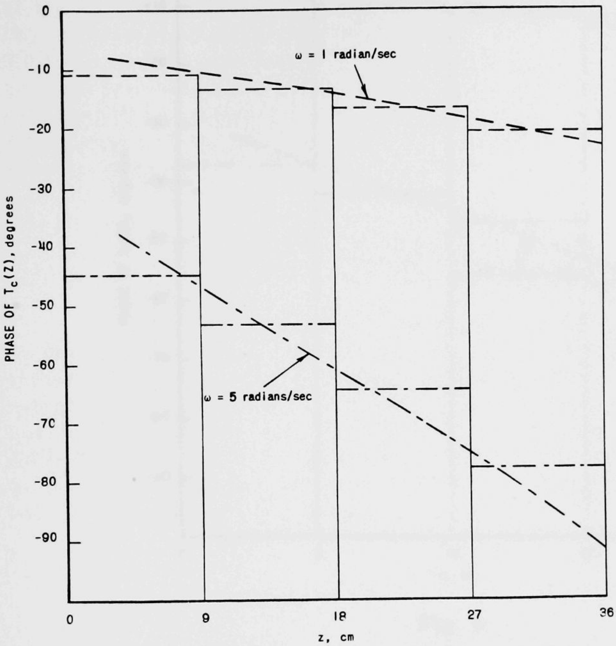


Fig. 4

Phase of $T_c(z)$ in EBR-II Core Relative to Power. Coolant Velocity 270 cm/sec.
112-809

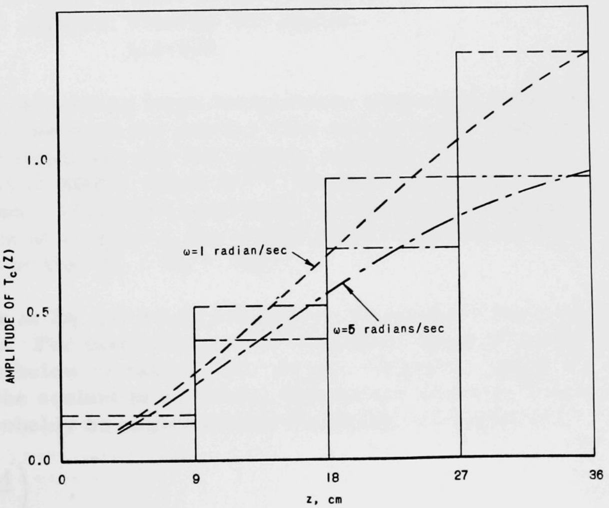


Fig. 5

Amplitude of $T_c(z)$ in EBR-II Core in °C for Oscillations of 1% of Full Power. Coolant Velocity 537 cm/sec.
112-811

Fig. 4

Phase of $T_c(s)$ in EBR-II Core Relative to Power. Coolant Velocity 270 cm/sec.

112-809



Fig. 5

Amplitude of $T_c(s)$ in EBR-II Core in °C for Oscillations of 1% of Full Power. Coolant Velocity 537 cm/sec.

112-811

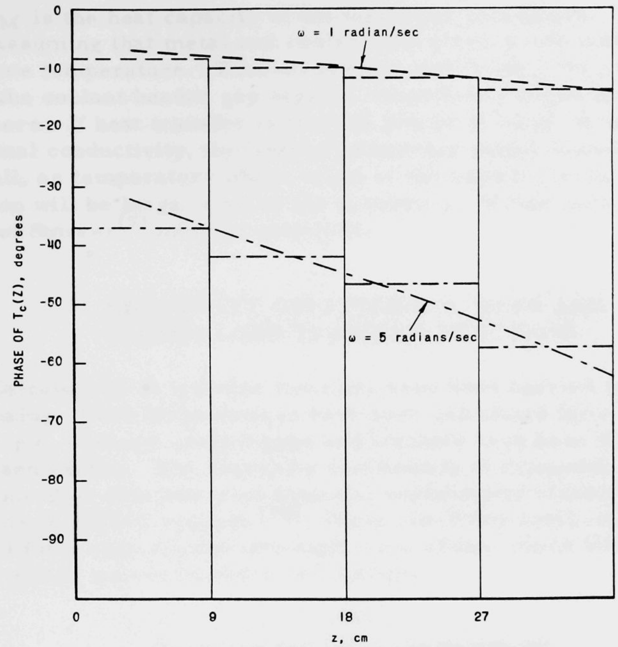


Fig. 6

Phase of $T_c(z)$ in EBR-II Core Relative to Power. Coolant Velocity 537 cm/sec.
112-810

The problem of calculating large temperature phase and amplitude changes which can occur between the reactor core and an upper supporting structure can be an important one for reactor stability. It has been found to be so in studies of EBR-I, Mark II.[9] The radially lumped parameter model is probably of limited usefulness in attacking this problem. In this case, since there is no source, the steady-state temperature distribution is a constant, so that $\overline{T}_1 = \overline{T}_2 = T(R)$.

The term $1/8\pi k$ in Eq. (3) would not apply in the case of a one-region pin with no heat source. For fast reactors the frequency range of greatest interest for stability is below ~1 radian/sec. In this frequency range, if heat transfer is from the coolant to relatively thin pieces of metal, amplitude attenuation can probably be neglected and the phase calculated from

$$\lambda = i\omega \left(1 + \frac{C_M}{C_c}\right) , \qquad (9)$$

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The term $i\omega K$ in Eq. (3) would not apply in the case of a one-region pin with no heat source. For fast reactors the frequency range of greatest interest for stability is below 1 radian/sec. In this frequency range, if heat transfer is from the coolant to relatively thin structures, the amplitude attenuation can probably be neglected and [illegible] from where $C_M$ is the heat capacity of the metal per unit length. This is equivalent to assuming that metal and coolant at a given z are instantaneously at the same temperature. This assumption was made in this paper in dealing with the coolant header gap regions immediately above and below the EBR-II core. If heat transfer is to thick pieces of metal or to material of low thermal conductivity, the lumped parameter model probably cannot be used at all, as temperature phase shifts in the solid material and amplitude attenuation will be large, even at low frequency. In this case, the exact solution of Storrer(7) must be employed.

## 3. REACTIVITY COEFFICIENTS, OPEN AND CLOSED LOOP TRANSFER FUNCTIONS

Calculations of transfer functions have been carried out so far only on the analog. Core temperatures have been calculated for a single representative pin, whereas coolant gaps and blankets have been lumped both radially and axially. The reactivity coefficients of expansion of fuel, steel, and sodium were obtained from material replacement measurements in ZPR-III on an EBR-II mockup.(10) These reactivity coefficients are given in Table I for a core divided into eight axial slices, which were lumped into four slices for the feedback calculations.

Table I

EBR-II REACTIVITY COEFFICIENTS FOR ISOTHERMAL EXPANSION[a]

$\left(-\frac{\delta k}{\delta T} \times 10^5/^\circ C\right)$

| | Axial Expansion | | Coolant | | Radial Structural Expansion | |
|---|---|---|---|---|---|---|
| | Fuel | Steel | Expansion | Displacement[b] by Steel Expansion | Fuel | Steel |
| Core-$\Delta z$, cm (from bottom) | | | | | | |
| 0- 4.52 | 0.017 | 0.008 | 0.135 | 0.017 | 0.100 | 0.019 |
| 4.52- 9.03 | 0.060 | 0.006 | 0.115 | 0.015 | 0.142 | 0.013 |
| 9.03-13.55 | 0.086 | 0.004 | 0.108 | 0.014 | 0.157 | 0.009 |
| 13.55-18.06 | 0.100 | 0.003 | 0.101 | 0.013 | 0.174 | 0.006 |
| 18.06-22.58 | 0.098 | 0.003 | 0.101 | 0.013 | 0.168 | 0.006 |
| 22.58-27.09 | 0.080 | 0.004 | 0.108 | 0.014 | 0.141 | 0.009 |
| 27.09-31.61 | 0.059 | 0.006 | 0.115 | 0.015 | 0.125 | 0.013 |
| 31.61-36.12 | 0.027 | 0.008 | 0.135 | 0.017 | 0.087 | 0.019 |
| Core Total | 0.527 | 0.042 | 0.918 | 0.118 | 1.094 | 0.094 |
| Gap, Unlagged | | 0.008 | 0.157 | | | 0.054 |
| Gap, Lagged[c] | | 0.005 | 0.115 | | | |
| Upper Blanket | 0.016 | 0.004 | 0.103 | 0.013 | 0.032 | 0.008 |
| Radial Blanket | 0.104 | 0.051 | 0.128 | 0.066 | 0.216 | 0.102 |

[a] Assumed coefficients of expansion - Sodium Volume $2.9 \times 10^{-5}/^\circ C$; Steel Linear $1.9 \times 10^{-5}/^\circ C$; Fuel Linear $1.8 \times 10^{-5}/^\circ C$

All expansions except axial fuel assumed proportional to local coolant temperature.

[b] This effect is cancelled if radial structure expansion occurs.

[c] Lag is 0.24 sec at full flow of 537 cm/sec and is inversely proportional to coolant velocity.

The lack of symmetry of fuel worth about the core midplane found in the measurements is presumably due to a similar dissymmetry in EBR-II geometry. The coefficients are divided into those which are based on no radial expansion and those which are associated with unrestrained radial expansion. Lumped coefficients are given for the upper coolant gap, upper blanket, and the radial blanket. The unlagged and lagged components of the gap feedback and the lag time were obtained from use of the temperature variation through the gap obtained as described above.

The amplitudes and phases of the reactivity feedbacks relative to the power are given in Tables II and III. Feedback for the radial blanket was found to be negligible above $\omega$ = 0.1 radian/sec. Large axial phase shifts are expected to occur in this case because of the low coolant flow rate. A more refined treatment is clearly desirable here.

Table II

AMPLITUDE OF REACTIVITY FEEDBACK OF EBR-II AT FULL POWER

| | v = 270 cm/sec | | | | | | | | v = 537 cm/sec | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Radial Expansion | | | | No Radial Expansion | | | | Radial Expansion | | | | No Radial Expansion | | | |
| Frequency $\omega$, radians/sec | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 |
| Feedback, $\delta k/\beta/(\delta n/n_0)$ | | | | | | | | | | | | | | | | |
| Na in Core | 0.391 | 0.377 | 0.322 | 0.211 | 0.200 | 0.192 | 0.164 | 0.112 | 0.219 | 0.214 | 0.197 | 0.156 | 0.111 | 0.110 | 0.101 | 0.080 |
| U in Core | 0.164 | 0.158 | 0.132 | 0.087 | 0.164 | 0.159 | 0.132 | 0.087 | 0.117 | 0.115 | 0.104 | 0.078 | 0.115 | 0.114 | 0.104 | 0.079 |
| Total Core | 0.555 | 0.534 | 0.455 | 0.308 | 0.362 | 0.350 | 0.297 | 0.199 | 0.336 | 0.330 | 0.300 | 0.236 | 0.229 | 0.224 | 0.205 | 0.159 |
| Upper Gap (Unlagged) | 0.080 | 0.077 | 0.062 | 0.037 | 0.060 | 0.057 | 0.047 | 0.028 | 0.048 | 0.047 | 0.042 | 0.031 | 0.036 | 0.035 | 0.031 | 0.023 |
| Upper Gap (Lagged)[a] | 0.042 | 0.039 | 0.032 | 0.018 | 0.042 | 0.041 | 0.032 | 0.018 | 0.024 | 0.024 | 0.021 | 0.015 | 0.024 | 0.024 | 0.021 | - |
| Upper Blanket | 0.059 | 0.050 | 0.035 | 0.017 | 0.049 | 0.043 | 0.029 | 0.015 | 0.035 | 0.031 | 0.026 | 0.016 | 0.029 | 0.025 | 0.021 | 0.014 |
| Radial Blanket | 0.033 | - | - | - | 0.013 | - | - | - | 0.021 | - | - | - | 0.009 | - | - | - |
| Total | 0.760 | 0.689 | 0.530 | 0.312 | 0.523 | 0.478 | 0.354 | 0.195 | 0.460 | 0.430 | 0.378 | 0.274 | 0.325 | 0.308 | 0.267 | 0.184 |

[a] Lag is 0.48 sec at 270 cm/sec, 0.24 sec at 537 cm/sec coolant velocity.

Table III

PHASE LAG IN DEGREES OF REACTIVITY FEEDBACK OF EBR-II RELATIVE TO POWER

| | v = 270 cm/sec | | | | | | | | v = 537 cm/sec | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Radial Expansion | | | | No Radial Expansion | | | | Radial Expansion | | | | No Radial Expansion | | | |
| Frequency $\omega$, radians/sec | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 | 0.1 | 1.0 | 2.5 | 5.0 |
| Feedback Phase | | | | | | | | | | | | | | | | |
| Na in Core | 1.8 | 17.3 | 40.1 | 65.4 | 1.9 | 17.7 | 40.7 | 66.2 | 1.2 | 12.0 | 28.7 | 50.4 | 1.1 | 11.9 | 29.0 | 51.2 |
| U in Core | 1.4 | 17.4 | 39.8 | 63.0 | 1.5 | 17.6 | 40.0 | 62.6 | 1.3 | 12.2 | 29.1 | 50.5 | 1.1 | 12.3 | 29.3 | 39.9 |
| Total Core | 1.8 | 17.4 | 40.1 | 64.7 | 1.8 | 17.5 | 40.3 | 64.8 | 1.2 | 12.2 | 28.9 | 50.4 | 1.3 | 12.3 | 29.1 | 50.6 |
| Gap (Unlagged) | 2.3 | 22.7 | 52.6 | 84.8 | 2.4 | 22.9 | 52.4 | 84.3 | 1.0 | 15.1 | 35.1 | 61.6 | 1.3 | 13.6 | 35.3 | 61.4 |
| Gap (Lagged) | 5.5 | 48.9 | 120.7 | 213.5 | 3.4 | 49.3 | 121.6 | 224.1 | 3.8 | 27.9 | 69.6 | 135.0 | 1.8 | 30.4 | 69.8 | - |
| Upper Blanket | 3.8 | 58.7 | 131.5 | 230.4 | 6.1 | 60.8 | 133.3 | 235.1 | 3.8 | 36.7 | 82.2 | 141.2 | 4.3 | 37.9 | 79.1 | 139.1 |
| Radial Blanket | 47.3 | - | - | - | 46.4 | - | - | - | 28.8 | - | - | - | 28.4 | - | - | - |
| Total | 4.1 | 22.9 | 48.9 | 69.9 | 3.4 | 24.4 | 51.5 | 70.1 | 2.4 | 13.9 | 34.9 | 58.5 | 2.5 | 15.8 | 36.3 | 60.5 |

The lack of symmetry of fuel worth about the core midplane found in the measurements is presumably due to a similar dissymmetry in EBR-II geometry. The coefficients are divided into those which are based on no radial expansion and those which are associated with unrestrained radial expansion. Lumped coefficients are given for the upper coolant gap, upper blanket, and the radial blanket. The unlagged and lagged components of the gap feedback and the lag time were obtained from use of the temperature variation through the gap obtained as described above.

The amplitudes and phases of the reactivity feedbacks relative to the power are given in Tables II and III. Feedback for the radial blanket was found to be negligible above ω = 0.1 radian/sec. Large axial phase shifts are expected to occur in this case because of the low coolant flow rate. A more refined treatment is clearly desirable here.

The 537-cm/sec case corresponds to average full flow in the core. Amplitudes for both velocities are given as $(\delta k_{FB}/\beta)/(\delta n/n_0)$, where $\delta k_{FB}/\beta$ is the amplitude in dollars of the oscillating reactivity feedback for oscillation at full power, and $\delta n/n_0$ is the fractional change in power. In calculating the closed loop transfer function at reduced power, which would be necessary at reduced flow, the amplitudes in Table II are to be multiplied by the fraction of full power at which the reactor operates.

In Figs. 7 and 8, the EBR-II closed loop transfer function phase and amplitude are given for the various assumptions made in the feedback. The amplitude in this case is $(\delta n/n_0)/(\delta k_{APP}/\beta)$, where $\delta k_{APP}/\beta$ is the applied oscillating reactivity in dollars. The phase is that of the power relative to the applied reactivity. As expected, there is no indication of the formation of a resonance.

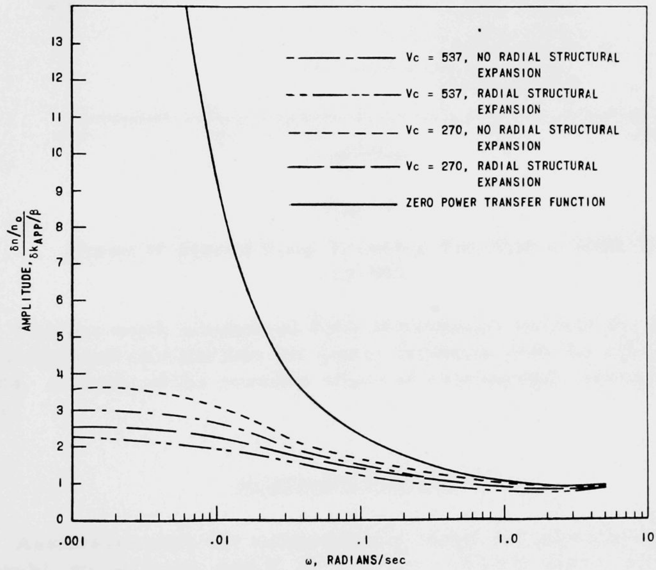


Fig. 7

Amplitude of Closed Loop Transfer Function of EBR-II. Curves for v = 537 cm/sec for Full Power. Curves for v = 270 cm/sec for Full Power x 270/537.

112-815

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112-815

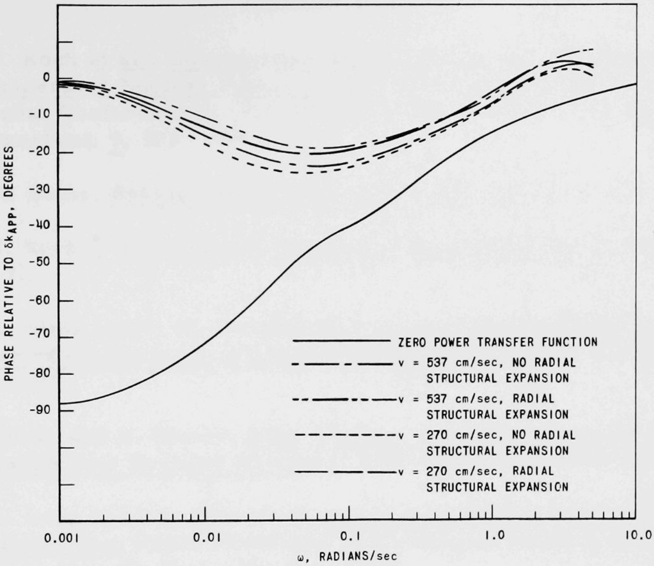


Fig. 8

Phase of Closed Loop Transfer Function of EBR-II
112-817

Further work planned on EBR-II dynamics include the performing of more detailed calculations for linear feedback with the aid of the IBM-704. A study of the possible effect of subassembly bowing is also planned.

## ACKNOWLEDGMENT

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