

Argonne National Laboratory

**THE THEORY AND MEASUREMENT
OF CRITICALITY**

by

Charles N. Kelber

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TABLE OF CONTENTS

PREFACE 1

I. INTRODUCTION 2

II. STANDARD EXPLOITATION 3

 A. The Total Source 3

 B. Production Capture 4

 C. Nonproduction Capture 5

 1. Losses 5

 2. All Other Particles 6

 D. Leakage 7

 E. The Critical Equation 8

III. METHODS OF MEASUREMENT 10

IV. USES OF OTHER DATA 14

V. CONNECTION WITH THE STANDARD THEORY
PARAMETERS 18

APPENDIX 22

REFERENCES 28

ACKNOWLEDGMENT 34

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TABLE OF CONTENTS

	<u>Page</u>
PREFACE	4
I. INTRODUCTION	5
II. DESIRED EXPLICIT MEASUREMENTS	5
A. The Total Source	6
B. Productive Capture	7
C. Nonproductive Capture	7
1. U^{235}	7
2. U^{238}	7
3. All Other Nuclides	8
D. Leakage	9
E. The Critical Equation	9
III. METHODS OF INFERENCE	10
IV. USES OF OTHER DATA	14
V. CONNECTION WITH THE STANDARD THEORY PARAMETERS	18
APPENDIX	22
REFERENCES	24
ACKNOWLEDGMENT	24

LIST OF TABLES

No.	Title	Page
I	Primary Quantities.	10
II	Secondary Quantities.	17

PREFACE

The work presented here started in early 1961. It was prompted by the difficulty experienced in correlating the results of critical experiments for presentation in the report of the Atomic Physics Constants Center. As the work progressed, it became clear that our approach was similar to that presented by S. M. Peaberg *et al.*, Fuel Burnup in Water-moderated Water-cooled Power Reactors and Nuclear Water Lattice Experiments, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy (Vol. 13, pp. 448-457).

Our viewpoint differs from Peaberg's however, in that we have in mind setting up a consistent system of measurements which not only permits a correlation of experiments, but which permits, through scientific design of experiments, a method of extrapolating into design conditions from critical experiments. Thus, the standard fuel-burnup formula is a functional relationship between the net rate of neutron production and the so-called lattice parameters. If the interdependence of the lattice parameters is known, it is possible to predict the effect of reactivity and conversion ratio of changing, say, the ratio of fuel to moderator in a given experiment having measured or inferred the lattice parameters of the original experiment.

The determination of the interdependence of lattice parameters is, however, the crux of the matter. By obtaining a functional relationship between quantities measurable in a critical system, which then defines completely the neutron economy, it will be possible to devise a systematic experimental strategy to determine the parameter interdependences. The problem of extrapolation to design conditions is then analogous to the extrapolation of a function whose value and partial derivatives are known at a point.

The contents of this report are an amplified version of a talk given by the author at the 1961 Winter Meeting, November 1, 2, 3, 1961, of the American Nuclear Society in Chicago, Illinois. Although this review is admittedly incomplete, it is hoped that it will answer some of the questions raised at the talk.

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PREFACE

The work presented here started in early 1961. It was prompted by the difficulty experienced in correlating the results of critical experiments for presentation in the report of the Reactor Physics Constants Center. As the work progressed, it became clear that our approach was similar to that presented by S. M. Feinberg et al., Fuel Burnup in Water-moderated Water-cooled Power Reactors and Uranium-Water Lattice Experiments, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy (Vol. 13, pp. 348-415).

Our viewpoint differs from Feinberg's however, in that we have in mind setting up a consistent scheme of measurements which not only permits a correlation of experiments, but which permits, through scientific design of experiments, a method of extrapolation to design conditions from critical experiments. Thus, the standard four-factor formula is a functional relationship between the net rate of neutron production and the so-called lattice parameters. If the interdependence of the lattice parameters is known, it is possible to predict the effect on reactivity and conversion ratio of changing, say, the ratio of fuel to moderator in a given experiment having measured or inferred the lattice parameters of the original experiment.

The determination of the interdependence of lattice parameters is, however, the crux of the matter. By obtaining a functional relationship between quantities measurable in a critical system, which also defines completely the neutron economy, it will be possible to devise a systematic experimental survey to determine the parametric interdependence. The problem of extrapolation to design conditions is then analogous to the extrapolation of a function whose value and partial derivatives are known at a point.

The contents of this report are an amplified version of a talk given by the author at the 1961 Winter Meeting, November 7, 8, 9, 1961, of the American Nuclear Society in Chicago, Illinois. Although this review is admittedly incomplete, it is hoped that it will answer some of the questions raised at the talk.

I. INTRODUCTION

The statement that a system is critical imposes stringent conditions on the neutron economy. Our aim is to establish a list of measurable quantities which define the neutron economy completely. In critical systems these measured quantities must be internally consistent. A valid theoretical computation of the same quantities will reproduce not only the total behavior of the neutron economy, but also the internal consistency of the experimental data. To avoid begging the question, we further require that the measurable quantities be derivable from raw data with a minimum of theoretical inference.

Associated with the preceding ideas is the assumption that all comparative calibrations of foils, chambers, etc., can be performed. We also assume that the relative neutron cross sections of materials whose cross-section energy response is the same are known. In what follows we describe some measurements not generally made, but these measurements are sufficiently like more common ones that the assumption is valid.

This work can be easily generalized in many respects. In this paper, however, we consider only thermal systems fueled with U^{235} and U^{238} . Part II of this report is devoted to defining and listing the desired measurements and to explaining those which are experimentally attainable. In Part III we discuss means of inferring those data required for description of criticality which do not appear to be directly measurable. In Part IV we discuss the use of other data, and in Part V the connection of measured quantities with the parameters of standard theory is discussed.

II. DESIRED EXPLICIT MEASUREMENTS

The neutron economy can be characterized by the way in which neutrons are removed from the system. In the crudest sense there are three possible fates for neutrons in a critical system:

- (1) capture which causes fission and the release of source neutrons - this we term productive capture;
- (2) capture which does not cause fission, or nonproductive capture; and
- (3) leakage from the system.

Although items (2) and (3) have similar effects on the neutron economy, they are qualitatively different, since the leakage rate is primarily an extensive property and nonproductive capture an intensive property. In the exact theory these properties interact, but in the lowest approximation the distinction between them is clear.

Since, in a critical system, the neutron population is not determined by an external source, it is necessary to define all rates with respect to some process. Arbitrarily, we choose to define all rates as rates per fission in U^{235} .

A. The Total Source

The total source is known if the average neutron yield per fission and the fission rate in U^{238} are known. The latter quantity is directly accessible from foil data and is usually listed as δ_{28} : the ratio of fission rate in U^{238} to fission rate in U^{235} . A typical procedure for obtaining δ_{28} is given in Ref. 1.

The values of ν for U^{235} (ν_{25}) and U^{238} (ν_{28}) are slightly dependent on the energy of the incident neutron. Thus, $d \ln \nu / dE$ is of the order of 5% per Mev. Thus in thermal reactors, where most capture occurs below a few hundred volts, ν_{25} is sensibly constant for all such systems. Similarly, ν_{28} is insensitive to the small changes in the spectrum above the threshold for U^{238} fission that may occur in thermal systems.

If there are $n,2n$ sources present, these will yield productive captures similar to fast fission in U^{238} . The rates at which such processes occur are not nearly as easy to measure as fast fission in U^{238} . One possible method is to compare the results of absolute determinations of the $n,2n$ reaction rate with the U^{235} fission rate. Fortunately, the contribution to the source from $n,2n$ reactions is likely to be small.

In the absence of $n,2n$ sources, the total source S per U^{235} fission is given by

$$S = \nu_{25} + \delta_{28} \nu_{28} \quad (1)$$

In this equation, ν_{25} and ν_{28} are constants, and δ_{28} is directly derivable from experimental data.

Although the values of ν are here taken as assignable, and in principle they may be obtained by direct measurement, it is true that they are not now known to high precision from direct measurements. However, the extensive literature of correlation of critical experiments can be quoted to lend credence to values of ν which are to be used in conjunction with assumed sets of cross section. This reasoning is not as circular as it might appear, since the body of such experiments has enough variables to justify " ν " values so obtained as being "best fit" data.

B. Productive Capture

Per fission in U^{235} there are $1 + \delta_{28}$ productive captures. If $n, 2n$ sources are present, terms analogous to δ_{28} for each nuclide having an $n, 2n$ cross section must be added. No measurements other than those already discussed need be made.

C. Nonproductive Capture

This topic can be divided into a discussion of nonproductive capture in: (1) U^{235} , (2) U^{238} , and (3) all other nuclides.

1. U^{235}

The ratio of capture to fission in U^{235} , α_{25} , is not easily arrived at except by mass-spectroscopic analysis: a common assumption is that such analysis is sensitive to about 1 ppm; thus a measurement of the atom ratio of U^{236} to U^{235} accurate to 10% would require an irradiation of U^{235} to about $2 \times 10^{-5} n/kb$. Fluxes in critical assemblies are rarely larger than $10^{-13} n/(kb)(sec)$, so that the required irradiation times are prohibitive. Internal exponential assemblies can be driven at higher powers, however, and in such cases α_{25} can be determined. With this exception, α_{25} will have to be regarded as not directly measurable.

2. U^{238}

The ratio of captures in U^{238} to fissions in U^{235} , β_{28} , can in principle be determined by direct measurement, since both processes lead to detectable activation. In the absence of accurate absolute counting techniques, a calibration is required. We are indebted to M. Brenner for help in the discussion of this matter (see Appendix A).

This method relies on a standard pile or thermal column for calibration of Np^{239} activity counting against counting fission product activity. The ratio β_{28} is given by

$$\beta_{28} = \left(\frac{N_F}{N_T} \right) \left(\frac{F_{25,T}}{F_{25,F}} \right) \left\langle \frac{\sigma_{28}}{\sigma_{f,25}} \right\rangle T \quad , \quad (2)$$

where N_F is the Np^{239} activity in the fuel, N_T the Np^{239} activity in the thermal column, $F_{25,T}$ the U^{235} fission product activity in the thermal column, $F_{25,F}$ is the U^{235} fission product activity in the fuel, and $\langle \sigma_{28}/\sigma_{f,25} \rangle T$ is the ratio of effective U^{238} capture cross section in the thermal column to the effective U^{235} fission cross section there.

An advantage of this method is that the inherent error in β_{28} is independent of which system is being measured. (Obviously all self-shielding corrections must be made or rendered unnecessary.)

3. All Other Nuclides

If all other nuclides have capture cross sections proportional to $1/v$, then the reaction rate in any $1/v$ absorber can be used to determine the reaction rate in all $1/v$ absorbers. The assumption that all other nuclides are $1/v$ absorbers is usually a good one. In those cases for which this is not so, special techniques, which are similar to those used with U^{238} , have to be used.

Let $\beta_{1/v}$ be the rate of capture in all other nuclides per fission in U^{235} . The subscript $1/v$ shows that we have assumed that only $1/v$ capture occurs. Since $\beta_{1/v}$ is defined with respect to fission in U^{235} , the effect of spatial flux (and spectral) variation between fuel and moderator regions in heterogeneous assemblies is included in $\beta_{1/v}$. Thus, in the lowest approximation, spatial variations in flux level and spectrum need not be determined.

The fact that these spatial heterogeneities do exist, however, leads to difficulty in determining $\beta_{1/v}$. If one is willing to assume that the activation rate of materials such as copper or sodium is a good approximation to the $1/v$ activation rate, then, conceptually, absolute counting methods can be used. Generally prevailing experimental difficulties usually argue against use of this method. The calibration method mentioned in connection with β_{28} can be used. In this case, however, there are some new difficulties.

These difficulties arise from the fact that insertion of a sample for activation perturbs the activating flux; determining the extent of this perturbation usually requires some knowledge of the nature of the activating flux and thus begs the question. In the case of β_{28} , the U^{238} can be inserted in such a way that the sample is essentially the same as the fuel. Usually, this will not be the case for the moderator and the inserted sample.

These difficulties are such that, if it cannot be demonstrated that flux perturbations from sample insertion are negligible, then we say that $\beta_{1/v}$ must be inferred from other data. The situation is not entirely bleak. A satisfactory estimate of the perturbation may often be obtained on the basis of information of a secondary nature. As an example, the activation arising from the presence of a high-lying resonance may be estimated from a measurement of the cadmium ratio. Under such circumstances, it is clear that one should not state that $\beta_{1/v}$ is not directly observable. Since more than a calibration is involved, however, we tentatively conclude that an accurate estimate of $\beta_{1/v}$ must, in general, be inferred from other data, but that a rough estimate of $\beta_{1/v}$ is, in fact, directly observable. We return to this point in Part III.

D. Leakage

Net leakage, L , per fission in U^{235} is usually inferred from buckling measurements. The latter, in turn, are inferred from flux plots. Since leakage probabilities $P(B^2)$ must also be used, there is a certain amount of question begging involved. There are other methods. In bare systems, long-counter techniques can be used. Effectively infinite liquid reflectors lend themselves to activation methods employing soluble indicators. Thin reflectors require a combination of techniques. In any event, a standard pile or thermal column is generally necessary to calibrate foils in order to compute the leakage rate per U^{235} fission. Once again, L must be inferred from the measurements.

In short, we determine the leakage by counting a representative sample of all the neutrons which leak from the core. Thus, in a thick (effectively infinite) reflector all the neutrons which leak from the core are captured in the reflector. The total capture rate in the reflector is equal to the total leakage rate from the core. If the reflector captures according to the $1/v$ law, techniques used to measure $\beta_{1/v}$ can be applied to measuring the reflector capture rate. In a bare system, the long-counter technique has been suggested. The difficulties of this method are well known, but the method has the advantage of counting directly a representative sample of the leaking neutrons.

The contribution of L to reactivity can be estimated in the case of thick reflectors by observing the change in reactivity caused by the uniform addition of $1/v$ poison to the reflector and extrapolating to zero capture in the reflector; that is, for a thick reflector, almost all neutrons emitted into a noncapturing reflector eventually return to the core. Hence, by extrapolating the reactivity vs reflector absorption area to zero absorption area one can estimate the reactivity held down by the reflector. This measurement is complicated by the well-known difficulty of obtaining absolute values of reactivity. Supposing this to be done and the reactivity held down by leakage found to be $\rho(L)$, then measurements of $\partial\rho/\partial L$ permit one to compute L from

$$\rho(L) = \int_0^L \left(\frac{\partial\rho}{\partial L} \right) dL \quad .$$

A more amenable use of $\rho(L)$ is in a description of the net neutron economy in terms of the relative rates of creation and destruction of importance, as suggested by Goertzel.⁽²⁾

E. The Critical Equation

The list of desired explicit measurements is summarized in Table I. These measurements characterize the neutron economy completely, so the critical equation can be written in terms of the measured quantities and constants.

Table I

PRIMARY QUANTITIES

Symbol	Definition
δ_{28}	Fission rate in U^{238} divided by fission rate in U^{235} .
α_{25}	Capture rate in U^{235} divided by fission rate in U^{235} .
β_{28}	Captures in U^{238} per fission in U^{235} .
$\beta_{1/v}$	Capture rate in all materials (assumed $1/v$) other than uranium divided by U^{235} fission rate.
L	Net leakage rate per U^{235} fission.
ν_{25}, ν_{28}	The number of neutrons per fission in U^{235} and U^{238} . In principle, these numbers are obtained by direct measurement.

The critical equation is

$$\nu_{25} + \nu_{28}\delta_{28} = 1 + \alpha_{25} + \beta_{28} + \beta_{1/v} + L + \delta_{28} \quad (3)$$

Now, ν_{25} and ν_{28} are known ($n, 2n$ sources are assumed absent); δ_{28} , β_{28} , and $\beta_{1/v}$ are derivable directly from experimental data or inferred (see Part II-C); L can be inferred from experimental data; α_{25} can sometimes be found by direct means, but, in general, is found by solving Equation (3). If all the terms in (3) are known, the equation serves to check the internal consistency of the data.

The quantities listed in Table I and used in Equation (3) are average values for the reactor. Just as L represents the net leakage rate for the entire reactor, so α_{25} represents the capture rate in U^{235} throughout the entire reactor; similarly for the other quantities. Hence, extensive sampling must be done in making measurements to get an average. In general, the argument that measurements are made at a representative point is unjustified. The various rate processes all compete in varying amounts spatially and energetically; although there may be cases where this variation is negligible, they are the exceptions.

III. METHODS OF INFERENCE

A difference between δ_{28} , β_{28} , and $\beta_{1/v}$, on the one hand, and L, on the other, is that the former are derived from the ratio of specific activities of the same type whereas the latter is not. Moreover, β_{28} and $\beta_{1/v}$ are distinct from δ_{28} since they require a separate foil calibration technique,

in general. Finally, various difficulties, especially arising from system complexity, often prevent accurate direct measurement even of such straightforwardly determined quantities as δ_{28} . When this is the case, less accessible data, such as β_{28} , $\beta_{1/v}$, and L , will usually be even harder to obtain with any accuracy.

The problem is how to avoid the necessity of making absolute determinations of the separate activities in all cases, and to infer these activities when they cannot be measured precisely relative to one another. There are at least three ways of solving this problem, all involving different types of error.

We want to obtain a measurement of the $1/v$ activity in units commensurable with the fission activity in U^{235} . Were it the case that U^{235} fission obeyed the $1/v$ law, then that activity could be used to find $\beta_{1/v}$. Although it is well known that U^{235} fission is far from $1/v$ in detail, the fact is that in many cases the behavior of the average U^{235} fission rate is remarkably like a $1/v$ detector. Thus, in a pure Maxwellian spectrum characterized by temperatures in the range from 300 to 400°K the average U^{235} fission response is proportional to that of a $1/v$ detector to within 2%. The fission activity averaged over a Westcott spectrum⁽³⁾ of moderate hardness ($r = 0.07$) shows an even smaller variation over the same range. In fact, over the very wide range of Maxwellian component temperatures, 300 to 1600°K, the total rate of fission activation varies by no more than 8.4% from that predicted by the $1/v$ law for Westcott's $r = 0.07$ spectrum.

Thus, in weakly capturing systems with little resonance capture in U^{238} , the fission rate in U^{235} can be used to determine $\beta_{1/v}$ with an error that is about of the same order as that associated with absolute counting methods. In applying this rule, the 0.0253-eV fission cross section must be multiplied by a correction factor, $K \doteq 0.98$. This behavior can be better understood when it is recalled that the infinitely dilute resonance integral for fission in U^{235} is about 269 b; this is less than +8% below that to be expected if U^{235} fission followed the $1/v$ law.

In systems with a large amount of capture, particularly resonance capture in U^{238} , the resonance energy flux may be severely depleted at the U^{235} resonances, and the total fission response cannot be reasonably expected to be a good indicator of $1/v$ response.

A. These considerations give rise to the first method of obtaining $\beta_{1/v}$, namely, the sector foil method. This method has been discussed at length in Ref. 1.

If the system is weakly capturing,

$$\beta_{1/v} \doteq \frac{F_{25,M}}{F_{25,F}} \frac{\sigma_{1/v}(0.0253)}{\sigma_{f,25}(0.0253)} \frac{1}{K} \quad (4a)$$

(K is approximately 0.98; since knowledge of its exact value involves detailed knowledge of the reactor spectrum, we do not consider it within the scope of this paper to give a more complete rule for its evaluation.)

When there is much capture, the assumption may be made that $\sigma_{f,25}$ is proportional to $1/v$ below the cadmium cutoff. Then, if the $1/v$ cadmium ratio,

$$\rho_{1/v} = \frac{\text{reaction rate in bare } 1/v \text{ detector}}{\text{reaction rate in cadmium-covered } 1/v \text{ detector}},$$

can be determined, $\beta_{1/v}$ can be estimated; that is,

$$\beta_{1/v} \doteq \left(\frac{\rho_{1/v}}{\rho_{1/v} - 1} \right) \left(\frac{F_{25,M}^*}{F_{25,F}} \right) \left(\frac{\sigma_{1/v}(0.0253)}{\sigma_{f,25}(0.0253)} \right) \left(\frac{1}{K} \right). \quad (4b)$$

In Equations (4a) and (4b), $F_{25,M}^*$ is the sub-cadmium fission rate in U^{235} in the moderator, $F_{25,F}$ is the total U^{235} fission rate in the fuel, $F_{25,M}$ the total fission rate in the moderator, and $\sigma_{1/v}(0.0253)/\sigma_{f,25}(0.0253)$ is just the ratio of the $1/v$ capture cross section to the U^{235} fission cross section at 0.0253 ev. In Part IV we will discuss cadmium ratios more fully. The sector foil technique given in Ref. 1 can be used to obtain the ratio $F_{25,M}^*/F_{25,F}$ or $F_{25,M}/F_{25,F}$. When there is considerable spatial variation of the flux within a cell or where the cell is small, it may be difficult to get an accurate average value of $\rho_{1/v}$. In systems which contain a large amount of U^{235} the spectral distortion in the moderator will be severe, but the error in the neutron economy will still be small; this has been pointed out in Ref. 1.

The same techniques can be employed to find β_{28} . The assumption is made that, below the cadmium cut-off, the U^{238} capture cross section is proportional to $1/v$, that is, the total capture rate in U^{238} is $\rho_{28}/(\rho_{28} - 1)$ times the sub-cadmium capture rate in U^{238} , by definition. But since the sub-cadmium capture rate in U^{238} is closely proportional to $1/v$, the same methods of determination may be used as for $\beta_{1/v}$. If the assumption that sub-cadmium U^{235} fission obeys the $1/v$ law were good in the fuel, one could use

$$\beta_{28} = \left(\frac{\rho_{28}}{\rho_{28} - 1} \right) \left(\frac{\rho_{25} - 1}{\rho_{25}} \right) \frac{1}{K} \frac{\sigma_{28}(0.0253)}{\sigma_{f,25}(0.0253)}. \quad (5a)$$

In general, this assumption is poor and it is better to refer the $1/v$ capture rates to those in the moderator:

$$\beta_{28} \doteq \left(\frac{\rho_{28}}{\rho_{28} - 1} \right) \left(\frac{F_{25,M}^*}{F_{25,F}} \right) \left(\frac{\sigma_{28}\phi_F}{\sigma_{28}\phi_M} \right) \left(\frac{\sigma_{28}(0.0253)}{\sigma_{f,25}(0.0253)} \right) \frac{1}{K}. \quad (5b)$$

The ratio $\sigma_{28} \phi_F / \sigma_{28} \phi_M$ is the ratio of sub-cadmium activation of U^{238} in the fuel to that in the moderator. The other terms are defined in the same way as the corresponding terms in Equation (4). Since U^{235} contamination may pose a problem in determining the activation rates, sub-cadmium gold or copper (for example) activation may be used.

This procedure is cumbersome as compared with the direct method discussed in Part II. The sub-cadmium copper or gold activation rates may be used to normalize the sub-cadmium U^{238} activity in the direct method as in the indirect whenever U^{235} contamination is a problem.

Since U^{235} fission is not $1/v$, the error in the measurements of $\beta_{1/v}$ and β_{28} will change with the degree and nature of spectral distortion in the reactor. This is a disadvantage in attempting to make correlations of data from critical experiments.

B. The second method, that of using a standard pile or thermal column to calibrate foils, falls in the class of direct measurements and has been described in Part II. If β_{28} , $\beta_{1/v}$, and L are determined by similar activation techniques, the errors in the right-hand side of Equation (3) reside in $\langle \sigma_{28} / \sigma_{f,25} \rangle_T$ and $\langle \sigma_{1/v} / \sigma_{f,25} \rangle_T$, except for the term $1 + \alpha_{25}$. If the errors are the same, their effect is to cause a uniform change in ν_{25} , ν_{28} , and $1 + \alpha_{25}$. Such a change leaves $\eta_{25} = \nu_{25} / (1 + \alpha_{25})$ unchanged in each experiment. If a monoenergetic beam of neutrons is available for irradiation instead of a thermal column, the inherent errors are reduced to those involved in finding the ratio of cross sections at a given energy.

In determining $\beta_{1/v}$, cadmium ratios in the reactor may be needed, as with the sector foil method. Otherwise the procedure is the same as with β_{28} .

The leakage L can, in the case of thick reflectors, be measured by activation techniques. In such a case, the remarks which apply to $\beta_{1/v}$ apply to L . When long-counter techniques are used, calibration becomes less obvious: it may be necessary to use absolute counting methods.

C. There exists a third method for the determination of β_{28} and $\beta_{1/v}$. The reactor can be used as its own detector. The danger coefficient for the introduction of $1/v$ absorber in a cell divided by the danger coefficient for U^{235} gives

$$D_{1/v} / D_{25} = \frac{\sigma_{1/v} \phi_M \phi_M^+}{\nu_{25} - (1 + \alpha_{25}) \phi_F^+ \phi_F (\sigma_{f25})} \quad (6)$$

where $D_{1/v} / D_{25}$ is the change in reactivity per per cent change in $1/v$ absorption area and D_{25} is the change in reactivity per per cent change

in U^{235} content, ϕ_M^+ is the adjoint flux in the moderator, ϕ_F^+ is the adjoint flux in the fuel, and the other quantities have been previously defined.

In cases such as highly enriched, nearly homogeneous light water systems, the adjoint flux is nearly constant locally in space and in energy in the range below the cadmium cut-off. Then $\phi_M^+ \doteq \phi_F^+$ and $\beta_{1/v}$ and β_{28} can be obtained from cadmium ratio determinations and sub-cadmium danger coefficients. For example,

$$\beta_{1/v} = [D_{1/v}^*/D_{25}^*] \frac{\rho_{1/v}}{\rho_{1/v} - 1} \frac{\rho_{25} - 1}{\rho_{25}} [\nu_{25} - (1 + \alpha_{25}^*)] \quad (7)$$

The asterisks indicate sub-cadmium quantities and the letter ρ indicates a cadmium ratio. (In U^{235} the ratio is with respect to fission.)

A new unknown, α_{25}^* , has been introduced. The capture-to-fission ratio below the cadmium cut-off can, in principle, be determined in the same way as α_{25} . Except in the special cases in which a direct determination of α_{25}^* can be made, α_{25}^* will have to be estimated. Fortunately, α_{25}^* shows little variation over a large range of spectra, so that the error introduced in this way is not sensitive to reactor types.

The danger coefficient technique is not applicable to the determination of L in any obvious way except as has been described in Part II.

IV. USES OF OTHER DATA

The primary data are those listed in Table I. In Part II we have seen that such secondary data as cadmium ratios may be necessary to infer values of $\beta_{1/v}$ and β_{28} . There are at least three uses for secondary data. One is to aid in the inference of primary data when such data are directly available, or to be used in conjunction with theoretical calculations to estimate primary data.

A second use is to furnish further checks on the internal consistency of the theory. In other words, a theory is not necessarily correct even if it yields correct values of the primary data. The secondary data give information about the details of the processes corresponding to each item of primary data.

A third use of secondary data is that, together with primary data, estimates of standard theory parameters may be obtained. This problem is discussed in more detail in Part V. We point out here, however, that standard theory definitions do not always correspond to the definitions used in gathering primary or secondary data. A prime example is shown

in the case of the fast effect and resonance escape probability, which, between them, involve most (but not all) epi-cadmium U^{238} capture and all U^{238} fission. Theory may assign some capture to the fast effect so that the resonance escape probability will correspond to single-rod measurements and calculations, or it may assign all capture to the resonance escape process, in which case the single-rod results become meaningless.

The general criteria for secondary quantities are:

1. The quantities are each related to a single primary process.
2. The quantities are associated with energy dependence in an identifiable way, one that is useful in theoretical analysis.
3. As is the case of primary quantities, secondary quantities are derivable from experimental data without theoretical interpretation.

The second criterion assures us that the secondary quantities yield information about the details of the primary processes in a useful way. Thus, when multigroup diffusion theory is used (for example), a sharp energy cut is necessary to match the observed details of primary processes with the calculated values.

The various cadmium ratios ($\rho_{1/v}$, ρ_{28} , ρ_{25}) mentioned in Part II satisfy these three criteria. The cadmium cut-off is not as sharp as we would like. It is possible to define an effective cut-off energy vs thickness relationship^(4,5) however, and if the cut-off is made in a region of smoothly varying cross section, the unprecision in the notion of cadmium cut-off causes little error.

In order to minimize the sensitivity of the cadmium cut-off to spectral shape, it is necessary to use a thickness in which the cut-off is nearly linear with thickness. The most prominent resonances at cadmium cut-off energies are the U^{235} resonances at 0.2 and 0.28 volt. To avoid these resonances and sensitivity to spectral shape, a cadmium cut-off of 0.7 ev (foils 40 mils thick)⁽⁵⁾ is recommended.

A popular theoretical cut-off, 0.625 ev, corresponds roughly to 30-mil foils, but the thicker foils are less sensitive to spectral shape.

A number of spectral indicators other than cadmium have been proposed. These include samarium, lutetium, and dysprosium. Samarium indication has been discussed at some length in Ref. 5. In general, it can be expected that reaction rates of samples shielded by a variety of indicators will give useful internal checks on primary processes. At present, techniques are not well enough developed to give a definitive list of measurements.

The importance of internal checks on the capture rate, especially at the lower end of the energy scale, should not be minimized. Many items of great practical importance, such as the conversion ratio, depend heavily on estimates of low-energy spectral effects. There is considerable theoretical basis for suspecting that these effects can be correlated in an adequate approximation by the use of indicators such as those mentioned above. It is too much to expect that such approximate correlations will permit equally good estimates of reactivity; the inability to predict both reactivity and conversion ratio on a common basis is a primary motivation for the more complex system of measurements described here.

Indicators such as gold and indium, which have prominent, single, low-lying resonances, have been frequently used with cadmium shielding to characterize the epithermal flux. The resonances in these indicators lie below the majority of the resonances in U^{235} and U^{238} . If these measurements could be coupled with estimates of fluxes in the few keV range, a check could be made of the calculation of capture in the resolved resonance region. Flux indication at such levels is difficult, however, so that it is unlikely the cadmium-covered gold or indium measurements can be made to give much information about epi-cadmium processes.

Single resonance or threshold indicators are useful, though, in connection with leakage processes and flux maps. Typically, the theoretical treatment of L involves the computation of leakage probability [$P(B^2)$] between specific energy limits and the effective buckling of the flux within these limits. Flux maps taken with threshold detectors provide information about buckling and offer checks on the theory of migration of neutrons between specific energy limits. Specifically, U^{238} fission activity is a good measure of the distribution of source neutrons. Neptunium fission, with a threshold at a few hundred keV, detects the flux at a point below which most scattering cross sections are very nearly constant.

Cadmium-covered gold and indium have already been mentioned as epi-cadmium flux indicators, and sub-cadmium gold and U^{235} (fission) activations are useful as thermal flux indicators.

With the various flux maps at hand, it is possible to set up a procedure for buckling determination in reflected systems.

If the description of multigroup diffusion theory is assumed and if it is also assumed that the indicated fluxes represent the fluxes within distinct, but otherwise unspecified, groups, then it follows that the measured fluxes can each be described by a fundamental mode associated with some buckling, β_0 , and a collection of transient terms arising from the reflector. If arbitrary linear combinations of all the measured fluxes are now taken, there is one combination which yields a minimum least-squares

error fit to a fundamental mode associated with some buckling β_0 . Since the U^{238} fission activation is probably least affected by the reflector, this plot may be used to get an initial estimate of β_0 .

It is an open question as to how rapidly edge effects in the various fluxes die off with distance from the interface. Thus it is not obvious that using many flux plots will give a better estimate of β_0 than just a few.

The secondary quantities we have discussed are listed in Table II; others are certainly possible, but increasing the amount of secondary data does not increase the precision of the experiment unless a check on the internal consistency is furnished.

Table II

SECONDARY QUANTITIES

Symbol	Definition
ρ (with various subscripts) (ρ_{25} refers to fission activity only)	Reaction rate in bare detector divided by reaction rate in cadmium-covered detector. Recommended thickness of cadmium: 0.040 in. (0.102 cm).
F* (with various subscripts)	Sub-cadmium fission rate: Total fission rate - fission rate when detector is covered with cadmium.
$D_{1/v}$	Danger coefficient for $1/v$ absorber: Reactivity change (in dollars) per per cent change in $1/v$ absorption area.
D_{25}	Danger coefficient for U^{235} : Reactivity change (in dollars) per per cent change in U^{235} content.
D^* (with subscripts)	The difference of danger coefficients taken with bare sample and cadmium-covered sample.
α_{25}^*	The difference between capture to fission ratio in bare and cadmium-covered U^{235} .

Note: In addition, the specific activity of U^{238} (fission), Np (fission), cadmium-covered Au, cadmium-covered In, and the sub-cadmium Au and U^{235} fission activities have been mentioned as flux indicators.

V. CONNECTION WITH THE STANDARD THEORY PARAMETERS

The standard four-factor formula for the multiplication process is typically based on considerations which lump all events that occur in a given region of space or energy range into escape or nonescape from a single process. It has long been realized that this approximation is crude; its success arises from the fact that in simple systems often only one capture process is dominant in any given energy range. There has been a natural tendency to improve the formulation by separating out processes (usually in the resonance region) which, on the energy scale, can be considered disjoint and then making some assumption as to neutron migration between the disjoint processes.

Such considerations are often helpful, but they suffer from the basic defect of the four-factor formula: the parameters are not directly related to primary and secondary data. We can, however, identify certain combinations of primary and secondary data which do, in the appropriate limits, approach the classical values.

Since the net leakage L is a separate quantity, it is possible to take the ratio of the source to all capture processes, productive or not. This ratio we term $k_{\infty}(L)$:

$$k_{\infty}(L) = \frac{\nu_{25} + \delta_{28}\nu_{28}}{1 + \alpha_{25} + \beta_{1/v} + \beta_{28} + \delta_{28}} \quad (8)$$

The ratio $k_{\infty}(L)$ is a function of the leakage L , since each of the primary data on the right-hand side of Equation (8) represents a rate process which competes with L over the entire energy range; moreover, the relative value of leakage rate to capture rate varies with energy. Thus the dependence of k_{∞} on L indicates that $k_{\infty}(L)$ is not the multiplication rate which would be observed if the reactor core were extended to infinity. As L tends to zero, however, $k_{\infty}(L)$ approaches the usual value.

In what follows the interdependence of the various data will not be explicitly stated. We note, however, that δ_{28} , α_{25} , $\beta_{1/v}$, and β_{28} are all average values over the reactor (see the discussion in Part II); the set used in Equation (8) do not necessarily refer to the set of values observed at any one point in the reactor core. By definition, L is a reactor-averaged value in any circumstance and has no significance when referred to a single point.

The number of neutrons emitted from U^{235} fission per neutron captured in U^{235} , " η_{25} ," is given by

$$" \eta_{25} " = \nu_{25} / (1 + \alpha_{25}) \quad (9)$$

Except for the internal dependence of α_{25} on the system parameters, this is the conventional definition. As mentioned in Part II, this value may be subject to less error than $\beta_{1/v}$, β_{28} , and L in appropriate circumstances.

The concept of thermal utilization is closely allied to the notion of $1/v$ utilization, that is, in a system with small capture, the capture in materials with a $1/v$ dependence can be assumed to take place only in the thermal group. This prescription leads to a familiar formula for the cut-off energy E_C for the thermal group:

$$(E_C/T)^2 \exp - (E_C/T) = \langle \Sigma_a \rangle / \xi \Sigma_s \quad .$$

(The conventional definitions are assumed.) Deviations from $1/v$ dependence can then be ascribed to resonance absorption. It is thus convenient to define a $1/v$ utilization, $f(1/v)$, which is just the capture rate in U^{235} divided by the total capture rate calculated as though all isotopes had a $1/v$ dependence. The utility of this notion arises from the fact that the gross behavior of U^{235} fission is almost as though it had a $1/v$ dependence (see Part III). For convenience, we introduce the value of the macroscopic absorption of a composition at 2200 m/sec: Σ_{a0} . Under these conditions, $f(1/v)$ is given by

$$f(1/v) = (1 + \alpha_0) / \left\{ \beta_{1/v} \left[1 + \left(\Sigma_{a0}(25) / \Sigma_{a0}(1/v) + \Sigma_{a0}(28) / \Sigma_{a0}(1/v) \right) \right] \right\} , \quad (10)$$

where α_0 is the value of $\alpha(25) = \Sigma_c(25) / \Sigma_f(25)$ at 2200 m/sec. Concurrent with the introduction of α_0 we introduce

$$\gamma_0 = \frac{\alpha - \alpha_0}{1 + \alpha_0} \quad .$$

The ratio $k_\infty(L) / \eta f(1/v)$ is then

$$k_\infty(L) / \eta f = \frac{\beta_{1/v} (1 + \gamma_0) \left(1 + \frac{\nu_{28}}{\nu_{25}} \delta_{28} \right) \left(1 + \frac{\Sigma_{a0}(28)}{\Sigma_{a0}(1/v)} + \frac{\Sigma_{a0}(25)}{\Sigma_{a0}(1/v)} \right)}{\beta_{1/v} + 1 + \alpha + \beta_{28} + \delta_{28}} \quad . \quad (11)$$

The right-hand side of Equation (11) is related to the conventional product ϵp . This relationship can be made somewhat clearer by a little algebraic manipulation which yields:

$$\text{Right-hand side of Equation (11)} = \frac{(1 + \gamma_0) \left(1 + \frac{\nu_{28}}{\nu_{25}} \delta_{28} \right)}{1 + \frac{\frac{1 + \alpha}{\beta_{1/v}} \frac{\Sigma_{a0}(25)}{\Sigma_{a0}(1/v)} + \left[\frac{\beta_{28} + \delta_{28}}{\beta_{1/v}} \frac{\Sigma_{a0}(28)}{\Sigma_{a0}(1/v)} \right]}{1 + \frac{\Sigma_{a0}(1/v)}{\Sigma_{a0}(1/v)} + \frac{\Sigma_{a0}(28)}{\Sigma_{a0}(1/v)}}} \quad . \quad (11a)$$

The terms in brackets clearly represent the capture in excess of $1/\nu$ absorption - the resonance capture. Since U^{235} is nearly $1/\nu$, the first bracketed term is small.

Just as the denominator of Equation (11a) contains terms which increase the total capture rate, so the numerator contains a term, $1 + \gamma_0$, which increases the capture rate in uranium. Had we introduced $\eta_0 = \nu_{25}/(1 + \alpha_0)$ instead of " η ," the term $1 + \gamma_0$ would be replaced by $(1 + \alpha_0)/(1 + \alpha)$. This redefinition removes η_0 to the realm of a constant independent of the system under study; arbitrarily we prefer not to do this.

More or less by default we identify $k_\infty(L)/\eta f(1/\nu)$ as " ϵ p." There is no obvious simple connection, however, between terms in the right-hand side of (11) or (11a) and classical definitions of ϵ or p . It is clear from arguments at the beginning of this section that such a connection is necessarily obscure. If the fission in U^{238} and non- $1/\nu$ behavior of U^{235} are ignored, as well as spatial disadvantage factors, then resonance capture in U^{238} leads to a correction term:

$$"p" = \frac{1}{1 + \left\{ \left[\frac{\beta_{28}}{\beta_{1/\nu}} - \frac{\Sigma_{a0}(28)}{\Sigma_{a0}(1/\nu)} \right] \left/ \left[1 + \frac{\Sigma_{a0}(25)}{\Sigma_{a0}(1/\nu)} + \frac{\Sigma_{a0}(28)}{\Sigma_{a0}(1/\nu)} \right] \right\}}. \quad (12)$$

If, still ignoring fast fission and non- $1/\nu$ behavior in U^{235} , we write

$$k_\infty(L)/\eta f(1/\nu) = \frac{S}{S_0},$$

where S_0 and S are the sources in the absence and presence of U^{238} resonance capture respectively, then

$$\frac{S}{S_0} = \left[\beta_{28} - \frac{\beta_{1/\nu}}{\Sigma_a(1/\nu)} \Sigma_{a0}(28) \right] \frac{"p"}{1 - "p"}. \quad (12a)$$

But in standard theory the term in brackets in Equation (12a) is (with appropriate normalization) just what is ordinarily computed as $1 - p$. Thus, " p " approaches p in the appropriate limit if we identify " p " as the resonance escape probability when all capture in excess of $1/\nu$ capture is included in the process of resonance capture.

If all capture processes are accounted for by either $1/\nu$ or resonance capture, a redefinition of ϵ is needed. Under these conditions, ϵ is simply the ratio of total fissions to fissions in U^{235} ; it is a source-magnification factor. Hence, we define

$$" \epsilon " = 1 + \frac{\nu_{28}}{\nu_{25}} \delta_{28} \quad . \quad (13)$$

This definition does not approach any classical limit. The reason is that all capture associated with fast fission has been lumped into "p," where "p" is the resonance escape probability observed in lattice-substitution experiments. If the capture associated with fast fission is included in the definition of ϵ , the p must be the resonance escape probability based on single-rod measurements. The product " ϵ " "p" defined here is not quite the same as the classical value of ϵp , even so, because all non- $1/v$ behavior is lumped into "p." If this contribution is ignored, then " ϵ " "p" approaches ϵp in the limit of large, dilute, weakly capturing systems.

Experiments reported in Refs. (1) and (6) show that when UO_2 rods are substituted for uranium rods the relative decrease in " ϵ " is twice that in ϵ ; the relative increase in U^{238} resonance capture, however, is only $\frac{3}{4}$ of that expected from single rod measurements. In the experiments reported in Ref. (6) all high energy capture was associated with resonance capture. Thus, if these measurements were used to obtain a resonance integral, the fast fission factor used should be " ϵ " not ϵ .

APPENDIX A
CALIBRATION TECHNIQUES

The calibration technique mentioned in Part II is described here in greater detail.

The procedure is to irradiate uranium foils and get the Np^{239} activity P . One foil is irradiated in the fuel (f) and the other in the thermal column (T). If the fraction of U^{235} in the foil is b , the fission product activity F is given by

$$F_f = bF_f^{25} + (1-b) F_f^{28} \quad (\text{A-1})$$

and

$$F_T = bF_T^{25} \quad (\text{A-2})$$

If we assume that all samples are measured with the same counter and the counter efficiency is constant, then

$$\langle \sigma_c^{28} \rangle_f = \frac{P_f}{P_T} \langle \sigma_c^{28} \rangle_T \quad (\text{A-3})$$

$$\langle \sigma_f^{25} \rangle_f = \frac{F_T^{25}}{F_f^{25}} \langle \sigma_f^{25} \rangle_T \quad (\text{A-4})$$

Then

$$\beta_{28} = \frac{P_f}{P_T} \frac{F_T^{25}}{F_f^{25}} \frac{\langle \sigma_c^{28} \rangle_T}{\langle \sigma_f^{25} \rangle_T}, \quad (\text{A-5})$$

which is Equation (2) in Part II.

To determine F_f^{25}/F_T^{25} , use foils of two different enrichments, b_1 and b_2 . Then

$$r_1 = \frac{b_1 F_f^{25} + (1-b_1) F_f^{28}}{b_1 F_T^{25}}$$

and

$$r_2 = \frac{b_2 F_f^{25} + (1-b_2) F_f^{28}}{b_2 F_T^{25}}$$

can be measured directly.

Solving for F_f^{25}/F_T^{25} , we get

$$R = F_f^{25}/F_T^{25} = \left[r_1 - \frac{b_2}{b_1} r_2 \left(\frac{1 - b_1}{1 - b_2} \right) \right] / \left(1 - \frac{b_2}{b_1} \frac{1 - b_1}{1 - b_2} \right) \quad (\text{A-6})$$

The relative error in R depends on $\delta_{28} = F_f^{28}/bF_f^{25}$; approximately,

$$\frac{dR}{R} \doteq \frac{b_1}{b_1 - b_2} (1 + \delta_{28}) \frac{dr_1}{r_1} + \left(\frac{b_2}{b_1 - b_2} \right) \left(1 + \frac{b_1}{b_2} \delta_{28} \right) \frac{dr_2}{r_1}$$

where b_1 is the fuel enrichment, b_2 is a lower enrichment and $b_1, b_2 \ll 1$.

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