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FEASIBILITY OF Pu^{239}-U^{235}-FUELED CORES TO PREDICT Pu^{239}-FUELED CORE DIMENSIONS

by

D. Meneghetti and H. Ishikawa

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Reactor Engineering Division

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ABSTRACT

Use of Pu^{239}-U^{235}-fueled fast critical assemblies to estimate properties of Pu^{239}-fueled assemblies is of interest because of safety considerations and limited plutonium availability.

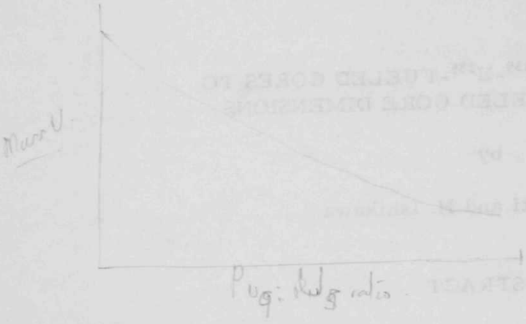
Bare and reflected homogeneous cores and reflected two-region cores are considered. The fuel, 5% by volume, is assumed to be Pu^{239} and U^{235} of various fuel composition ratios for the homogeneous cores. For the two-region cores the 5% fuel volume is Pu^{239} in the central region and U^{235} in the outer core region. Core diluents, simulating fertile, structural, and coolant materials, are assumed identical in all cases.

It is estimated that construction of the reflected two-region core with ratio of central core region volume to total core volume of 0.1 will theoretically decrease the calculated error in prediction of the critical size of a corresponding solely Pu^{239}-fueled assembly by a factor of about 10 to 20.

I. INTRODUCTION

Critical assemblies representing Pu^{239}-fueled dilute fast breeder reactors require large quantities of Pu^{239}. Because of safety considerations and limited Pu^{239} availability it is desirable to determine the feasibility of employing composite Pu^{239}-U^{235}-fueled criticals to predict properties of analogous Pu^{239}-fueled criticals.

For prediction of critical size, from an operational point of view, cross-section parameters for an analogous U^{235}-fueled critical can first be adjusted to force agreement of calculated with experimental critical size. A cross-section parameter of Pu^{239} can subsequently be adjusted to force agreement of calculation with the observed size of an analogous Pu^{239}-U^{235}-fueled critical. The critical size of an analogous Pu^{239}-fueled



critical might then be predicted with improved accuracy by use of the adjusted cross-section parameters. The improvement in accuracy depends upon the extent to which compensation adjustment at the intermediate $Pu^{239}/(Pu^{239} + U^{235})$ core composition ratio continues to compensate for the unknown cross-section errors as Pu^{239} replaces the U^{235}. Ideally, if the functional relations of the deviations in critical size as a function of $Pu^{239}/(Pu^{239} + U^{235})$ ratio are proportional for all cross-section variations then the compensation adjustment will remain exact upon extrapolation to the all Pu^{239}-fueled core.

A calculational study to determine the extent to which compensation occurs upon extrapolating to the Pu^{239} system has been carried out. Bare and reflected homogeneous cores and reflected two-region cores were considered. The fuel, 5% by volume, was Pu^{239} and U^{235} of various composition ratios in the homogeneous cores. For the regional cores the 5% fuel volume was Pu^{239} in the central core region and U^{235} in the outer core region. Equal volumes of U^{235} and Pu^{239} are assumed to contain equal numbers of atoms. Exchange of a volume of U^{235} by an equal volume of Pu^{239} is then equivalent to an exchange of equal number of atoms. Core diluents were 45% U^{238}, $16\text{-}\frac{2}{3}$% iron and 19.3% aluminum in all cases. The reflectors, 58 cm thick, contained 40% U^{238}, 20% iron and 23.2% aluminum by volume. Effects of Pu^{239} parameter variations of capture, inelastic transfer, i.e., $\sigma_{1\rightarrow 2}$, transport cross-section parameters, and number of neutrons per fission were considered. The multigroup analyses employed four neutron energy groups having lower energy limits 1.35, 0.3, 0.067, and 0 Mev. Except for buckling calculations the discrete SN, i.e., DSN, IBM-704 code(1,2) in S_4 approximation was used for the analyses.

It should be noted that the exchange of Pu^{239} and U^{235} throughout this study has been made assuming that the average fuel density in the cores is always 5% by volume. Criticality in the calculations were obtained by varying the dimensions of the core regions. An effect of this is the large inventory of U^{235} required for criticality of both the composite system and the corresponding U^{235}-fueled system. For example consider 10% of the total fuel volume in the core of 5% is the Pu^{239} in the composite system. Then the reflected two-region core of the present study would require about 60 kg of Pu^{239}. This composite system would be used to extrapolate to a fully Pu^{239}-system of about 200 kg of Pu^{239}. The fully U^{235}-fueled system, which would also be required, would require about 1240 kg of U^{235}.

II. THEORETICAL CONSIDERATIONS FOR HOMOGENEOUS MIXTURES

An orientation into the effects of compensation for the unknown cross section errors by ad hoc adjustment of some suitable cross-section parameter may be obtained by one-group considerations. The material buckling $B^2 = (\nu\Sigma_F - \Sigma_{C+F})\, 3\Sigma_{TR}$. Σ_{C+F} is the sum of capture and fission cross sections.

critical might then be predicted with improved accuracy by use of the adjusted cross-section parameters. The improvement in accuracy depends upon the extent to which compensation adjustment at the intermediate $Pu^{239}/(Pu^{239} + U^{235})$ core composition ratio continues to compensate for the unknown cross-section errors as Pu^{239} replaces U^{235}. Ideally, if the functional relations of the deviations in critical size as a function of $Pu^{239}/(Pu^{239} + U^{235})$ ratio are proportional for all cross-section variations then the compensation adjustment will remain exact upon extrapolation to the all Pu^{239}-fueled core.

A calculational study to determine the extent to which compensation occurs upon extrapolating to the Pu^{239} system has been carried out. Bare and reflected homogeneous cores and reflected two-region cores were considered. The fuel, 5% by volume, was Pu^{239} and U^{235} at various composition ratios in the homogeneous cores. For the regional cores the 5% fuel volume was Pu^{239} in the central core region and U^{235} in the outer core region. Equal volumes of U^{235} and Pu^{239} are assumed to contain equal numbers of atoms. Exchange of a volume of U^{235} by an equal volume of Pu^{239} is then equivalent to an exchange of equal number of atoms. Core diluents were 45% U^{238}, 16-2/3% iron and 33-1/3% aluminum in all cases. The reflectors, 9" thick, contained 40% U^{238}, 20% iron and 22.2% aluminum by volume. Eighteen Pu^{239} parameter variations of capture, inelastic transfer, fission, $\nu\sigma_f$, transport cross-section parameters, and number of neutrons per fission were considered. The multigroup analyses employed four neutron energy groups having lower energy limits 1.35, 0.4, 0.067 and 0 Mev. Except for buckling calculations the discrete SN, i.e., DSN, IBM-704 code in S_4 approximation was used for the analyses.

It should be noted that the exchange of Pu^{239} and U^{235} throughout this study has been made, assuming that the average fuel density in the cores is always 5% by volume. Criticality in the calculations were obtained by varying the dimensions of the core regions. An effect of this is the large inventory of U^{235} required for criticality of both the composite system and the corresponding U^{235}-fueled system. For example consider 10% of the total fuel volume in the core as Pu^{239} as the Pu^{239} in the composite system. Then the reflected two-region core of the present study would require about 50 kg of Pu^{239}. This composite system would be used to extrapolate to a fully Pu^{239}-system of about 200 kg of Pu^{239}. The critical U^{235}-fueled system, which would also be required, would require about 170 kg of U^{235}.

II. THEORETICAL CONSIDERATIONS FOR HOMOGENEOUS MIXTURES

An orientation into the effects of compensation for the unknown cross section errors by a long adjustment of some suitable cross-section parameter may be obtained by one-group considerations. The material buckling $B^2 = (\nu\Sigma_f - \Sigma_{c+f})/D$. Σ_{c+f} is the sum of capture and fission cross sections.

With N^P the atomic density of Pu^{239} in the core, N^U the atomic density of U^{235} in the core, and $N^P + N^U = N \equiv$ a constant, the buckling becomes

$$B^2 = \frac{\left[\nu^U\sigma_F^U(N - N^P) + \nu^P\sigma_F^P N^P - \sigma_{C+F}^U(N - N^P) - \sigma_{C+F}^P N^P - \Sigma_C^D\right] X}{3\left[\sigma_{TR}^U(N - N^P) + \sigma_{TR}^P N^P + \Sigma_{TR}^D\right]}$$

where Σ^D refers to the cross sections of all materials other than U^{235} and Pu^{239}. This may be rewritten as the product of two linear functions:

$$B^2 = (ax + b)(cx + d) \quad ,$$

where

$$x = N^P \quad , \quad a = -\nu^U\sigma_F^U + \nu^P\sigma_F^P + \sigma_{C+F}^U - \sigma_{C+F}^P \quad ,$$

$$b = \nu^U\sigma_F^U N - \sigma_{C+F}^U N - \Sigma_C^D \quad , \quad c = -3\sigma_{TR}^U + 3\sigma_{TR}^P \quad ,$$

and

$$d = 3\sigma_{TR}^U N + 3\Sigma_{TR}^D \quad .$$

If $c = 0$, then $B^2 = \text{const.}\ (ax + b)$. Thus if $\sigma_{TR}^U = \sigma_{TR}^P$, then B^2 is a linear function of N^P.

Variations of cross sections give $B'^2 = (a'x + b')(c'x + d')$. With $\Delta a = a' - a$, $\Delta b = b' - b$, $\Delta c = c' - c$, and $\Delta d = d' - d$ is obtained

$$\Delta(B^2) \equiv B'^2 - B^2 = (c\Delta a + a\Delta c + \Delta a\Delta c)x^2$$
$$+ (d\Delta a + c\Delta b + b\Delta c + a\Delta d + \Delta a\Delta d + \Delta b\Delta c)x$$
$$+ (d\Delta b + b\Delta d + \Delta b\Delta d) \quad .$$

In order that $\Delta(B^2) = 0$ when $x = 0$, non-Pu^{239} cross sections are adjusted so that $b' = b$ and $d' = d$. $B'^2 - B^2$ then becomes

$$\Delta(B^2) = (c\Delta a + a\Delta c + \Delta a\Delta c)x^2 + (d\Delta a + b\Delta c)x \quad .$$

Now if $c = 0$, and $\Delta c = 0$, then $\Delta(B^2) = (d\Delta a)x$. Therefore if $\sigma_{TR}^U = \sigma_{TR}^P$ and if $\Delta\sigma_{TR}^U = \Delta\sigma_{TR}^P$, then $\Delta(B^2)$ is a linear function of N^P for variations in the Pu^{239} cross sections in the quantity Δa.

With N^P the atomic density of Pu^{239} in the core, N^U the atomic density of U^{238} in the core, and $N^P + N^U = N$ = a constant, the buckling becomes

$$B^2 = \left[\nu^U\sigma_F^U(N - N^P) + \nu^P\sigma_F^P N^P - \sigma_{C+F}^U(N - N^P) - \sigma_{C+F}^P N^P - \Sigma_C^D\right] \times$$

$$\left[3\sigma_{TR}^U(N - N^P) + 3\sigma_{TR}^P N^P + 3\Sigma_{TR}^D\right]$$

where Σ^D refers to the cross sections of all materials other than U^{238} and Pu^{239}. This may be rewritten as the product of two linear functions:

$$B^2 = (ax + b)(cx + d)$$

where:

$$x = N^P, \quad a = -\nu^U\sigma_F^U + \nu^P\sigma_F^P + \sigma_{C+F}^U - \sigma_{C+F}^P,$$

$$b = \nu^U\sigma_F^U N - \sigma_{C+F}^U N - \Sigma_C^D, \quad c = -3\sigma_{TR}^U + 3\sigma_{TR}^P,$$

and

$$d = 3\sigma_{TR}^U N + 3\Sigma_{TR}^D$$

If $c = 0$, then B^2 = const. $(ax + b)$. Thus if $\sigma_{TR}^U = \sigma_{TR}^P$, then B^2 is a linear function of N^P.

Variations of cross sections give $B'^2 = (a'x + b')(c'x + d')$. With $\Delta a = a' - a$, $\Delta b = b' - b$, $\Delta c = c' - c$, and $\Delta d = d' - d$ is obtained

$$\Delta(B^2) = B'^2 - B^2 = (c\Delta a + a\Delta c + \Delta a\Delta c)x^2$$

$$+ (d\Delta a + c\Delta b + b\Delta c + a\Delta d + \Delta a\Delta d + \Delta b\Delta c)x$$

$$+ (d\Delta b + b\Delta d + \Delta b\Delta d)$$

In order that $\Delta(B^2) = 0$ when $x = 0$, non-Pu^{239} cross sections are adjusted so that $b' = b$ and $d' = d$. $B'^2 - B^2$ then becomes

$$\Delta(B^2) = (c\Delta a + a\Delta c + \Delta a\Delta c)x^2 + (d\Delta a + b\Delta c)x \quad .$$

Now if $c = 0$, and $\Delta c = 0$, then $\Delta(B^2) = (d\Delta a)x$. Therefore if $\sigma_{TR}^U = \sigma_{TR}^P$ and if $\Delta\sigma_{TR}^U = \Delta\sigma_{TR}^P$, then $\Delta(B^2)$ is a linear function of N^P for variations in the Pu^{239} cross sections in the quantity Δa.

If instead c = 0, and $\Delta a = 0$, then

$\Delta(B^2) = (a\Delta c)x^2 + (b\Delta c)x = x\Delta c(ax + b)$ so that the quantity

$$\frac{\Delta(B^2)}{B^2} = \frac{x\Delta c(ax+b)}{(ax+b)(cx+d)} = \frac{x\Delta c}{d} \quad .$$

As

$$\frac{\Delta(B^2)}{B^2} \equiv \frac{B^{2\prime} - B^2}{B^2} = \frac{(B' - B)(B' + B)}{B^2} \text{ and } B' \cong B \quad ,$$

then

$$2\,\frac{(B' - B)}{B} \cong \frac{x\Delta c}{d}$$

so that

$$\frac{\Delta B}{B} \cong \frac{x\Delta c}{2d} \quad .$$

Therefore for variation of only the transport cross section of Pu^{239} from the case of $\sigma_{TR}^{U^{235}} = \sigma_{TR}^{Pu^{239}}$ the quantities $\Delta(B^2)/B^2$ and $\Delta B/B$ are approximately linear functions of N^P. Furthermore the one-group analyses shows that if errors in transport cross section are negligible in addition to the requirement $\sigma_{TR}^{U^{235}} = \sigma_{TR}^{Pu^{239}}$ then compensation adjustment for other cross-section errors will be exact when subsequently extrapolated to the all Pu^{239}-fueled system. The extent to which this will occur for actual U^{235} and Pu^{239} fueled cores has been carried out in the present study by multi-group analyses of representative cores.

III. BARE AND REFLECTED HOMOGENEOUS CORES

A. Bare Homogeneous Cores

Material bucklings for various $Pu^{239}/(Pu^{239} + U^{235})$ fuel composition ratios were obtained by multigroup fundamental mode calculations using the four-group fast neutron cross-section parameters given in Appendix I. The material bucklings, B^2, using diffusion theory leakage are shown in Fig. 1. Values of B^2 by transport theory are about 1 to 2% larger than by diffusion theory. The various curves of Fig. 1 represent assumed changes in various cross-section parameters of Pu^{239}. The curves referred to as standard, high capture, and low capture correspond to the three sets of values of capture to fission of Pu^{239} given in the eleven group cross section set of Loewenstein and Okrent(3) from which the four-group set used herein was obtained by collapsing of groups. The changes from the standard $\sigma_C^{Pu^{239}}$

are -34.8%, -46.5%, -44.9%, and -17.3% for the respective groups in case of low $\sigma_C^{Pu^{239}}$. In the case of high $\sigma_C^{Pu^{239}}$ the standard values are increased by 0%, 15.0%, 25.9%, and 9.8% respectively.

Critical masses obtained using the standard Pu^{239} cross sections are shown in Fig. 2. These were derived from the B values calculated by the transport theory fundamental mode calculations and were corrected by subtraction of extrapolation distances.

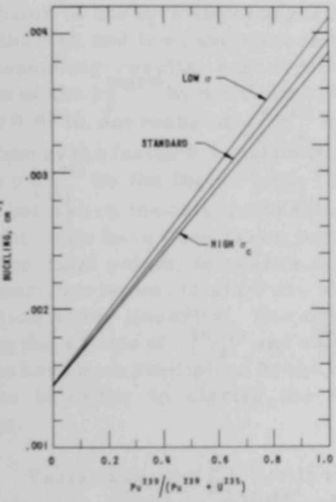


Fig. 1

Buckling as Function of Pu^{239} Concentration

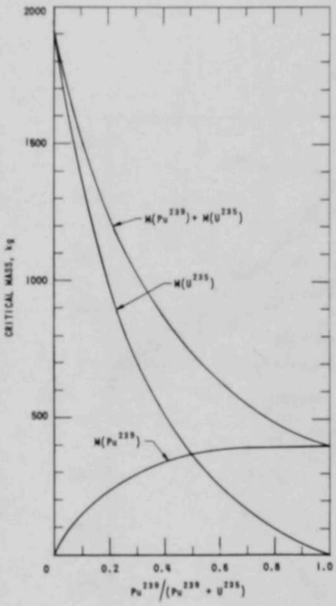


Fig. 2

Critical Mass as Function of $Pu^{239}/(Pu^{239} + U^{235})$ for Homogeneous Bare Cores

Changes in the distribution of group fluxes with plutonium concentration are shown in Fig. 3. Effects upon spectra of the cross-section variations of the magnitudes assumed in the present study do not affect the curves of Fig. 3 sufficiently to be noticed on the indicated graph.

The quantities, $\Delta(B^2)$, representing the differences between the diffusion theory material bucklings calculated using cross-section variations of Pu^{239} and those denoted as standard cross sections are shown in Fig. 4.

are [illegible], [illegible] and [illegible] for the respective groups in case of [illegible]. In the case of high [illegible] the standard values are increased by [illegible], [illegible], 26.5%, and [illegible] respectively.

Critical mass was obtained using the standard Pu^{239} cross sections are shown in Fig. [illegible]. These were obtained from the B values calculated by the [illegible] theory (uniform [illegible]) calculations and were corrected by [illegible] of extrapolation distance.

Fig. [illegible]

Fig. [illegible]

Critical Mass as Function of $Pu^{239}/(Pu^{239}+U^{235})$ for Homogeneous Bare Cores

[illegible] as Function of Pu^{239} Concentration

Changes in the distribution of group fluxes with plutonium concentration are shown in Fig. 3. Effects upon spectra of the fission-section [illegible] of the [illegible] as used in the present study do not affect the curves of Fig. 3 sufficiently to be noticed on the indicated graph.

The quantities, $\Delta(\delta^*)$, representing the differences between the [illegible] theory material [illegible] calculated using cross-section values of Pu^{239} and those obtained as [illegible] cross sections are shown in Fig. [illegible].

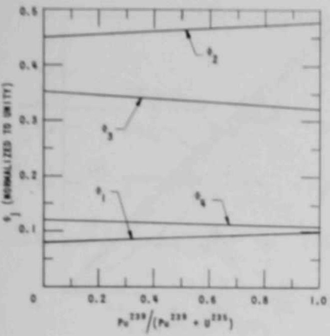


Fig. 3

Distribution of Group Fluxes as Function of Pu^{239} Concentration

In addition to the buckling changes resulting from the high and low capture variations are corresponding results representing: decrease of the $\nu_j^{Pu^{239}}$ by multiplication by the factor 0.9736, decrease of $\sigma_{1\rightarrow 2}^{Pu^{239}}$ by multiplication by the factor 0.9, and multiplication of the $\sigma_{TRj}^{Pu^{239}}$ by the factor 1.08. The indicated points are the calculated differences. Straight lines have been drawn between initial and final points, for cases other than transport variation, to show the amount of deviation from linearity. The curves depicting the effects of $\sigma_{1\rightarrow 2}^{Pu^{239}}$ and $\sigma_{TRj}^{Pu^{239}}$ variations have been multiplied by the indicated factors in order to clarify the resulting shapes.

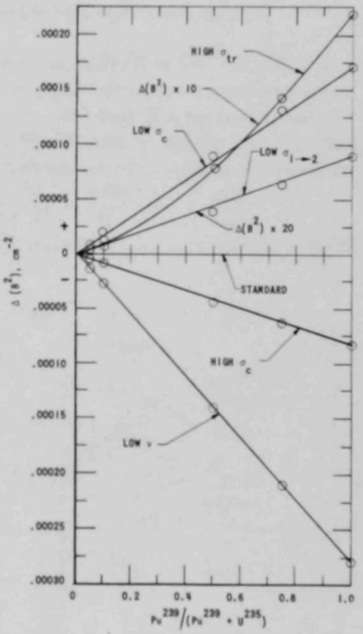


Fig. 4

Buckling Deviations as Function of Pu^{239} Concentration

Variations in $\sigma_{F_j}^{Pu^{239}}$ will vary both the composite quantities $\nu\sigma_{F_j}^{Pu^{239}}$ and $\sigma_{C_j+F_j}^{Pu^{239}}$. Deviation curves for fission cross-section variations will consist of linear combinations of deviation curves derived from appropriate variations in $\nu_j^{Pu^{239}}$ and $\sigma_{C_j}^{Pu^{239}}$. Thus modification of σ_{F_j} to $\epsilon\sigma_{F_j}$ is equivalent to modification of ν_j to $\epsilon\nu_j$ and σ_{C_j} to $\sigma_{C_j} + (\epsilon - 1)\sigma_{F_j}$.

The reduction of error in calculation of material buckling for the extrapolated plutonium-fueled core by cross-section compensating adjustment at a Pu^{239}-U^{235} composition may be estimated using Fig. 5. The $|\Delta(B^2)|$ were normalized to the value unity at the Pu^{239} core. The deviations from linearity as well as the range of spread of the curves from each other are evident.

(1) 5% U_5 Pu9 — difficult in practice to get say 1% P_0, 5% U_5

without marked heterogeneity effect

(2) Can we not check this technique using the available ZPR III assemblies

ie take an assembly with ~~various~~ x columns out of say 16

in a module containing C + Al (y of C $x - y$ of Al) & other assemblies with

different values of y

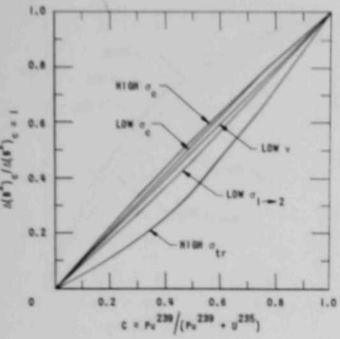


Fig. 5

Relative Values of Normalized Buckling Deviations

For example suppose the unknown inherent cross-section errors are to be compensated by adjustment of the $\nu_j^{Pu^{239}}$ for an assembly corresponding to $Pu^{239}/(Pu^{239} + U^{235} = 0.1$. The relative values of $|\Delta(B^2)|$ are about 0.11, 0.09, 0.045, and 0.10 for Pu^{239} capture, inelastic transfer, transport, and ν variation respectively. The error in calculated B^2 for the extrapolated Pu^{239}-fueled system is thus reduced to about 10%, 10%, 55%, and 0%, respectively, of the error which would have been obtained if no compensation adjustment had been made, depending upon the respective assumed inherent error.

The ratios, $\Delta B/B \equiv (B' - B)/B$, for the assumed cross-section variations are shown in Fig. 6. B' and B are material bucklings obtained using asymptotic transport fundamental mode calculations. The primed quantities refer to the perturbed cross section and unprimed to the unperturbed. As $\frac{\Delta R}{R} \equiv \frac{R' - R}{R} = -\left(\frac{\Delta B}{B}\right)\Big/\left(1 + \frac{\Delta B}{B}\right) \cong -\frac{\Delta B}{B}$, the corresponding fractional changes in extrapolated bare core radii, $\Delta R/R$ may be estimated from the curves of Fig. 6.

The relative values of $|\Delta R/R|$ normalized to unity for the Pu^{239}-fueled system are shown in Fig. 7, for the various cases. From these curves the fractions of error remaining, relative to the error otherwise obtained using the standard Pu^{239} set, in extrapolated bare core radius of the Pu^{239}-fueled system due to compensation adjustment can be estimated. Thus if the $\nu_j^{Pu^{239}}$ are adjusted at the composition ratio $Pu^{239}/(Pu^{239} + U^{235}) = 0.1$ the fractions of the errors remaining are about 0.18 for σ_c error, 0.14 for $\sigma_{1\rightarrow 2}$ error, 0.54 for σ_{TR} error, and zero for ν error. The error in prediction of the critical mass is also then reduced by the same factors. From the point of view of plutonium inventory a composition ratio 0.5 would necessitate about 93% of the plutonium necessary for the analogous plutonium-fueled assembly. At a composition ratio 0.1 only about 38% of the plutonium would be required. The U^{235}

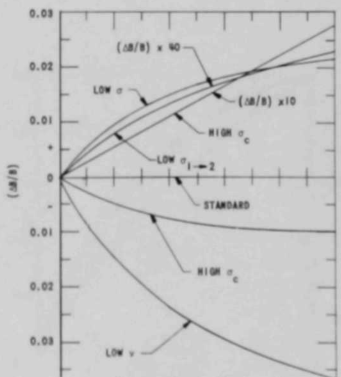


Fig. 6

Fractional Deviations of Square Roots of Bucklings as Functions of Pu^{239} Concentration

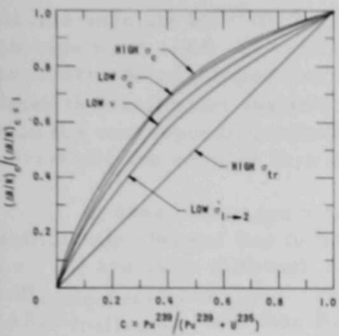


Fig. 7

Relative Deviations of Extrapolated Radii for Homogeneous Bare Cores

required at this ratio would be about 70% of that necessary for the analogous U^{235}-fueled system. In any case the construction of a bare Pu-U^{235}-fueled assembly is hardly the way of reducing the plutonium requirement; however, the previous statement on inventory does have relevance also for a corresponding reflected homogeneous assembly.

B. Reflected Homogeneous Cores

Curves of $\Delta R/R$ for the cores of corresponding reflected homogeneous cores are shown in Fig. 8 for the parameter variations. Similarly the normalized $|\Delta R/R|$ are shown in Fig. 9. (These two sets of curves were obtained by decreasing the preceding extrapolated bare core radii by reflector savings. The reflector savings of the fully U^{235}-fueled system and of the various fully plutonium-fueled systems were determined by differences of reflected core radii and corresponding extrapolated bare core radii.

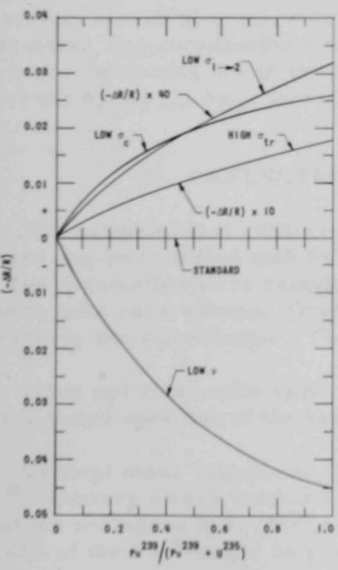


Fig. 8

Deviations of Core Radii for Reflected Homogeneous Cores

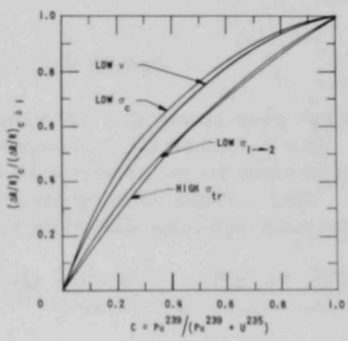


Fig. 9

Relative Deviations of Core Radii for Reflected Homogeneous Cores reactance at this ratio would be about 10% of that necessary for the analogous U²³³-fueled reactors. In any case the construction of a pure Pu²³⁹-fueled assembly is hardly the way of verifying the parametric requirements; however, the previous statement on inventory once have relevance also for a corresponding reflected homogeneous assembly.

D. Reflected Homogeneous Cores

Curves of [illegible] for the cores of corresponding reflected homogeneous [illegible] are shown in Fig. [illegible] for the parameter variations. Similarly the [illegible] are shown in Fig. [illegible]. (These two sets of curves were obtained by [illegible] the previously extrapolated bare core radii by reflector savings. The reflector savings of the fully U²³³-fueled system and of the various fully plutonium-fueled systems were approximated by differences of reflected core radii and corresponding extrapolated bare core radii.

Fig. 7

Relative Deviations of Extrapolated Radii for Homogeneous Bare Cores

Fig. [illegible]

Relative Deviations of Core Radii for Reflected Homogeneous Cores

Fig. 8

Deviations of Core Radii for Reflected Homogeneous Cores

At intermediate Pu^{239}-U^{235} compositions linear interpolations for reflector savings were used. The validity of the linear interpolation was verified at an intermediate composition for the $\nu_j^{Pu^{239}}$ parameter variation.) For other than transport variation the values of $\Delta R/R$ in Fig. 8 are greater than the corresponding values for the bare cores largely because reflected core radii are smaller than the corresponding bare core radii.

In case of transport variation the reflector savings in addition is sufficiently changed due to the transport variation so that the numerators, i.e., ΔR are quite different. For the assumed transport variation $|\Delta R_{reflected}| < |\Delta R_{bare}|$. This is more important in decreasing the $(\Delta R/R)_{reflected}$ ratio than $R_{reflected} < R_{bare}$ is in increasing it. Inherent errors in transport are thus less important for the reflected cores. In addition the normalized $|\Delta R/R|$ ratios in Fig. 9 show the curve of transport variation to follow somewhat more closely the curves of the other variations. This is in contrast to the transport curve for the bare cores shown in Fig. 7. A cross section compensation adjustment by, for example, adjustment of $\nu_j^{Pu^{239}}$ should then better compensate for transport errors.

The shape and range of the deviations of the normalized curves for the reflected cores and the bare cores for other than transport variation are similar. The fraction of error remaining upon extrapolation to the Pu^{239} system should be of the order of magnitude as for the corresponding bare system. For compensation at $Pu^{239}/(Pu^{239} + U^{235}) = 0.1$ the fractions of the error remaining are in the reflected case about 11%, 27%, 42%, and zero for the σ_c, $\sigma_{1\rightarrow 2}$, σ_{TR}, and ν variations respectively.

IV. REFLECTED TWO-REGION CORES

The reflected fast criticals consisted of two concentric core regions. The inner region is fueled with Pu^{239} and the outer region is fueled with U^{235}. The assemblies were calculated for a series of ratios of inner core volume to total core volume. Criticality was achieved by varying both core radii by the same factor. The reflector thickness was kept constant.

Inner and outer core radii at criticality are shown in Fig. 10. These radii are based upon use of the standard Pu^{239} cross-section parameters.

Critical mass values are shown in Fig. 11. From the point of view of Pu^{239} inventory a core volume ratio of 0.5 requires about 67% of the Pu^{239} of the analogous fully Pu^{239}-fueled core. At a volume ratio of 0.1 about 28% of the Pu^{239} would be required. These values are considerably smaller than for the equivalent composition homogeneous cores.

At intermediate Pu^{239}-U^{238} compositions linear interpolations for reflector savings were used. The validity of the linear interpolation was verified at an intermediate composition for the $\nu_f Pu^{239}$ parameter variation. For other than transport variation the values of $\Delta R/R$ in Fig. 8 are greater than the corresponding values for the bare cores largely because reflected core radii are smaller than the corresponding bare core radii.

In case of transport variation the reflector savings in addition is sufficiently changed due to the transport variation so that the numerators, i.e., ΔR are quite different. For the assumed transport variation $|\Delta R_{reflected}| < |\Delta R_{bare}|$. This is more important in decreasing the $|\Delta R/R|_{reflected}$ ratio than $R_{reflected} < R_{bare}$ is in increasing it. Inherent errors in transport are thus less important for the reflected cores. In addition the normalized $|\Delta R/R|$ ratios in Fig. 9 show the curve of transport variation to follow somewhat more closely the curves of the other variations. This is in contrast to the transport curve for the bare cores shown in Fig. 7. A cross section compensation adjustment by, for example, adjustment of $\nu_f^{Pu^{239}}$ should then better compensate for transport errors.

The shape and range of the deviations of the normalized curves for the reflected cores and the bare cores for other than transport variations are similar. The fraction of error remaining upon extrapolation to the Pu^{244} system should be of the order of magnitude as for the corresponding bare system. For compensation at $Pu^{239}/(Pu^{239} + U^{238}) = 0.1$ the fractions of the error remaining are in the reflected case about 11%, 27%, 45%, and 22% for the ν_f, σ_f, σ_{TR}, and ν variations respectively.

IV. REFLECTED TWO-REGION CORES

The reflected last criticals consisted of two concentric core regions. The inner region is fueled with Pu^{239} and the outer region is fueled with U^{235}. The reactivities were calculated for a series of ratios of inner core volume to total core volume. Criticality was achieved by varying both core radii by the same factor. The reflector thickness was kept constant.

Inner and outer core radii at criticality are shown in Fig. 10. These radii are based upon use of the standard Pu^{239} cross-section parameters.

Critical mass values are shown in Fig. 11. From the point of view of Pu^{239} inventory a core volume ratio of 0.5 requires about 67% of the Pu^{239} of the analogous fully Pu^{239}-fueled core. At a volume ratio of 0.1, about 28% of the Pu^{239} would be required. These values are considerably smaller than for the equivalent composition homogeneous cores.

The fractional distributions of the group fluxes are shown in Fig. 12 for a Pu^{239}-fueled core and for a two-region core. The $Pu^{239}/(Pu^{239} + U^{235})$ ratio for the entire core region is 0.1. The U^{235}-fueled core region contains about 560 kg U^{235}.

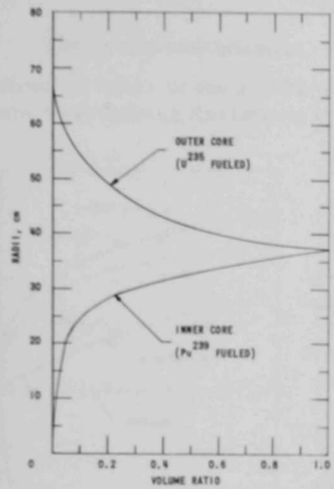


Fig. 10

Radii of Two-region Spherical Cores as Functions of Volume Ratio of Pu^{239} Fueled Core to Total Core

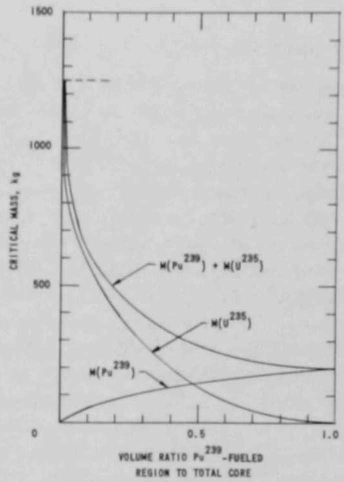


Fig. 11

Critical Masses of Reflected Spherical Two-region Cores Fueled by Pu^{239} and U^{235}

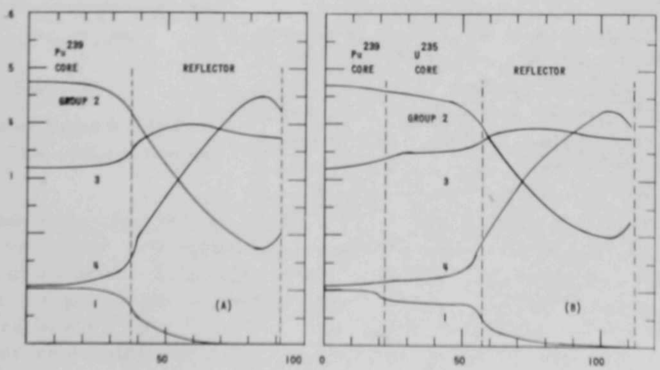


Fig. 12

Radial Fractional Distribution of Group Fluxes for (A) Pu^{239}-fueled Core and (B) Pu^{239} U^{235}-fueled Two-region Core

The corresponding all U^{235}-fueled core requires about 1240 kg U^{235}. The Pu^{239} fuel core contains about 200 kg Pu^{239}. The central region of the two-region core by comparison has about 60 kg of Pu^{239}. Within about a 20-cm radius the spectrum in the central region approximates the spectrum of the Pu^{239}-fueled assembly.

The fractional changes, $\frac{-\Delta R}{R} \equiv -\left(\frac{R' - R}{R}\right)$, for outer core radii as functions of inner to total core volumes are shown in Fig. 13, for the various assumed parameter variations of Pu^{239}. The ratio of inner to total core volumes is identical with the ratio of Pu^{239} atoms to (Pu^{239} + U^{235}) atoms in the overall core. These are also the fractional changes in inner core radii. (The negative of the $\Delta R/R$ values are plotted so as to allow comparison with the $\Delta B/B$ curves of the homogeneous bare cores.) Except for transport variation the deviation curves initially rise more rapidly for low Pu^{239} concentrations in comparison to the corresponding curves for the homogeneous cores. This greater sensitivity of core size to cross section variations should allow better detection of inherent cross section errors at the smaller Pu^{239} concentrations with subsequent improved compensation of the errors. The deviation curve for the transport variation follows essentially the corresponding curve obtained for the reflected homogeneous core.

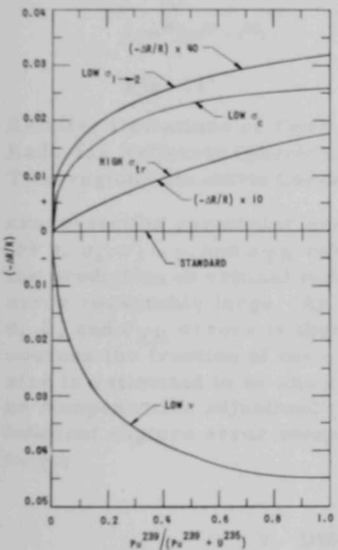


Fig. 13

Deviations of Core Radii for Reflected Two-region Cores

The quantities, $|\Delta R/R|$, normalized to unity at the Pu^{239}-fueled core are shown in Fig. 14. These curves may be used to estimate the decrease in the error of extrapolation to the reflected fully Pu^{239}-fueled assembly in a manner analogous to that previously described for the homogeneous cores. The relative displacements of the curves from each other at the small Pu^{239} concentrations, for other than transport variation, are within the accuracy with which these deviations have been calculated. It is estimated, assuming no transport error, that parameter compensation at core volume ratio 0.1 (corresponding to overall core fuel ratio $Pu^{239}/(Pu^{239} + U^{235}) = 0.1$) results in about half the fraction of the error remaining for the corresponding reflected homogeneous core. The fraction of the error remaining is about 65% for transport error.

The corresponding all U^{235}-fueled core requires about 1240 kg U^{235}. The Pu^{239}-fueled core contains about 700 kg Pu^{239}. The central region of the two-region core by comparison has about 60 kg of Pu^{239}. Within about a 20-cm radius, the spectrum in the central region approximates the spectrum of the Pu^{239}-fueled assembly.

The fractional changes, $\frac{\Delta R}{R} = \left(\frac{R - R}{R}\right)$, for outer core radii as functions of inner to total core volumes are shown in Fig. 13, for the various assumed parameter variations of Pu^{239}. The ratio of inner to total core volumes is identical with the ratio of Pu^{239} atoms to $(Pu^{239} + U^{235})$ atoms in the overall core. These are also the fractional changes in inner core radii. (The negative of the $\Delta R/R$ values are plotted so as to allow comparison with the $\Delta B/B$ curves of the homogeneous bare cores.) Except for transport variation the deviation curves initially rise more rapidly for low Pu^{239} concentrations in comparison to the corresponding curves for the homogeneous cores. This greater sensitivity of core size to cross section variations should allow better detection of inherent cross section errors at the smaller Pu^{239} concentrations with subsequent improved compensation of the errors. The deviation curve for the transport variation follows essentially the corresponding curve obtained for the reflected homogeneous core.

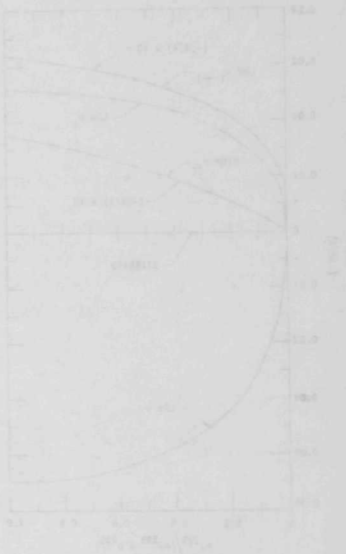


Fig. 13

Deviations of Core Radii for Reflected Two-region Cores

The quantities $|\Delta R/R|$, normalized to unity at the Pu^{239}-fueled core are shown in Fig. 14. These curves were used to estimate the decrease in the error of extrapolation to the reflected fully Pu^{239}-fueled assembly in a manner analogous to that previously described for the homogeneous cores. The relative displacements of the curves from each other at the small Pu^{239} concentrations, for other than transport variation, are within the accuracy with which these deviations have been calculated. It is estimated, assuming no transport error, that parameter compensation at core volume ratio 0.1 (corresponding to overall core fuel ratio $Pu^{239}/Pu^{239} + U^{235}$ = 0.1) results in about half the fraction of the error remaining for the corresponding reflected homogeneous core. The fraction of the error remaining is about 60% for transport error.

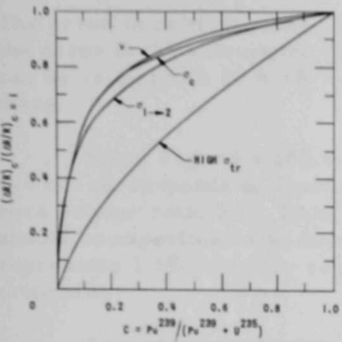


Fig. 14

Relative Deviations of Core Radii for Reflected Spherical Two-region Concentric Cores

With increasing volume of Pu^{239} core region the detection of the effect of inherent errors can be more easily detected and, hence, adjusted for. The decrease of the error in the extrapolated Pu^{239} system is not much improved, however, over that compensated at the smaller volume ratios of Pu^{239}-fueled region to total core until large volume ratios are reached. In any case the savings in Pu^{239} inventory at these larger volume ratios are not substantial.

From the core radii deviation curves of Fig. 13 and from the values of the cross-section variations producing these deviations the magnitude of the variation in a given cross section to produce a given deviation may be estimated. For 1% radius deviations at the fully Pu^{239} core composition inherent cross-section parameter errors of the order of 0.6%, 16%, 118%, and 44% for ν, σ_a, $\sigma_{1\rightarrow 2}$, and σ_{TR} respectively are required. Thus for affecting the prediction of critical mass, $\sigma_{1\rightarrow 2}$ error must be very large and σ_{TR} error reasonably large. An assumption of negligible size error due to $\sigma_{1\rightarrow 2}$ and σ_{TR} errors is therefore reasonable. Neglecting these error sources the fraction of the error in the extrapolated Pu^{239}-fueled core size is estimated to be about 5% to 10% of that which would be obtained if no compensation adjustment were made. This estimate is for the case of inherent capture error compensated by adjustment of the ν_j by the necessary factor.

V. DISCUSSION AND CONCLUSIONS

Success of the described use of Pu^{239}-U^{235}-fueled systems ultimately depends upon the accuracy with which core radii can be experimentally determined.

Consider the reflected two-region core method for cross-section adjustment. Suppose it is desired to estimate the critical mass of the fully Pu^{239}-fueled assembly to an accuracy of 3%. The corresponding accuracy in spherical core radius would be 1%. Suppose that the volume ratio of the central Pu^{239} fueled core region to the total core is 0.1 for the composite Pu^{239}-U^{235}-fueled assembly. Assume that errors of transport cross section are negligible in affecting the core size in comparison to other parameters. Then compensation adjustment of, for example, $\nu_1^{Pu^{239}}$, will decrease the radius error of the fully Pu^{239}-fueled system by at least a factor of ten.

The error in critical mass will also be decreased by the same factor. Thus the error in the calculation of the Pu^{239}-fueled system before compensation can be as large as $10 \times 1\% = 10\%$ in core radius or $10 \times 3\%$ in critical mass.

From Fig. 14 a 10% error in core radius for the fully Pu^{239}-fueled system corresponds to about a 5% → 7% error range for the assembly having core volume ratio 0.1. Thus the core radius of the two-region core assembly should be experimentally determined to an accuracy of about 0.5%. This represents 1.5% accuracy requirement upon the Pu^{239} and U^{235} masses at criticality.

As pointed out in the introduction, the varying of the core sizes to attain criticality requires a large U^{235} inventory. In practice it might be better to retain a constant core volume and attain criticality by varying the total fuel volume fraction. A calculational study based on the latter method is therefore also of interest.

ACKNOWLEDGEMENT

The authors wish to thank Dr. R. Avery for suggesting the study and for helpful suggestions and advice. Mr. J. R. White assisted with many of the computations and with preparations of graphs.

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3. W. B. Loewenstein, and D. Okrent, "The Physics of Fast Power Reactors - A Status Report." Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 12, 16. United Nations, New York (1958).

The error in critical mass will also be decreased by the same factor. Thus the error in the calculation of the Pu^{239}-fueled system before compensation can be as large as 10 × 1% = 10% in core radius or 10 × 3% in critical mass.

From Fig. 14 a 10% error in core radius for the fully Pu^{239} fueled system corresponds to about a 4% error range for the assembly having core volume ratio 4:1. Thus the core radius of the two-region core assembly should be experimentally determined to an accuracy of about 0.4%. This represents a 5% accuracy requirement upon the Pu^{239} and U^{235} masses at criticality.

As pointed out in the introduction, the variance of the core sizes to attain criticality requires a large U^{235} inventory. In practice it might be better to retain a constant core volume and attain criticality by varying the total fuel volume fraction. A calculational study based on the latter method is therefore also of interest.

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APPENDIX

FOUR GROUP CONSTANTS[a]

j	β	E_L	ΔU
1	0.574	1.35	-
2	0.360	0.3	1.5
3	0.066	0.067	1.5
4	0	0	-

Pu^{239} (standard) ($N = 0.048 \times 10^{24}$)

j	Σ_A	$3\,\Sigma_{TR}$	$\nu\Sigma_F$	$\Sigma_{IN}(j \to j+1)$
1	0.1427	0.6624	0.2964	0.04513
2	0.1089	0.8958	0.2493	0.01833
3	0.1045	1.3485	0.2424	0.00297
4	0.1453	1.9260	0.2987	-

U^{235} ($N = 0.048 \times 10^{24}$)

j	Σ_A	$3\,\Sigma_{TR}$	$\nu\Sigma_F$	$\Sigma_{IN}(j \to j+1)$	ν
1	0.1468	0.6480	0.1673	0.08009	2.70
2	0.0975	0.7912	0.1511	0.03025	2.53
3	0.0973	1.3131	0.1917	0.00380	2.48
4	0.1765	1.8207	0.3281	-	2.47

U^{238} ($N = 0.048 \times 10^{24}$)

j	Σ_A	$3\,\Sigma_{TR}$	$\nu\Sigma_F$	$\Sigma_{IN}(j \to j+1)$	ν
1	0.1308	0.6605	0.06396	0.10428	2.60
2	0.0214	0.8426	0.00006	0.01487	2.29
3	0.0176	1.3398	-	0.00765	-
4	0.0228	1.8207	-	-	-

Fe ($N = 0.0847 \times 10^{24}$)

j	Σ_A	$3\,\Sigma_{TR}$	$\nu\Sigma_F$	$\Sigma_{IN}(j \to j+1)$
1	0.06455	0.5138	-	0.06215
2	0.01032	0.6128	-	0.00986
3	0.00957	1.0501	-	0.00893
4	0.00178	1.4700	-	-

Al ($N = 0.0603 \times 10^{24}$)

j	Σ_A	$3\,\Sigma_{TR}$	$\nu\Sigma_F$	$\Sigma_{IN}(j \to j+1)$
1	0.02550	0.2941	-	0.02547
2	0.01245	0.4946	-	0.01239
3	0.00999	0.8036	-	0.00981
4	0.00054	0.6610	-	-

[a] $\Sigma_A = \Sigma_c + \Sigma_f + \Sigma_{IN}$