# Argonne National Laboratory

FEASIBILITY OF Pu<sup>239</sup>-U<sup>235</sup>-FUELED

CORES TO PREDICT Pu<sup>239</sup>-FUELED

CORE DIMENSIONS

by

D. Meneghetti and H. Ishikawa

#### LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

ANL-6559 Reactor Technology (TID-4500, 17th Ed.) AEC Research and Development Report

## ARGONNE NATIONAL LABORATORY 9700 South Cass Avenue Argonne, Illinois

## FEASIBILITY OF Pu<sup>239</sup>-U<sup>235</sup>-FUELED CORES TO PREDICT Pu<sup>239</sup>-FUELED CORE DIMENSIONS

by

D. Meneghetti and H. Ishikawa\*

Reactor Engineering Division

June 1962

\*Formerly on loan from JAERI, Tokai-Mura, Japan

Operated by The University of Chicago under Contract W-31-109-eng-38



#### TABLE OF CONTENTS

		Page
ABS	STRACT	4
I.	INTRODUCTION	4
II.	THEORETICAL CONSIDERATIONS FOR HOMOGENEOUS MIXTURES	5
III.	BARE AND REFLECTED HOMOGENEOUS CORES	7
	A. Bare Homogeneous Cores	
IV.	REFLECTED TWO-REGION CORES	12
v.	DISCUSSION AND CONCLUSIONS	15
RE	FERENCES	16
ACI	KNOWLEDGEMENT	16
API	PENDIX	17

## ABLE OF CONTENTS

ABSTRACT

IN THEORETICAL COMMUNICATIONS FOR ROMOGENEOUS

MARGUEST AND REFERENCES CORES

A SET INDEPENDENCES CORES

A SET INDEPENDENCES CORES

TO DISCUSSION AND CONCLUSIONS

ACKNOWLEDGEMENT

A SET INCIDENCES

THEORETICAL COMMUNICATIONS

THEORETICAL COMMUN

#### LIST OF FIGURES

No.	Title	Page
1.	Buckling as Function of Pu <sup>239</sup> Concentration	8
2.	Critical Mass as Function of $Pu^{239}/(Pu^{239}+U^{235})$ for Homogeneous Bare Cores	8
3.	Distribution of Group Fluxes as Function of Pu <sup>239</sup> Concentration	9
4.	Buckling Deviations as Function of Pu <sup>239</sup> Concentration	9
5.	Relative Values of Normalized Buckling Deviations	10
6.	Fractional Deviations of Square Roots of Bucklings as Functions of Pu <sup>239</sup> Concentration	10
7.	Relative Deviations of Extrapolated Radii for Homogeneous Bare Cores	11
8.	Deviations of Core Radii for Reflected Homogeneous Cores	11
9.	Relative Deviations of Core Radii for Reflected Homogeneous Cores	11
10.	Radii of Two-region Spherical Cores as Functions of Volume Ratio of Pu <sup>239</sup> Fueled Core to Total Core	13
11.	Critical Masses of Reflected Spherical Two-region Cores Fueled by $Pu^{239}$ and $U^{235}$	13
12.	Radial Fractional Distribution of Group Fluxes for (A) $Pu^{239}$ -fueled Core and (B) $Pu^{239}$ -U <sup>235</sup> -fueled Two-region Core	13
13.	Deviations of Core Radii for Reflected Two-region Cores	14
14.	Relative Deviations of Core Radii for Reflected Spherical Two-region Concentric Cores	15

#### SERVICE SECTIONS

## FEASIBILITY OF Pu<sup>239</sup>-U<sup>235</sup>-FUELED CORES TO PREDICT Pu<sup>239</sup>-FUELED CORE DIMENSIONS

by

### D. Meneghetti and H. Ishikawa

#### ABSTRACT

Use of  $Pu^{239}$ - $U^{235}$ -fueled fast critical assemblies to estimate properties of  $Pu^{239}$ -fueled assemblies is of interest because of safety considerations and limited plutonium availability.

Bare and reflected homogeneous cores and reflected two-region cores are considered. The fuel, 5% by volume, is assumed to be  $Pu^{239}$  and  $U^{235}$  of various fuel composition ratios for the homogeneous cores. For the two-region cores the 5% fuel volume is  $Pu^{239}$  in the central region and  $U^{235}$  in the outer core region. Core diluents, simulating fertile, structural, and coolant materials, are assumed identical in all cases.

It is estimated that construction of the reflected two-region core with ratio of central core region volume to total core volume of 0.1 will theoretically decrease the calculated error in prediction of the critical size of a corresponding solely  $Pu^{239}$ -fueled assembly by a factor of about 10 to 20.

#### I. INTRODUCTION

Critical assemblies representing Pu<sup>239</sup>-fueled dilute fast breeder reactors require large quantities of Pu<sup>239</sup>. Because of safety considerations and limited Pu<sup>239</sup> availability it is desirable to determine the feasibility of employing composite Pu<sup>239</sup>-U<sup>235</sup>-fueled criticals to predict properties of analogous Pu<sup>239</sup>-fueled criticals.

For prediction of critical size, from an operational point of view, cross-section parameters for an analogous U<sup>235</sup>-fueled critical can first be adjusted to force agreement of calculated with experimental critical size. A cross-section parameter of Pu<sup>239</sup> can subsequently be adjusted to force agreement of calculation with the observed size of an analogous Pu<sup>239</sup>-U<sup>235</sup>-fueled critical. The critical size of an analogous Pu<sup>239</sup>-fueled

Hei

PERSONAL PROPERTY OF PARENTS CORES TO

oul -

D. Meneghetti and M. Ishibawa

Pug: Nog wis TEARTERA

Hare and esticited hardogeneous cores and reliented two-region cores ere considered. The first, 5% by relating to assemble to be Put and Utt of various has communicated relient the hormogeneous cores. For the two-region cores the 2% fust volume is for "in the central region and Can in the central region and Can in the central region and the first the same attacking terification attacking a color materials, are essumed adentical to all cases.

It is extincted that construction of the reflected proregion volume of 0 1 will theoretically decrease the extralated error in prediction of the critical size of a corresponding solety Full boundary by a factor of about 10 to 20.

#### NETTO DECEMBE

Critical assembling representing but "-incled diduct fact breader executry require large confidence of Pat". Because of safety considers tions and limited by "basicalists in the desirable to determine the feasibility of employing chargosite Fath. U. V. incled controls to predict properties of analyzing Path. Incled criticals.

For prediction of critical size, drain an operational point of view cross-section parameters for an analogous U<sup>222</sup>. (welst critical can first be adjusted to large aptenment of calculated with experimental critical critical core systematic parameter of Pa<sup>429</sup> car adbacquently be adjusted to lorge systematic of calculation with the observed size of an analogous pa<sup>429</sup>. Delegant palacing critical Tip extract size of an analogous Pa<sup>429</sup>-incled

Murl

critical might then be predicted with improved accuracy by use of the adjusted cross-section parameters. The improvement in accuracy depends upon the extent to which compensation adjustment at the intermediate  $Pu^{239}/(Pu^{239}+U^{235})$  core composition ratio continues to compensate for the unknown cross-section errors as  $Pu^{239}$  replaces the  $U^{235}$ . Ideally, if the functional relations of the deviations in critical size as a function of  $Pu^{239}/(Pu^{239}+U^{235})$  ratio are proportional for all cross-section variations then the compensation adjustment will remain exact upon extrapolation to the all  $Pu^{239}$ -fueled core.

A calculational study to determine the extent to which compensation occurs upon extrapolating to the Pu239 system has been carried out. Bare and reflected homogeneous cores and reflected two-region cores were considered. The fuel, 5% by volume, was Pu<sup>239</sup> and U<sup>235</sup> of various composition ratios in the homogeneous cores. For the regional cores the 5% fuel volume was Pu239 in the central core region and U235 in the outer core region. Equal volumes of U235 and Pu239 are assumed to contain equal numbers of atoms. Exchange of a volume of U235 by an equal volume of Pu239 is then equivalent to an exchange of equal number of atoms. Core diluents were 45% U238,  $16-\frac{2}{3}\%$  iron and 19.3% aluminum in all cases. The reflectors, 58 cm thick, contained 40% U238, 20% iron and 23.2% aluminum by volume. Effects of Pu<sup>239</sup> parameter variations of capture, inelastic transfer, i.e., σ<sub>1→2</sub>, transport cross-section parameters, and number of neutrons per fission were considered. The multigroup analyses employed four neutron energy groups having lower energy limits 1.35, 0.3, 0.067, and 0 Mev. Except for buckling calculations the discrete SN, i.e., DSN, IBM-704 code (1,2) in S4 approximation was used for the analyses.

It should be noted that the exchange of Pu<sup>239</sup> and U<sup>235</sup> throughout this study has been made assuming that the average fuel density in the cores is always 5% by volume. Criticality in the calculations were obtained by varying the dimensions of the core regions. An effect of this is the large inventory of U<sup>235</sup> required for criticality of both the composite system and the corresponding U<sup>235</sup>-fueled system. For example consider 10% of the total fuel volume in the core of 5% is the Pu<sup>239</sup> in the composite system. Then the reflected two-region core of the present study would require about 60 kg of Pu<sup>239</sup>. This composite system would be used to extrapolate to a fully Pu<sup>239</sup>-system of about 200 kg of Pu<sup>239</sup>. The fully U<sup>235</sup>-fueled system, which would also be required, would require about 1240 kg of U<sup>235</sup>.

#### II. THEORETICAL CONSIDERATIONS FOR HOMOGENEOUS MIXTURES

An orientation into the effects of compensation for the unknown cross section errors by ad hoc adjustment of some suitable cross-section parameter may be obtained by one-group considerations. The material buckling  $B^2 = (\nu \Sigma_F - \Sigma_{C+F}) \ 3\Sigma_{TR}. \ \Sigma_{C+F} \ is the sum of capture and fission cross sections.$ 

ortifical might then be readloted with induced accounty by var of the and parted or one of the and parted or one accounts and accounts are accounts are accounts and accounts and accounts are accounts and accounts and accounts accounts and accounts accounts and accounts and accounts and accounts and accounts accounts and accounts accounts and accounts and accounts and accounts and accounts and accounts account accounts accounts accounts account account accounts account account accounts account accounts account account accounts account accounts account account account account account account accounts account account accounts account account accounts account account accounts account account account account accounts account account account account account account ac

A cateful stong and you determine the extent to which congeneration out one attent of our flarest out of the Pa" System has been cornered on the water the and relianced. The nucl. 5% by vidums, way Pa" and A. A. carons composition ratios at the cursons composition was Pa" and A. A. carons composition was Pa" in the central core region and U" in the central core region. Equal was part of the cordinate of U" in the continues of U" in the central core region. Equal violents of U" in the central core region. Equal both and U" in the continues of U" in the continues of U" in the central core region. Equal because of unitarity of the continues of U" in the continues of U". A continue of U" in the continues of the con

It should be noted that the exchange of Lee "and the eleganest tills always been made as always been made of the contract of the contract of the contract of the eleganest and been made as a superior of the contract of the

THE ORIGINAL CONSIDERATIONS FOR HONOGENEOUS MARTIRES

na orientation into the effects of compensation for the introval crues ascition excused by at box adjustment of come withinke your execution garantadiest that he chained by one-group completitions. The interist midling is a following Topy of the first and capture and fission cross With  $N^{\hbox{\it P}}$  the atomic density of  $Pu^{239}$  in the core, NU the atomic density of  $U^{235}$  in the core, and NP + NU = N  $\equiv$  a constant, the buckling becomes

$$\begin{split} B^2 &= \left[ \nu^{\mathbf{U}} \sigma_{\mathbf{F}}^{\mathbf{U}} (\mathbf{N} - \mathbf{N^P}) + \nu^{\mathbf{P}} \sigma_{\mathbf{F}}^{\mathbf{P}} \mathbf{N^P} - \sigma_{\mathbf{C} + \mathbf{F}}^{\mathbf{U}} (\mathbf{N} - \mathbf{N^P}) - \sigma_{\mathbf{C} + \mathbf{F}}^{\mathbf{P}} \mathbf{N^P} - \Sigma_{\mathbf{C}}^{\mathbf{D}} \right] \mathbf{X} \\ 3 \left[ \sigma_{\mathbf{TR}}^{\mathbf{U}} (\mathbf{N} - \mathbf{N^P}) + \sigma_{\mathbf{TR}}^{\mathbf{P}} \mathbf{N^P} + \Sigma_{\mathbf{TR}}^{\mathbf{D}} \right] \end{split}$$

where  $\Sigma^D$  refers to the cross sections of all materials other than  $U^{235}$  and  $Pu^{239}$ . This may be rewritten as the product of two linear functions:

$$B^2 = (ax + b)(cx + d)$$

where

$$\mathbf{x} = \mathbf{N}^{\mathbf{P}} \quad , \quad \mathbf{a} = -\nu^{\mathbf{U}} \sigma_{\mathbf{F}}^{\mathbf{U}} + \nu^{\mathbf{P}} \sigma_{\mathbf{F}}^{\mathbf{P}} + \sigma_{\mathbf{C}+\mathbf{F}}^{\mathbf{U}} - \sigma_{\mathbf{C}+\mathbf{F}}^{\mathbf{P}} \quad ,$$

$$\mathbf{b} = \nu^{\mathbf{U}} \sigma_{\mathbf{F}}^{\mathbf{U}} \mathbf{N} - \sigma_{\mathbf{C}+\mathbf{F}}^{\mathbf{U}} \mathbf{N} - \Sigma_{\mathbf{C}}^{\mathbf{D}} \quad , \quad \mathbf{c} = -3\sigma_{\mathbf{T}\mathbf{R}}^{\mathbf{U}} + 3\sigma_{\mathbf{T}\mathbf{R}}^{\mathbf{P}} \quad ,$$

and

$$d = 3\sigma_{TR}^{U}N + 3\Sigma_{TR}^{D}$$

If c = 0, then  $B^2$  = const. (ax + b). Thus if  $\sigma^U_{TR}$  =  $\sigma^P_{TR}$ , then  $B^2$  is a linear function of  $N^P$ .

Variations of cross sections give  $B'^2 = (a'x + b')(c'x + d')$ . With  $\Delta a = a' - a$ ,  $\Delta b = b' - b$ ,  $\Delta c = c' - c$ , and  $\Delta d = d' - d$  is obtained

$$\begin{split} \Delta \left( B^2 \right) & \equiv B^{\, 12} \, - \, B^2 \, = \, \left( c \triangle \, a \, + \, a \triangle \, c \, + \, \triangle \, a \triangle \, c \right) \, x^2 \\ \\ & + \, \left( d \triangle a \, + \, c \triangle \, b \, + \, b \triangle \, c \, + \, a \triangle \, d \, + \, \triangle \, a \triangle \, d \, + \, \triangle \, b \triangle \, c \right) \, x \\ \\ & + \, \left( d \triangle \, b \, + \, b \triangle \, d \, + \, \Delta \, b \triangle \, d \right) \quad . \end{split}$$

In order that  $\Delta(B^2) = 0$  when x = 0, non-Pu<sup>239</sup> cross sections are adjusted so that b' = b and d' = d.  $B'^2 - B^2$  then becomes

$$\Delta(B^2) = (c\Delta a + a\Delta c + \Delta a\Delta c)x^2 + (d\Delta a + b\Delta c)x$$

Now if c = 0, and  $\Delta c$  = 0, then  $\Delta(B^2)$  =  $(d\Delta a)x$ . Therefore if  $\sigma_{TR}^U$  =  $\sigma_{TR}^P$  and if  $\Delta\sigma_{TR}^U$  =  $\Delta\sigma_{TR}^P$ , then  $\Delta(B^2)$  is a linear function of  $N^P$  for variations in the  $Pu^{239}$  cross sections in the quantity  $\Delta a$ .

With Mr the storile density of Pull in the core, All the alamic stories of Utte an energy of Utte and the total and the buckling stories.

where the refers to the cross sections of all materials other than 1000 and

He = 0, then E = const. (sx + b). Thus if up = 0.7 then B [st ]

Variations of core acctions give BR = (a'g + b')(c'g + a') With

weddan to ab the sale to ad a Abach

## BAND LLAGT GODLY

In order that A(B\*) = 0 when x = 0, non-10 cross sections are educated so that h = b and d' = d. h d - L' then becomes

## A(SAS+SAB) + Propagation + SAC + SAC + SAC)

cowil or 0, and he = 0, then A(B") = (notice. Therefore if of a second of the variations. It is referred to the provider of the termination of the provider to the provider has been considered to the provider has the provider has been considered to the provider has been considered t

If instead c = 0, and  $\Delta a = 0$ , then

$$\triangle(B^2) = (a\triangle c)x^2 + (b\triangle c)x = x\triangle c(ax + b)$$
 so that the quantity

$$\frac{\triangle(B^2)}{B^2} = \frac{x\triangle c(ax+b)}{(ax+b)(cx+d)} = \frac{x\triangle c}{d}$$

As

$$\frac{\Delta(B^2)}{B^2} = \frac{B^{2_1} - B^2}{B^2} = \frac{(B' - B)(B' + B)}{B^2} \text{ and } B' \cong B$$

then

$$2 \frac{(B'-B)}{B} \cong \frac{x \triangle c}{d}$$

so that

$$\frac{\triangle B}{B} \cong \frac{x\triangle c}{2d} .$$

Therefore for variation of only the transport cross section of  $Pu^{239}$  from the case of  $\sigma_{TR}^{TR} = \sigma_{TR}^{Pu^{239}}$  the quantities  $\Delta(B^2)/B^2$  and  $\Delta B/B$  are approximately linear functions of  $N^P$ . Furthermore the one-group analyses shows that if errors in transport cross section are negligible in addition to the requirement  $\sigma_{TR}^{U^{239}} = \sigma_{TR}^{Pu^{239}}$  then compensation adjustment for other cross-section errors will be exact when subsequently extrapolated to the all  $Pu^{239}$ -fueled system. The extent to which this will occur for actual  $U^{235}$  and  $Pu^{239}$  fueled cores has been carried out in the present study by multigroup analyses of representative cores.

#### III. BARE AND REFLECTED HOMOGENEOUS CORES

## A. Bare Homogeneous Cores

Material bucklings for various  $Pu^{239}/(Pu^{239} + U^{235})$  fuel composition ratios were obtained by multigroup fundamental mode calculations using the four-group fast neutron cross-section parameters given in Appendix I. The material bucklings,  $B^2$ , using diffusion theory leakage are shown in Fig. 1. Values of  $B^2$  by transport theory are about 1 to 2% larger than by diffusion theory. The various curves of Fig. 1 represent assumed changes in various cross-section parameters of  $Pu^{239}$ . The curves referred to as standard, high capture, and low capture correspond to the three sets of values of capture to fission of  $Pu^{239}$  given in the eleven group cross section set of Loewenstein and Okrent<sup>(3)</sup> from which the four-group set used herein was obtained by collapsing of groups. The changes from the standard  $\sigma^2$ 

Helo

noar 0 - az bus 0 a sastant 1

william of tall only - any day of all Advis and all a

and a

5 4

Table of

Therefore for gravitating only the remaindrates are accounted for a property of the second of the first engrous.

In case of the property of the second seco

ed "-fueled sesson, the mater to which this will use for negative use the contract use to the property of the contract use the province of the contract of the

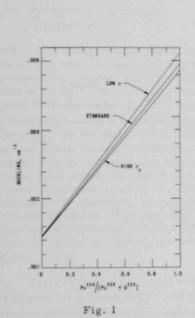
## THE PART AND I STILLET THE HOLD GET BOUS CORES

## trail amenegumoti ousil

Material bucklings for which any first troods calculations using a store were obtained by multiprotection for roods calculations using the form service for the composition of the first security for the complete service in Appendix I. The water of bucklings of the curvature of t

are -34.8%, -46.5%, -44.9%, and -17.3% for the respective groups in case of low  $\sigma_{\rm C}^{\rm Qu^{239}}$ . In the case of high  $\sigma_{\rm C}^{\rm Pu^{239}}$  the standard values are increased by 0%, 15.0%, 25.9%, and 9.8% respectively.

Critical masses obtained using the standard  $Pu^{239}$  cross sections are shown in Fig. 2. These were derived from the B values calculated by the transport theory fundamental mode calculations and were corrected by subtraction of extrapolation distances.



Buckling as Function of Pu<sup>239</sup> Concentration

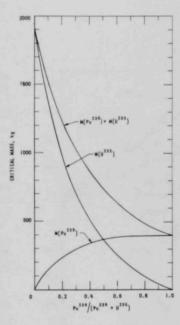


Fig. 2

Critical Mass as Function of Pu<sup>239</sup>/(Pu<sup>239</sup> + U<sup>235</sup>) for Homogeneous Bare Cores

Changes in the distribution of group fluxes with plutonium concentration are shown in Fig. 3. Effects upon spectra of the cross-section variations of the magnitudes assumed in the present study do not affect the curves of Fig. 3 sufficiently to be noticed on the indicated graph.

The quantities,  $\Delta(B^2)$ , representing the differences between the diffusion theory material bucklings calculated using cross-section variations of  $Pu^{239}$  and those denoted as standard cross sections are shown in Fig. 4.

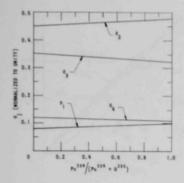
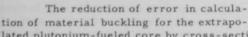


Fig. 3

Distribution of Group Fluxes
as Function of Pu<sup>239</sup> Concentration

In addition to the buckling changes resulting from the high and low capture variations are corresponding results representing: decrease of the v Pu239 by multiplication by the factor 0.9736, decrease of of Pu239 by multiplication by the factor 0.9, and multiplication of the o Pu239 by the factor 1.08. The indicated points are the calculated differences. Straight lines have been drawn between initial and final points, for cases other than transport variation, to show the amount of deviation from linearity. The curves depicting the effects of o Pu239 and o Pu239 variations have been multiplied by the indicated factors in order to clarify the resulting shapes.

Variations in  $\sigma_{F_j}^{Pu^{239}}$  will vary both the composite quantities  $\nu_{\sigma}^{P_j}P_j^{u^{239}}$  and  $\sigma_{C_j+F_j}^{Pu^{239}}$ . Deviation curves for fission cross-section variations will consist of linear combinations of deviation curves derived from appropriate variations in  $\nu_{F_j}^{Pu^{239}}$  and  $\sigma_{C_j}^{Pu^{239}}$ . Thus modification of  $\sigma_{F_j}$  to  $\varepsilon\sigma_{F_j}$  is equivalent to modification of  $\nu_j$  to  $\varepsilon\nu_j$  and  $\sigma_{C_j}$  to  $\sigma_{C_j}^{-+}$  ( $\varepsilon$ -1) $\sigma_{F_j}^{--}$ .



lated plutonium-fueled core by cross-section compensating adjustment at a Pu<sup>239</sup>-U<sup>235</sup> composition may be estimated using Fig. 5. The  $\left|\Delta(B^2)\right|$  were normalized to the value unity at the Pu<sup>239</sup> core. The deviations from linearity as well as the range of spread of the curves from each other are evident.

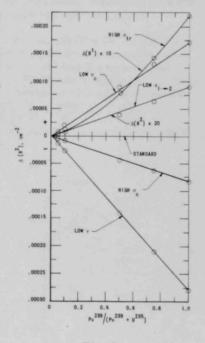


Fig. 4

Buckling Deviations as Function of Pu<sup>239</sup> Concentration

5/0 U5 Lug introck marked beliegenedy effect (2) Commend check this technique using the available ZPTI II assemblies le take an assembly with some a column, out of say 16 ma module centaming C+Al (yol C n-yol M) + etter assembles with deferent values of 9

dyfull in practice to get say 1/0 to 1 5/0 Us

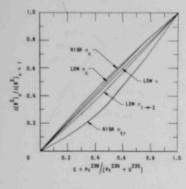


Fig. 5

Relative Values of Normalized Buckling Deviations For example suppose the unknown inherent cross-section errors are to be compensated by adjustment of the  $\nu_{\rm corresponding}^{\rm Pu^{239}}$  for an assembly corresponding to  ${\rm Pu^{239}}^{\rm J}/({\rm Pu^{239}}+{\rm U^{235}}=0.1.$  The relative values of  $|\Delta({\rm B^2})|$  are about 0.11, 0.09, 0.045, and 0.10 for  ${\rm Pu^{239}}$  capture, inelastic transfer, transport, and  $\nu$  variation respectively. The error in calculated  ${\rm B^2}$  for the extrapolated  ${\rm Pu^{239}}$ -fueled system is thus reduced to about 10%, 10%, 55%, and 0%, respectively, of the error which would have been obtained if no compensation adjustment had been made, depending upon the respective assumed inherent error.

The ratios,  $\Delta B/B \equiv (B'-B)/B$ , for the assumed cross-section variations are shown in Fig. 6. B' and B are material

bucklings obtained using asymptotic transport fundamental mode calculations. The primed quantities refer to the perturbed cross section and unprimed to the unperturbed. As  $\frac{\Delta R}{R} \equiv \frac{R'-R}{R} = -\left(\frac{\Delta B}{B}\right) / \left(1+\frac{\Delta B}{B}\right) \stackrel{\sim}{=} -\frac{\Delta B}{B}$ , the corresponding fractional changes in extrapolated bare core radii,  $\Delta R/R$  may be estimated from the curves of Fig. 6.

The relative values of |AR/R| normalized to unity for the Pu239-fueled system are shown in Fig. 7, for the various cases. From these curves the fractions of error remaining, relative to the error otherwise obtained using the standard Pu239 set, in extrapolated bare core radius of the Pu<sup>239</sup>-fueled system due to compensation adjustment can be estimated. Thus if the v Pu239 are adjusted at the composition ratio  $Pu^{239}/(Pu^{239}+U^{235}) = 0.1$  the fractions of the errors remaining are about 0.18 for o error, 0.14 for o error, 0.54 for oTR error, and zero for verror. The error in prediction of the critical mass is also then reduced by the same factors. From the point of view of plutonium inventory a composition ratio 0.5 would necessitate about 93% of the plutonium necessary for the analogous plutonium-fueled assembly. At a composition ratio 0.1 only about 38% of the plutonium would be required. The U235

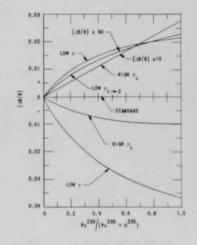


Fig. 6

Fractional Deviations of Square Roots of Bucklings as Functions of Pu<sup>239</sup> Concentration

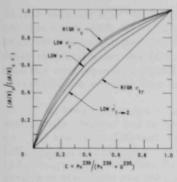


Fig. 7

Relative Deviations of Extrapolated Radii for Homogeneous Bare Cores required at this ratio would be about 70% of that necessary for the analogous U<sup>235</sup>-fueled system. In any case the construction of a bare Pu-U<sup>235</sup>-fueled assembly is hardly the way of reducing the plutonium requirement; however, the previous statement on inventory does have relevance also for a corresponding reflected homogeneous assembly.

### B. Reflected Homogeneous Cores

Curves of  $\Delta R/R$  for the cores of corresponding reflected homogeneous cores are shown in Fig. 8 for the parameter variations. Similarly the normalized  $|\Delta R/R|$  are shown in Fig. 9. (These two sets of curves were obtained by decreasing the preceding extrapolated bare core radii by reflector savings. The reflector savings of

the fully U<sup>235</sup>-fueled system and of the various fully plutonium-fueled systems were determined by differences of reflected core radii and corresponding extrapolated bare core radii.

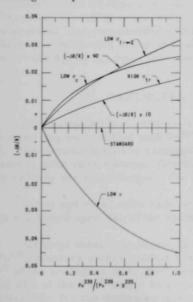


Fig. 8

Deviations of Core Radii for Reflected Homogeneous Cores

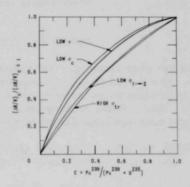


Fig. 9

Relative Deviations of Core Radii for Reflected Homogeneous Cores

At intermediate Pu<sup>239</sup>-U<sup>235</sup> compositions linear interpolations for reflector savings were used. The validity of the linear interpolation was verified at an intermediate composition for the  $\nu_{J}^{Pu^{239}}$  parameter variation.) For other than transport variation the values of  $\Delta\,R/R$  in Fig. 8 are greater than the corresponding values for the bare cores largely because reflected core radii are smaller than the corresponding bare core radii.

In case of transport variation the reflector savings in addition is sufficiently changed due to the transport variation so that the numerators, i.e.,  $\Delta R$  are quite different. For the assumed transport variation  $|\Delta R_{reflected}| < |\Delta R_{bare}|$ . This is more important in decreasing the  $(\Delta R/R)_{reflected}$  ratio than  $R_{reflected} < R_{bare}$  is in increasing it. Inherent errors in transport are thus less important for the reflected cores. In addition the normalized  $|\Delta R/R|$  ratios in Fig. 9 show the curve of transport variation to follow somewhat more closely the curves of the other variations. This is in contrast to the transport curve for the bare cores shown in Fig. 7. A cross section compensation adjustment by, for example, adjustment of  $\nu_1^{\rm Pu^{239}}$  should then better compensate for transport errors.

The shape and range of the deviations of the normalized curves for the reflected cores and the bare cores for other than transport variation are similar. The fraction of error remaining upon extrapolation to the Pu<sup>239</sup> system should be of the order of magnitude as for the corresponding bare system. For compensation at Pu<sup>239</sup>/(Pu<sup>239</sup> + U<sup>235</sup>) = 0.1 the fractions of the error remaining are in the reflected case about 11%, 27%, 42%, and zero for the  $\sigma_{\rm C}$ ,  $\sigma_{\rm 1 \rightarrow 2}$ ,  $\sigma_{\rm TR}$ , and  $\nu$  variations respectively.

#### IV. REFLECTED TWO-REGION CORES

The reflected fast criticals consisted of two concentric core regions. The inner region is fueled with  $Pu^{239}$  and the outer region is fueled with  $U^{235}$ . The assemblies were calculated for a series of ratios of inner core volume to total core volume. Criticality was achieved by varying both core radii by the same factor. The reflector thickness was kept constant.

Inner and outer core radii at criticality are shown in Fig. 10. These radii are based upon use of the standard Pu<sup>239</sup> cross-section parameters.

Critical mass values are shown in Fig. 11. From the point of view of Pu<sup>239</sup> inventory a core volume ratio of 0.5 requires about 67% of the Pu<sup>239</sup> of the analogous fully Pu<sup>239</sup>-fueled core. At a volume ratio of 0.1 about 28% of the Pu<sup>239</sup> would be required. These values are considerably smaller than for the equivalent composition homogeneous cores.

An interconduct Pot . Uff compositions linear interpolations for reflected at an interpolation was very linear and taken an interpolation was very linear an interpolation was very linear and interpolation and for the interpolation and for the interpolation and for the interpolation and interpolation and interpolation of the burn core and interpolation was resulted than the corresponding water core spin.

It case of transport carringes the reflection savings in addition fall miditaries and increasity changed due to the transport variation as that the authorations of the first of the state of the carriadion of the first case of the carriage of the first case of the carriage of the first case of the carriage of the carriage of the carried case of the case of

The shape and rarge of the deviations of the normalized curves for the referred object of varieties. Our reflected object than inquaprist varieties are similar. The fraction of error remaining upon extrapolation to the constitution of error mended as for the tor responding the extent about the consequence of mended as for the tor responding to the consequence of the consequence of the error fractions are in the reflected case about 11%, 27, 21%, and the error fractions are in the reflected case about 11%, 27, 21%, and careful to the main of an upon the variations are second respectively.

## REFLECTED TWO-REGION CORES

The relative to the criticals constited of two concepts a core content.

The two responses to the water Porth and the outs: region is togled with

"The responsive to coloring water delcoloring dot a sortes of ration of anner core
relating to total ore relation. Critically was relatived by residing both
correct add by the orang lation. The relation thickness was kept constant.

There and outer and outer radii at criticality are shown in Figure 10. (There

College and the second of the

The fractional distributions of the group fluxes are shown in Fig. 12 for a  $Pu^{239}$ -fueled core and for a two-region core. The  $Pu^{239}/(Pu^{239} + U^{235})$  ratio for the entire core region is 0.1. The  $U^{235}$ -fueled core region contains about 560 kg  $U^{235}$ .

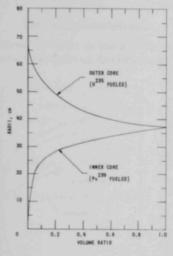


Fig. 10

Radii of Two-region Spherical Cores as Functions of Volume Ratio of Pu<sup>239</sup> Fueled Core to Total Core

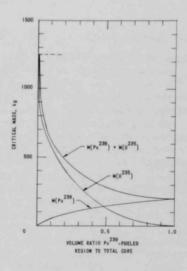


Fig. 11

Critical Masses of Reflected Spherical Two-region Cores Fueled by Pu<sup>239</sup> and U<sup>235</sup>

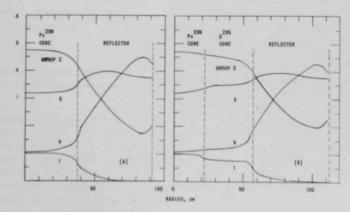
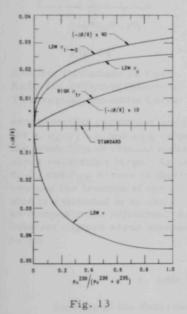


Fig. 12

Radial Fractional Distribution of Group Fluxes for (A) Pu<sup>239</sup>fueled Core and (B) Pu<sup>239</sup> U<sup>235</sup>-fueled Two-region Core

The corresponding all  $U^{235}$ -fueled core requires about 1240 kg  $U^{235}$ . The  $Pu^{239}$  fuel core contains about 200 kg  $Pu^{239}$ . The central region of the two-region core by comparison has about 60 kg of  $Pu^{239}$ . Within about a 20-cm radius the spectrum in the central region approximates the spectrum of the  $Pu^{239}$ -fueled assembly.

The fractional changes,  $\frac{-\Delta R}{R} \equiv -\left(\frac{R'-R}{R}\right)$ , for outer core radii as functions of inner to total core volumes are shown in Fig. 13, for the various assumed parameter variations of Pu<sup>239</sup>. The ratio of inner to total core



Deviations of Core Radii for Reflected Two-region Cores

volumes is identical with the ratio of Pu239 atoms to (Pu239 + U235) atoms in the overall core. These are also the fractional changes in inner core radii. (The negative of the ΔR/R values are plotted so as to allow comparison with the \DB/B curves of the homogeneous bare cores.) Except for transport variation the deviation curves initially rise more rapidly for low Pu239 concentrations in comparison to the corresponding curves for the homogeneous cores. This greater sensitivity of core size to cross section variations should allow better detection of inherent cross section errors at the smaller Pu239 concentrations with subsequent improved compensation of the errors. The deviation curve for the transport variation follows essentially the corresponding curve obtained for the reflected homogeneous core.

The quantities,  $|\Delta R/R|$ , normalized to unity at the  $Pu^{239}$ -fueled core are shown in Fig. 14. These curves may be used to estimate the decrease in the error of extrapolation to the reflected fully  $Pu^{239}$ -fueled assembly in a manner analogous to that previously described for the homogeneous cores. The relative displacements of the

curves from each other at the small  $Pu^{239}$  concentrations, for other than transport variation, are within the accuracy with which these deviations have been calculated. It is estimated, assuming no transport error, that parameter compensation at core volume ratio 0.1 (corresponding to overall core fuel ratio  $Pu^{239}/(Pu^{239}+U^{235})=0.1$ ) results in about half the fraction of the error remaining for the corresponding reflected homogeneous core. The fraction of the error remaining is about 65% for transport error.

The control of the co

the firm harm serve no. (2 - 11) = 111 | respects limitative that

as direct on the contract of t

relations of the state of the letter of the term of the term of the state of the st

The culture of 18 to 18

here as a second second

green from each liber at the small is a considerable interiors in the second or against the expension are within the second of the solution of the expensions of the each consideration of the expensions at the flat of the expension of the expens

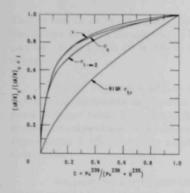


Fig. 14

Relative Deviations of Core Radii for Reflected Spherical Two-region Concentric Cores With increasing volume of Pu<sup>239</sup> core region the detection of the effect of inherent errors can be more easily detected and, hence, adjusted for. The decrease of the error in the extrapolated Pu<sup>239</sup> system is not much improved, however, over that compensated at the smaller volume ratios of Pu<sup>239</sup>-fueled region to total core until large volume ratios are reached. In any case the savings in Pu<sup>239</sup> inventory at these larger volume ratios are not substantial.

From the core radii deviation curves of Fig. 13 and from the values of the cross-section variations producing these deviations the magnitude of the variation in a given cross section to produce a given deviation may be estimated. For 1% radius deviations at the fully Pu<sup>239</sup> core composition inherent

cross-section parameter errors of the order of 0.6%, 16%, 118%, and 44% for  $\nu$ ,  $\sigma_a$ ,  $\sigma_1 \rightarrow 2$ , and  $\sigma_{TR}$  respectively are required. Thus for affecting the prediction of critical mass,  $\sigma_1 \rightarrow 2$  error must be very large and  $\sigma_{TR}$  error reasonably large. An assumption of negligible size error due to  $\sigma_1 \rightarrow 2$  and  $\sigma_{TR}$  errors is therefore reasonable. Neglecting these error sources the fraction of the error in the extrapolated Pu^239-fueled core size is estimated to be about 5% to 10% of that which would be obtained if no compensation adjustment were made. This estimate is for the case of inherent capture error compensated by adjustment of the  $\nu_j$  by the necessary factor.

#### V. DISCUSSION AND CONCLUSIONS

Success of the described use of Pu<sup>239</sup>-U<sup>235</sup>-fueled systems ultimately depends upon the accuracy with which core radii can be experimentally determined.

Consider the reflected two-region core method for cross-section adjustment. Suppose it is desired to estimate the critical mass of the fully Pu<sup>239</sup>-fueled assembly to an accuracy of 3%. The corresponding accuracy in spherical core radius would be 1%. Suppose that the volume ratio of the central Pu<sup>239</sup> fueled core region to the total core is 0.1 for the composite Pu<sup>239</sup>-U<sup>235</sup>-fueled assembly. Assume that errors of transport cross section are negligible in affecting the core size in comparison to other parameters. Then compensation adjustment of, for example,  $\nu^{\text{Pu}^{239}}$ , will decrease the radius error of the fully Pu<sup>239</sup>-fueled system by at least a factor of ten.

The error in critical mass will also be decreased by the same factor. Thus the error in the calculation of the  $Pu^{239}$ -fueled system before compensation can be as large as  $10 \times 1\% = 10\%$  in core radius or  $10 \times 3\%$  in critical mass.

From Fig. 14 a 10% error in core radius for the fully  $Pu^{239}$ -fueled system corresponds to about a  $5\% \rightarrow 7\%$  error range for the assembly having core volume ratio 0.1. Thus the core radius of the two-region core assembly should be experimentally determined to an accuracy of about 0.5%. This represents 1.5% accuracy requirement upon the  $Pu^{239}$  and  $U^{235}$  masses at criticality.

As pointed out in the introduction, the varying of the core sizes to attain criticality requires a large U<sup>235</sup> inventory. In practice it might be better to retain a constant core volume and attain criticality by varying the total fuel volume fraction. A calculational study based on the latter method is therefore also of interest.

#### ACKNOWLEDGEMENT

The authors wish to thank Dr. R. Avery for suggesting the study and for helpful suggestions and advice. Mr. J. R. White assisted with many of the computations and with preparations of graphs.

#### REFERENCES

- B. J. Carlson, and G. I. Bell, "Solution of the Transport Equation by the S<sub>N</sub> Method." Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 16, 535, United Nations, New York (1958).
- B. Carlson, C. Lee, and J. Worlton. "The DSN and TDC Neutron Transport Codes." LAMS-2346 (1960).
- W. B. Loewenstein, and D. Okrent, "The Physics of Fast Power Reactors - A Status Report." Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 12, 16. United Nations, New York (1958).

The arter of the exhaulation of the Politicities were the same finish. This can be tree or the exhaulation of the Politicities were to be before communities can be as large as 10 x 15 at 10 ft. or the contract or the contract of the contract.

restand corresponds to show with a street cadron for the satisfied having seeks volume south of Taxas the core range for the satisfied having seeks volume south of Taxas the core range of the two region and a satisfied having about the two region and a satisfied to as according to the core of the satisfied to the correspond to the treet of the satisfied to the satisfied to

an author of the introduction the veryinger the conjugation to a prince of the conjugation to an author of the conjugation of t

#### AND MEDGEN WON THE WAY

The authors week to loant Dr. R. A. sty for suggesting the guide and for helpful suggestions and advice. Mr. J. R. White-assisted with row-w of the toerputations and with preparations of graphs

## REFERENCES

P. J. Garlson, and G. I. bell, "Solution to the Transport Equation by "The py Method." Frace-diags of the Second Claims National hundralists of Conference on the Percent Claim of Atomic Energy Carove, State (1998), "16, 535 United Nations, New Yorks (1958), "Third Nations, New Yorks (1958)."

B. Carlson, G. Lee, and J. Wurden, The Day and TDC Neutron Trans. port Codes, " IAMS 4 Mail 1950).

W. B. Liebergerede, "notif Chront, "The Thysics of East Rower of Recipies (A. Srains Report," Protectings of the Sommit United Martens of the County Control of the Posterior of Martin Control of South States (Martin Control of South States). Control of South States of Martin Control of South States (Martin South South States).

## APPENDIX

## FOUR GROUP CONSTANTS a

j	β	EL	ΔU		
1	0.574	1.35	7		
2	0.360	0.3	1.5		
3	0.066	0.067	1.5		
4	0	0			
		Pu239 (stand	ard) (N = 0.048	$8 \times 10^{24}$	
j	ΣΑ	3 Σ <sub>TR</sub>	νΣϝ	$\Sigma_{IN}(j \rightarrow j + 1)$	
1	0.1427	0.6624	0.2964	0.04513	
2	0.1089	0.8958	0.2493	0.01833	
3	0.1045	1.3485	0.2424	0.00297	
4	0.1453	1.9260	0.2987		
		U <sup>235</sup> (	$N = 0.048 \times 10^{\circ}$	24)	
j	$\Sigma_{\mathbf{A}}$	3 Σ <sub>TR</sub>	$\nu \Sigma_{\mathbf{F}}$	$\Sigma_{\text{IN}}(j \rightarrow j + 1)$	ν
1	0.1468	0.6480	0.1673	0.08009	2.70
2	0.0975	0.7912	0.1511	0.03025	2.53
3	0.0973	1.3131	0.1917	0.00380	2.48
4	0.1765	1.8207	0.3281		2.47
		U <sup>238</sup> (	$N = 0.048 \times 10$	(24)	
j	$\Sigma_{\mathbf{A}}$	3 Σ <sub>TR</sub>	$\nu \Sigma_{\mathbf{F}}$	$\Sigma_{\text{IN}}(j \rightarrow j + 1)$	ν
1	0.1308	0.6605	0.06396	0.10428	2.60
2	0.0214	0.8426	0.00006	0.01487	2.29
3	0.0176	1.3398		0.00765	
4	0.0228	1.8207			
		Fe (N	$t = 0.0847 \times 10^{-1}$	)24)	
j	ΣΑ	3 Σ <sub>TR</sub>	νΣΕ	$\Sigma_{\text{IN}}(j \rightarrow j + 1)$	
1	0.06455	0.5138	-	0.06215	
2	0.01032	0.6128		0.00986	
3	0.00957	1.0501		0.00893	
4	0.00178	1.4700			
		Al (N	$= 0.0603 \times 10^{3}$	24)	
j	ΣΑ	3 ETR	$\nu \Sigma_{\mathbf{F}}$	$\Sigma_{\text{IN}}(j \rightarrow j + 1)$	
1	0.02550	0.2941		0.02547	
2	0.01245	0.4946		0.01239	
3	0.00999	0.8036		0.00981	
4	0.00054	0.6610	Maria Maria		
a	$\Sigma_{\mathbf{A}} = \Sigma_{\mathbf{c}} + \Sigma_{\mathbf{f}}$	+ Σ <sub>IN</sub>			

## SETULATION GROUP SUCT

