ANL-6903

Argonne National Laboratory

A FORTRAN PROGRAM TO ANALYZE MASS SPECTROMETER ION OPTICS

by

Kurt Kaiser

Printed in USA. Price $1.00. Available from the Clearinghouse for Federal Scientific and Technical Information, National Bureau of Standards, U. S. Department of Commerce, Springfield, Virginia

ANL-6903
Mathematics and
Computers
(TID-4500, 35th Ed.)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60440

A FORTRAN PROGRAM TO ANALYZE MASS SPECTROMETER ION OPTICS

by

Kurt Kaiser

Chemistry Division

August 1964

Operated by The University of Chicago
under
Contract W-31-109-eng-38
with the
U. S. Atomic Energy Commission

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A FORTRAN PROGRAM TO ANALYZE MASS SPECTROMETER ION OPTICS

by

Kurt Kaiser

ABSTRACT

A Fortran program is described for the computation of ion-optical aberration coefficients through second order for any number of tandem electric and magnetic sector fields, such as those used in multistaged mass spectrometers. Also computed are the image positions and their magnifications, the system mass dispersion, and the inclinations of the direction- and energy-focusing planes with respect to the optic axis.

INTRODUCTION

Since 1950 extensive studies have been made of the ion-optical properties of tandem electric and magnetic sector fields by Johnson and Nier,[1] Voorhies,[2] Hintenberger and König,[3] Ewald and Liebl,[4] Tasman, Boerboom, and Wachsmuth,[5] and others. These studies formed the basis for the computer program described herein, although other results were derived when needed.

This program considers ions having the following initial conditions different from those of ions which traverse the systems optic axis (see Fig. 1):

(1) an initial angle to the optic axis

α in the radial direction;
α_z in the axial direction;

(2) a different velocity or energy

$V = V_0(1 + \beta)$ or
$E = E_0(1 + \eta)$;

(3) a different mass

$M = M_0(1 + \gamma)$;

(4) an initial displacement to the optic axis

δ in the radial direction;
δ_z in the axial direction.

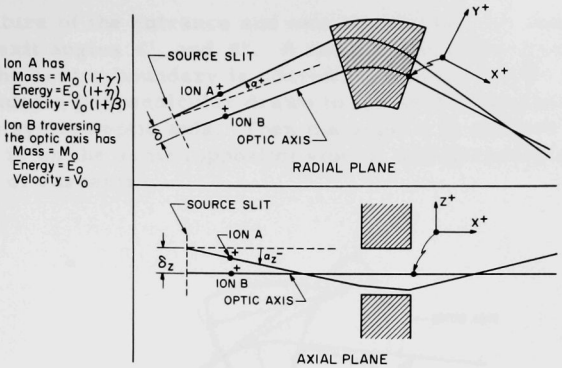


Fig. 1. Figure Showing the Trajectories of Two Ions through an Electromagnetic Sector Field. The initial displacement of ion A to the optic axis is resolved into the radial component δ and an axial component δ_z. Also, its initial direction to the optic axis is resolved into radial and axial components α and α_z, respectively.

The program calculates ion trajectories as functions of α, β, γ, δ, α_z, and δ_z. In the radial plane the trajectories are given as power series to second-order terms, and in the axial direction to first-order terms. In field-free regions the trajectories are straight lines given directly by the program output. Points along the trajectories in the sector fields can be found by subdividing the field into a number of tandem sector fields, zero distance apart, and reading out computed data for each edge of the subfields.

The electric sectors may have curved boundaries and toroidal shape, which includes cylindrical and spherical analyzers as special cases. The magnetic fields are assumed homogeneous, but can have non-normal entrance and exit, as well as circularly curved boundaries.

INPUT DATA

The input data to this program are punched on cards, one card for each field in the system. For convenience these data are divided into three types, each of which is discussed below.

TYPE I DATA: A set of parameters for each field specifying its configuration:

(1) For a homogeneous magnetic field, as shown in Fig. 2, the relevant parameters are the field radius r_m, the beam-deflection angle ϕ_m, the

radii of curvature of the entrance and exit boundaries, R', and R", and the entrance and exit angles ϵ', and ϵ''. A radius of curvature is positive (negative) if the sector boundary is convex (concave) towards the field-free region. Consider a perpendicular drawn to the sector boundary where it intersects the system optic axis. Then the angle ϵ is positive (negative) if the optic axis is on the same (opposite) side of this perpendicular as the sector center of curvature.

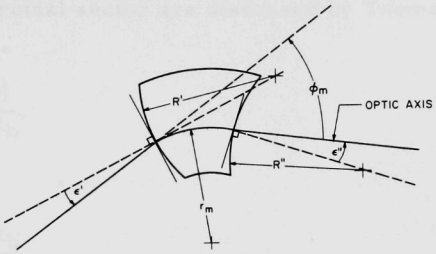


Fig. 2. Radial Section of a Magnetic Sector Field. In this figure R' is positive, R" is negative, ϵ' is positive, and ϵ'' is negative.

(2) For a toroidal electric field, as shown in Fig. 3, the parameters are the radius r_e, described by the optic axis, the beam-deflection angle ϕ_e, the radii of curvature of the entrance and exit boundaries, R_I and R_{II}, and two parameters, c and R'_e, describing approximately the electric field in the neighborhood of the optic axis. Again, the radius of curvature of a boundary is positive (negative) if the sector boundary is convex (concave) towards the field-free region.

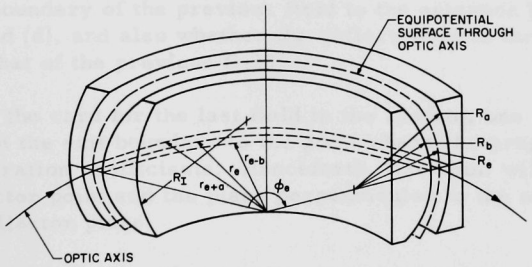


Fig. 3. Toroidal Electrostatic Sector Field. The entrance boundary has a positive curvature with its center of curvature in the Median plane.

The two parameters c and R'_e are defined as follows:

$$c = r_e/R_e,$$

and

$$R'_e = \left(\frac{\partial R}{\partial r}\right)_{r=r_e,\, z=0},$$

where R is the radius of curvature of an equipotential surface near the optic axis and the other symbols are as defined in Fig. 3. The values of c and R'_e for a general toroidal sector are discussed by Tasman[6] and Albrecht.[7] To first order,

$$c = \frac{r_e(a+b)}{bR_a + aR_b},$$

and

$$R'_e = \frac{R_a - R_b}{a+b}.$$

Two special types of toroidal electric fields are frequently used: the cylindrical analyzer which has c = 0 and $R'_e = 1$, and the spherical analyzer which has c = 1 and $R'_e = 1$.

TYPE II DATA: A set of parameters describing the relation of each field to its neighbors (see Fig. 4):

(1) On the card for the first field, one enters the distance from the source slit to the first field entrance boundary (ℓ').

(2) On the rest of the cards for the system, one enters the distance from the exit boundary of the previous field to the entrance boundary of the field considered (d), and also whether the deflection is in the same or opposite sense as that of the previous field.

(3) On the card for the last field in the system, one enters the distance (ℓ'') from the exit boundary to the point where the program is to calculate the aberration coefficients. Henceforth, this point will be referred to as the collector point and the plane perpendicular to the optic axis at this point as the collector plane.

(4) On each field card enter the sequence number of each field along with the total number of fields in the system.

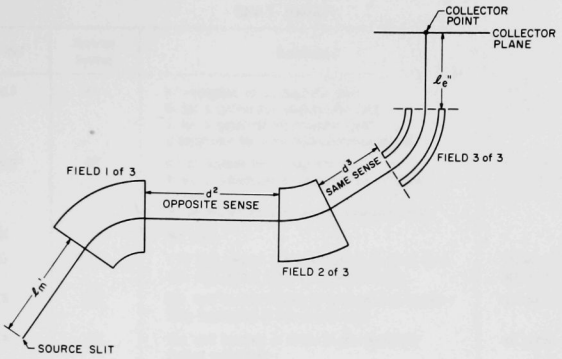


Fig. 4. A Tandem Sector Field System

TYPE III DATA: Digits for controlling the following program options:

(1) For a single symmetric field, the program can calculate both the entrance and the exit distances (ℓ' and ℓ''), so the point of first-order angle (α) focus is the collector point.

(2) For any system of fields, the program can calculate the distance (ℓ'') from the last field to the point of first-order angle (α) focus and use it as the collector point.

(3) The aberration coefficients can be calculated in terms of energy differences η rather than velocity differences β.

(4) The coefficient and ray trace arrays, described under output, may be read out if desired.

A detailed listing of the data card format is given in Table I.

Table I

INPUT DATA CARD FORMAT

Card Columns	Format	Fortran Symbol	Explanation	Punched on
1-6	F6.0	Q	Field deflection angle in degrees (ϕ_e or ϕ_m)	all cards.
7-12	F6.0	R	Field radius (r_e or r_m)	all cards.
13-18	F6.0	DA	The distance from the source slit to the first field (ℓ_e' or ℓ_m')	card for the first field only. Not necessary at all if 1 is punched in CC77.
19-24	F6.0	DB	The distance from the previous field to the present field (d)	all cards except the one for the first field.
25-30	F6.0	DC	The distance from the last field to the point where an aberration coefficient summary is desired (collector point) (ℓ_e'' or ℓ_m'')	card for the last field only. Not necessary at all if 1 is punched in CC76.
31-36	F6.0	R1	The radius of curvature of the entrance field boundary (R' or R_I)	all cards that do not have a 2 or 3 punched in CC74.
37-42	F6.0	R2	The radius of curvature of the exit field boundary (R" or R_{II})	all cards that do not have a 2 or 4 punched in CC74.

Table I (Contd.)

Card Columns	Format	Fortran Symbol	Explanation	Punched on
43-48	F6.0	E1	ε' in degrees for a magnetic field 0. for a cylindrical electrostatic field 1. for a spherical electrostatic field c parameter for a toroidal electrostatic field	all cards except those magnetic field cards that have $\varepsilon' = 0$.
49-54	F6.0	E2	ε'' in degrees for a magnetic field 1. for a cylindrical or spherical electrostatic field $R_{\hat{e}}$ parameter for a toroidal electrostatic field	all cards except those magnetic field cards that have $\varepsilon'' = 0$.
55-59	5X		Not used	
60	I1	I9	1 in this column if the printer is to bring up a new page before doing any more printing	those cards where this option is desired.
61-66	I6	K1	The field number (fields to be numbered sequentially beginning at the source end)	all cards.
67-72	I6	K2	The total number of fields in the particular system considered	all cards.
73	I1	I1	1 for a magnetic field, blank for an electric field	all cards.
74	I1	I2	1 if R1 and R2 (CC 31-42) are both not equal to infinity. 2 if R1 and R2 (CC 31-42) are both equal to infinity. 3 if R1 (CC 31-36) is equal to infinity and R2 (CC 37-42) is not equal to infinity. 4 if R1 (CC 31-36) is not equal to infinity and R2 (CC 37-42) is equal to infinity.	all cards.
75	I1	I3	1 if the deflection is in the opposite sense as the previous field, blank if it is in the same sense	all cards except the one for the first field.
76	I1	I4	1 if the program is to calculate the distance from the last field to the image plane which gives first-order angle focusing - this distance will be printed out (labeled DC) and used in subsequent calculations.	card for the last field when this option is desired.
77	I1	I5	1 if the program is to calculate the distance from the source to the first field - this feature can be used for a single symmetric field only.	field card on which this option is wanted.
78	I1	I6	1 if the aberrations are to be obtained in the η (energy) representation, blank for aberrations in the β (velocity) representation	all cards, blanks or ones must be used consistently throughout a system.
79	I1	I7	1 for the coefficient print feature	card for field for which this option is desired.
80	I1	I8	1 for the ray trace print feature	card for field for which this option is desired.

Notes:

1. CC abbreviation for card column(s).
2. The distances required above may be measured in any consistent unit.

PROGRAM OUTPUT

For each field-free space this program calculates the radial displacement y to second order and the axial displacement z to first order as a function of α, β, γ, δ, α_z, and δ_z. This is accomplished by calculating certain constants for each field, then altering the aberration coefficients by use of these constants with a subprogram.

After each field is calculated the coefficients may be read out through a program option. This feature is called the coefficient print.

If this option is elected, thirty-six coefficients are read out for each field, in the order C_1 to C_{18}, then D_1 to D_{18}. This output is labeled by C followed by the field number. Specifically, the C's and D's are defined by the following equations, where the superscript

1 refers to the first field of a system,

ℓ refers to the last field of a system, and

N refers to the Nth field of a system:

$$Y^N = r^N\left[C_1^N\alpha + C_2^N\beta + C_3^N\gamma + C_4^N\left(\frac{\delta}{r^1}\right) + C_7^N\alpha^2 + C_8^N\alpha\beta + C_9^N\beta^2 + C_{10}^N\alpha\gamma + C_{11}^N\beta\gamma + C_{12}^N\gamma^2\right.$$
$$\left. + C_{13}^N\alpha\left(\frac{\delta}{r^1}\right) + C_{14}^N\beta\left(\frac{\delta}{r^1}\right) + C_{15}^N\left(\frac{\delta}{r^1}\right)^2 + C_{16}^N\alpha_z^2 + C_{17}^N\alpha_z\left(\frac{\delta_z}{r^1}\right) + C_{18}^N\left(\frac{\delta_z}{r^1}\right)^2\right]$$
$$+ X^N\left[D_1^N\alpha + D_2^N\beta + D_3^N\gamma + D_4^N\left(\frac{\delta}{r^1}\right) + D_7^N\alpha^2 + D_8^N\alpha\beta + D_9^N\beta^2 + D_{10}^N\alpha\gamma + D_{11}^N\beta\gamma\right.$$
$$\left. + D_{12}^N\gamma^2 + D_{13}^N\alpha\left(\frac{\delta}{r^1}\right)^2 + D_{14}^N\beta\left(\frac{\delta}{r^1}\right) + D_{15}^N\left(\frac{\delta}{r^1}\right)^2 + D_{16}^N\alpha_z^2 + D_{17}^N\alpha_z\left(\frac{\delta_z}{r^1}\right) + D_{18}^N\left(\frac{\delta_z}{r^1}\right)^2\right],$$

$$Z^N = r^N\left[C_5^N\alpha_z + C_6^N\left(\frac{\delta_z}{r^1}\right)\right] + X^N\left[D_5^N\alpha_z + D_6^N\left(\frac{\delta_z}{r^1}\right)\right];$$

In these equations r is a field radius, X^N is the distance along the optic axis measured from the exit boundary of the Nth field, and Y^N and Z^N are the displacements measured in a plane located at X^N perpendicular to the optic axis. Note that Y^N is positive on the opposite side of the optic axis from the field center of curvature, Z^N is positive if above the optic axis, and(X^ℓ, Y^ℓ, Z^ℓ) are the coordinates in image space.

At this time twelve first-order coefficients: W_1 to W_6 for the entrance boundary, and W_1 to W_6 for the exit boundary, may be printed out. These twelve are labeled by R followed by the field number. This option is called the ray trace. The W's are defined by the equations

$$Y^N = W_1^N\alpha + W_2^N\beta + W_3^N\gamma + W_4^N\delta$$

and

$$Z^N = W_5^N\alpha_z + W_6^N\delta_z.$$

Just before the program summary a line labeled RS is read out, which contains W_1 to W_6 calculated at the collector point.

After the last field calculation has been completed, the program begins a summary calculation. This summary includes the aberration coefficients for the collector plane and other constants of the system. Eighteen coefficients B_1 to B_{18}, labeled S1, are read out. They are

defined by the equations

$$\frac{Y^N}{r^\ell} = B_1\alpha + B_2\beta + B_3\gamma + B_4\left(\frac{\delta}{r^1}\right) + B_7\alpha^2 + B_8\alpha\beta + B_9\beta^2 + B_{10}\alpha\gamma + B_{11}\beta\gamma + B_{12}\gamma^2$$

$$+ B_{13}\alpha\left(\frac{\delta}{r^1}\right) + B_{14}\beta\left(\frac{\delta}{r^1}\right) + B_{15}\left(\frac{\delta}{r^1}\right)^2 + B_{16}\alpha_Z^2 + B_{17}\alpha_Z\left(\frac{\delta_Z}{r^1}\right) + B_{18}\left(\frac{\delta_Z}{r^1}\right)^2$$

and

$$\frac{Z^N}{r^\ell} = B_5\alpha_Z + B_6\left(\frac{\delta_Z}{r^1}\right).$$

Following the S1 readout is the last output record, labeled S2, which contains the following six constants in order;

(1) the position of the axial focusing plane, i.e., the distance X^ℓ to the point where $B_5 = 0$;

(2) the system mass dispersion per $\Delta M/M$;

(3) the radial image magnification (measured at the point of α focus, i.e., where $B_1 = 0$);

(4) the axial image magnification (measured at the point of α_Z focus, i.e., where $B_5 = 0$);

(5) the angle ψ_α that the direction-focusing curve makes with the optic axis;

(6) the angle ψ_β (or ψ_η) that the velocity (energy)-focusing curve makes with the optic axis.

The angles ψ_α and ψ_β (or ψ_η) (see Fig. 5) are defined as follows: the locus of points of direction or α focus described as an ion mass or γ is varied will be a surface in image space. The intersection of this surface with the system median plane is the direction-focusing curve χ_α. The slope, in degrees, of the tangent to χ_α at its intersection with the optic axis is ψ_α. The X^ℓ coordinate of this point is the one for which $B_1 = 0$. In a completely analogous manner, the velocity or energy parameters χ_β and ψ_β (or χ_η and ψ_η) may be defined. The X^ℓ coordinate for the intersection of χ_β or χ_η with the optic axis is that value which makes $B_2 = 0$. The angle ψ_β (or ψ_η) is of most interest for systems which are "double focusing," that is, where B_1 and B_2 both equal zero for the same value of X^ℓ.

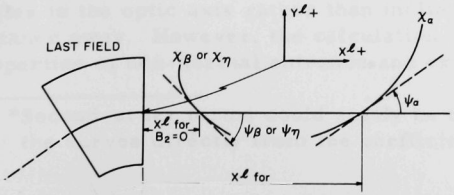


Fig. 5

Figure Illustrating the Direction and Energy Focusing Curves and the Angles These Curves Make with the Optic Axis. In this figure ψ_α is positive and ψ_β is negative.

SAMPLE PROBLEM

As an example of the use of this program consider the Argonne 100-in. mass spectrometer, shown pictorially in Fig. 6. This instrument consists of a 100-in.-radius, 75° spherical electrostatic analyzer, followed by a 100-in.-radius, 110° magnetic analyzer.

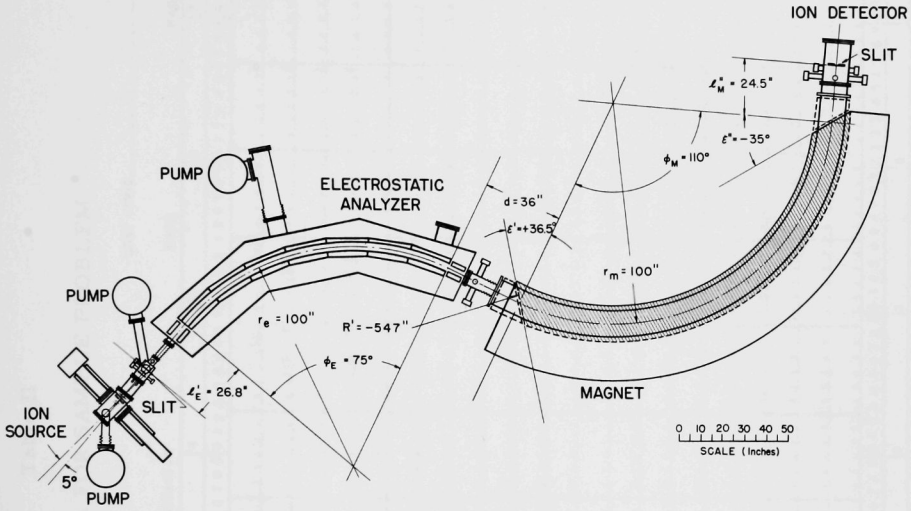


Fig. 6. 100-in. Radius Double Focusing Mass Spectrometer

The parameters to be used in this example were read off Fig. 6 and punched on data cards as in Table II. The program output listing is shown in Table III, in which the second S1 and S2 outputs are for the energy (η) rather than the velocity (β) representation used in the preceding output records.

The ray trace feature was used to calculate the curves plotted in Figs. 7A and 7B. For this calculation the electric and magnetic sectors were each divided into ten subfields zero distance apart. The ray trace output for these twenty fields gave enough points to plot the curves. This output takes into account first-order* terms only, i.e., rays an infinitesimal distance away from the optic axis. In the figures the dimensions measured perpendicular to the optic axis can be considered greatly magnified. For this reason the magnetic sector boundaries are shown at right angles to the optic axis rather than inclined as they look to rays at a finite distance away. However, the calculation does take into account the focal properties of non-normal entrance and exit from the magnetic field.

*Second-order terms could easily be taken into account by calculating the curves directly from the coefficient print feature.

Table II

INPUT DATA FOR SAMPLE PROBLEM

COST CODE _____

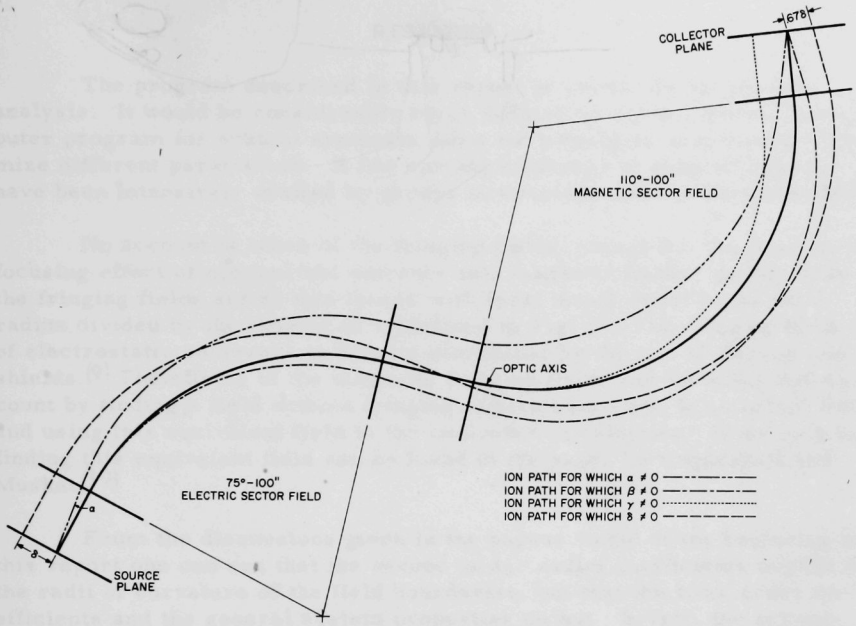
PROGRAM	PROBLEM	ORIGINATOR	DATE	PAGE OF
	1	2	3	4	5	6	7	8
1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0
7 5 1.	1 0 0 1.	2 6 1. 8				1.	1.	1	1	2	2	1 1
1 1 0 1.	1 0 0 1.		3 6 1.	2 4 1. 5	- 5 4 7 1.		3 6 1. 5	- 3 5 1.		2	2	1 4 1 1	1 1
7 5 1.	1 0 0 1.	2 6 1. 8				1.	1.		1	2	2	1	
1 1 0 1.	1 0 0 1.		3 6 1.	2 4 1. 5	- 5 4 7 1.		3 6 1. 5	- 3 5 1.	2	2	1 4 1 1	1	
1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0
	1	2	3	4	5	6	7	8

AMD-9 (9-60)

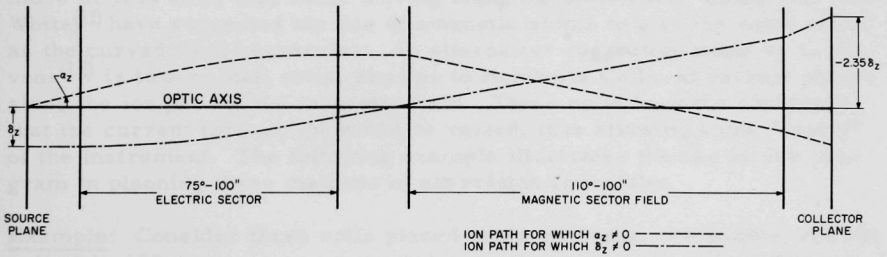
Table III

SAMPLE PROBLEM OUTPUT

C 1	1.03528933	1.48236198	0.00000000	0.25881901
	1.03528933	0.25881901	-0.00004912	3.86389310
	-0.02614593	0.00000000	0.00000000	0.00000000
	-0.96602072	3.73205088	-0.93301272	-0.26774917
	0.76733677	0.09591587		
	-0.00004911	1.93185167	0.00000000	-0.96592584
	-0.00004911	-0.96592584	-1.03534018	3.03542511
	-0.03407404	0.00000000	0.00000000	0.00000000
	-1.25884443	2.93185155	-0.24999997	0.03472601
	0.99997631	-0.23296295		
R 1	26.80000000	0.00000000	0.00000000	1.00000000
	26.80000000	1.00000000	103.52893304	148.23619794
	0.00000000	0.25881901	103.52893304	0.25881901
C 2	-0.36573166	-1.24278774	0.67101010	0.93908818
	-0.43555374	-1.81704510	0.19775253	-9.78742093
	-10.52564577	-0.70047860	-2.61798160	-0.16775250
	1.82385587	-2.17157112	0.57892024	-0.33628085
	-2.26343107	-0.32902210		
	1.49091663	5.06829209	-0.00000004	-1.09397747
	-1.07108789	-2.17244144	-0.60531209	12.63628107
	0.81198440	-0.74548292	-1.56822031	0.00000003
	-3.41156817	9.73563290	-1.72109583	-0.33287384
	2.62351919	-0.21561004		
R 2	-103.52716495	-217.78285810	-0.00000000	0.08891429
	103.52716495	-0.08891429	-36.57316597	-124.27877381
	67.10100965	0.93908818	-43.55537433	-1.81704510
RS	-0.04570846	-0.10561756	67.10100859	0.67106371
	-69.79702763	-2.34929325		
S1	-0.00045708	-0.00105618	0.67101009	0.67106371
	-0.69797028	-2.34929325	0.04945107	-6.69153207
	-10.32670959	-0.88312192	-3.00219557	-0.16775250
	0.98802167	0.21365894	0.15725176	-0.41783494
	-1.62066887	-0.38184657		
S2	-40.66461282	67.10100859	0.67072831	-0.93363020
	48.56350975	48.56285248		
S1	-0.00045708	-0.00052809	0.67101009	0.67106371
	-0.69797028	-2.34929325	0.04945107	-3.34576604
	-2.66548765	-0.88312192	-1.50109779	-0.16775250
	0.98802167	0.10682947	0.15725176	-0.41783494
	-1.62066887	-0.38184657		
S2	-40.66461282	67.10100859	0.67072831	-0.93363020
	48.56350975	48.56285248		



A. Radial Plane



B. Axial Plane

Fig. 7. First-order Approximation to the Ion Trajectories for the Argonne 100-in. Mass Spectrometer. These plots were found by use of the ray trace feature of the program. Note that the distance in the radial and axial directions are shown greatly magnified.

REMARKS

The program described in this report is primarily for system analysis. It would be considerably more difficult to write a general computer program for system synthesis since each designer may wish to optimize different parameters. A few special syntheses of general interest have been intensively studied by groups in Germany and the Netherlands.[8]

No account is taken of the fringing fields, except for the Z or axial focusing effect of non-normal entrance into magnetic fields. In this case the fringing fields act as thin lenses with focal length equal to the field radius divided by the tangent of ϵ (defined in Fig. 2). The fringing field of electrostatic analyzers is usually eliminated by the use of Herzog end shields.[9] The effects of the magnetic fringing fields can be taken into account by finding a field without fringing effects equivalent to an actual field and using this equivalent field in the computer calculations. Some help in finding this equivalent field can be found in the paper by Coggeshall and Muskat.[10]

From the discussions given in the papers listed at the beginning of this report one can see that the second-order radial coefficients depend on the radii of curvature of the field boundaries, but that the first-order coefficients and the general system properties do not. In fact, the second-order coefficients are linear functions of the reciprocals of these radii. With curved field boundaries, the ions traveling off the optic axis see slightly more or less field than those moving along the optic axis. Balestrini and White[11] have suggested the use of magnetic shims to give the same effect as the curved field boundaries. An alternative suggestion made by C. Stevens[12] is to use small coils, similar to Hemholtz Coils, at various places along the ion path in the magnetic field. These coils have the advantage that the current through them can be varied, thus allowing some "tuning" of the instrument. The following example illustrates the use of this program in planning these methods of aberration correction.

Example: Consider three coils placed at the entrance, the middle, and the exit of the 100-in. mass spectrometer magnet used in the sample problem (p. 12). We wish to find the coil field strengths which will make $B_{11} = 0$ and $|B_{12}| + |B_{22}|$ a minimum. It is convenient to measure the coil field strengths in units of effective boundary radii of curvature, which can be related to the coil currents for a given coil construction. A practical upper limit of coil field strength or minimum radius of curvature may be set at 10 in.

Solution: Since B_{11}, B_{12}, and B_{22} are linear functions of the reciprocals of the coil strengths, they can be written as

$$B_{11} = \frac{C_1}{R_1} + \frac{C_2}{R_2} + \frac{C_3}{R_3} + C_4;$$

$$B_{12} = \frac{C_5}{R_1} + \frac{C_6}{R_2} + \frac{C_7}{R_3} + C_8;$$

$$B_{22} = \frac{C_9}{R_1} + \frac{C_{10}}{R_2} + \frac{C_{11}}{R_3} + C_{12},$$

where the C's are constants, and R_1, R_2, and R_3 are the coil field strengths. To find the C's, set all three R's equal to infinity and obtain C_4, C_8, and C_{12} by use of this program. Next, take any three linearly independent sets of (R_1, R_2, R_3) and calculate the B coefficients for each set. This procedure gives three sets of three linear simultaneous equations which can be solved for the nine remaining C's. At this point we have,

$$B_{11} = \frac{71.67}{R_1} + \frac{61.09}{R_2} + \frac{2.981}{R_3} + 0.1805;$$

$$B_{12} = \frac{301.5}{R_1} + \frac{372.7}{R_2} + \frac{20.26}{R_3} - 6.140;$$

$$B_{22} = \frac{317.2}{R_1} + \frac{568.4}{R_2} + \frac{34.42}{R_3} - 9.747.$$

The magnetic field is broken into two separate fields, zero distance apart, for calculating the effects of the middle coil. Setting the B's equal to zero and solving for the R's yields a physically unrealizable result. However, the stated problem of making $B_{11} = 0$ and $|B_{12}| + |B_{22}|$ a minimum, subject to the conditions $|R_i| \geq 10$, can be solved by straightforward linear programming methods.[13] The solution is $R_1 = -28.912$, $R_2 = 23.528$, $R_3 = -10$, and minimum of $|B_{12}| + |B_{22}| = 2.755$. Other simple synthesis problems can be handled by similar methods.

THE FORTRAN PROGRAM

The program was written in "basic" Fortran through use of the most elementary forms of Fortran statements. All input-output statements are written for cards and are enclosed by lines of asterisks. This program should be easy to adapt to any computer installation. The program running time is negligible, the CDC 3600 machine at Argonne compiled the program in 35 sec and calculated about 120 fields per minute. A program flowchart (Fig. 8) and listing (Table IV) follow.

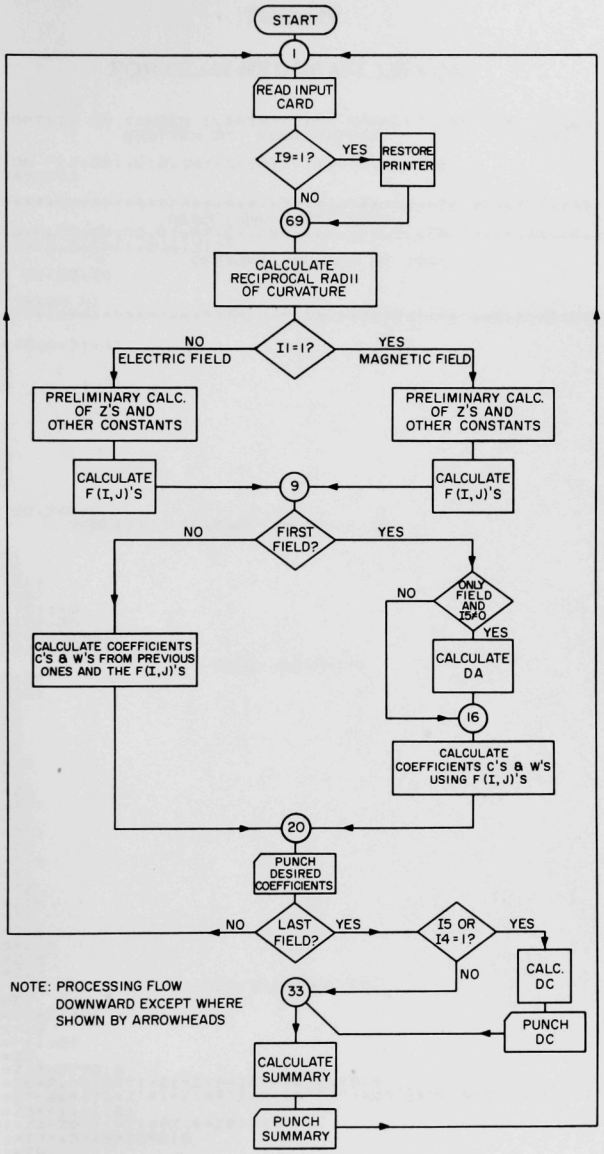


Fig. 8. Fortran Program Flowchart

Table IV

FORTRAN PROGRAM LISTING

```
C                                                                                IOP  001
C          ION OPTICS OF TANDEM ELECTRIC AND MAGNETIC SECTOR FIELDS              IOP  002
C                    WRITTEN BY  KURT KAISER              9-63                   IOP  003
C                                                                                IOP  004
      DIMENSION F(2,20),C(2,18),S(18),T(2,18),W(2,6)                             IOP  005
      PI=.017453293                                                              IOP  006
C                                                                                IOP  007
C*****************************************************************************  IOP  008
C                          READ INPUT DATA CARD                                  IOP  009
    1 READ 2,Q,R,DA,DB,DC,R1,R2,E1,E2,I9,K1,K2,I1,I2,I3,I4,I5,I6,I7,I8           IOP  010
    2 FORMAT (9F6.0,5XI1,2I6,8I1)                                                IOP  011
C                          RESTORE PRINTER IF I9=1                               IOP  012
   71 IF (I9) 50,69,70                                                           IOP  013
   70 PUNCH 3                                                                    IOP  014
    3 FORMAT(78X2H -)                                                            IOP  015
C*****************************************************************************  IOP  016
C                                                                                IOP  017
   69 GO TO(4,5,6,7),I2                                                          IOP  018
    4 P1=R/R1                                                                    IOP  019
      P2=R/R2                                                                    IOP  020
      GO TO 8                                                                    IOP  021
    5 P1=0.                                                                      IOP  022
      P2=0.                                                                      IOP  023
      GO TO 8                                                                    IOP  024
    6 P1=0.                                                                      IOP  025
      P2=R/R2                                                                    IOP  026
      GO TO 8                                                                    IOP  027
    7 P1=R/R1                                                                    IOP  028
      P2=0.                                                                      IOP  029
    8 IF(I1) 50,39,40                                                            IOP  030
C                  MAGNETIC  SECTOR  CALCULATION                                 IOP  031
   40 Q=PI*Q                                                                     IOP  032
      E1=PI*E1                                                                   IOP  033
      E2=PI*E2                                                                   IOP  034
      G=COSF(E1)                                                                 IOP  035
      D=COSF(E2)                                                                 IOP  036
      T1=SINF(E1)/G                                                              IOP  037
      T2=SINF(E2)/D                                                              IOP  038
      P1=.5*P1/(G*G*G)                                                           IOP  039
      P2=.5*P2/(D*D*D)                                                           IOP  040
C                                ZEES  MAGNETIC                                  IOP  041
      Z1=SINF(Q)                                                                 IOP  042
      Z2=COSF(Q)                                                                 IOP  043
      Z3=Z1*Z1                                                                   IOP  044
      Z4=Z2*Z2                                                                   IOP  045
      Z5=1.-Z2                                                                   IOP  046
      Z6=Z1*Z2                                                                   IOP  047
      Z7=-.5*Z3                                                                  IOP  048
      Z8=Z1*Z5                                                                   IOP  049
      Z9=-.5*Z5                                                                  IOP  050
      Z10=-.5*Z1                                                                 IOP  051
      Z11=Z2-Z4                                                                  IOP  052
      Z12=.5*Z11                                                                 IOP  053
      Z13=Z4-Z3                                                                  IOP  054
      Z14=Z2-Z13                                                                 IOP  055
      Z15=T1*T1                                                                  IOP  056
      Z16=T2*T2                                                                  IOP  057
      Z17=T2*Z16                                                                 IOP  058
      Z18=T1*T2                                                                  IOP  059
      Z19=T1*Z16                                                                 IOP  060
      Z20=Z15*Z16                                                                IOP  061
      Z21=T2*Z15                                                                 IOP  062
      Z22=T1*Z17                                                                 IOP  063
C                                F(I,J) MAGNETIC                                 IOP  064
      F(1,1)=Z1                                                                  IOP  065
      F(1,2)=Z2+Z1*T1                                                            IOP  066
      F(1,3)=-Z9                                                                 IOP  067
      F(1,4)=F(1,2)                                                              IOP  068
      F(1,5)=Z5                                                                  IOP  069
      F(1,6)=Z12-Z7*Z16                                                          IOP  070
      F(1,7)=Z6+2.*Z12*T1+Z1*Z15+Z6*Z16+Z3*Z19                                   IOP  071
      F(1,8)=Z7+Z6*(T1+Z19)+Z15*(Z12-.5*Z2)+.5*(Z4*Z16+Z3*Z20)+P1*Z1             IOP  072
      F(1,9)=Z8*(1.+Z16)                                                         IOP  073
      F(1,10)=Z3-Z6*T1+Z11*Z16+Z8*Z19                                            IOP  074
      F(1,11)=Z7+.5*Z5*Z5*Z16                                                    IOP  075
      F(1,12)=Z9                                                                 IOP  076
      F(1,13)=Z5*T1                                                              IOP  077
      F(1,14)=Z9*Z15                                                             IOP  078
      F(1,15)=Q                                                                  IOP  079
      F(1,16)=1.-Q*T1                                                            IOP  080
```

Table IV (Contd.)

```
      F(2,1)=Z2+Z1*T2                                                          IOP  081
      F(2,2)=-Z1+Z2*(T1+T2)+Z1*Z18                                             IOP  082
      F(2,5)=Z1+Z5*T2                                                          IOP  083
      F(2,4)=F(2,2)                                                            IOP  084
      F(2,3)=.5*F(2,5)                                                         IOP  085
      F(2,6)=Z10+Z12*T2-Z6*Z16+Z3*(P2-.5*Z17)                                  IOP  086
      F(2,7)=2.*Z10*T1+Z6*(T2-Z17-2.*Z19)+Z2*Z15+2.*Z12*Z18-Z13*Z16            IOP  087
     1+Z1*Z21-Z3*Z22+2.*P2*(Z6+Z3*T1)                                          IOP  088
      F(2,8)=Z7*(T2+Z15*Z17)-.5*Z4*(Z17+Z21)+Z6*(Z16+Z18-Z20-Z22)              IOP  089
     1-Z13*Z19+P2*(Z4+2.*Z6*T1+Z3*Z15)+P1*(Z2+Z1*T2)                           IOP  090
      F(2,9)=-Z6*T2-Z14*Z16+Z8*(2.*P2-Z17)                                     IOP  091
      F(2,10)=Z1-Z2*T1+(Z3-Z2)*T2-Z16*(2.*Z6-Z1)-Z18*(Z1+Z6)-Z11*Z17           IOP  092
     1-Z14*Z19-Z8*Z22+2.*P2*(Z11+Z8*T1)                                        IOP  093
      F(2,11)=-Z1+(Z7-Z5)*T2-Z8*Z16+Z5*Z5*(P2-.5*Z17)                          IOP  094
      F(2,12)=Z10+Z9*T2                                                        IOP  095
      F(2,13)=-2.*(Z10*T1+Z9*Z18)                                              IOP  096
      F(2,14)=Z10*Z15+Z9*Z21                                                   IOP  097
      F(2,15)=1.-Q*T2                                                          IOP  098
      F(2,16)=Q*Z18-T1-T2                                                      IOP  099
      DO 55 I=1,2                                                              IOP  100
      F(I,17)=.5*F(I,9)                                                        IOP  101
      F(I,18)=.5*F(I,10)                                                       IOP  102
      F(I,19)=.5*F(I,5)+F(I,11)                                                IOP  103
      F(I,20)=.25*F(I,11)-.125*F(I,5)                                          IOP  104
      IF(16) 50,55,58                                                          IOP  105
   58 F(I,5)=.5*F(I,5)                                                         IOP  106
      F(I,9)=F(I,17)                                                           IOP  107
      F(I,10)=F(I,18)                                                          IOP  108
      F(I,11)=F(I,20)                                                          IOP  109
      F(I,19)=.5*F(I,19)                                                       IOP  110
   55 CONTINUE                                                                 IOP  111
      GO TO 9                                                                  IOP  112
C                                ELECTRIC  SECTOR  CALCULATION                 IOP  113
   39 A=SQRTF(2.-E1)                                                           IOP  114
      Q=PI*Q                                                                   IOP  115
      G=SQRTF(E1)                                                              IOP  116
      F1=Q*G                                                                   IOP  117
      E2=E1*(1.+E2)                                                            IOP  118
      RE=E1*E2                                                                 IOP  119
      E2=1.+E2                                                                 IOP  120
      B=3.*(E1-1.)-.5*RE                                                       IOP  121
      D=E1+RE                                                                  IOP  122
      F2=Q*A                                                                   IOP  123
C                                  ZEES  ELECTRIC                            IOP  124
      Z1=SINF(F2)                                                              IOP  125
      Z2=COSF(F2)                                                              IOP  126
      Z3=SINF(F1)                                                              IOP  127
      Z4=COSF(F1)                                                              IOP  128
      Z5=Z2*Z2                                                                 IOP  129
      Z6=Z4*Z4                                                                 IOP  130
      Z7=Z1*Z2                                                                 IOP  131
      Z8=Z3*Z4                                                                 IOP  132
      Z9=A*A                                                                   IOP  133
      Z10=3.*Z9                                                                IOP  134
      Z11=A*Z10                                                                IOP  135
      Z12=Z9*Z9                                                                IOP  136
      Z13=3.*Z12                                                               IOP  137
      Z14=A*Z13                                                                IOP  138
      Z15=B+Z9                                                                 IOP  139
      Z16=2.*B-Z9                                                              IOP  140
      Z17=B-2.*Z9                                                              IOP  141
      Z18=2.-3.*E1                                                             IOP  142
      Z19=Z18-2.*E1                                                            IOP  143
      Z20=Z5*Z17                                                               IOP  144
      Z21=3.*F2*(2.*B+Z9+Z12)                                                  IOP  145
      Z22=Z1*Z21                                                               IOP  146
      Z21=Z2*Z21                                                               IOP  147
C                                  F(I,J) ELECTRIC                           IOP  148
      F(1,1)=Z1/A                                                              IOP  149
      F(1,2)=Z2                                                                IOP  150
      F(1,3)=0.                                                                IOP  151
      F(1,4)=Z2                                                                IOP  152
      F(1,5)=2.*(1.-Z2)/Z9                                                     IOP  153
      F(1,6)=(Z15-Z2*Z16+Z20)/Z13                                              IOP  154
      F(1,7)=(Z1*Z16-2.*Z7*Z17)/Z11                                            IOP  155
      F(1,8)=(Z16-Z2*Z15-Z5*Z17)/Z10                                           IOP  156
      F(1,9)=((Z13+4.*Z10+Z16)*Z1+4.*Z7*Z17-Z21)/Z14                           IOP  157
      F(1,10)=(4.*(Z20+Z2*Z15-Z16)+Z22)/Z13                                    IOP  158
      F(1,11)=(20.*Z15+Z13-(16.*Z15+4.*Z10+Z13)*Z2-4.*Z20-2.*Z22)              IOP  159
     1/(A*Z14)                                                                 IOP  160
      F(1,12)=(E2*(2.*E1*Z2+Z18-Z6*Z9)/(2.*Z19)+Z2-1.)/Z9                      IOP  161
      F(1,13)=G*E2*Z8/Z19-Z1*D/(A*Z19)                                         IOP  162
      F(1,14)=(D*(Z6*Z9-Z2*Z18-2.*E1)/(2.*Z19)+E1*(Z2-1.))/Z9                  IOP  163
     1+.5*F(1,1)*P1                                                            IOP  164
      IF(E1) 10,11,10                                                          IOP  165
```

Table IV (Contd.)

```
   11 F(1,15)=Q                                                                  IOP  166
      GO TO 12                                                                   IOP  167
   10 F(1,15)=Z3/G                                                               IOP  168
   12 F(1,16)=Z4                                                                 IOP  169
      F(2,1)=Z2                                                                  IOP  170
      F(2,2)=-A*Z1                                                               IOP  171
      F(2,3)=0.                                                                  IOP  172
      F(2,4)=F(2,2)                                                              IOP  173
      F(2,5)=2.*Z1/A                                                             IOP  174
      F(2,6)=Z16*(Z1-Z7)/Z11                                                     IOP  175
      F(2,7)=Z16*(1.+Z2-2.*Z5)/Z10                                               IOP  176
      Z7=Z7*Z16                                                                  IOP  177
      F(2,8)=(Z1*Z15+Z7)/(3.*A)                                                  IOP  178
      F(2,9)=(2.*Z16*(2.*Z5-Z2-1.)+Z22)/Z13                                      IOP  179
      F(2,10)=((2.*Z15+Z10+Z13)*Z1-4.*Z7+Z21)/Z11                                IOP  180
      F(2,11)=((4.*Z15+2.*Z10-Z13)*Z1+4.*Z7-2.*Z21)/Z14                          IOP  181
      F(2,12)=F(1,13)-F(1,1)+.5*F(1,15)*F(1,15)*P2                               IOP  182
      F(2,13)=D*(2.*Z6-Z2-1.)/Z19+F(1,15)*F(1,16)*P2                             IOP  183
      F(2,14)=D*(.5*Z1*Z18/A-G*Z8)/Z19-E1*Z1/A+.5*(F(2,1)*P1                     IOP  184
     1+F(1,16)*F(1,16)*P2)                                                       IOP  185
      F(2,15)=Z4                                                                 IOP  186
      F(2,16)=-G*Z3                                                              IOP  187
      DO 56 I=1,2                                                                IOP  188
      IF(16) 50,59,60                                                            IOP  189
   60 F(I,5)=.5*F(I,5)                                                           IOP  190
      F(I,9)=.5*F(I,9)                                                           IOP  191
      F(I,10)=.5*F(I,10)                                                         IOP  192
      F(I,11)=.25*F(I,11)-.125*F(I,5)                                            IOP  193
   59 F(I,17)=0.                                                                 IOP  194
      F(I,18)=0.                                                                 IOP  195
      F(I,19)=0.                                                                 IOP  196
   56 F(I,20)=0.                                                                 IOP  197
    9 IF(K1-1) 50,13,14                                                         IOP  198
C                                FIRST FIELD SUBPROGRAM                         IOP  199
   13 IF(K2-1) 50,15,16                                                         IOP  200
   15 IF(15) 50,16,17                                                           IOP  201
   17 A=F(1,2)+F(2,1)                                                           IOP  202
      DA=R*(-A-SQRTF(A*A-4.*F(1,1)*F(2,2)))/(2.*F(2,2))                         IOP  203
   16 DA=DA/R                                                                   IOP  204
      DS=DA*DA                                                                  IOP  205
      DO 18 I=1,2                                                               IOP  206
      C(I,1)=F(I,1)+F(I,2)*DA                                                   IOP  207
      C(I,2)=F(I,5)                                                             IOP  208
      C(I,3)=F(I,3)                                                             IOP  209
      C(I,4)=F(I,4)                                                             IOP  210
      C(I,5)=F(I,15)+F(I,16)*DA                                                 IOP  211
      C(I,6)=F(I,16)                                                            IOP  212
      C(I,7)=F(I,6)+F(I,7)*DA+F(I,8)*DS                                         IOP  213
      C(I,8)=F(I,9)+F(I,10)*DA                                                  IOP  214
      C(I,9)=F(I,11)                                                            IOP  215
      C(I,10)=F(I,17)+DA*F(I,18)                                                IOP  216
      C(I,11)=F(I,19)                                                           IOP  217
      C(I,12)=F(I,20)                                                           IOP  218
      C(I,13)=F(I,7)+2.*F(I,8)*DA                                               IOP  219
      C(I,14)=F(I,10)                                                           IOP  220
      C(I,15)=F(I,8)                                                            IOP  221
      C(I,16)=F(I,12)+F(I,13)*DA+F(I,14)*DS                                     IOP  222
      C(I,17)=F(I,13)+2.*F(I,14)*DA                                             IOP  223
   18 C(I,18)=F(I,14)                                                           IOP  224
      RB=R                                                                      IOP  225
      W(1,1)=DA*R                                                               IOP  226
      W(1,2)=0.                                                                 IOP  227
      W(1,3)=0.                                                                 IOP  228
      W(1,4)=1.                                                                 IOP  229
      W(1,5)=DA*R                                                               IOP  230
      W(1,6)=1.                                                                 IOP  231
      DO 61 J=1,6                                                               IOP  232
   61 W(2,J)=R*C(1,J)                                                           IOP  233
      W(2,4)=W(2,4)/R                                                           IOP  234
      W(2,6)=W(2,6)/R                                                           IOP  235
C                                                                               IOP  236
C*****************************************************************            IOP  237
C                    PUNCH COEFFICIENT PRINT OR RAY TRACE                       IOP  238
   30 FORMAT(///)                                                               IOP  239
   19 FORMAT(/4X2H RI2,4F18.8/(8X4F18.8))                                       IOP  240
   65 PUNCH 30                                                                  IOP  241
   20 IF(I7) 50,62,37                                                           IOP  242
   37 PUNCH 52,K1,((C(I,J),J=1,18),I=1,2)                                       IOP  243
   52 FORMAT(/4X2H CI2,4F18.8,3(/8X4F18.8)/8X2F18.8,/(8X4F18.8))                IOP  244
   62 IF(I8) 50,21,41                                                           IOP  245
   41 PUNCH 19,K1,((W(I,J),J=1,6),I=1,2)                                        IOP  246
C*****************************************************************            IOP  247
C                                                                               IOP  248
   21 RA=R                                                                      IOP  249
      IF(K2-K1) 50,23,1                                                         IOP  250
```

Table IV (Contd.)

```
C                    MORE THAN 1 FIELD COMBINATION SUBPROGRAM                    IOP  251
   14 DO 44 J=1,6                                                                IOP  252
      W(1,J)=R*C(1,J)+DB*C(2,J)                                                  IOP  253
      IF(I3) 50,44,82                                                            IOP  254
   82 IF(J-5) 83,44,44                                                           IOP  255
   83 W(1,J)=-W(1,J)                                                             IOP  256
   44 CONTINUE                                                                   IOP  257
      W(1,4)=W(1,4)/RB                                                           IOP  258
      W(1,6)=W(1,6)/RB                                                           IOP  259
      U=RA/R                                                                     IOP  260
      V=DB/R                                                                     IOP  261
      DO 24 J=1,18                                                               IOP  262
   24 S(J)=U*C(1,J)+V*C(2,J)                                                     IOP  263
      IF(I3) 50,25,26                                                            IOP  264
   26 DO 27 I=1,2                                                                IOP  265
      F(I,1)=-F(I,1)                                                             IOP  266
      F(I,2)=-F(I,2)                                                             IOP  267
      F(I,9)=-F(I,9)                                                             IOP  268
      F(I,10)=-F(I,10)                                                           IOP  269
      F(I,17)=-F(I,17)                                                           IOP  270
   27 F(I,18)=-F(I,18)                                                           IOP  271
   25 D1=C(2,1)                                                                  IOP  272
      D2=C(2,2)                                                                  IOP  273
      D3=C(2,3)                                                                  IOP  274
      D4=C(2,4)                                                                  IOP  275
      H3=C(2,5)                                                                  IOP  276
      H4=C(2,6)                                                                  IOP  277
      DO 28 I=1,2                                                                IOP  278
      DO 28 J=1,18                                                               IOP  279
   28 T(I,J)=F(I,1)*C(2,J)+F(I,2)*S(J)                                           IOP  280
      DO 29 I=1,2                                                                IOP  281
      C(I,1)=T(I,1)                                                              IOP  282
      C(I,2)=T(I,2)+F(I,5)                                                       IOP  283
      C(I,3)=T(I,3)+F(I,3)                                                       IOP  284
      C(I,4)=T(I,4)                                                              IOP  285
      C(I,5)=F(I,15)*H3+F(I,16)*S(5)                                             IOP  286
      C(I,6)=F(I,15)*H4+F(I,16)*S(6)                                             IOP  287
      C(I,7)=T(I,7)+F(I,6)*D1*D1+F(I,7)*D1*S(1)+F(I,8)*S(1)*S(1)                 IOP  288
      C(I,8)=T(I,8)+2.*F(I,6)*D1*D2+F(I,7)*(D1*S(2)+D2*S(1))                     IOP  289
     1+2.*F(I,8)*S(1)*S(2)+F(I,9)*D1+F(I,10)*S(1)                                IOP  290
      C(I,9)=T(I,9)+F(I,6)*D2*D2+F(I,7)*D2*S(2)+F(I,8)*S(2)*S(2)                 IOP  291
     1+F(I,9)*D2+F(I,10)*S(2)+F(I,11)                                            IOP  292
      C(I,10)=T(I,10)+2.*F(I,6)*D1*D3+F(I,7)*(D1*S(3)+D3*S(1))                   IOP  293
     1+2.*F(I,8)*S(1)*S(3)+F(I,17)*D1+F(I,18)*S(1)                               IOP  294
      C(I,11)=T(I,11)+2.*F(I,6)*D2*D3+F(I,7)*(D2*S(3)+D3*S(2))                   IOP  295
     1+2.*F(I,8)*S(2)*S(3)+F(I,9)*D3+F(I,10)*S(3)+F(I,19)                        IOP  296
     2+F(I,17)*D2+F(I,18)*S(2)                                                   IOP  297
      C(I,12)=T(I,12)+F(I,6)*D3*D3+F(I,7)*D3*S(3)+F(I,8)*S(3)*S(3)               IOP  298
     1+F(I,17)*D3+F(I,18)*S(3)+F(I,20)                                           IOP  299
      C(I,13)=T(I,13)+2.*F(I,6)*D1*D4+F(I,7)*(D1*S(4)+D4*S(1))                   IOP  300
     1+2.*F(I,8)*S(1)*S(4)                                                       IOP  301
      C(I,14)=T(I,14)+2.*F(I,6)*D2*D4+F(I,7)*(D2*S(4)+D4*S(2))                   IOP  302
     1+2.*F(I,8)*S(2)*S(4)+F(I,9)*D4+F(I,10)*S(4)                                IOP  303
      C(I,15)=T(I,15)+F(I,6)*D4*D4+F(I,7)*D4*S(4)+F(I,8)*S(4)*S(4)               IOP  304
      C(I,16)=T(I,16)+F(I,12)*H3*H3+F(I,13)*H3*S(5)+F(I,14)*S(5)*S(5)            IOP  305
      C(I,17)=T(I,17)+2.*F(I,12)*H3*H4+F(I,13)*(H3*S(6)+H4*S(5))                 IOP  306
     1+2.*F(I,14)*S(5)*S(6)                                                      IOP  307
   29 C(I,18)=T(I,18)+F(I,12)*H4*H4+F(I,13)*H4*S(6)+F(I,14)*S(6)*S(6)            IOP  308
      DO 47 J=1,6                                                                IOP  309
   47 W(2,J)=R*C(1,J)                                                            IOP  310
      W(2,4)=W(2,4)/RB                                                           IOP  311
      W(2,6)=W(2,6)/RB                                                           IOP  312
      GO TO 20                                                                   IOP  313
C                                  SUMMARY                                       IOP  314
   23 S(1)=-RA*C(1,5)/C(2,5)                                                     IOP  315
      S(3)=1./C(2,1)                                                             IOP  316
      S(4)=1./C(2,5)                                                             IOP  317
      IF(I5) 50,32,31                                                            IOP  318
   32 IF(I4) 50,33,31                                                            IOP  319
   31 DC=-RA*C(1,1)/C(2,1)                                                       IOP  320
C                                                                                IOP  321
C  ***************************************************************             IOP  322
C                              PUNCH DC                                          IOP  323
      PUNCH 34,DC                                                                IOP  324
   34 FORMAT(/4X4H DC F18.8)                                                     IOP  325
C  ***************************************************************             IOP  326
C                                                                                IOP  327
   33 DO 64 J=1,6                                                                IOP  328
   64 T(1,J)=R*C(1,J)+DC*C(2,J)                                                  IOP  329
      T(1,4)=T(1,4)/RB                                                           IOP  330
      T(1,6)=T(1,6)/RB                                                           IOP  331
      DC=DC/RA                                                                   IOP  332
      S(7)=C(1,3)+DC*C(2,3)                                                      IOP  333
      S(8)=C(1,10)+DC*C(2,10)                                                    IOP  334
      S(9)=C(1,11)+DC*C(2,11)                                                    IOP  335
```

Table IV (Contd.)

```
      IF(S(8)) 53,54,53                                                    IOP  336
   54 S(5)=90.                                                             IOP  337
      S(6)=90.                                                             IOP  338
      GO TO 57                                                             IOP  339
   53 S(5)=ATANF(-C(2,1)*S(7)/S(8))/PI                                     IOP  340
      S(6)=ATANF(-C(2,2)*S(7)/S(9))/PI                                     IOP  341
   57 DO 35 J=1,18                                                         IOP  342
   35 C(1,J)=C(1,J)+DC*C(2,J)                                              IOP  343
      S(2)=RA*C(1,3)                                                       IOP  344
C                                                                          IOP  345
CC ********************************************************************  IOP  346
C                                PUNCH SUMMARY                             IOP  347
   66 FORMAT(/4X4H RS 4F18.8/8X2F18.8)                                     IOP  348
   67 FORMAT(/4X4H S1 4F18.8/(8X4F18.8))                                   IOP  349
   68 FORMAT(/4X4H S2 4F18.8/8X2F18.8)                                     IOP  350
      IF(I8) 50,51,49                                                      IOP  351
   49 PUNCH 66,(T(1,J),J=1,6)                                              IOP  352
   51 PUNCH 67,((C(1,J),J=1,18)                                            IOP  353
      PUNCH 68,(S(J),J=1,6)                                                IOP  354
C  ********************************************************************  IOP  355
C                                                                          IOP  356
      GO TO 1                                                              IOP  357
   50 STOP                                                                 IOP  358
      END                                                                  IOP  359
```

REFERENCES

1. E. G. Johnson and A. O. Nier, Angular Aberrations in Sector Shaped Electromagnetic Lenses for Focusing Beams of Charged Particles, Phys. Rev. 91, 10 (1953).

2. H. G. Voorhies, Second-order Aberrations in Sector Shaped Electromagnetic Analyzers, Rev. Sci. Instr. 26, 716-717 (1955); 27, 58 (1956).

3a. L. A. König and H. Hintenberger, Über die Abbildungsfehler von beliebig begrenzten homogenen magnetischen Sektorenfeldern, Z. Naturforsch. 12a, 377-385 (1957).

3b. H. Hintenberger and L. A. König, Mass Spectrometers and Mass Spectrographs Corrected for Image Defects, Advances in Mass Spectrometry (Ed. by J. D. Waldron), Pergamon Press (1959).

4a. H. Ewald and H. Liebl, Die Bildfehler des Toroidkondensators, Z. Naturforsch. 12a, 28-33, 588 (1957).

4b. H. Liebl, Die Schnittwinkel der Richtungs- und Energiefokussierungskurven bei doppelfokussierenden Massenspektrographen, Optik 16, 19-26 (1959).

5a. H. A. Tasman, A. J. H. Boerboom, and H. Wachsmuth, Calculation of the Ion Optical Properties of Inhomogeneous Magnetic Sector Fields, Z. Naturforsch. 14a, 121-129, 816-827 (1959).

5b. H. A. Tasman, Ion Optics of Mass Spectrometers with Virtually Enlarged Radius, PhD. Thesis, Leiden (1961).

6. H. A. Tasman, op. cit., 5b, 64-66.

7. R. Albrecht, Das Potential in doppelt gekrümmten Kondensatoren, Z. Naturforsch. 11a, 156-163 (1956).

8. H. A. Tasman, op cit., 5b, Ch. 7. H. Hintenberger and L. A. König, op. cit., 3b: H. Liebl and H. Ewald, Z. Naturforsch. 14a, 199-200 (1959).

9. R. Herzog, Ablekung von Kathoden- und Kanalstrahlen am Rande eines Kondensators, dessen Streufeld durch eine Blende begrenzt ist, Z. Physik 97, 596-602 (1935).

10. N. D. Coggeshall and M. Muskat, The Paths of Ions and Electrons in Non-uniform Magnetic Fields, Phys. Rev. 66, 187-198 (1944).

11. S. J. Balestrini and F. A. White, Workable Magnetic Shim to Correct Second-order Aberration in a Mass Spectrometer, Rev. Sci. Instr. 31, 633-636 (1960).

12. C. Stevens, Correction Coils for Second-order Focusing with the Argonne 100 inch Radius Mass Spectrometer, Paper No. 48, meeting of ASTM Committee E-14 on Mass Spectrometry, June 1962.

13. A good introduction to this subject is given in the book by G. Hadley, Linear Programming, Addision-Wesley (1962).