ANL-7416
Supplement 1

Argonne National Laboratory

ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK

Numerical Determination of the Space, Time, Angle, or Energy Distribution of Particles in an Assembly

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ANL-7416 Supplement 1
Mathematics and Computers

ARGONNE NATIONAL LABORATORY
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Numerical Determination of the Space, Time, Angle, or Energy Distribution of Particles in an Assembly

Prepared by the
Benchmark Problem Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

Revised December 1972

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PREFACE

This book is an outgrowth of activities of the Benchmark Problem Committee of the Mathematics and Computation Division of the American Nuclear Society. After much discussion and consideration of sample benchmark problems, the committee decided to restrict this publication to a specific area which encompasses a major portion of reactor-physics computations.

The objectives and their implementation are described in the introductory material. A few sample benchmark problems prepared by committee members were included in the initial book skeleton. The revision consists of additional problems.

The structure and mechanism for maintaining this book were designed to facilitate use of the benchmark problems in a variety of ways without excessive paperwork. This is an attempt to establish a workable framework upon which this and other benchmark problem books may be developed.

February 1968

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DEDICATION

This supplement is dedicated to the memory of Harold Greenspan, who passed away suddenly while working on its publication. Harold served as a member of the Benchmark Problem Committee from its inception and has contributed his time, talent, and effort to this activity. His contributions are recognized by the Committee, the Division, the Society, and the nuclear community, all of whom have suffered a loss of a colleague, mentor, and friend.

Constraints

Boundary conditions

Approximations and simplifications

3. Configuration

Description

Reduction from source situation, if applicable

Sketch

4. Data

5. Specific Problems of Interest

Reference problem

Other problems

6. Expected Results

Primary results

Auxiliary results

7. Summary of Available Solutions

Most accurate known solution with the estimated accuracy

Other solutions

8. Documentation of Solutions

References

Sufficient data on techniques to enable duplication of results

Computer characteristics, including essential hardware and software data for the program with which the problem was solved, and significant figures carried in the computation

Solution details

Rigid adherence to any form might hinder rather than aid the specification of benchmark problems. For this reason, format guidelines rather than standard forms have been presented for general use.

IV. BENCHMARK PROBLEMS

Source Situations

1. Small Spherical Critical Experiment
2. A High-temperature Gas-cooled Reactor Configuration
3. An Analytical 2-D Multigroup Diffusion Problem
4. A Simple Highly Nonseparable Reactor
5. Two-dimensional Isolated Source in an Absorbing Medium
6. Infinite Slab Reactor Model
7. Monoenergetic Point Reactor Model

Expected Primary Results

1. Multiplication factor

2. Apparent asymptotic convergence rate

3. Number of iterations, total and outer

4. Machine time (total and iteration only)

5. Ratio of surface leakage to total losses by energy group

Possible Additional Results

1. Dependence of results on space and/or angular quadrature scheme

2. Dependence of execution time on numerical refinement of solution

3. Scalar and angular fluxes

Best Solution Available: Converged integral equation solution

Reference

1. G. E. Hansen and W. H. Roach, Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies, Los Alamos Scientific Laboratory Report LAMS-2543 (Dec 1961).

Solutions

1. Discrete Ordinates: 1-A1-1, 1-A1-2, 1-A1-3, and 1-A1-4

2. Monte Carlo: 1-A1-5

3. Integral Transport Theory: 1-A1-6

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-1 Benchmark Problem ID.1-A1

Date Submitted: July 1966 By: K. D. Lathrop (GGA)
(Name and Organization)

Date Accepted: August 1, 1966 By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Multigroup Discrete Ordinates Calculation

Mathematical Model: Discrete ordinates,[1] diamond difference scheme, multigroup solution of transport equation

Pertinent Features of Techniques Used

Gauss Legendre quadrature,[2] uniform space mesh. Group rebalancing. Relative change in pointwise scalar flux required to be less than 10^{-6} over entire system for convergence.

Computer: IBM-7030 Date Solved: July 1966

at: Los Alamos
(Installation)

Program: DTF-IV

References

1. K. D. Lathrop, DTF-IV, A Fortran-IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering, Los Alamos Scientific Laboratory Report LA-3373 (Nov 1965).

2. M. Abramowitz and I. A. Stegun, Eds., Handbook of Mathematical Functions, NBS, Appl. Math. Sci. 55, USGPO, Washington, pp. 916-919 (1964).

Primary Results (Space intervals 40, angular quadrature order 16)

1. Multiplication factor 0.996679

2. Apparent asymptotic convergence rate (reciprocal of the number of iterations required to reduce apparent absolute error by a factor of e) 0.80 based on outer iterations

3. Number of iterations, 761 total, 17 outer, where total counts each group individually

Results

CASE	5A	5B	5C	5D
Neutron Histories Followed	32,565	187,875	467,100	3,290,000
Calculated Multiplication Factor	0.99724	0.991116	0.994576	0.995246
Statistical Error Bound (1 Standard Deviation)	0.00690	0.00286	0.00193	0.00072
Surface Leakage Group				
1	0.078102	0.080570	0.080009	0.079898
2	0.14799	0.14798	0.14672	0.14686
3	0.089853	0.090335	0.091038	0.09125
4	0.14564	0.14842	0.14740	0.14771
5	0.094199	0.092197	0.092545	0.091483
6	0.010560	0.010742	0.011102	0.011301
Total	0.56635	0.57025	0.56882	0.56851
Absorption Group				
1	0.050351	0.049825	0.049341	0.049843
2	0.096839	0.096216	0.096611	0.096384
3	0.062037	0.061383	0.060525	0.060585
4	0.10283	0.10215	0.10338	0.10389
5	0.094746	0.094268	0.095655	0.095242
6	0.026850	0.025911	0.025670	0.025550
Total	0.43365	0.42975	0.43118	0.43149
Computer	360/50	360/75	360/75	360/75
Machine Time, min	10	10	19	120

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-6 Benchmark Problem ID.1-A1

Date Submitted: December 1968 By: I. Carlvik (AE)
(Name and Organization)

Date Accepted: October 1972 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Discrete Integral Transport Calculation

Mathematical Model

Discrete integral transport theory using point-to-point transport kernels.[1]

Pertinent Features of Solution Methods

1. The elements of the transport kernels are calculated by means of Gaussian quadrature. The accuracy of this quadrature may be tested by varying the number of points used by the integration routine.

2. Reflective, white, or vacuum boundary conditions may be applied at the outer cell boundary.

Computer: IBM-7044 Date Solved: December 1968

at: AE

Program: FLUBAG[2]

References

1. I. Carlvik, Integral Transport Theory in One-Dimensional Geometries, Nukleonik 10, 104-119 (1967).

2. Å. Ahlin, Instructions for the Use of the FLUBAG and FLUP Programmes, AB Atomenergi Report RFN-219 (1966).

Results

EXHIBIT A Eigenvalues and computing times for FLUBAG calculation. The two-region results were obtained with a region boundary placed arbitrarily at a radius of 6.5 cm. "Number of iterations" refers to the number required for calculation of three successive eigenvalues within $\varepsilon = 10^{-5}$.

EXHIBIT B Average fluxes and ratio of surface leakage to total losses (in all groups) for the 40-point case of EXHIBIT A.

EXHIBIT C Influence of varying quadrature limits {IA, IM} and eigenvalue convergence criterion, ϵ.

EXHIBIT D Plot of eigenvalue vs $(8/N)^3$, where N is the number of space points in the problem. Linear extrapolation to $N = \infty$ results in an eigenvalue estimate of 0.995971. The best extimate, considering numerical and plotting uncertainties, is 0.995970 ± 0.000003.

EXHIBIT E Eigenvalue vs computing time for a number of different methods applied to the benchmark problem.

EXHIBIT A

k_{eff} and Computing Times

Number of Space Points	One Region				Two Regions			
	k_{eff}	No. of Iterations	Total Time (min)	Iteration Time (min)	k_{eff}	No. of Iterations	Total Time (min)	Iteration Time (min)
8	.997480	64	0.244	0.127				
12	.996455	64	0.391	0.233				
16 (8 + 8)	.996184	64	0.581	0.370	.996208	60	0.597	0.347
20	.996083	64	0.818	0.536				
24 (12 + 12)	.996037	64	1.094	0.733	.996042	60	1.075	0.687
32 (16 + 16)					.996000	60	1.755	1.140
40 (20 + 20)					.995986	60	2.608	1.705

EXHIBIT B

Average Flux and Ratio of Surface Leakage to Total Losses by Energy Group for the 40-point Case

Group	Average Flux	Ratio of Surface Leakage to Total Losses
1	3.586603	0.080049
2	6.714589	0.146978
3	4.212351	0.091289
4	7.080458	0.147314
5	5.289965	0.091788
6	0.794566	0.011233
		0.568651

EXHIBIT C

Comparison with Calculations with Increased Numerical Accuracy

Number of Space Points	k_{eff} Standard IA = 2, IM = 5 $\varepsilon = 10^{-5}$	IA = 3, IM = 7 $\varepsilon = 10^{-5}$	IA = 3, IM = 7 $\varepsilon = 10^{-6}$
8	0.997480	0.997477	0.997480
12	0.996455	0.996456	0.996454
16	0.996184	0.996185	0.996183
20	0.996083	0.996084	0.996082
24	0.996037	0.996038	0.996037

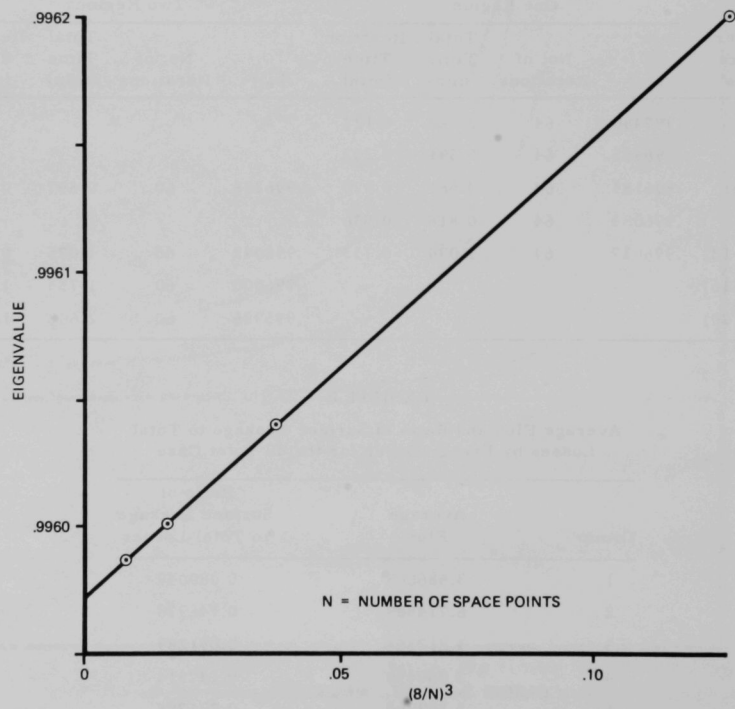


EXHIBIT D

Extrapolation to N = ∞

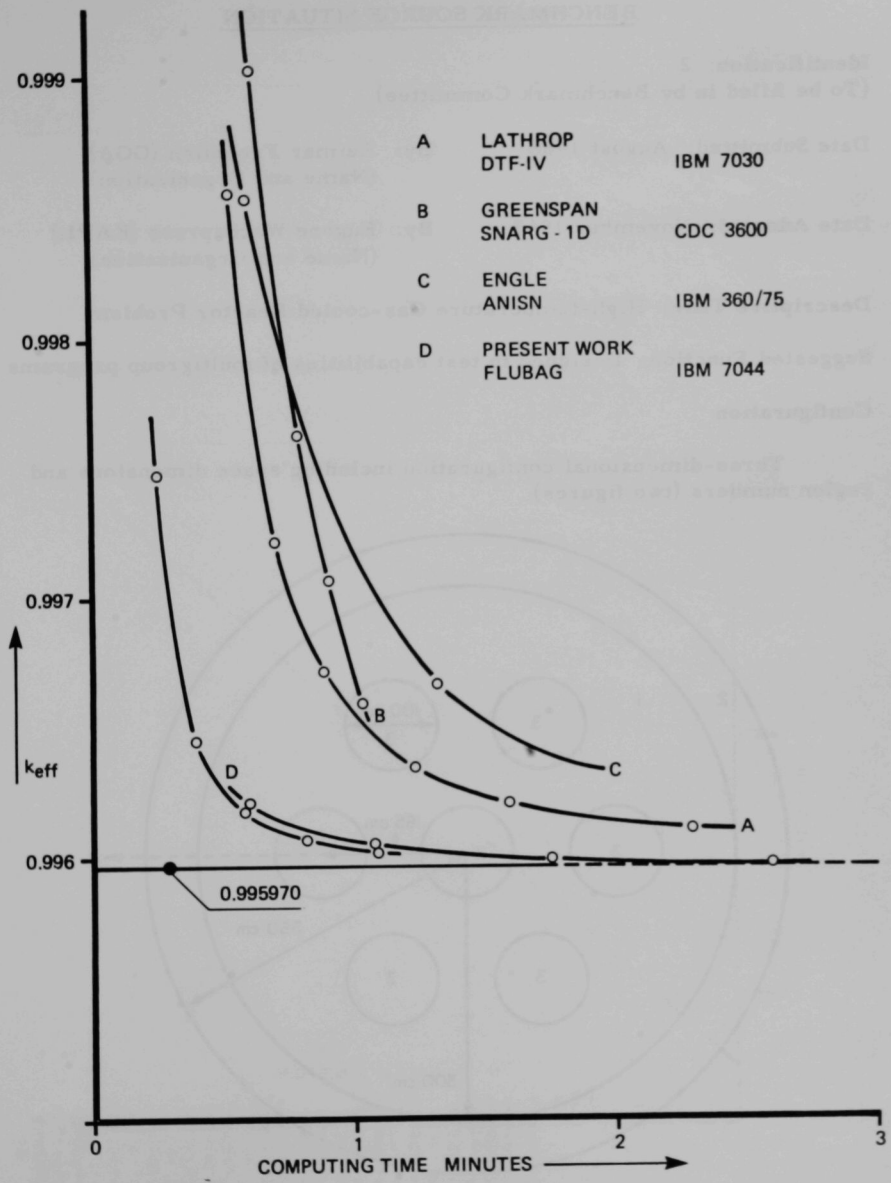


EXHIBIT E

k_{eff} vs Computing Time

BENCHMARK SOURCE SITUATION

Identification: 2
(To be filled in by Benchmark Committee)

Date Submitted: August 1966 By: Reimar Froehlich (GGA)
(Name and Organization)

Date Adopted: November 1967 By: Eugene Wachspress (KAPL)
(Name and Organization)

Descriptive Title: High-temperature Gas-cooled Reactor Problem

Suggested Function: Designed to test capabilities of multigroup programs

Configuration

Three-dimensional configuration including space dimensions and region numbers (two figures)

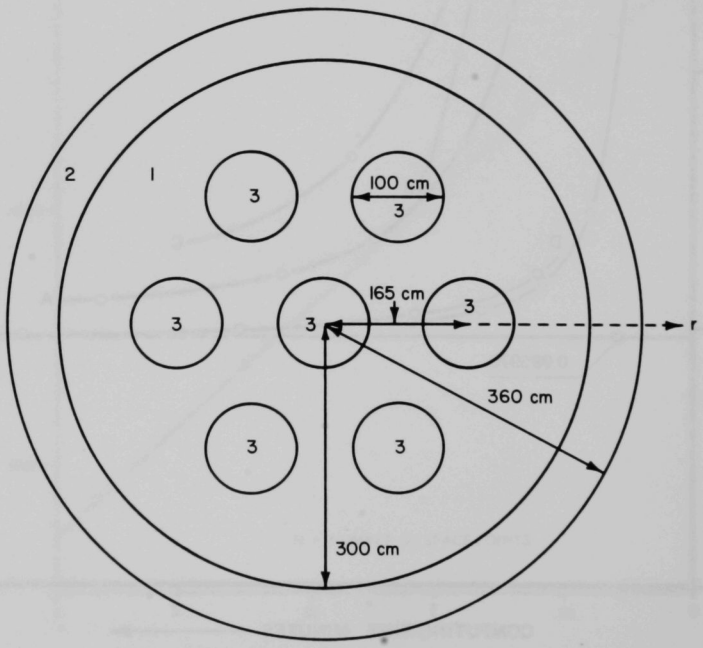


Fig. 1. (r, θ) Picture

J	I = 61
27	8.45828
26	8.54130
25	8.74519
24	9.23379
23	9.87783
22	10.75032
21	11.86746
20	13.22177
19	14.75127
18	16.28465
17	17.44637
16	17.49988
15	15.11344
14	8.07337
13	3.95253
12	3.53218
11	3.76235
10	4.08555
9	4.39807
8	4.67973
7	4.92603
6	5.13560
5	5.30776
4	5.44209
3	5.53826
2	5.59606
1	5.61534

BENCHMARK SOURCE SITUATION

Identification: 5
(To be filled in by Benchmark Committee)

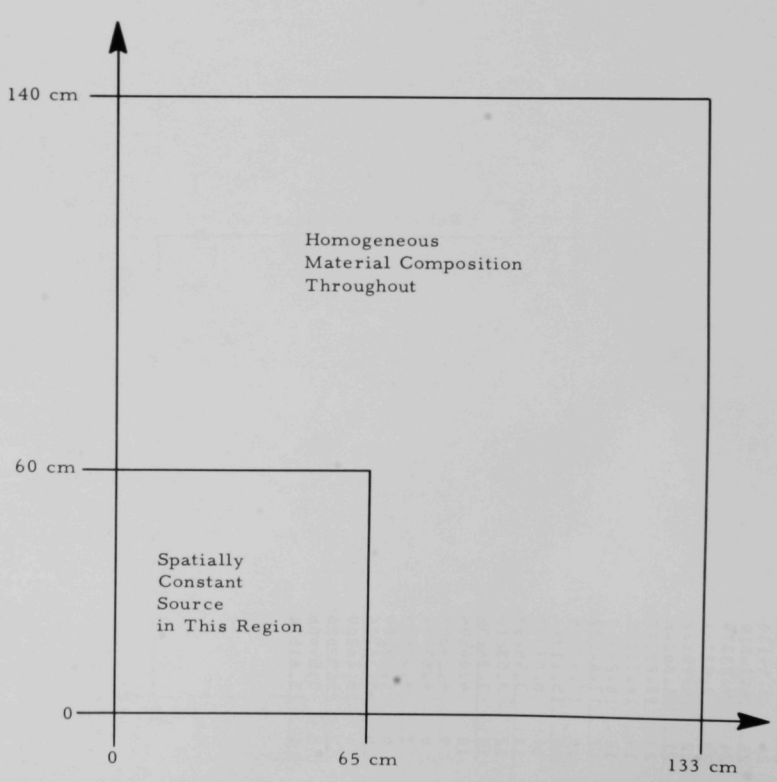
Date Submitted: April 1970 By: E. M. Gelbard (BAPL)
(Name and Organization)

Date Adopted: July 1970 By: R. Froehlich (GGA)
(Name and Organization)

Descriptive Title: Two-dimensional Isolated Source in an Absorbing Medium

Suggested Function: Provide Severe Test of Two-dimensional Transport Programs

Configuration



BENCHMARK PROBLEM

Identification: 5-A1 Source Situation ID. 5

Date Submitted: April 1970 By: E. M. Gelbard (BAPL) and B. Crawford (KAPL) (Name and Organization)

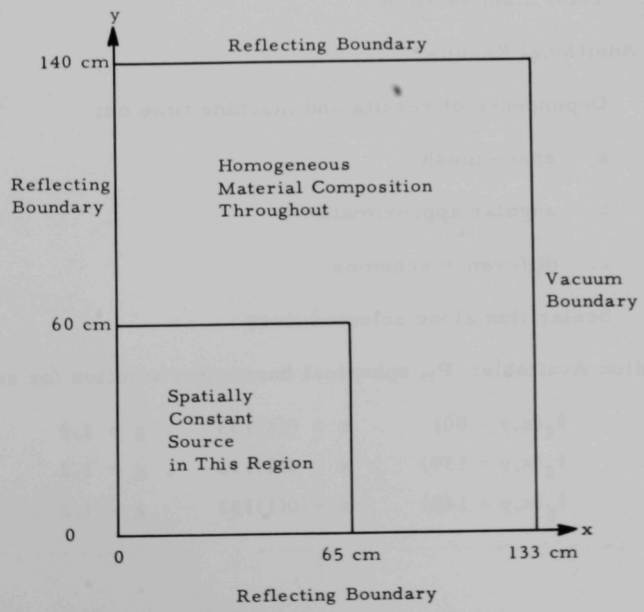
Date Accepted: July 1970 By: R. Froehlich (GGA) (Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport

Reduction of Source Problem:

1. Multigroup approximation made
2. Isotropic scattering assumed
3. x,y geometry
4. Boundary conditions as shown

Configuration



Data

Isotropic cross sections (cm^{-1}) and source density ($neutrons/cm^3$)

	Group 1	Group 2
Σ_a	0.061723	0.096027
$\nu\Sigma_f$	0.0	0.0
Σ_t	0.092104	0.100877
$\Sigma_{s0}^{g\to g}$	0.006947	0.004850
$\Sigma_{s0}^{g-1\to g}$	0.0	0.023434
Source density	0.0065460	0.017701

Expected Primary Results

1. Scalar flux in each group along horizontal and vertical lines
2. Total right leakage in each group
3. Number of iterations in each group
4. Total machine time

Possible Additional Results

1. Dependence of results and machine time on:
   a. space mesh
   b. angular approximation
   c. difference schemes
2. Scalar flux along selected lines

Best Solution Available: P_{19} spherical harmonics solution for scalar flux

$\Phi_g(x,y = 80)$	$x = 0(1)133$	$g = 1,2$
$\Phi_g(x,y = 139)$	$x = 0(1)133$	$g = 1,2$
$\Phi_g(x,y = 140)$	$x = 0(1)133$	$g = 1,2$

Solutions

1. Spherical Harmonics: 5-A1-1

2. Discrete Ordinates: 5-A1-2, 5-A1-3

BENCHMARK PROBLEM SOLUTION

Identification: 5-A1-1 Benchmark Problem ID.5-A1

Date Submitted: April 1970 By: E. M. Gelbard (BAPL) and B. Crawford (KAPL) (Name and Organization)

Date Accepted: July 1970 By: R. Froehlich (GGA) (Name and Organization)

Descriptive Title: Two-group Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Green's functions are calculated from a spherical harmonics (P_{19}) approximation. The solution at each point of interest is obtained by integration over source regions.

Pertinent Features of Solution Method

1. Each group-kernel is tabulated as a function of the mean free path of group 1 with a table spacing of 0.01. Linear interpolation is performed in this table given the distance from the flux point to the source point. Trapezoidal integration of source point contributions is carried out over each source domain to obtain the total flux.

2. Contributions to the flux are computed from only the four nearest sources.

3. The problem was solved with a reflective right boundary. Therefore, fluxes at those points which are within a few mean free paths of the right boundary should not be considered as reference values for the benchmark problem as stated.

Computer: CDC-6600 Date Solved: April 1970

at: BAPL

Program: BE27

Reference

R. B. Fischer and E. M. Gelbard, BE27--A Two-Group X-Y Geometry Transport Program for Deep Penetrating Homogeneous Media, Bettis Atomic Power Laboratory Report WAPD-TM-949 (1970).

Results

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines y = 80, y = 139, and y = 140 cm for x = 0(1)133 cm.

EXHIBIT A

Y=80. EXACT SOLUTION GROUP 1 P0 XY

.19547E-02	.19546E-02	.19545E-02	.19544E-02	.19541E-02	.19538E-02
.19535E-02	.19530E-02	.19525E-02	.19519E-02	.19512E-02	.19504E-02
.19495E-02	.19485E-02	.19474E-02	.19462E-02	.19448E-02	.19432E-02
.19415E-02	.19396E-02	.19376E-02	.19353E-02	.19327E-02	.19299E-02
.19269E-02	.19235E-02	.19198E-02	.19157E-02	.19112E-02	.19063E-02
.19009E-02	.18951E-02	.18886E-02	.18815E-02	.18738E-02	.18654E-02
.18562E-02	.18461E-02	.18352E-02	.18233E-02	.18103E-02	.17962E-02
.17809E-02	.17643E-02	.17463E-02	.17269E-02	.17060E-02	.16834E-02
.16592E-02	.16332E-02	.16053E-02	.15756E-02	.15440E-02	.15104E-02
.14750E-02	.14375E-02	.13982E-02	.13571E-02	.13143E-02	.12699E-02
.12241E-02	.11770E-02	.11289E-02	.10799E-02	.10305E-02	.98076E-03
.93104E-03	.88158E-03	.83267E-03	.78456E-03	.73747E-03	.69163E-03
.64722E-03	.60440E-03	.56329E-03	.52399E-03	.48657E-03	.45108E-03
.41752E-03	.38590E-03	.35620E-03	.32837E-03	.30236E-03	.27811E-03
.25555E-03	.23460E-03	.21519E-03	.19723E-03	.18064E-03	.16533E-03
.15122E-03	.13824E-03	.12631E-03	.11536E-03	.10531E-03	.96092E-04
.87653E-04	.79929E-04	.72864E-04	.66406E-04	.60507E-04	.55121E-04
.50206E-04	.45723E-04	.41637E-04	.37915E-04	.34524E-04	.31438E-04
.28631E-04	.26078E-04	.23759E-04	.21652E-04	.19741E-04	.18007E-04
.16437E-04	.15016E-04	.13731E-04	.12572E-04	.11528E-04	.10590E-04
.97487E-05	.89972E-05	.83286E-05	.77368E-05	.72164E-05	.67629E-05
.63719E-05	.60400E-05	.57642E-05	.55418E-05	.53710E-05	.52500E-05
.51779E-05	.51540E-05				

Y=80. EXACT SOLUTION GROUP 2 P0 XY

.45559E-02	.45558E-02	.45556E-02	.45553E-02	.45549E-02	.45543E-02
.45536E-02	.45528E-02	.45518E-02	.45506E-02	.45492E-02	.45477E-02
.45460E-02	.45440E-02	.45418E-02	.45393E-02	.45365E-02	.45335E-02
.45300E-02	.45262E-02	.45220E-02	.45173E-02	.45121E-02	.45064E-02
.45000E-02	.44930E-02	.44853E-02	.44767E-02	.44673E-02	.44569E-02
.44454E-02	.44328E-02	.44189E-02	.44036E-02	.43868E-02	.43684E-02
.43482E-02	.43260E-02	.43017E-02	.42751E-02	.42461E-02	.42144E-02
.41798E-02	.41421E-02	.41012E-02	.40567E-02	.40086E-02	.39565E-02
.39003E-02	.38398E-02	.37748E-02	.37052E-02	.36309E-02	.35517E-02
.34678E-02	.33790E-02	.32855E-02	.31875E-02	.30852E-02	.29789E-02
.28690E-02	.27559E-02	.26403E-02	.25226E-02	.24036E-02	.22838E-02
.21641E-02	.20450E-02	.19274E-02	.18117E-02	.16986E-02	.15887E-02
.14824E-02	.13801E-02	.12821E-02	.11886E-02	.10999E-02	.10159E-02
.93675E-03	.86241E-03	.79280E-03	.72782E-03	.66731E-03	.61111E-03
.55903E-03	.51087E-03	.46643E-03	.42548E-03	.38781E-03	.35322E-03
.32150E-03	.29244E-03	.26586E-03	.24156E-03	.21938E-03	.19914E-03
.18070E-03	.16390E-03	.14861E-03	.13471E-03	.12208E-03	.11060E-03
.10019E-03	.90735E-04	.82163E-04	.74394E-04	.67356E-04	.60982E-04
.55214E-04	.49996E-04	.45279E-04	.41016E-04	.37168E-04	.33696E-04
.30565E-04	.27746E-04	.25210E-04	.22932E-04	.20889E-04	.19061E-04
.17429E-04	.15976E-04	.14689E-04	.13554E-04	.12559E-04	.11694E-04
.10951E-04	.10322E-04	.97999E-05	.93800E-05	.90577E-05	.88299E-05
.86941E-05	.86490E-05				

EXHIBIT A (continued)

Y=139. EXACT SOLUTION GROUP 1 P0 XY

.61185E-05	.61179E-05	.61163E-05	.61135E-05	.61096E-05	.61046E-05
.60985E-05	.60912E-05	.60828E-05	.60732E-05	.60624E-05	.60505E-05
.60373E-05	.60228E-05	.60071E-05	.59901E-05	.59718E-05	.59521E-05
.59311E-05	.59086E-05	.58848E-05	.58595E-05	.58327E-05	.58044E-05
.57746E-05	.57432E-05	.57102E-05	.56756E-05	.56393E-05	.56014E-05
.55618E-05	.55205E-05	.54775E-05	.54327E-05	.53862E-05	.53379E-05
.52879E-05	.52361E-05	.51825E-05	.51272E-05	.50701E-05	.50113E-05
.49508E-05	.48886E-05	.48247E-05	.47591E-05	.46920E-05	.46233E-05
.45531E-05	.44814E-05	.44083E-05	.43339E-05	.42582E-05	.41813E-05
.41033E-05	.40242E-05	.39441E-05	.38631E-05	.37813E-05	.36989E-05
.36158E-05	.35322E-05	.34482E-05	.33639E-05	.32793E-05	.31947E-05
.31101E-05	.30255E-05	.29412E-05	.28572E-05	.27735E-05	.26904E-05
.26079E-05	.25261E-05	.24451E-05	.23650E-05	.22858E-05	.22077E-05
.21307E-05	.20550E-05	.19805E-05	.19073E-05	.18355E-05	.17652E-05
.16964E-05	.16291E-05	.15634E-05	.14994E-05	.14370E-05	.13763E-05
.13173E-05	.12600E-05	.12044E-05	.11506E-05	.10985E-05	.10482E-05
.99963E-06	.95279E-06	.90767E-06	.86426E-06	.82254E-06	.78250E-06
.74411E-06	.70735E-06	.67220E-06	.63862E-06	.60660E-06	.57609E-06
.54708E-06	.51953E-06	.49340E-06	.46867E-06	.44531E-06	.42327E-06
.40255E-06	.38309E-06	.36487E-06	.34787E-06	.33205E-06	.31739E-06
.30386E-06	.29144E-06	.28010E-06	.26982E-06	.26058E-06	.25236E-06
.24516E-06	.23894E-06	.23370E-06	.22943E-06	.22612E-06	.22376E-06
.22234E-06	.22187E-06				

Y=139. EXACT SOLUTION GROUP 2 P0 XY

.99238E-05	.99229E-05	.99204E-05	.99161E-05	.99101E-05	.99024E-05
.98930E-05	.98817E-05	.98687E-05	.98538E-05	.98371E-05	.98186E-05
.97980E-05	.97756E-05	.97511E-05	.97246E-05	.96960E-05	.96652E-05
.96322E-05	.95970E-05	.95595E-05	.95197E-05	.94774E-05	.94327E-05
.93855E-05	.93356E-05	.92832E-05	.92281E-05	.91703E-05	.91097E-05
.90463E-05	.89800E-05	.89109E-05	.88388E-05	.87638E-05	.86858E-05
.86048E-05	.85209E-05	.84339E-05	.83439E-05	.82509E-05	.81549E-05
.80560E-05	.79542E-05	.78495E-05	.77419E-05	.76316E-05	.75186E-05
.74030E-05	.72848E-05	.71642E-05	.70412E-05	.69160E-05	.67887E-05
.66594E-05	.65282E-05	.63954E-05	.62610E-05	.61252E-05	.59881E-05
.58500E-05	.57110E-05	.55712E-05	.54309E-05	.52903E-05	.51494E-05
.50086E-05	.48679E-05	.47276E-05	.45878E-05	.44488E-05	.43106E-05
.41735E-05	.40377E-05	.39032E-05	.37703E-05	.36391E-05	.35097E-05
.33823E-05	.32570E-05	.31339E-05	.30132E-05	.28949E-05	.27791E-05
.26659E-05	.25554E-05	.24477E-05	.23428E-05	.22408E-05	.21416E-05
.20454E-05	.19521E-05	.18618E-05	.17745E-05	.16902E-05	.16088E-05
.15304E-05	.14550E-05	.13824E-05	.13128E-05	.12459E-05	.11819E-05
.11207E-05	.10622E-05	.10063E-05	.95308E-06	.90241E-06	.85423E-06
.80851E-06	.76517E-06	.72416E-06	.68542E-06	.64889E-06	.61451E-06
.58222E-06	.55197E-06	.52371E-06	.49737E-06	.47290E-06	.45026E-06
.42940E-06	.41027E-06	.39284E-06	.37706E-06	.36289E-06	.35031E-06
.33927E-06	.32977E-06	.32176E-06	.31524E-06	.31019E-06	.30659E-06
.30443E-06	.30371E-06				

EXHIBIT A (continued)

Y=140. EXACT SOLUTION GROUP 1 P0 XY

.60863E-05	.60857E-05	.60841E-05	.60813E-05	.60775E-05	.60725E-05
.60664E-04	.60591E-05	.60507E-05	.60412E-05	.60305E-05	.60185E-05
.60054E-05	.59910E-05	.59754E-05	.59584E-05	.59402E-05	.59206E-05
.58997E-05	.58773E-05	.58536E-05	.58284E-05	.58017E-05	.57736E-05
.57439E-05	.57126E-05	.56798E-05	.56453E-05	.56093E-05	.55715E-05
.55321E-05	.54910E-05	.54482E-05	.54037E-05	.53574E-05	.53094E-05
.52596E-05	.52081E-05	.51548E-05	.50998E-05	.50430E-05	.49845E-05
.49243E-05	.48624E-05	.47988E-05	.47337E-05	.46669E-05	.45986E-05
.45288E-05	.44575E-05	.43848E-05	.43108E-05	.42356E-05	.41591E-05
.40815E-05	.40028E-05	.39232E-05	.38427E-05	.37614E-05	.36794E-05
.35968E-05	.35137E-05	.34302E-05	.33464E-05	.32623E-05	.31782E-05
.30941E-05	.30100E-05	.29262E-05	.28426E-05	.27595E-05	.26769E-05
.25948E-05	.25135E-05	.24330E-05	.23533E-05	.22746E-05	.21969E-05
.21204E-05	.20450E-05	.19709E-05	.18982E-05	.18268E-05	.17569E-05
.16885E-05	.16216E-05	.15562E-05	.14925E-05	.14305E-05	.13701E-05
.13114E-05	.12544E-05	.11991E-05	.11456E-05	.10938E-05	.10437E-05
.99542E-06	.94881E-06	.90391E-06	.86072E-06	.81921E-06	.77936E-06
.74116E-06	.70457E-06	.66959E-06	.63617E-06	.60430E-06	.57393E-06
.54505E--6	.51762E-06	.49162E-06	.46700E-06	.44374E-06	.42180E-06
.40116E-06	.38179E-06	.36365E-06	.34672E-06	.33097E-06	.31637E-06
.30290E-06	.29058E-06	.27923E-06	.26899E-06	.25979E-06	.25161E-06
.24443E-06	.23824E-06	.23303E-06	.22877E-06	.22547E-06	.22312E-06
.22171E-06	.22124E-06				

Y=140. EXACT SOLUTION GROUP 2 P0 XY

.98655E-05	.98647E-05	.98621E-05	.08579E-05	.98519E-05	.98442E-05
.98348E-05	.98236E-05	.98107E-05	.97959E-05	.97792E-05	.97607E-05
.97403E-05	.97180E-05	.96936E-05	.96672E-05	.96387E-05	.96081E-05
.95753E-05	.95403E-05	.95030E-05	.94633E-05	.94213E-05	.93768E-05
.93298E-05	.92803E-05	.92281E-05	.91733E-05	.91158E-05	.90555E-05
.89924E-05	.89266E-05	.88578E-05	.87861E-05	.87115E-05	.86340E-05
.85535E-05	.84700E-05	.83835E-05	.82940E-05	.82016E-05	.81062E-05
.80079E-05	.79067E-05	.78026E-05	.76957E-05	.75861E-05	.74737E-05
.73588E-05	.72414E-05	.71215E-05	.69993E-05	.68749E-05	.67484E-05
.66199E-05	.64896E-05	.63576E-05	.62240E-05	.60891E-05	.59529E-05
.58157E-05	.56776E-05	.55387E-05	.53994E-05	.52596E-05	.51197E-05
.49797E-05	.48400E-05	.47006E-05	.45617E-05	.44236E-05	.42863E-05
.41501E-05	.40151E-05	.38815E-05	.37494E-05	.36191E-05	.34905E-05
.33639E-05	.32394E-05	.31171E-05	.29971E-05	.28795E-05	.27645E-05
.26520E-05	.25422E-05	.24351E-05	.23308E-05	.22294E-05	.21308E-05
.20352E-05	.19425E-05	.18527E-05	.17659E-05	.16821E-05	.16012E-05
.15232E-05	.14482E-05	.13760E-05	.13067E-05	.12403E-05	.11766E-05
.11157E-05	.10575E-05	.10019E-06	.94898E-06	.89857E-06	.85064E-06
.80515E-06	.76203E-06	.72122E-06	.68267E-06	.64632E-06	.61211E-06
.57998E-06	.54988E-06	.52174E-06	.49553E-06	.47118E-06	.44864E-06
.42788E-06	.40884E-06	.39149E-06	.37577E-06	.36167E-06	.34914E-06
.33816E-06	.32870E-06	.32073E-06	.31424E-06	.30920E-06	.30562E-06
.30347E-06	.30275E-06				

BENCHMARK PROBLEM SOLUTION

Identification: 5-A1-2 Benchmark Problem ID.5-A1

Date Submitted: October 1971 By: K. D. Lathrop (LASL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Discrete Ordinates in two modifications:

1. Diamond Difference Scheme with Set-to-Zero Negative Flux Control[1]

2. Variable-weighted positive difference scheme[2]

Pertinent Features of Techniques Used

All calculations were executed with the TWOTRAN program,[1] or in the case of option 2 a modification thereof.

Computer: CDC-6600 and 7600 Date Solved: January 1971

at: LASL

References

1. K. D. Lathrop and F. W. Brinkley, Theory and Use of the General Geometry TWOTRAN Program, Los Alamos Scientific Laboratory Report LA-4432 (1970).

2. K. D. Lathrop, J. Comput. Phys. 4, 475 (1969).

Details of the Calculation

Three different spatial meshes were used. In each case, a equally-spaced intervals between 0.0 and 65.0 and b equally-spaced intervals between 65.0 and 133.0 in x, and c equally-spaced intervals between 0.0 and 60.0 and d equally-spaced intervals between 60.0 and 140.0 in y were used. The three meshes are given by

	a	b	c	d	Total Number of Cells
A	13	14	12	16	756
B	26	28	24	32	3024
C	39	42	36	48	6804

Three different S_n orders were used and two types of calculations done. The calculations are labeled ND for normal diamond and VD for variable weight schemes. Using this key, the symbol AS8ND means a small mesh, order S_8, normal diamond calculation.

Results

EXHIBIT A Total right leakage by group

EXHIBIT B Execution time in minutes

EXHIBIT C The group 1 relative percentage error with respect to the P_{19} infinite-medium solution. A negative deviation indicates an approximate solution smaller than the P_{19} value at that point. Deviations near the right boundary are not significant because of the different boundary conditions used. In addition to the Los Alamos CS12VD solution, a Bettis Atomic Power Laboratory discrete ordinate solution and a Knolls Atomic Power Laboratory solution are compared with the P_{19} solution. The P_{19} solutions were extrapolated or interpolated linearly to the point at which the approximate solution was given.

Results

EXHIBIT A

Total Right Leakage by Group

Problem	Group 1	Group 2
AS8ND	5.10-4	8.00-4
BS8ND	5.64-4	9.01-4
CS8ND	5.74-4	9.21-4
AS12ND	4.96-4	7.76-4
BS12ND	5.47-4	8.72-4
CS12ND	5.57-4	8.91-4
BS16ND	5.40-4	8.62-4
CS16ND	5.51-4	8.81-4
AS8VD	5.18-4	8.22-4
BS8VD	5.67-4	9.09-4
CS8VD	5.76-4	9.25-4
AS12VD	5.06-4	7.99-4
BS12VD	5.49-4	8.78-4
CS12VD	5.58-4	8.95-4

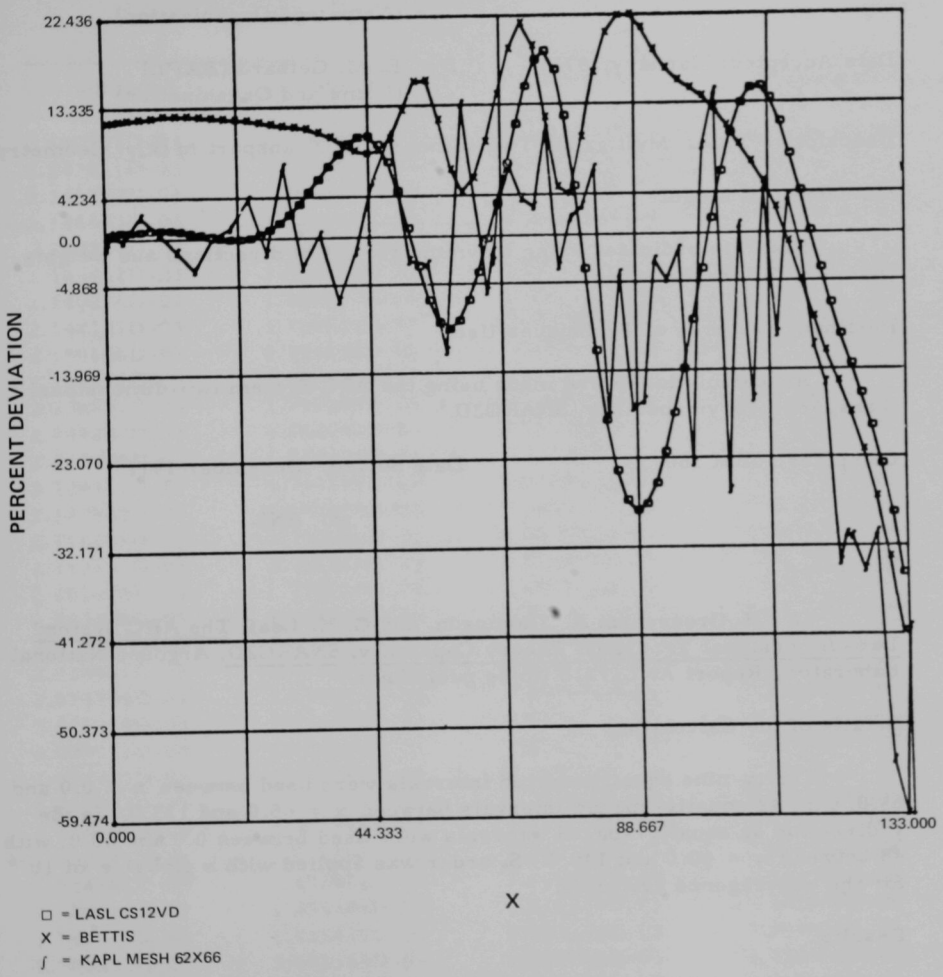
EXHIBIT B

Execution Time in Minutes

Problem	CDC-6600	CDC-7600
AS8ND	1.64	
BS8ND	6.41	
CS8ND	14.85	
AS12ND	3.13	
BS12ND	13.02	
CS12ND	28.36	
BS16ND	17.88	
CS16ND	44.95	
AS8VD		0.45
BS8VD		1.43
CS8VD		3.15
AS12VD		0.74
BS12VD		2.51
CS12VD		5.73

EXHIBIT C

Comparison of First Group Errors in Isotropic Scattering Test Problem near y = 140. The KAPL results involve stochastic integration and are not expected to be smooth.



BENCHMARK PROBLEM SOLUTION

Identification: 5-A1-3 Source Situation ID.5

Date Submitted: December 1971 By: H. Greenspan (ANL)
(Name and Organization)

Date Accepted: January 1972 By: E. M. Gelbard (BAPL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Discrete ordinates using a symmetric set of directions and weights, Diamond Difference Scheme.

Pertinent Features of Techniques Used

All calculations were made using the ARC System two-dimensional transport theory capability, SNARC2D.[1]

Computer: IBM 360/75 Date Solved: December 1971

at: ANL

Reference

1. H. Greenspan, R. Thompson, and G. K. Leaf, The ARC System Two-dimensional Transport Theory Capability, SNARC2D, Argonne National Laboratory Report ANL-7718 (to be published).

Details of the Calculation

Thirty-nine equally-spaced intervals were used between x = 0.0 and 65.0, with 42 equally-spaced intervals between x = 65.0 and 133.0. In the y direction 36 equally-spaced intervals were used between 0.0 and 60.0, with 48 between y = 60.0 and 140.0. S_8 order was applied with a global ϵ of 10^{-6} for the convergence criterion.

Results

Execution times were 21.79 and 21.81 minutes for the vacuum and reflective right boundary problems, respectively.

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines y = 79.16667, 80.83333, 139.16667, and the interpolated value at y = 80 cm. The values are given for x = 0.833(1.667)64.167 and 65.810(1.619)132.190. The vacuum boundary condition was used at x = 133.0 cm.

EXHIBIT B Scalar flux solutions for groups 1 and 2 on lines y = 79.16667, 80.83333, and 139.16667 cm for the same values of x as in EXHIBIT A. The reflective boundary condition was used at x = 133.0 cm.

EXHIBIT A

Group 1

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
2.147914D-03	1.760324D-03	1.954119D-03	5.453132D-06
2.147011D-03	1.759346D-03	1.953179D-03	5.625431D-06
2.146357D-03	1.761005D-03	1.953681D-03	5.401984D-06
2.146682D-03	1.757411D-03	1.952047D-03	5.295525D-06
2.143869D-03	1.760953D-03	1.952411D-03	5.442424D-06
2.145452D-03	1.755614D-03	1.950533D-03	5.425199D-06
2.140216D-03	1.760197D-03	1.950207D-03	5.469453D-06
2.144121D-03	1.754841D-03	1.949481D-03	5.437408D-06
2.139186D-03	1.754634D-03	1.946910D-03	5.246000D-06
2.139776D-03	1.754608D-03	1.947192D-03	5.367665D-06
2.136637D-03	1.750285D-03	1.943461D-03	5.389423D-06
2.135240D-03	1.752052D-03	1.943880D-03	5.363096D-06
2.134750D-03	1.744009D-03	1.939380D-03	5.581783D-06
2.128317D-03	1.743289D-03	1.935803D-03	5.416907D-06
2.127925D-03	1.736468D-03	1.932197D-03	5.342413D-06
2.115348D-03	1.730106D-03	1.922727D-03	5.450468D-06
2.112857D-03	1.727045D-03	1.919951D-03	5.241096D-06
2.101408D-03	1.708904D-03	1.905156D-03	5.280846D-06
2.087108D-03	1.698324D-03	1.892716D-03	5.409810D-06
2.077395D-03	1.674293D-03	1.875844D-03	5.367401D-06
2.038931D-03	1.665718D-03	1.852325D-03	5.379116D-06
2.029799D-03	1.671117D-03	1.850458D-03	5.286475D-06
2.023374D-03	1.642094D-03	1.837917D-03	5.259014D-06
2.006011D-03	1.628290D-03	1.817151D-03	5.325996D-06
1.977729D-03	1.596840D-03	1.787285D-03	5.050462D-06
1.927537D-03	1.587284D-03	1.757411D-03	4.426341D-06
1.928900D-03	1.577764D-03	1.753332D-03	3.877987D-06
1.902141D-03	1.518106D-03	1.710123D-03	3.525424D-06
1.824092D-03	1.472444D-03	1.648268D-03	3.328385D-06
1.780995D-03	1.455304D-03	1.618150D-03	3.202471D-06
1.743843D-03	1.423425D-03	1.583634D-03	3.045613D-06
1.708817D-03	1.403353D-03	1.556085D-03	2.928498D-06
1.693213D-03	1.385198D-03	1.539206D-03	2.998199D-06
1.610768D-03	1.293526D-03	1.452147D-03	3.160326D-06
1.428372D-03	1.133741D-03	1.281057D-03	3.030211D-06

[a]Interpolated values.

EXHIBIT A (Contd.)

Group 1 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
1.243559D-03	9.956505D-04	1.119605D-03	2.935283D-06
1.136778D-03	9.236938D-04	1.030236D-03	3.005266D-06
1.096411D-03	8.965666D-04	9.964888D-04	2.928323D-06
1.080740D-03	8.862299D-04	9.834850D-04	2.965906D-06
1.071274D-03	8.777819D-04	9.745280D-04	2.926461D-06
1.056948D-03	8.683286D-04	9.626383D-04	2.894413D-06
1.020582D-03	8.442113D-04	9.323697D-04	2.933867D-06
9.232565D-04	7.794019D-04	8.513292D-04	2.861735D-06
7.453555D-04	6.503231D-04	6.978393D-04	2.887683D-06
5.615489D-04	4.901157D-04	5.258323D-04	2.838089D-06
4.615391D-04	3.839952D-04	4.227672D-04	2.814204D-06
4.464389D-04	3.603404D-04	4.033897D-04	2.841921D-06
4.208290D-04	3.471438D-04	3.839864D-04	2.755282D-06
3.908544D-04	3.262077D-04	3.585311D-04	2.751612D-06
3.405520D-04	3.043158D-04	3.224339D-04	2.632509D-06
2.821605D-04	2.599611D-04	2.710608D-04	2.405608D-06
2.516491D-04	2.124204D-04	2.320348D-04	1.965414D-06
2.332982D-04	1.930112D-04	2.131547D-04	1.180795D-06
2.024693D-04	1.779563D-04	1.920213D-04	9.194976D-07
1.768143D-04	1.554894D-04	1.661519D-04	8.849262D-07
1.484556D-04	1.392101D-04	1.438329D-04	7.522018D-07
1.215711D-04	1.175885D-04	1.195798D-04	6.477273D-07
1.015345D-04	9.682111D-05	9.917781D-05	5.352258D-07
8.232815D-05	8.163105D-05	8.197960D-05	5.311728D-07
6.864761D-05	6.570825D-05	6.717793D-05	5.080930D-07
5.728426D-05	5.402511D-05	5.565469D-05	4.888762D-07
4.752363D-05	4.490349D-05	4.621356D-05	4.802503D-07
3.813569D-05	3.731509D-05	3.772539D-05	4.654882D-07
3.154722D-05	3.080701D-05	3.117712D-05	4.603703D-07
2.626919D-05	2.577399D-05	2.602159D-05	4.431015D-07
2.247652D-05	2.076515D-05	2.162084D-05	4.308088D-07
1.905163D-05	1.751944D-05	1.828554D-05	4.308885D-07
1.578280D-05	1.417553D-05	1.497917D-05	4.271917D-07
1.399622D-05	1.216649D-05	1.308136D-05	4.224880D-07
1.219469D-05	1.053701D-05	1.136585D-05	4.083682D-07
9.993607D-06	9.148371D-06	9.570989D-06	4.026062D-07
8.254236D-06	7.503745D-06	7.878991D-06	3.990047D-07
6.918153D-06	6.622696D-06	6.770425D-06	3.983523D-07
5.637405D-06	5.613636D-06	5.625521D-06	3.986738D-07
4.939579D-06	4.768213D-06	4.579353D-06	3.954215D-07
4.315187D-06	4.047267D-06	4.181227D-06	3.849154D-07

EXHIBIT A (Contd.)

Group 1 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
3.573294D-06	3.423756D-06	3.498525D-06	3.623727D-07
2.895823D-06	2.902913D-06	2.899368D-06	3.109934D-07
2.666726D-06	2.467074D-06	2.566900D-06	1.963686D-07
2.534188D-06	2.117457D-06	2.325823D-06	9.086628D-08
2.238235D-06	1.905471D-06	2.071853D-06	9.498285D-08

EXHIBIT A (Contd.)

Group 2

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
5.023820D-03	4.074512D-03	4.549166D-03	8.918321D-06
5.022723D-03	4.071748D-03	4.547236D-03	9.081085D-06
5.022041D-03	4.075820D-03	4.548931D-03	8.818589D-06
5.024601D-03	4.065977D-03	4.545289D-03	8.682707D-06
5.018847D-03	4.073870D-03	4.546359D-03	8.809199D-06
5.024191D-03	4.058981D-03	4.541587D-03	8.718896D-06
5.010718D-03	4.069238D-03	4.539978D-03	8.868962D-06
5.019145D-03	4.055847D-03	4.537496D-03	8.858264D-06
5.004352D-03	4.056777D-03	4.530565D-03	8.472087D-06
5.003981D-03	4.058718D-03	4.531350D-03	8.622216D-06
4.995673D-03	4.049588D-03	4.522631D-03	8.769117D-06
4.991503D-03	4.057048D-03	4.524276D-03	8.678316D-06
4.990521D-03	4.043854D-03	4.517188D-03	8.924685D-06
4.980977D-03	4.044193D-03	4.512585D-03	8.672583D-06
4.990341D-03	4.024248D-03	4.507295D-03	8.559124D-06
4.965965D-03	4.002553D-03	4.484259D-03	8.815348D-06
4.954747D-03	3.999296D-03	4.477022D-03	8.579200D-06
4.921920D-03	3.965649D-03	4.443785D-03	8.430525D-06
4.884986D-03	3.965959D-03	4.425473D-03	8.591243D-06
4.872227D-03	3.899356D-03	4.385792D-03	8.625386D-06
4.792612D-03	3.871714D-03	4.332163D-03	8.642543D-06
4.759355D-03	3.878504D-03	4.318930D-03	8.391257D-06
4.711928D-03	3.830066D-03	4.270997D-03	8.293096D-06
4.713092D-03	3.821305D-03	4.267199D-03	8.473111D-06
4.692302D-03	3.735871D-03	4.214087D-03	8.043106D-06
4.568012D-03	3.681644D-03	4.124824D-03	7.244105D-06
4.528012D-03	3.643008D-03	4.085510D-06	6.506956D-06
4.443314D-03	3.537957D-03	3.990636D-06	5.877447D-06
4.294126D-03	3.430464D-03	3.862295D-06	5.560097D-06

EXHIBIT A (Contd.)

Group 2 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
4.196506D-03	3.377336D-03	3.786921D-03	5.280528D-06
4.091394D-03	3.310450D-03	3.700922D-03	5.128539D-06
4.011106D-03	3.265072D-03	3.638089D-03	5.111628D-06
3.985211D-03	3.223928D-03	3.604570D-03	4.981857D-06
3.807302D-03	3.023422D-03	3.415362D-03	5.151462D-06
3.395699D-03	2.665267D-03	3.030483D-03	4.990764D-06
2.965530D-03	2.347974D-03	2.656752D-03	4.769256D-06
2.703581D-03	2.174569D-03	2.439075D-03	4.986933D-06
2.591792D-03	2.098212D-03	2.345002D-03	4.697752D-06
2.537811D-03	2.056990D-03	2.297401D-03	4.776992D-06
2.495960D-03	2.022153D-03	2.259057D-03	4.718228D-06
2.440665D-03	1.986727D-03	2.213696D-03	4.595200D-06
2.337939D-03	1.915159D-03	2.126549D-03	4.634204D-06
2.102711D-03	1.753111D-03	1.927911D-03	4.506884D-06
1.687133D-03	1.458725D-03	1.572929D-03	4.531032D-06
1.275588D-03	1.096577D-03	1.186083D-03	4.410863D-06
1.062994D-03	8.644651D-04	9.637295D-04	4.326683D-06
1.022031D-03	8.178388D-04	9.199349D-04	4.335989D-06
9.522908D-04	7.811239D-04	8.667074D-04	4.219412D-06
8.871974D-04	7.292893D-04	8.082434D-04	4.184321D-06
7.701729D-04	6.754385D-04	7.228057D-04	3.826836D-06
6.269048D-04	5.767494D-04	6.018271D-04	3.523548D-06
5.526864D-04	4.628360D-04	5.077612D-04	2.828152D-06
5.157767D-04	4.119754D-04	4.638761D-04	1.942848D-06
4.409565D-04	3.849073D-04	4.129319D-04	1.762384D-06
3.820057D-04	3.316934D-04	3.568496D-04	1.328923D-06
3.193552D-04	2.961414D-04	3.077483D-04	1.212550D-06
2.583446D-04	2.454879D-04	2.519163D-04	1.054588D-06
2.152294D-04	2.009588D-04	2.080941D-04	9.748350D-07
1.733102D-04	1.709742D-04	1.721422D-04	9.075578D-07
1.425554D-04	1.359504D-04	1.392529D-04	8.221176D-07
1.175021D-04	1.115880D-04	1.145451D-04	8.288699D-07
9.654892D-05	9.086594D-05	9.370743D-05	7.685352D-07
7.846658D-05	7.529652D-05	7.688155D-05	7.380404D-07
6.515275D-05	6.194944D-05	6.355110D-05	7.282028D-07
5.331335D-05	5.150199D-05	5.240767D-05	6.783057D-07
4.434739D-05	4.210197D-05	4.322468D-05	6.769982D-07
3.585149D-05	3.377586D-05	3.481368D-05	6.483390D-07
3.108650D-05	2.823335D-05	2.965993D-05	6.165067D-07
2.663277D-05	2.459391D-05	2.561334D-05	6.048143D-07
2.248626D-05	2.100226D-05	2.174426D-05	5.949188D-07
1.793361D-05	1.759301D-05	1.776331D-05	5.856983D-07

EXHIBIT A (Contd.)

Group 2 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
1.493352D-05	1.466399D-05	1.479876D-05	5.713787D-07
1.265938D-05	1.222623D-05	1.244281D-05	5.553968D-07
1.109268D-05	1.017024D-05	1.063146D-05	5.285766D-07
9.696385D-06	8.329253D-06	9.012819D-06	5.116878D-07
8.633433D-06	6.880716D-06	7.757075D-06	5.046827D-07
7.091233D-06	6.077438D-06	6.584336D-06	4.808122D-07
5.800826D-06	5.324939D-06	5.562883D-06	3.635163D-07
4.906135D-06	4.670986D-06	4.788561D-06	2.129141D-07
4.297404D-06	3.852284D-06	4.074844D-06	1.678852D-07
3.627762D-06	3.085362D-06	3.356562D-06	1.490733D-07

EXHIBIT B

Group 1

y = 79.16667	y = 80.83333	y = 139.16667
2.147914D-03	1.760324D-03	5.451170D-06
2.147011D-03	1.759346D-03	5.630156D-06
2.146358D-03	1.761004D-03	5.410233D-06
2.146680D-03	1.757413D-03	5.299725D-06
2.143871D-03	1.760951D-03	5.439562D-06
2.145450D-03	1.755615D-03	5.418188D-06
2.140218D-03	1.760196D-03	5.465841D-06
2.144119D-03	1.754843D-03	5.437254D-06
2.139189D-03	1.754632D-03	5.260805D-06
2.139774D-03	1.754609D-03	5.373417D-06
2.136638D-03	1.750285D-03	5.385170D-06
2.135239D-03	1.752052D-03	5.362537D-06
2.134750D-03	1.744010D-03	5.582727D-06
2.128317D-03	1.743287D-03	5.416736D-06
2.127924D-03	1.736471D-03	5.345650D-06
2.115351D-03	1.730103D-03	5.453771D-06
2.112853D-03	1.727049D-03	5.240755D-06
2.101412D-03	1.708901D-03	5.271100D-06
2.087105D-03	1.698326D-03	5.408998D-06
2.077398D-03	1.674293D-03	5.372342D-06
2.038930D-03	1.665717D-03	5.381172D-06
2.029799D-03	1.671118D-03	5.290559D-06
2.023375D-03	1.642092D-03	5.253108D-06
2.006010D-03	1.628292D-03	5.337604D-06

EXHIBIT B (Contd.)

Group 1 (Contd.)

y = 79.16667	y = 80.83333	y = 139.16667
1.977731D-03	1.596840D-03	5.057720D-06
1.927537D-03	1.587284D-03	4.411611D-06
1.928901D-03	1.577765D-03	3.883763D-06
1.902142D-03	1.518106D-03	3.523709D-06
1.824092D-03	1.472444D-03	3.326764D-06
1.780996D-03	1.455306D-03	3.206951D-06
1.743845D-03	1.423424D-03	3.049548D-06
1.708816D-03	1.403358D-03	2.931061D-06
1.693219D-03	1.385196D-03	2.996654D-06
1.610767D-03	1.293530D-03	3.153381D-06
1.428376D-03	1.133742D-03	3.031932D-06
1.243561D-03	9.956532D-04	2.942374D-06
1.136782D-03	9.236994D-04	3.004133D-06
1.096417D-03	8.965656D-04	2.929789D-06
1.080740D-03	8.862395D-04	2.957042D-06
1.071284D-03	8.777818D-04	2.931569D-06
1.056949D-03	8.683440D-04	2.900547D-06
1.020598D-03	8.442093D-04	2.926591D-06
9.232567D-04	7.794159D-04	2.864431D-06
7.453699D-04	6.503344D-04	2.891788D-06
5.615616D-04	4.901274D-04	2.841382D-06
4.615550D-04	3.840031D-04	2.815051D-06
4.464512D-04	3.603645D-04	2.846778D-06
4.208471D-04	3.471708D-04	2.758432D-06
3.908896D-04	3.262131D-04	2.757579D-06
3.405697D-04	3.043454D-04	2.630445D-06
2.821974D-04	2.599899D-04	2.408892D-06
2.516844D-04	2.124604D-04	1.977993D-06
2.333398D-04	1.930598D-04	1.193384D-06
2.025237D-04	1.780017D-04	9.245372D-07
1.768717D-04	1.555616D-04	8.871932D-07
1.485428D-04	1.392658D-04	7.669692D-07
1.216468D-04	1.176754D-04	6.568548D-07
1.016218D-04	9.691429D-05	5.563736D-07
8.245601D-05	8.173868D-05	5.465524D-07
6.879686D-05	6.588461D-05	5.274246D-07
5.742354D-05	5.415486D-05	5.080255D-07
4.768582D-05	4.501686D-05	4.936101D-07
3.840613D-05	3.757631D-05	4.712466D-07
3.173713D-05	3.109711D-05	4.736130D-07
2.654690D-05	2.598549D-05	4.568663D-07
2.292861D-05	2.113447D-05	4.447307D-07

EXHIBIT B (Contd.)

Group 1 (Contd.)

y = 79.16667	y = 80.83333	y = 139.16667
1.944186D-05	1.793357D-05	4.490839D-07
1.628371D-05	1.460847D-05	4.424720D-07
1.457425D-05	1.265658D-05	4.422498D-07
1.279661D-05	1.110127D-05	4.412679D-07
1.076811D-05	9.815951D-06	4.365529D-07
9.139203D-06	8.229747D-06	4.480707D-07
7.909900D-06	7.399882D-06	4.486119D-07
6.788922D-06	6.486069D-06	4.454115D-07
6.226674D-06	5.710868D-06	4.458992D-07
5.771134D-06	5.035397D-06	4.559544D-07
5.194619D-06	4.531373D-06	4.463988D-07
4.780623D-06	4.028172D-06	3.990195D-07
4.725280D-06	3.536843D-06	3.004254D-07
4.725255D-06	3.321437D-06	1.912721D-07
4.430631D-06	3.440849D-06	1.965719D-07

EXHIBIT B (Contd.)

Group 2

y = 79.16667	y = 80.83333	y = 139.16667
5.023821D-03	4.074512D-03	8.886863D-06
5.022723D-03	4.071747D-03	9.080503D-06
5.022042D-03	4.075820D-03	8.800542D-06
5.024600D-03	4.065976D-03	8.675987D-06
5.018848D-03	4.073870D-03	8.814283D-06
5.024190D-03	4.058981D-03	8.734344D-06
5.010720D-03	4.069238D-03	8.868351D-06
5.019144D-03	4.055845D-03	8.866105D-06
5.004354D-03	4.056780D-03	8.464885D-06
5.003980D-03	4.058716D-03	8.606101D-06
4.995676D-03	4.049587D-03	8.787201D-06
4.991499D-03	4.057053D-03	8.657308D-06
4.990527D-03	4.043847D-03	8.985549D-06
4.980970D-03	4.044202D-03	8.652305D-06
4.990349D-03	4.024237D-03	8.560583D-06
4.965956D-03	4.002567D-03	8.821965D-06
4.954759D-03	3.999280D-03	8.522622D-06
4.921907D-03	3.965665D-03	8.417736D-06
4.885000D-03	3.965943D-03	8.611735D-06

EXHIBIT B (Contd.)

Group 2 (Contd.)

y = 79.16667	y = 80.83333	y = 139.16667
4.872212D-03	3.899371D-03	8.617937D-06
4.792625D-03	3.871701D-03	8.622745D-06
4.759344D-03	3.878517D-03	8.404377D-06
4.711937D-03	3.830056D-03	8.295117D-06
4.713083D-03	3.821315D-03	8.475612D-06
4.692312D-03	3.735865D-03	8.074239D-06
4.568007D-03	3.681645D-03	7.315579D-06
4.528012D-03	3.643014D-03	6.531111D-06
4.443320D-03	3.537944D-03	5.755230D-06
4.294113D-03	3.430486D-03	5.432452D-06
4.196531D-03	3.377313D-03	5.339559D-06
4.091371D-03	3.310472D-03	5.276976D-06
4.011130D-03	3.265058D-03	5.157078D-06
3.985195D-03	3.223938D-03	4.880749D-06
3.807314D-03	3.023426D-03	5.131040D-06
3.395701D-03	2.665260D-03	4.881834D-06
2.965526D-03	2.347990D-03	4.936031D-06
2.703595D-03	2.174561D-03	5.001243D-06
2.591786D-03	2.098228D-03	4.668709D-06
2.537828D-03	2.056981D-03	4.779821D-06
2.495949D-03	2.022178D-03	4.719716D-06
2.440693D-03	1.986717D-03	4.594313D-06
2.337927D-03	1.915181D-03	4.645401D-06
2.102740D-03	1.753106D-03	4.504569D-06
1.687125D-03	1.458753D-03	4.532128D-06
1.275623D-03	1.096578D-03	4.407551D-06
1.062986D-03	8.644766D-04	4.332810D-06
1.022061D-03	8.178758D-04	4.341020D-06
9.523286D-04	7.811197D-04	4.214546D-06
8.872330D-04	7.293086D-04	4.193599D-06
7.701851D-04	6.754941D-04	3.910996D-06
6.269425D-04	5.767795D-04	3.459596D-06
5.527542D-04	4.628641D-04	2.820856D-06
5.157989D-04	4.120432D-04	1.983275D-06
4.410285D-04	3.849864D-04	1.715536D-06
3.821094D-04	3.317403D-04	1.348022D-06
3.194257D-04	2.962287D-04	1.285827D-06
2.584441D-04	2.456199D-04	1.044974D-06
2.153867D-04	2.010471D-04	9.937721D-07
1.734264D-04	1.711571D-04	9.159223D-07
1.427550D-04	1.361291D-04	8.568769D-07

EXHIBIT B (Contd.)

Group 2 (Contd.)

y = 79.16667	y = 80.83333	y = 139.16667
1.177350D-04	1.117676D-04	8.377588D-07
9.677619D-05	9.111740D-05	7.935240D-07
7.878498D-05	7.554576D-05	7.580757D-07
6.546545D-05	6.228063D-05	7.314733D-07
5.369656D-05	5.191596D-05	7.143289D-07
4.484979D-05	4.258184D-05	6.818581D-07
3.647024D-05	3.431771D-05	6.671050D-07
3.180010D-05	2.882882D-05	6.712790D-07
2.741688D-05	2.527410D-05	6.503463D-07
2.337015D-05	2.185773D-05	6.303401D-07
1.903486D-05	1.854617D-05	6.279367D-07
1.623341D-05	1.569878D-05	6.220808D-07
1.418110D-05	1.342211D-05	6.185171D-07
1.279526D-05	1.146982D-05	6.061437D-07
1.160085D-05	9.756293D-06	5.831003D-07
1.090379D-05	8.378381D-06	5.817198D-07
9.664573D-06	7.809635D-06	5.974830D-07
8.769913D-06	7.094103D-06	5.384843D-07
8.136323D-06	6.722153D-06	3.600671D-07
7.699022D-06	6.295131D-06	2.885438D-07
7.303640D-06	6.150369D-06	3.200539D-07

BENCHMARK PROBLEM

Identification: 5-A2 Source Situation ID.5

Date Submitted: April 1970 By: E. M. Gelbard (BAPL) and B. Crawford (KAPL) (Name and Organization)

Date Accepted: July 1970 By: R. Froehlich (GGA) (Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport

Reduction of Source Problem

1. Multigroup approximation made
2. Linearly anisotropic scattering assumed
3. x,y geometry
4. Boundary conditions as shown

Configuration: Same as for ID.5-A1.

Data

Cross sections (cm^{-1}) and source density ($neutrons/cm^3$)

	Group 1	Group 2
Σ_a	0.061723	0.096027
$\nu\Sigma_f$	0.0	0.0
Σ_t	0.10108	0.108529
$\Sigma_{s0}^{g\to g}$	0.015923	0.012502
$\Sigma_{s0}^{g-1\to g}$	0.0	0.023434
$\Sigma_{s1}^{g\to g}$	0.008976	0.003914
$\Sigma_{s1}^{g-1\to g}$	0.0	0.009016
Source density	0.006546	0.017701

Expected Primary Results

1. Scalar flux in each group along horizontal and vertical lines

2. Total right leakage in each group

3. Number of iterations in each group

4. Total machine time

Possible Additional Results

1. Dependence of results and machine time on:

   a. space mesh

   b. angular approximation

   c. difference schemes

2. Scalar flux along selected lines

Best Solution Available: P_{19} spherical harmonics solution for scalar flux

$\Phi_g(x, y = 80)$	$x = 0(1)133$	$g = 1,2$
$\Phi_g(x, y = 139)$	$x = 0(1)133$	$g = 1,2$
$\Phi_g(x, y = 140)$	$x = 0(1)133$	$g = 1,2$

Solutions

1. Spherical Harmonics: 5-A2-1

2. Discrete Ordinates: 5-A2-2, 5-A2-3

BENCHMARK PROBLEM SOLUTION

Identification: 5-A2-1 Benchmark Problem ID.5-A2

Date Submitted: April 1970 By: E. M. Gelbard (BAPL) and B. Crawford (KAPL) (Name and Organization)

Date Accepted: July 1970 By: R. Froehlich (GGA) (Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Green's functions are calculated from a spherical harmonics (P_{19}) approximation. The solution at each point of interest is obtained by integration over source regions.

Pertinent Features of Solution Method

1. Each group-kernel is tabulated as a function of the mean free path of group 1 with a table spacing of 0.01. Linear interpolation is performed in this table given the distance from the flux point to the source point. Trapezoidal integration of source point contributions is carried out over each source domain to obtain the total flux.

2. Contributions to the flux are computed from only the four nearest sources.

3. The problem was solved with a reflective right boundary. Therefore, fluxes at those points which are within a few mean free paths of the right boundary should not be considered as reference values for the benchmark problem as stated.

Computer: CDC-6600 Date Solved: June 1970

at: BAPL

Program: BE27

Reference

R. B. Fischer and E. M. Gelbard, BE27--A Two-Group X-Y Geometry Transport Program for Deep Penetrating Homogeneous Media, Bettis Atomic Power Laboratory Report WAPD-TM-949 (1970).

Results

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines y = 80, y = 139, and y = 140 cm for x = 0(1)133 cm.

EXHIBIT A

Y=80. EXACT SOLUTION GROUP 1 P1 XY

.19664E-02	.19664E-02	.19663E-02	.19662E-02	.19659E-02	.19657E-02
.19653E-02	.19649E-02	.19644E-02	.19639E-02	.19632E-02	.19625E-02
.19616E-02	.19607E-02	.19596E-02	.19584E-02	.19571E-02	.19556E-02
.19540E-02	.19522E-02	.19502E-02	.19480E-02	.19455E-02	.19428E-02
.19399E-02	.19366E-02	.19330E-02	.19290E-02	.19246E-02	.19198E-02
.19145E-02	.19087E-02	.19024E-02	.18954E-02	.18878E-02	.18794E-02
.18703E-02	.18603E-02	.18494E-02	.18375E-02	.18245E-02	.18104E-02
.17951E-02	.17785E-02	.17604E-02	.17409E-02	.17199E-02	.16972E-02
.16727E-02	.16465E-02	.16184E-02	.15884E-02	.15565E-02	.15226E-02
.14867E-02	.14489E-02	.14091E-02	.13675E-02	.13241E-02	.12791E-02
.12327E-02	.11850E-02	.11362E-02	.10867E-02	.10366E-02	.98618E-03
.93579E-03	.88568E-03	.83612E-03	.78737E-03	.73967E-03	.69323E-03
.64826E-03	.60490E-03	.56328E-03	.52351E-03	.48566E-03	.44977E-03
.41585E-03	.38391E-03	.35392E-03	.32584E-03	.29962E-03	.27519E-03
.25248E-03	.23142E-03	.21191E-03	.19389E-03	.17725E-03	.16192E-03
.14782E-03	.13486E-03	.12296E-03	.11205E-03	.10206E-03	.92915E-04
.84553E-04	.76913E-04	.69938E-04	.63574E-04	.57773E-04	.52486E-04
.47673E-04	.43292E-04	.39307E-04	.35684E-04	.32392E-04	.29403E-04
.26690E-04	.24229E-04	.21998E-04	.19977E-04	.18147E-04	.16492E-04
.14997E-04	.13646E-04	.12429E-04	.11332E-04	.10347E-04	.94639E-05
.86737E-05	.79692E-05	.73436E-05	.67910E-05	.63061E-05	.58840E-05
.55207E-05	.52128E-05	.49571E-05	.47513E-05	.45932E-05	.44814E-05
.44147E-05	.43926E-05				

Y=80. EXACT SOLUTION GROUP 2 P1 XY

.45689E-02	.45688E-02	.45687E-02	.45684E-02	.45680E-02	.45674E-02
.45668E-02	.45659E-02	.45650E-02	.45639E-02	.45626E-02	.45611E-02
.45595E-02	.45576E-02	.45555E-02	.45531E-02	.45505E-02	.45475E-02
.45442E-02	.45405E-02	.45364E-02	.45319E-02	.45269E-02	.45213E-02
.45151E-02	.45083E-02	.45007E-02	.44923E-02	.44831E-02	.44729E-02
.44616E-02	.44492E-02	.44355E-02	.44204E-02	.44038E-02	.43855E-02
.43655E-02	.43434E-02	.43193E-02	.42929E-02	.42639E-02	.42323E-02
.41977E-02	.41601E-02	.41192E-02	.40747E-02	.40264E-02	.39742E-02
.39178E-02	.38571E-02	.37918E-02	.37218E-02	.36471E-02	.35675E-02
.34830E-02	.33936E-02	.32995E-02	.32007E-02	.30976E-02	.29905E-02
.28797E-02	.27657E-02	.26491E-02	.25305E-02	.24105E-02	.22897E-02
.21690E-02	.20489E-02	.19303E-02	.18137E-02	.16997E-02	.15889E-02
.14818E-02	.13787E-02	.12800E-02	.11858E-02	.10965E-02	.10120E-02
.93233E-03	.85758E-03	.78762E-03	.72234E-03	.66159E-03	.60521E-03
.55299E-03	.50475E-03	.46025E-03	.41930E-03	.38166E-03	.34713E-03
.31549E-03	.28654E-03	.26008E-03	.23593E-03	.21391E-03	.19384E-03
.17558E-03	.15897E-03	.14388E-03	.13017E-03	.11773E-03	.10645E-03
.96230E-04	.86968E-04	.78584E-04	.70997E-04	.64135E-04	.57933E-04
.52329E-04	.47269E-04	.42704E-04	.38586E-04	.34874E-04	.31532E-04
.28524E-04	.25820E-04	.23393E-04	.21216E-04	.19267E-04	.17526E-04
.15975E-04	.14596E-04	.13377E-04	.12302E-04	.11362E-04	.10546E-04
.98458E-05	.92532E-05	.87621E-05	.83673E-05	.80646E-05	.78507E-05
.77233E-05	.76809E-05				

EXHIBIT A (continued)

Y=139. EXACT SOLUTION GROUP 1 P1 XY

.50875E-05	.50871E-05	.50858E-05	.50836E-05	.50804E-05	.50764E-05
.50715E-05	.50657E-05	.50589E-05	.50512E-05	.50426E-05	.50329E-05
.50223E-05	.50107E-05	.49980E-05	.49842E-05	.49694E-05	.49535E-05
.49365E-05	.49183E-05	.48989E-05	.48783E-05	.48564E-05	.48334E-05
.48090E-05	.47833E-05	.47562E-05	.47278E-05	.46980E-05	.46668E-05
.46342E-05	.46001E-05	.45646E-05	.45275E-05	.44890E-05	.44489E-05
.44073E-05	.43642E-05	.43196E-05	.42734E-05	.42258E-05	.41766E-05
.41259E-05	.40738E-05	.40202E-05	.39652E-05	.39088E-05	.38510E-05
.37919E-05	.37315E-05	.36698E-05	.36070E-05	.35431E-05	.34781E-05
.34121E-05	.33452E-05	.32774E-05	.32088E-05	.31396E-05	.30697E-05
.29992E-05	.29283E-05	.28571E-05	.27856E-05	.27139E-05	.26420E-05
.25702E-05	.24985E-05	.24270E-05	.23557E-05	.22848E-05	.22144E-05
.21445E-05	.20752E-05	.20066E-05	.19388E-05	.18718E-05	.18058E-05
.17407E-05	.16767E-05	.16139E-05	.15522E-05	.14917E-05	.14325E-05
.13747E-05	.13181E-05	.12630E-05	.12093E-05	.11571E-05	.11063E-05
.10570E-05	.10092E-05	.96285E-06	.91805E-06	.87477E-06	.83299E-06
.79271E-06	.75393E-06	.71663E-06	.68079E-06	.64640E-06	.61345E-06
.58190E-06	.55174E-06	.52295E-06	.49549E-06	.46933E-06	.44446E-06
.42084E-06	.39845E-06	.37725E-06	.35721E-06	.33831E-06	.32051E-06
.30379E-06	.28812E-06	.27347E-06	.25982E-06	.24713E-06	.23539E-06
.22456E-06	.21463E-06	.20558E-06	.19738E-06	.19002E-06	.18348E-06
.17775E-06	.17281E-06	.16864E-06	.16525E-06	.16262E-06	.16075E-06
.15963E-06	.15926E-06				

.1833

Y=139. EXACT SOLUTION GROUP 2 P1 XY

.86522E-05	.86514E-05	.86493E-05	.86457E-05	.86406E-05	.86341E-05
.86260E-05	.86165E-05	.86055E-05	.85928E-05	.85787E-05	.85629E-05
.85454E-05	.85263E-05	.85055E-05	.84828E-05	.84584E-05	.84322E-05
.84040E-05	.83739E-05	.83418E-05	.83076E-05	.82713E-05	.82329E-05
.81923E-05	.81494E-05	.81042E-05	.80567E-05	.80067E-05	.79544E-05
.78995E-05	.78421E-05	.77821E-05	.77195E-05	.76544E-05	.75865E-05
.75160E-05	.74428E-05	.73669E-05	.72884E-05	.72071E-05	.71232E-05
.70366E-05	.69474E-05	.68556E-05	.67612E-05	.66644E-05	.65651E-05
.64635E-05	.63595E-05	.62533E-05	.61450E-05	.60347E-05	.59225E-05
.58085E-05	.56928E-05	.55755E-05	.54569E-05	.53369E-05	.52159E-05
.50938E-05	.49710E-05	.48475E-05	.47235E-05	.45991E-05	.44746E-05
.43501E-05	.42257E-05	.41017E-05	.39782E-05	.38553E-05	.37333E-05
.36122E-05	.34922E-05	.33735E-05	.32562E-05	.31405E-05	.30264E-05
.29141E-05	.28037E-05	.26953E-05	.25891E-05	.24850E-05	.23833E-05
.22839E-05	.21869E-05	.20924E-05	.20004E-05	.19111E-05	.18243E-05
.17402E-05	.16587E-05	.15799E-05	.15038E-05	.14303E-05	.13595E-05
.12914E-05	.12258E-05	.11629E-05	.11025E-05	.10447E-05	.98935E-06
.93646E-06	.88597E-06	.83784E-06	.79202E-06	.74845E-06	.70708E-06
.66787E-06	.63074E-06	.59565E-06	.56253E-06	.53135E-06	.50203E-06
.47453E-06	.44879E-06	.42476E-06	.40239E-06	.38164E-06	.36246E-06
.34479E-06	.32861E-06	.31388E-06	.30055E-06	.28859E-06	.27798E-06
.26868E-06	.26067E-06	.25393E-06	.24844E-06	.24418E-06	.24115E-06
.23934E-06	.23873E-06				

EXHIBIT A (continued)

Y=140. EXACT SOLUTION GROUP 1 P1 XY

.50585E-05	.50580E-05	.50567E-05	.50545E-05	.50514E-05	.50474E-05
.50425E-05	.50367E-05	.50300E-05	.50223E-05	.50137E-05	.50041E-05
.49935E-05	.49819E-05	.49693E-05	.49557E-05	.49409E-05	.49251E-05
.49081E-05	.48900E-05	.48707E-05	.48502E-05	.48285E-05	.48055E-05
.47812E-05	.47557E-05	.47288E-05	.47005E-05	.46709E-05	.46399E-05
.46074E-05	.45735E-05	.45381E-05	.45013E-05	.44629E-05	.44231E-05
.43817E-05	.43389E-05	.42945E-05	.42486E-05	.42012E-05	.41523E-05
.41020E-05	.40501E-05	.39969E-05	.39422E-05	.38861E-05	.38286E-05
.37699E-05	.37098E-05	.36486E-05	.35862E-05	.35226E-05	.34580E-05
.33924E-05	.33259E-05	.32586E-05	.31904E-05	.31216E-05	.30521E-05
.29821E-05	.29117E-05	.28409E-05	.27698E-05	.26985E-05	.26272E-05
.25558E-05	.24846E-05	.24135E-05	.23427E-05	.22722E-05	.22022E-05
.21327E-05	.20639E-05	.19957E-05	.19283E-05	.18618E-05	.17961E-05
.17315E-05	.16679E-05	.16054E-05	.15441E-05	.14840E-05	.14252E-05
.13676E-05	.13115E-05	.12567E-05	.12033E-05	.11513E-05	.11008E-05
.10518E-05	.10043E-05	.95823E-06	.91369E-06	.87064E-06	.82910E-06
.78904E-06	.75047E-06	.71337E-06	.67773E-06	.64353E-06	.61075E-06
.57937E-06	.54936E-06	.52071E-06	.49339E-06	.46737E-06	.44263E-06
.41913E-06	.39684E-06	.37574E-06	.35580E-06	.33699E-06	.31928E-06
.30264E-06	.28705E-06	.27246E-06	.25887E-06	.24624E-06	.23455E-06
.22377E-06	.21389E-06	.20488E-06	.19672E-06	.18939E-06	.18288E-06
.17717E-06	.17225E-06	.16811E-06	.16473E-06	.16211E-06	.16025E-06
.15913E-06	.15876E-06				

Y=140. EXACT SOLUTION GROUP 2 P1 XY

.85984E-05	.85977E-05	.85955E-05	.85920E-05	.85869E-05	.85804E-05
.85724E-05	.85629E-05	.85519E-05	.85394E-05	.85253E-05	.85096E-05
.84922E-05	.84732E-05	.84524E-05	.84300E-05	.84057E-05	.83795E-05
.83515E-05	.83215E-05	.82896E-05	.82556E-05	.82195E-05	.81813E-05
.81409E-05	.80983E-05	.80533E-05	.80061E-05	.79564E-05	.79043E-05
.78498E-05	.77927E-05	.77331E-05	.76709E-05	.76061E-05	.75387E-05
.74686E-05	.73958E-05	.73204E-05	.72423E-05	.71616E-05	.70782E-05
.69921E-05	.69035E-05	.68123E-05	.67185E-05	.66223E-05	.65237E-05
.64227E-05	.63194E-05	.62139E-05	.61063E-05	.59968E-05	.58853E-05
.57720E-05	.56571E-05	.55406E-05	.54228E-05	.53037E-05	.51834E-05
.50622E-05	.49402E-05	.48175E-05	.46944E-05	.45709E-05	.44472E-05
.43236E-05	.42001E-05	.40769E-05	.39542E-05	.38322E-05	.37109E-05
.35907E-05	.34715E-05	.33536E-05	.32371E-05	.31221E-05	.30088E-05
.28973E-05	.27876E-05	.26800E-05	.25744E-05	.24710E-05	.23699E-05
.22712E-05	.21748E-05	.20809E-05	.19896E-05	.19008E-05	.18146E-05
.17310E-05	.16500E-05	.15717E-05	.14960E-05	.14230E-05	.13526E-05
.12849E-05	.12197E-05	.11572E-05	.10971E-05	.10396E-05	.98461E-06
.93202E-06	.88181E-06	.83395E-06	.78838E-06	.74505E-06	.70391E-06
.66490E-06	.62797E-06	.59307E-06	.56013E-06	.52910E-06	.49994E-06
.47257E-06	.44697E-06	.42306E-06	.40080E-06	.38015E-06	.36106E-06
.34349E-06	.32738E-06	.31272E-06	.29945E-06	.28755E-06	.27699E-06
.26773E-06	.25976E-06	.25305E-06	.24759E-06	.24335E-06	.24034E-06
.23853E-06	.23793E-06				

BENCHMARK PROBLEM SOLUTION

Identification: 5-A2-2 Benchmark Problem ID.5-A2

Date Submitted: October 1971 By: K. D. Lathrop (LASL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Discrete Ordinates in three modifications

1. Diamond Difference Scheme with Set-to-Zero Negative Flux Control[1]

2. Variable-weighted positive difference scheme[2]

3. Formulation convertible to spherical harmonics-like equations[3]

Pertinent Features of Techniques Used

All calculations were executed with the TWOTRAN program,[1] or in the case of options 2 and 3, modifications thereof.

Computer: CDC-6600 and 7600 Date Solved: January 1971

at: LASL

References

1. K. D. Lathrop and F. W. Brinkley, Theory and Use of the General Geometry TWOTRAN Program, Los Alamos Scientific Laboratory Report LA-4432 (1970).

2. K. D. Lathrop, J. Comput. Phys. 4, 475 (1969).

3. K. D. Lathrop, Remedies for Ray Effects, Nucl. Sci. Eng. 45, 255 (1971).

Details of the Calculation

Mesh spacing variations were the same as those used for ID.5-A1-2. Problem identification conventions are described under ID.5-A1-2.

Three different S_n orders were used and three types of calculations done. We will label the calculations ND for normal diamond, VD for variable-weight, and FD for fixed-weight difference schemes. For those solutions obtained by conversion to spherical harmonics we append the order of approximation. Using this key, the symbol AS8ND means a small mesh, order S_8, normal diamond calculation, while BS8FDP5 is a medium mesh, S_8, fixed-weight, converted to P5, calculation.

Results

EXHIBIT A

Total Right Leakage by Group

Problem	Group 1	Group 2
AS8ND	4.19-4	6.96-4
BS8ND	4.66-4	7.86-4
CS8ND	4.75-4	8.04-4
AS12ND	4.06-4	6.75-4
BS12ND	4.51-4	7.60-4
CS12ND	4.60-4	7.77-4
BS12VD	4.56-4	7.69-4
CS12VD	4.63-4	7.83-4
BS16ND	4.46-4	7.51-4
CS16ND	4.55-4	7.68-4
BS16VD	4.51-4	7.61-4
AS6FDP3	4.63-4	7.72-4
BS8FDP5	4.57-4	7.73-4

EXHIBIT B

Execution Time in Minutes

Problem	CDC-6600	CDC-7600[a]
AS8ND	2.30	
BS8ND	8.83	
CS8ND	19.56	
AS12ND	4.43	
BS12ND	17.81	3.60
CS12ND	39.54	
BS12VD	15.74	
CS12VD	34.90	
BS16ND	25.58	
CS16ND	56.76	
BS16VD		6.55
AS6FDP3	6.54	
BS8FDP5	26.88	

[a]These problems were run before the 7600 became available. The 7600 times shown are typical. All times include time for periodic and final dumps and printing.

EXHIBIT C The group 1 relative error for CS16ND, CS12VD, and BS8FDP5, with respect to the P_{19} infinite medium solution along y = 80 cm. A negative deviation indicates an approximate solution smaller than the P_{19} value at that point. Comparisons near the right boundary are not significant because of the different boundary conditions used. The approximate solutions were interpolated linearly in x and y to the y = 80 line at the x values for which the P_{19} solution was given.

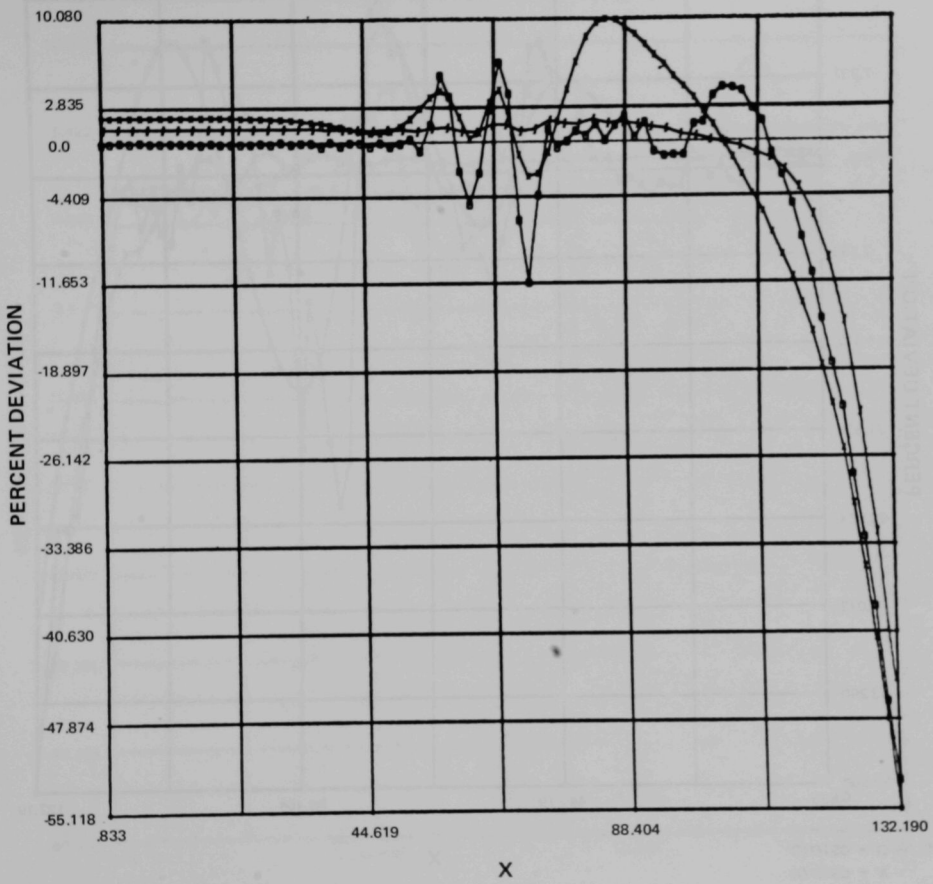
Note that the P_n-like discrete-ordinate computations give the smoothest fluxes. The VD calculations show a pronounced ray effect which is also visible in the ND calculations. The ray effect excites a spatial oscillation in the CS16ND fluxes.

EXHIBIT D Same plot as EXHIBIT C, except near y = 140. In this plot, the P_{19} solutions were extrapolated or interpolated to the y-value at which the approximate solution was given.

EXHIBIT E Same plot as EXHIBIT C, for group 2. The BS8FDP5 solution was obtained with a reflecting boundary condition on the right.

EXHIBIT C

Group 1 Relative Error at y = 80. Vacuum boundary condition used in all solutions.



□ = CS16ND
X = CS12VD
∫ = BS8FDP5

EXHIBIT D

Group 1 Relative Error near y = 140

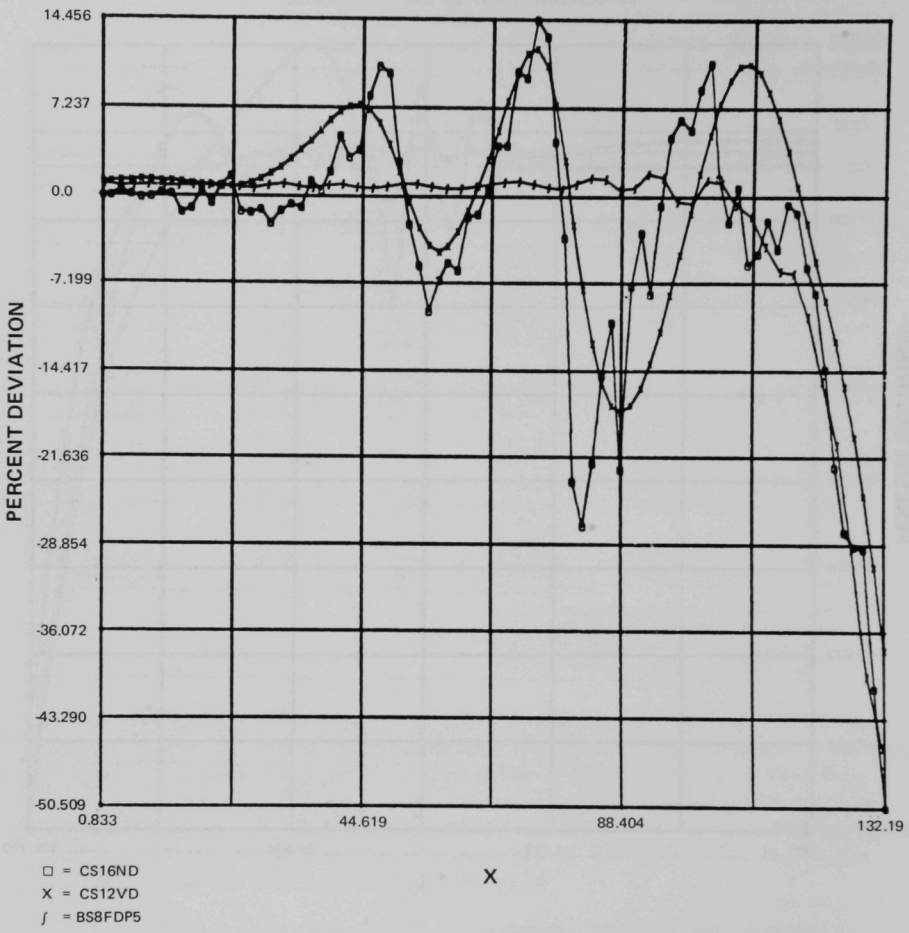
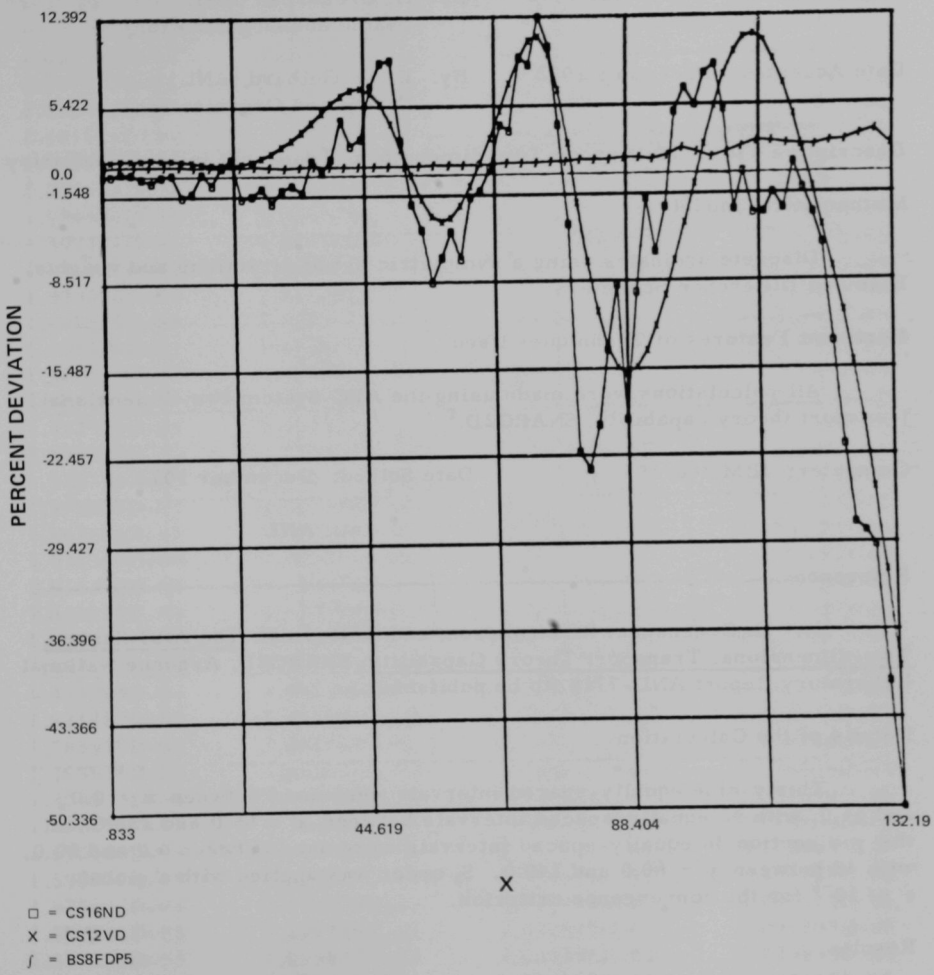


EXHIBIT E

Group 2 Relative Error near y = 140. Same plot as the group 1 plot of EXHIBIT D, except that BS8FDP5 used a reflecting boundary condition on the right.



BENCHMARK PROBLEM SOLUTION

Identification: 5-A2-3 Benchmark Problem ID.5-A2

Date Submitted: December 1972 By: H. Greenspan (ANL)
(Name and Organization)

Date Accepted: December 1972 By: E. M. Gelbard (ANL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (x,y) Geometry

Mathematical Model

Discrete ordinates using a symmetric set of directions and weights, Diamond Difference Scheme.

Pertinent Features of Techniques Used

All calculations were made using the ARC System two-dimensional transport theory capability, SNARC2D.[1]

Computer: IBM 360/195 Date Solved: December 1972

at: ANL

Reference

1. H. Greenspan, R. Thompson, and G. K. Leaf, The ARC System Two-dimensional Transport Theory Capability, SNARC2D, Argonne National Laboratory Report ANL-7718 (to be published).

Details of the Calculation

Thirty-nine equally-spaced intervals were used between x = 0.0 and 65.0, with 42 equally-spaced intervals between x = 65.0 and 133.0. In the y direction 36 equally-spaced intervals were used between 0.0 and 60.0, with 48 between y = 60.0 and 140.0. S_8 order was applied with a global ϵ of 10^{-6} for the convergence criterion.

Results

Execution time was 5.31 minutes.

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines 79.16667, 80.83333, the interpolated value at y = 80.0, and y = 139.16667 cm for x = 0.833(1.667)64.167 and 65.810(1.619)132.190.

EXHIBIT A

Group 1

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
2.001469D-03	1.627102D-03	1.814286D-03	3.549456D-06
2.001045D-03	1.626072D-03	1.813559D-03	3.654250D-06
2.000790D-03	1.627559D-03	1.814175D-03	3.545921D-06
2.001686D-03	1.623893D-03	1.812790D-03	3.493487D-06
1.999530D-03	1.626784D-03	1.813157D-03	3.549045D-06
2.001433D-03	1.621222D-03	1.811328D-03	3.540072D-06
1.996376D-03	1.624984D-03	1.810680D-03	3.572496D-06
1.999401D-03	1.619958D-03	1.809680D-03	3.549923D-06
1.993821D-03	1.620243D-03	1.807032D-03	3.427383D-06
1.993588D-03	1.620841D-03	1.807215D-03	3.491421D-06
1.990387D-03	1.617314D-03	1.803851D-03	3.500240D-06
1.988725D-03	1.619854D-03	1.804290D-03	3.461970D-06
1.988166D-03	1.614693D-03	1.801430D-03	3.636423D-06
1.984399D-03	1.614485D-03	1.799442D-03	3.489909D-06
1.987529D-03	1.606629D-03	1.797079D-03	3.399701D-06
1.978249D-03	1.597655D-03	1.787952D-03	3.524340D-06
1.974149D-03	1.593971D-03	1.784060D-03	3.482470D-06
1.959850D-03	1.580385D-03	1.770118D-03	3.399345D-06
1.943888D-03	1.583395D-03	1.763642D-03	3.400200D-06
1.938357D-03	1.565533D-03	1.751945D-03	3.374925D-06
1.916330D-03	1.547026D-03	1.731678D-03	3.358434D-06
1.896219D-03	1.537764D-03	1.716992D-03	3.349214D-06
1.875528D-03	1.531952D-03	1.703740D-03	3.387642D-06
1.877131D-03	1.531495D-03	1.704313D-03	3.395431D-06
1.873704D-03	1.492489D-03	1.683097D-03	3.181113D-06
1.821487D-03	1.455404D-03	1.638446D-03	2.906674D-06
1.785972D-03	1.443269D-03	1.614621D-03	2.658226D-06
1.755575D-03	1.413634D-03	1.584605D-03	2.378639D-06
1.709873D-03	1.372948D-03	1.541411D-03	2.252070D-06
1.666135D-03	1.350646D-03	1.508391D-03	2.177667D-06
1.630124D-03	1.318661D-03	1.474393D-03	2.123464D-06
1.595332D-03	1.297607D-03	1.446470D-03	2.101185D-06
1.578682D-03	1.280830D-03	1.429756D-03	2.039615D-06
1.508923D-03	1.202726D-03	1.355825D-03	2.095805D-06
1.351320D-03	1.065672D-03	1.208496D-03	2.023839D-06
1.186989D-03	9.437428D-04	1.065366D-03	1.937375D-06
1.085007D-03	8.751516D-04	9.800793D-04	2.012146D-06
1.038607D-03	8.424028D-04	9.405049D-04	1.902583D-06
1.013069D-03	8.231911D-04	9.181301D-04	1.918753D-06
9.925418D-04	8.060939D-04	8.993179D-04	1.867202D-06
9.658077D-04	7.897318D-04	8.777698D-04	1.832424D-06

EXHIBIT A (Contd.)

Group 1 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
9.248966D-04	7.593816D-04	8.421391D-04	1.843981D-06
8.336389D-04	6.949977D-04	7.643183D-04	1.780179D-06
6.771993D-04	5.805636D-04	6.288815D-04	1.784665D-06
5.155570D-04	4.449907D-04	4.802739D-04	1.756242D-06
4.342741D-04	3.593301D-04	3.968021D-04	1.710659D-06
4.060134D-04	3.354091D-04	3.707113D-04	1.694638D-06
3.771479D-04	3.126138D-04	3.448809D-04	1.634262D-06
3.586446D-04	2.877495D-04	3.155585D-04	1.622515D-06
3.060188D-04	2.724723D-04	2.892456D-04	1.537987D-06
2.498888D-04	2.299727D-04	2.399308D-04	1.394353D-06
2.250452D-04	1.878122D-04	2.064287D-04	1.136261D-06
2.064635D-04	1.679189D-04	1.871912D-04	7.646382D-07
1.787712D-04	1.540737D-04	1.664225D-04	6.566986D-07
1.550864D-04	1.341322D-04	1.446093D-04	5.869904D-07
1.265244D-04	1.202719D-04	1.233982D-04	4.994963D-07
1.019036D-04	1.016404D-04	1.017720D-04	4.383654D-07
8.479527D-05	8.209084D-05	8.344306D-05	3.865517D-07
6.898750D-05	6.731519D-05	6.815135D-05	3.693979D-07
5.823644D-05	5.392353D-05	5.607999D-05	3.423007D-07
4.807798D-05	4.516522D-05	4.662160D-05	3.452801D-07
3.979903D-05	3.725493D-05	3.852698D-05	3.284595D-07
3.262787D-05	3.071256D-05	3.167022D-05	3.035142D-07
2.732641D-05	2.525017D-05	2.628829D-05	2.916747D-07
2.253819D-05	2.113486D-05	2.183653D-05	2.826035D-07
1.873888D-05	1.764423D-05	1.819156D-05	2.667329D-07
1.516441D-05	1.459790D-05	1.488116D-05	2.572303D-07
1.264113D-05	1.228604D-05	1.246359D-05	2.505929D-07
1.073426D-05	1.001282D-05	1.037354D-05	2.424421D-07
9.403200D-06	8.415031D-06	8.909116D-06	2.348729D-07
7.201202D-06	7.090543D-06	7.145873D-06	2.284419D-07
6.339252D-06	5.798104D-06	6.068678D-06	2.191207D-07
5.385424D-06	4.863145D-06	5.124285D-06	2.123933D-07
4.605304D-06	4.120356D-06	4.362830D-06	2.071681D-07
3.882391D-06	3.537408D-06	3.709900D-06	1.989596D-07
3.341176D-06	2.940686D-06	3.140931D-06	1.893857D-07
2.756357D-06	2.510808D-06	2.633583D-06	1.717827D-07
2.172642D-06	2.111212D-06	2.141927D-06	1.445471D-07
1.926137D-06	1.779935D-06	1.853036D-06	8.388830D-08
1.796682D-06	1.463748D-06	1.630215D-06	6.762190D-08
1.604686D-06	1.276550D-06	1.440618D-06	6.323535D-08

EXHIBIT A (Contd.)

Group 2

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
4.722835D-03	3.800795D-03	4.261815D-03	6.147741D-06
4.722008D-03	3.798112D-03	4.260060D-03	6.336581D-06
4.720963D-03	3.802234D-03	4.261599D-03	6.134945D-06
4.723245D-03	3.793179D-03	4.258212D-03	6.022309D-06
4.717451D-03	3.801213D-03	4.259332D-03	6.127439D-06
4.722723D-03	3.787879D-03	4.255301D-03	6.111912D-06
4.710109D-03	3.798440D-03	4.254275D-03	6.120777D-06
4.718715D-03	3.787083D-03	4.252899D-03	6.066946D-06
4.705956D-03	3.788994D-03	4.247475D-03	5.833043D-06
4.707511D-03	3.790951D-03	4.249231D-03	5.966277D-06
4.701658D-03	3.781863D-03	4.241761D-03	6.168332D-06
4.699014D-03	3.786929D-03	4.242972D-03	5.926314D-06
4.698184D-03	3.772772D-03	4.235478D-03	5.941656D-06
4.686980D-03	3.772096D-03	4.229538D-03	6.036126D-06
4.691996D-03	3.756507D-03	4.224252D-03	5.957926D-06
4.667578D-03	3.741130D-03	4.204354D-03	5.917844D-06
4.661010D-03	3.739753D-03	4.200382D-03	5.790044D-06
4.635838D-03	3.708900D-03	4.172369D-03	5.857716D-06
4.606609D-03	3.715898D-03	4.161254D-03	6.009557D-06
4.601224D-03	3.674378D-03	4.137801D-03	5.893730D-06
4.559772D-03	3.645991D-03	4.102882D-03	5.749774D-06
4.544995D-03	3.603774D-03	4.074137D-03	5.648509D-06
4.467766D-03	3.532600D-03	4.000183D-03	5.765124D-06
4.385755D-03	3.563243D-03	3.974499D-03	5.770130D-06
4.387396D-03	3.523170D-03	3.955283D-03	5.307046D-06
4.324480D-03	3.440363D-03	3.882422D-03	5.059443D-06
4.265568D-03	3.394748D-03	3.830158D-03	4.707374D-06
4.161410D-03	3.320955D-03	3.741183D-03	4.252585D-06
4.052533D-03	3.236180D-03	3.644357D-03	3.988442D-06
3.980207D-03	3.172927D-03	3.576567D-03	3.883074D-06
3.871506D-03	3.099662D-03	3.485584D-03	3.702709D-06
3.783899D-03	3.043659D-03	3.413779D-03	3.748959D-06
3.741417D-03	3.012601D-03	3.377009D-03	3.549974D-06
3.591609D-03	2.834170D-03	3.212890D-03	3.574309D-06
3.225521D-03	2.522016D-03	2.873769D-03	3.453220D-06
2.840629D-03	2.235161D-03	2.537895D-03	3.480971D-06
2.592631D-03	2.066371D-03	2.329501D-03	3.445005D-06
2.467451D-03	1.983539D-03	2.225495D-03	3.258541D-06
2.396330D-03	1.927510D-03	2.161920D-03	3.298288D-06
2.335331D-03	1.877828D-03	2.106580D-03	3.177022D-06
2.259112D-03	1.830596D-03	2.044854D-03	3.093347D-06

EXHIBIT A (Contd.)

Group 2 (Contd.)

y = 79.16667	y = 80.83333	y = 80.0[a]	y = 139.16667
2.151677D-03	1.750131D-03	1.951494D-03	3.125055D-06
1.929578D-03	1.592082D-03	1.760830D-03	3.000364D-06
1.559622D-03	1.324337D-03	1.441980D-03	3.018329D-06
1.185520D-03	1.012993D-03	1.099257D-03	2.932069D-06
1.003793D-03	8.201179D-04	9.119555D-04	2.847648D-06
9.430083D-04	7.709776D-04	8.569930D-04	2.803006D-06
8.561989D-04	7.166892D-04	7.864441D-04	2.682276D-06
8.186897D-04	6.536832D-04	7.361865D-04	2.622296D-06
6.987154D-04	6.198329D-04	6.592742D-04	2.503798D-06
5.624091D-04	5.129801D-04	5.376946D-04	2.294163D-06
5.040688D-04	4.119667D-04	4.580178D-04	1.811259D-06
4.618966D-04	3.700326D-04	4.159646D-04	1.333693D-06
3.916943D-04	3.405514D-04	3.661229D-04	1.202928D-06
3.394085D-04	2.932947D-04	3.163516D-04	9.493772D-07
2.751051D-04	2.607647D-04	2.679349D-04	9.016316D-07
2.214743D-04	2.154423D-04	2.184583D-04	7.304310D-07
1.836098D-04	1.752320D-04	1.794209D-04	7.011084D-07
1.473154D-04	1.431899D-04	1.452327D-04	6.469371D-07
1.230125D-04	1.144255D-04	1.187190D-04	5.847411D-07
9.892470D-05	9.668825D-05	9.780648D-05	5.650618D-07
8.260762D-05	7.780630D-05	8.020696D-05	5.450379D-07
6.791698D-05	6.286702D-05	6.539200D-05	5.204371D-07
5.642885D-05	5.112834D-05	5.377860D-05	4.927532D-07
4.604554D-05	4.337735D-05	4.471145D-05	4.521012D-07
3.702753D-05	3.582055D-05	3.642404D-05	4.497325D-07
3.100025D-05	2.893029D-05	2.996527D-05	4.114592D-07
2.655523D-05	2.431917D-05	2.543720D-05	4.109258D-07
2.201963D-05	2.026584D-05	2.114274D-05	3.719022D-07
1.865951D-05	1.702136D-05	1.784044D-05	3.606390D-07
1.451499D-05	1.408056D-05	1.429778D-05	3.566153D-07
1.214741D-05	1.141206D-05	1.177974D-05	3.285397D-07
1.049411D-05	9.828743D-06	1.016143D-05	3.263041D-07
8.977187D-06	8.226479D-06	8.601833D-06	3.218885D-07
7.513291D-06	6.720978D-06	7.117135D-06	2.926214D-07
6.428657D-06	5.655577D-06	6.042117D-06	2.770520D-07
5.299891D-06	4.838325D-06	5.069108D-06	2.616916D-07
4.076739D-06	4.220389D-06	4.148564D-06	1.959424D-07
3.401178D-06	3.381979D-06	3.391579D-06	1.119969D-07
3.204541D-06	2.768785D-06	2.986663D-06	1.004464D-07
2.667998D-06	2.327367D-06	2.497683D-06	9.346458D-08

BENCHMARK PROBLEM

Identification: 5-B1 Source Situation ID.5

Date Submitted: December 1970 By: B. Crawford (KAPL)
(Name and Organization)

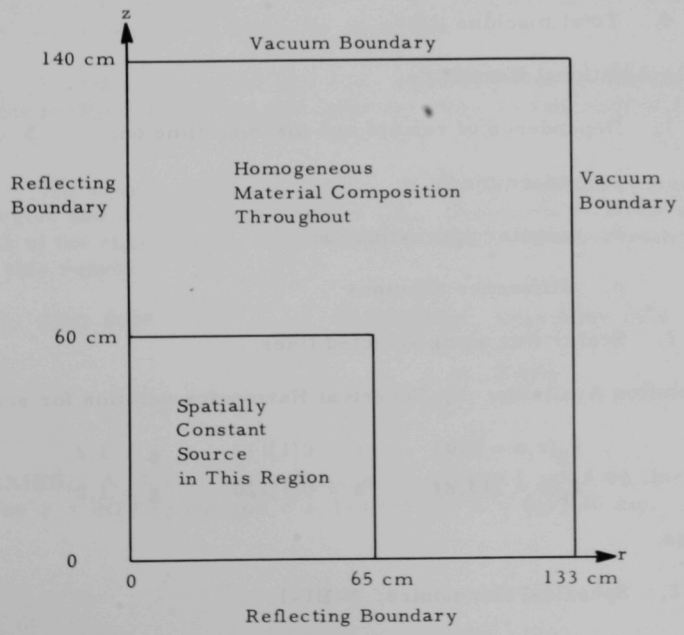
Date Accepted: December 1970 By: K. Lathrop (LASL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport

Reduction of Source Problem

1. Multigroup approximation made
2. Isotropic scattering assumed
3. r,z geometry
4. Boundary conditions as shown

Configuration



Data

Isotropic cross sections (cm^{-1}) and source density (neutrons/cm^3)

	Group 1	Group 2
Σ_a	0.061723	0.096027
$\nu\Sigma_f$	0.0	0.0
Σ_t	0.092104	0.100877
$\Sigma_{s0}^{g\to g}$	0.006947	0.004850
$\Sigma_{s0}^{g-1\to g}$	0.0	0.023434
Source density	0.006546	0.017701

Expected Primary Results

1. Scalar flux in each group along horizontal and vertical lines
2. Total right and top leakage in each group
3. Number of iterations in each group
4. Total machine time

Possible Additional Results

1. Dependence of results and machine time on:
   a. space mesh
   b. angular approximation
   c. difference schemes
2. Scalar flux along selected lines

Best Solution Available: P_{19} Spherical Harmonics solution for scalar flux

$\Phi_g(r,z = 120)$ $\quad r = 0(1)113 \quad g = 1,2$

$\Phi_g(r = 113,z)$ $\quad z = 0(1)120 \quad g = 1,2$

Solutions

1. Spherical Harmonics: 5-B1-1
2. Discrete Ordinates: 5-B1-2, 5-B1-3

BENCHMARK PROBLEM SOLUTION

Identification: 5-B1-1 Benchmark Problem ID.5-B1

Date Submitted: December 1970 By: B. Crawford (KAPL)
(Name and Organization)

Date Accepted: December 1970 By: K. D. Lathrop (LASL)
(Name and Organization)

Descriptive Title: Two-group Two-dimensional Transport in (r,z) Geometry

Mathematical Model

Green's functions are calculated from a spherical harmonics (P_{19}) approximation. The solution at each point of interest is obtained by integration over source regions.

Pertinent Features of Solution Method

1. Each group-kernel is tabulated as a function of the mean free path of group 1 with a table spacing of 0.01. Linear interpolation is performed in this table given the distance from the flux point to the source point. Trapezoidal integration of source point contributions is carried out over each source domain to obtain the total flux.

2. Contributions to the flux are computed from the source region given in the problem statement and from the image of this source region about z = 0.

3. The source geometry implies an infinite medium around the source region and its reflection about z = 0. Fluxes within a few mean free paths of the right and top boundaries have been excluded from the results for this reason.

Computer: CDC-6600 Date Solved: December 1970

at: KAPL

Results

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines z = 120 cm for r = 0(1)113 cm and r = 113 cm for z = 0(1)120 cm.

EXHIBIT A

EXACT SOLUTION Z=120 GRP 1 PHI(R,120) P0 RZ

.22794E-04	.22792E-04	.22785E-04	.22773E-04	.22757E-04	.22736E-04
.22710E-04	.22679E-04	.22644E-04	.22603E-04	.22558E-04	.22507E-04
.22451E-04	.22390E-04	.22323E-04	.22251E-04	.22173E-04	.22090E-04
.22000E-04	.21904E-04	.21802E-04	.21694E-04	.21579E-04	.21458E-04
.21330E-04	.21195E-04	.21053E-04	.20903E-04	.20747E-04	.20582E-04
.20411E-04	.20232E-04	.20045E-04	.19850E-04	.19647E-04	.19436E-04
.19218E-04	.18991E-04	.18757E-04	.18515E-04	.18265E-04	.18007E-04
.17741E-04	.17468E-04	.17188E-04	.16900E-04	.16605E-04	.16304E-04
.15997E-04	.15683E-04	.15364E-04	.15039E-04	.14709E-04	.14375E-04
.14036E-04	.13694E-04	.13349E-04	.13001E-04	.12651E-04	.12299E-04
.11946E-04	.11593E-04	.11239E-04	.10887E-04	.10535E-04	.10185E-04
.98373E-05	.94925E-05	.91509E-05	.88131E-05	.84796E-04	.81509E-05
.78274E-05	.75095E-05	.71976E-05	.68920E-05	.65932E-05	.63013E-05
.60167E-05	.57395E-05	.54701E-05	.52085E-05	.49548E-05	.47093E-05
.44720E-05	.42428E-05	.40219E-05	.38092E-05	.36047E-05	.34083E-05
.32200E-05	.30396E-05	.28670E-05	.27020E-05	.25446E-05	.23946E-05
.22517E-05	.21158E-05	.19866E-05	.18640E-05	.17478E-05	.16377E-05
.15336E-05	.14351E-05	.13421E-05	.12543E-05	.11716E-05	.10936E-05
.10203E-05	.95132E-06	.88652E-06	.82569E-06	.76861E-06	.71512E-06

EXACT SOLUTION Z=120 GRP 2 PHI(R,120) P0 RZ

.41873E-04	.41869E-04	.41857E-04	.41837E-04	.41809E-04	.41772E-04
.41728E-04	.41675E-04	.41613E-04	.41543E-04	.41464E-04	.41376E-04
.41279E-04	.41172E-04	.41056E-04	.40929E-04	.40793E-04	.40646E-04
.40489E-04	.40321E-04	.40141E-04	.39950E-04	.39747E-04	.39531E-04
.39303E-04	.39063E-04	.38809E-04	.38542E-04	.38261E-04	.37966E-04
.37657E-04	.37334E-04	.36995E-04	.36642E-04	.36274E-04	.35891E-04
.35492E-04	.35078E-04	.34649E-04	.34205E-04	.33745E-04	.33270E-04
.32780E-04	.32275E-04	.31756E-04	.31223E-04	.30676E-04	.30116E-04
.29543E-04	.28958E-04	.28361E-04	.27754E-04	.27136E-04	.26510E-04
.25875E-04	.25232E-04	.24584E-04	.23929E-04	.23270E-04	.22608E-04
.21944E-04	.21278E-04	.20612E-04	.19947E-04	.19284E-04	.18625E-04
.17969E-04	.17319E-04	.16676E-04	.16040E-04	.15412E-04	.14794E-04
.14186E-04	.13588E-04	.13003E-04	.12430E-04	.11870E-04	.11324E-04
.10792E-04	.10275E-04	.97729E-05	.92861E-05	.88150E-05	.83596E-05
.79201D-05	.74967E-05	.70892E-05	.66977E-05	.63221E-05	.59621E-05
.56177E-05	.52885E-05	.49743E-05	.46749E-05	.43898E-05	.41188E-05
.38614E-05	.36172E-05	.33858E-05	.31669E-05	.29599E-05	.27644E-05
.25800E-05	.24062E-05	.22426E-05	.20887E-05	.19441E-05	.18084E-05
.16811E-05	.15618E-05	.14500E-05	.13455E-05	.12479E-05	.11566E-05

EXHIBIT A (continued)

EXACT SOLUTION R=113 GRP 1 PHI(113,Z) P0 RZ

.59910E-04	.59906E-04	.59892E-04	.59870E-04	.59838E-04	.59797E-04
.59746E-04	.59686E-04	.59615E-04	.59535E-04	.59443E-04	.59341E-04
.59228E-04	.59102E-04	.58964E-04	.58814E-04	.58650E-04	.58472E-04
.58280E-04	.58072E-04	.57849E-04	.57609E-04	.57352E-04	.57077E-04
.56783E-04	.56469E-04	.56136E-04	.55782E-04	.55406E-04	.55007E-04
.54586E-04	.54141E-04	.53671E-04	.53177E-04	.52657E-04	.52110E-04
.51538E-04	.50939E-04	.50312E-04	.49659E-04	.48978E-04	.48270E-04
.47535E-04	.46773E-04	.45985E-04	.45171E-04	.44332E-04	.43469E-04
.42583E-04	.41674E-04	.40745E-04	.39796E-04	.38829E-04	.37846E-04
.36848E-04	.35838E-04	.34816E-04	.33787E-04	.32750E-04	.31710E-04
.30667E-04	.29624E-04	.28583E-04	.27546E-04	.26516E-04	.25495E-04
.24484E-04	.23486E-04	.22502E-04	.21535E-04	.20585E-04	.19655E-04
.18746E-04	.17858E-04	.16994E-04	.16154E-04	.15339E-04	.14549E-04
.13786E-04	.13049E-04	.12339E-04	.11656E-04	.11000E-04	.10371E-04
.97693E-05	.91937E-05	.86443E-05	.81206E-05	.76222E-05	.71483E-05
.66985E-05	.62720E-05	.58681E-05	.54860E-05	.51251E-05	.47845E-05
.44635E-05	.41612E-05	.38768E-05	.36095E-05	.33586E-05	.31233E-05
.29027E-05	.26962E-05	.25030E-05	.23224E-05	.21537E-05	.19962E-05
.18494E-05	.17125E-05	.15851E-05	.14664E-05	.13561E-05	.12535E-05
.11583E-05	.10698E-05	.98771E-06	.91159E-06	.84102E-06	.77565E-06
.71512E-06					

EXACT SOLUTION R=113 GRP 2 PHI(113,Z) P0 RZ

.11802E-03	.11802E-03	.11799E-03	.11795E-03	.11789E-03	.11782E-03
.11773E-03	.11762E-03	.11749E-03	.11734E-03	.11717E-03	.11699E-03
.11678E-03	.11655E-03	.11630E-03	.11602E-03	.11571E-03	.11538E-03
.11503E-03	.11464E-03	.11422E-03	.11377E-03	.11329E-03	.11277E-03
.11221E-03	.11162E-03	.11098E-03	.11030E-03	.10958E-03	.10882E-03
.10801E-03	.10714E-03	.10623E-03	.10527E-03	.10426E-03	.10319E-03
.10207E-03	.10089E-03	.99660E-04	.98370E-04	.97023E-04	.95619E-04
.94159E-04	.92642E-04	.91070E-04	.89443E-04	.87764E-04	.86033E-04
.84253E-04	.82426E-04	.80554E-04	.78641E-04	.76689E-04	.74703E-04
.72686E-04	.70641E-04	.68574E-04	.66488E-04	.64388E-04	.62278E-04
.60164E-04	.58050E-04	.55940E-04	.53840E-04	.51754E-04	.49686E-04
.47641E-04	.45623E-04	.43636E-04	.41684E-04	.39770E-04	.37897E-04
.36068E-04	.34287E-04	.32555E-04	.30873E-04	.29245E-04	.27671E-04
.26152E-04	.24689E-04	.23282E-04	.21933E-04	.20639E-04	.19402E-04
.18221E-04	.17095E-04	.16023E-04	.15004E-04	.14037E-04	.13120E-04
.12253E-04	.11433E-04	.10659E-04	.99298E-05	.92429E-05	.85970E-05
.79902E-05	.74208E-05	.68872E-05	.63876E-05	.59202E-05	.54835E-05
.50759E-05	.46957E-05	.43413E-05	.40115E-05	.37046E-05	.34194E-05
.31544E-05	.29085E-05	.26805E-05	.24692E-05	.22734E-05	.20923E-05
.19247E-05	.17698E-05	.16267E-05	.14946E-05	.13727E-05	.12603E-05
.11566E-05					

BENCHMARK PROBLEM SOLUTION

Identification: 5-B1-2 Benchmark Problem ID.5-B1

Date Submitted: October 1971 By: K. D. Lathrop (LASL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (r,z) Geometry

Mathematical Model

Discrete Ordinates

Pertinent Features of Techniques Used

Los Alamos calculations were performed with the TWOTRAN program[1] or modifications thereof.

Computer: CDC-6600 and 7600 Date Solved: January and September 1971

at: LASL

Reference

1. K. D. Lathrop and F. W. Brinkley, Code Abstract: General-Geometry TWOTRAN, Nucl. Sci. Eng. 44, 276 (1971).

Details of the Calculation

Three different spatial meshes were used. In each case, a equally-spaced intervals between 0.0 and 65.0 and b equally-spaced intervals between 65.0 and 133.0 in r, and c equally-spaced intervals between 0 and 60.0 and d equally-spaced intervals between 60.0 and 140.0 in z were used. The three meshes are given by

	a	b	c	d	Total Number of Cells
A	13	14	12	16	756
B	26	28	24	32	3024
C	39	42	36	48	6804

Two different S_n orders were used and two types of calculations done. Normal diamond (ND) and variable-diamond (VD) difference schemes were

used. The calculations are labeled by mesh, S_n order, and difference scheme. For example, BS12VD is a B mesh, order S_{12}, variable-diamond scheme.

Results

The relative percentage error with respect to the P_{19} infinite-medium solution is given. In addition to the Los Alamos CS12VD solution, a Bettis Atomic Power Laboratory discrete ordinate solution and a Knolls Atomic Power Laboratory solution are compared with the P_{19} solution.

EXHIBIT A The group 1 radial error traverse near $z = 120$ cm.

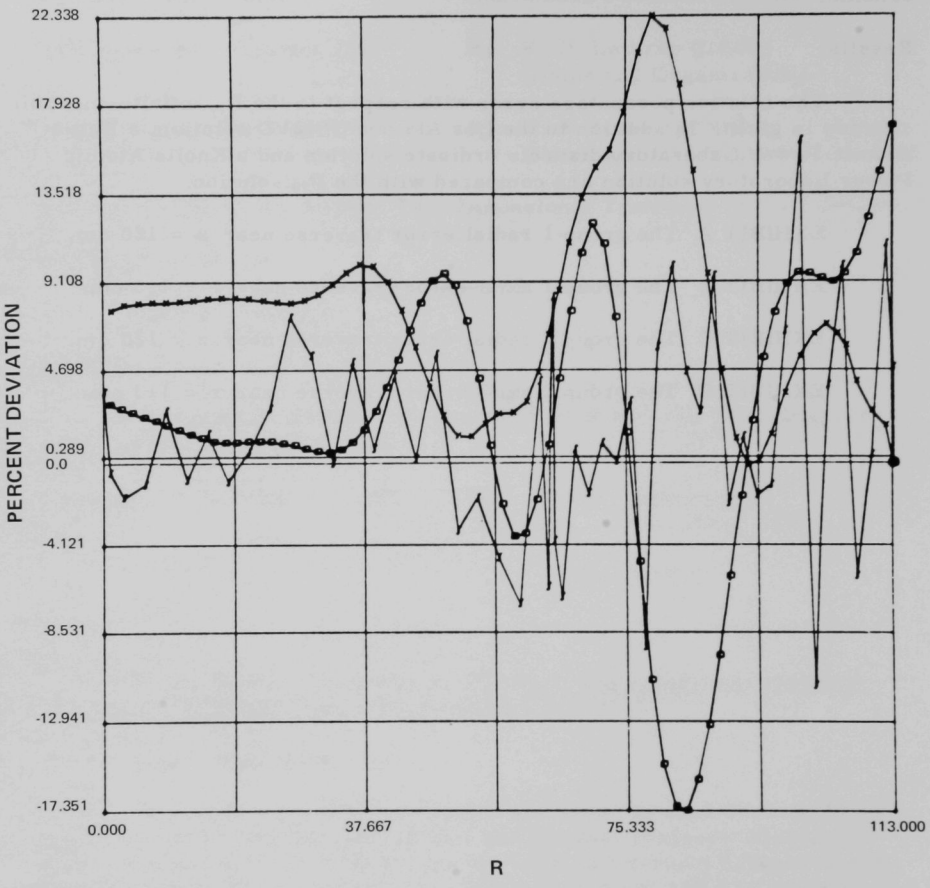
EXHIBIT B The group 1 axial error traverse near $r = 113$ cm.

EXHIBIT C The group 2 radial error traverse near $z = 120$ cm.

EXHIBIT D The group 2 axial error traverse near $r = 113$ cm.

EXHIBIT A

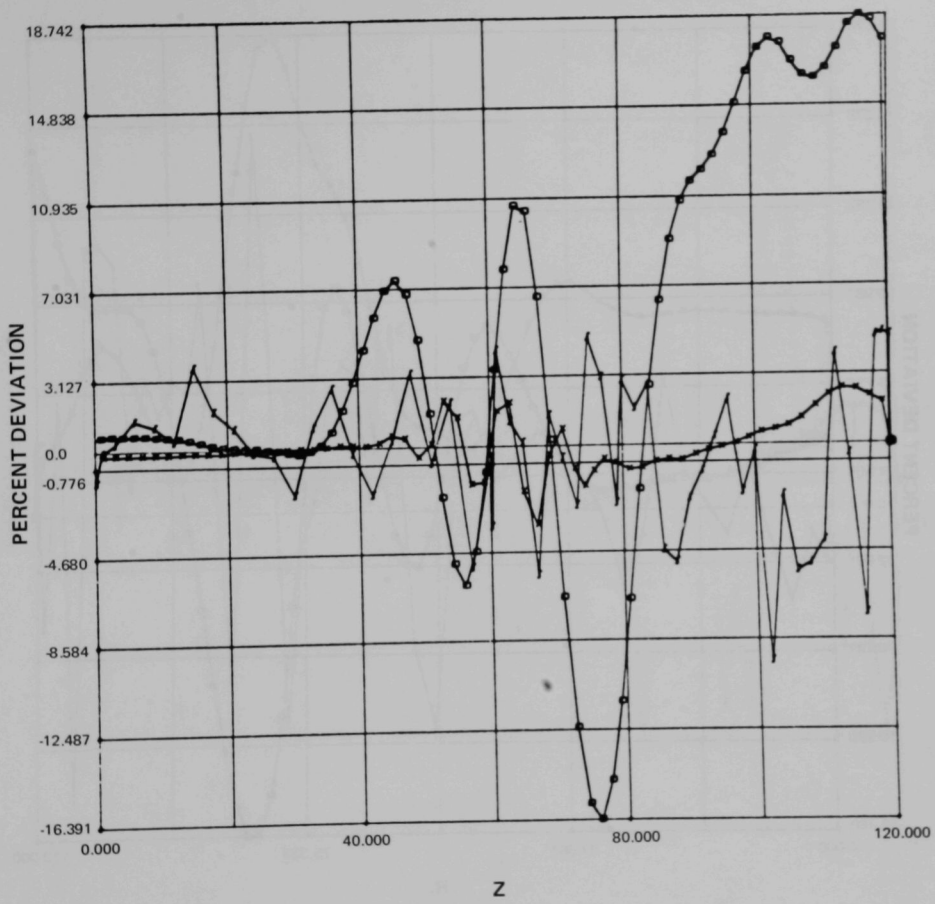
Group 1 Radial Error Traverse near z = 120 cm



□ = LASL CS12VD
X = BETTIS
∫ = KAPL MESH 62X66

EXHIBIT B

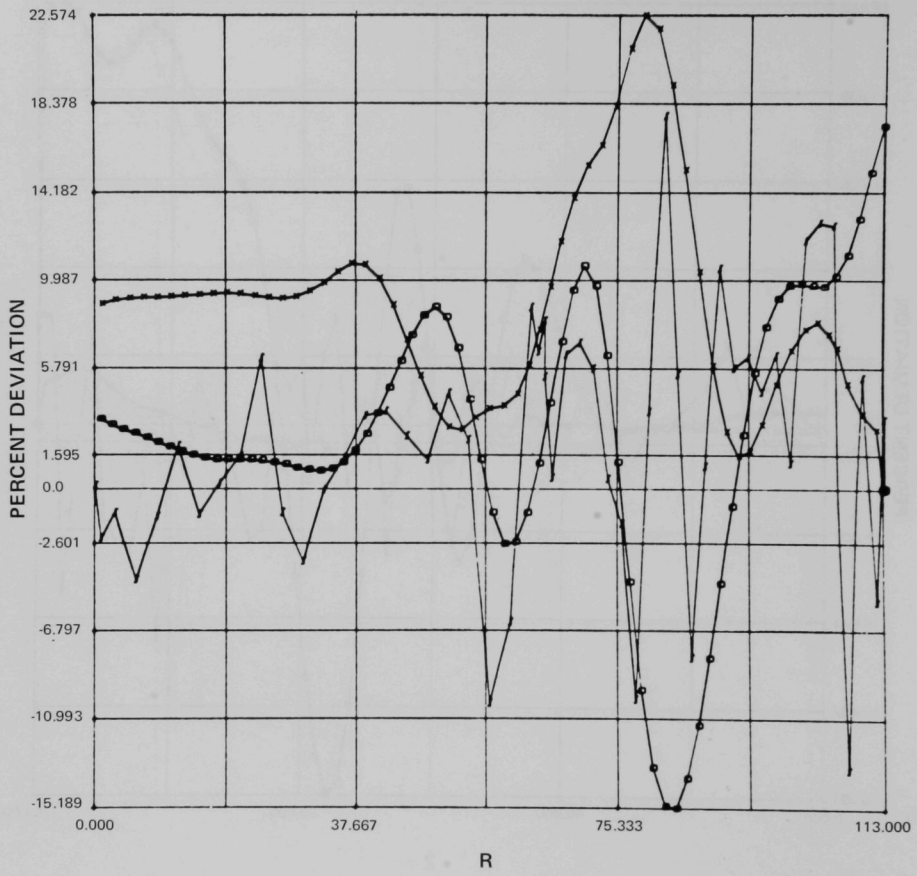
Group 1 Axial Error Traverse near r = 113 cm



□ = LASL CS12VD
X = BETTIS
∫ = KAPL MESH 62X66

EXHIBIT C

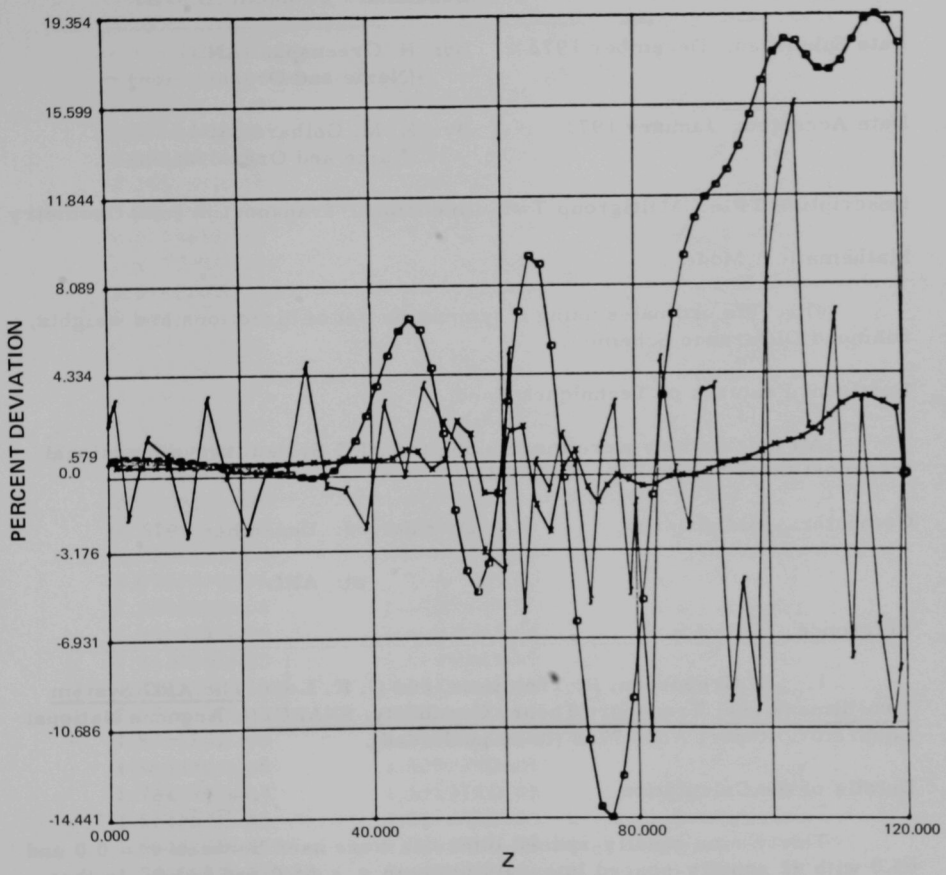
Group 2 Radial Error Traverse near z = 120 cm



□ = LASL CS12VD
X = BETTIS
∫ = KAPL MESH 62X66

EXHIBIT D

Group 2 Axial Error Traverse near r = 113 cm



□ = LASL CS12VD
X = BETTIS
∫ = KAPL MESH 62X66

md-2

BENCHMARK PROBLEM SOLUTION

Identification: 5-B1-3 Benchmark Problem ID.5-B1

Date Submitted: December 1972 By: H. Greenspan (ANL)
(Name and Organization)

Date Accepted: January 1973 By: E. M. Gelbard (ANL)
(Name and Organization)

Descriptive Title: Multigroup Two-dimensional Transport in (r,z) Geometry

Mathematical Model

Discrete ordinates using a symmetric set of directions and weights, Diamond Difference Scheme.

Pertinent Features of Techniques Used

All calculations were made using the ARC System two-dimensional transport theory capability, SNARC2D.[1]

Computer: IBM 360/195 Date Solved: December 1972

at: ANL

Reference

1. H. Greenspan, R. Thompson, and G. K. Leaf, The ARC System Two-dimensional Transport Theory Capability, SNARC2D, Argonne National Laboratory Report ANL-7718 (to be published).

Details of the Calculation

Thirty-nine equally-spaced intervals were used between r = 0.0 and 65.0 with 42 equally-spaced intervals between r = 65.0 and 133.0. In the z direction 36 equally-spaced intervals were used between 0.0 and 60.0 with 48 between z = 60.0 and 140.0. S_{12} order was applied with a global ϵ of 10^{-6} for the convergence criterion.

Results

Execution time was 11.35 minutes.

EXHIBIT A Scalar flux solutions for groups 1 and 2 on lines z = 119.167, 120.833, and the interpolated value at z = 120.0. The values are given for r = 0.833(1.667)64.167 and 65.810(1.619)132.190.

EXHIBIT A

Group 1

z = 119.16667	z = 120.83333	z = 120.0[a]
2.491060D-05	2.055263D-05	2.273162D-05
2.458020D-05	2.056456D-05	2.257238D-05
2.331381D-05	1.978747D-05	2.155064D-05
2.355154D-05	1.990274D-05	2.172714D-05
2.356846D-05	2.010267D-05	2.183557D-05
2.412991D-05	1.980581D-05	2.196786D-05
2.367298D-05	1.969566D-05	2.168432D-05
2.317196D-05	1.927473D-05	2.122335D-05
2.304615D-05	1.932672D-05	2.118644D-05
2.276916D-05	1.941803D-05	2.109360D-05
2.269952D-05	1.893046D-05	2.081499D-05
2.279314D-05	1.899790D-05	2.089552D-05
2.226052D-05	1.890167D-05	2.058110D-05
2.221103D-05	1.858822D-05	2.039963D-05
2.243945D-05	1.893005D-05	2.068475D-05
2.185955D-05	1.833180D-05	2.009568D-05
2.172468D-05	1.776740D-05	1.974604D-05
2.078744D-05	1.777527D-05	1.928136D-05
2.071752D-05	1.672224D-05	1.871988D-05
1.978203D-05	1.688091D-05	1.833147D-05
1.961900D-05	1.638718D-05	1.800309D-05
1.957436D-05	1.636986D-05	1.797211D-05
1.882338D-05	1.597891D-05	1.740115D-05
1.899921D-05	1.579958D-05	1.739940D-05
1.871164D-05	1.607931D-05	1.739548D-05
1.863430D-05	1.525079D-05	1.694255D-05
1.784744D-05	1.531913D-05	1.658329D-05
1.814104D-05	1.511291D-05	1.662698D-05
1.804423D-05	1.535331D-05	1.669877D-05
1.749694D-05	1.444823D-05	1.597259D-05
1.586650D-05	1.315017D-05	1.450834D-05
1.430971D-05	1.175826D-05	1.303399D-05
1.300548D-05	1.086936D-05	1.193742D-05
1.233093D-05	1.030136D-05	1.131615D-05
1.182204D-05	9.990604D-06	1.090632D-05
1.150493D-05	9.672094D-06	1.058851D-05
1.132055D-05	9.495794D-06	1.040817D-05
1.091163D-05	9.226801D-06	1.006922D-05
1.091133D-05	9.118818D-06	1.001507D-05
1.042698D-05	8.844472D-06	9.635726D-06

[a]Interpolated values.

EXHIBIT A (Contd.)

Group 1 (Contd.)

z = 119.16667	z = 120.83333	z = 120.0[a]
1.040646D-05	8.643894D-06	9.525177D-06
9.999392D-06	8.534492D-06	9.266942D-06
9.915618D-06	8.288546D-06	9.102082D-06
9.568201D-06	8.244851D-06	8.906526D-06
9.284359D-06	7.754369D-06	8.519364D-06
8.588659D-06	7.414854D-06	8.001757D-06
7.284991D-06	6.374865D-06	6.829928D-06
5.323357D-06	4.655490D-06	4.989424D-06
4.883058D-06	4.137845D-06	4.510452D-06
4.344502D-06	3.768569D-06	4.056536D-06
4.187243D-06	3.516081D-06	3.851662D-06
3.952069D-06	3.211843D-06	3.581956D-06
3.747242D-06	3.161833D-06	3.454538D-06
3.605115D-06	3.044103D-06	3.324609D-06
3.423133D-06	2.902839D-06	3.162986D-06
3.213847D-06	2.808525D-06	3.011186D-06
3.155513D-06	2.577120D-06	2.866317D-06
3.044316D-06	2.570249D-06	2.807283D-06
2.824380D-06	2.448491D-06	2.636436D-06
2.568662D-06	2.260916D-06	2.414789D-06
2.239994D-06	2.001540D-06	2.120767D-06
1.847881D-06	1.680015D-06	1.763948D-06
1.396702D-06	1.373806D-06	1.385254D-06
1.243981D-06	1.037993D-06	1.140987D-06
1.263334D-06	9.551035D-07	1.109219D-06
1.170802D-06	9.810568D-07	1.075929D-06
1.046039D-06	9.375236D-07	9.917813D-07
9.546569D-07	8.135953D-07	8.841261D-07
9.445554D-07	7.853456D-07	8.649505D-07
9.194017D-07	7.470554D-07	8.332286D-07
8.224097D-07	7.314186D-07	7.769142D-07
5.844626D-07	6.588601D-07	6.216614D-07
4.252916D-07	4.345718D-07	4.299317D-07
3.955206D-07	3.253622D-07	3.604414D-07
3.392006D-07	3.176826D-07	3.284416D-07
3.581693D-07	2.679897D-07	3.130795D-07
2.539491D-07	3.121011D-07	1.799489D-07
1.731778D-07	1.890131D-07	1.810955D-07
2.665867D-07	1.668179D-07	2.167023D-07
2.793622D-07	1.992124D-07	2.394223D-07
2.339988D-07	2.094527D-07	2.217258D-07

EXHIBIT A (Contd.)

Group 2

z = 119.16667	z = 120.83333	z = 120.0[a]
4.410663D-05	3.798102D-05	4.104383D-05
4.538022D-05	3.645401D-05	4.091712D-05
4.304431D-05	3.626111D-05	3.965271D-05
4.402604D-05	3.607823D-05	4.005214D-05
4.299722D-05	3.689225D-05	3.994474D-05
4.318618D-05	3.566646D-05	3.942632D-05
4.340710D-05	3.558324D-05	3.949517D-05
4.249726D-05	3.532345D-05	3.891036D-05
4.269174D-05	3.515580D-05	3.892377D-05
4.186430D-05	3.539284D-05	3.862857D-05
4.170778D-05	3.434279D-05	3.802529D-05
4.201017D-05	3.450067D-05	3.825542D-05
4.121006D-05	3.462503D-05	3.791754D-05
4.090062D-05	3.368237D-05	3.729150D-05
4.161759D-05	3.467093D-05	3.814426D-05
4.002208D-05	3.363442D-05	3.682825D-05
4.008278D-05	3.212677D-05	3.610478D-05
3.827020D-05	3.271314D-05	3.549167D-05
3.706118D-05	3.029887D-05	3.368003D-05
3.727100D-05	3.031911D-05	3.379506D-05
3.722807D-05	3.089365D-05	3.406086D-05
3.564761D-05	3.013134D-05	3.288948D-05
3.506603D-05	2.874201D-05	3.190402D-05
3.505289D-05	2.925014D-05	3.214252D-05
3.431583D-05	2.907100D-05	3.169342D-05
3.436568D-05	2.782795D-05	3.109682D-05
3.272311D-05	2.821878D-05	3.047095D-05
3.331048D-05	2.721528D-05	3.026288D-05
3.277273D-05	2.778914D-05	3.028094D-05
3.199469D-05	2.604971D-05	2.902220D-05
2.916286D-05	2.406676D-05	2.661481D-05
2.669169D-05	2.155605D-05	2.412387D-05
2.440117D-05	2.015316D-05	2.227717D-05
2.315995D-05	1.899166D-05	2.107581D-05
2.214921D-05	1.841277D-05	2.028099D-05
2.134530D-05	1.781004D-05	1.957767D-05
2.101024D-05	1.743696D-05	1.922360D-05
1.992733D-05	1.683135D-05	1.837934D-05
1.995905D-05	1.637796D-05	1.816851D-05
1.883459D-05	1.599375D-05	1.746417D-05
1.887266D-05	1.546017D-05	1.716642D-05

EXHIBIT A (Contd.)

Group 2 (Contd.)

z = 119.16667	z = 120.83333	z = 120.0[a]
1.798364D-05	1.508605D-05	1.653485D-05
1.799339D-05	1.478543D-05	1.638941D-05
1.692343D-05	1.435276D-05	1.563810D-05
1.635448D-05	1.358611D-05	1.497030D-05
1.522392D-05	1.288531D-05	1.405487D-05
1.279173D-05	1.107085D-05	1.193129D-05
9.888080D-06	8.398830D-06	9.143455D-06
8.087922D-06	7.424005D-06	8.115964D-06
7.981456D-06	6.774025D-06	7.377741D-06
7.597172D-06	6.378555D-06	6.987864D-06
7.030327D-06	5.842807D-06	6.436567D-06
6.570342D-06	5.574242D-06	6.072292D-06
6.320202D-06	5.358819D-06	5.839511D-06
5.977710D-06	4.961667D-06	5.469689D-06
5.785563D-06	4.681286D-06	5.233425D-06
5.569232D-06	4.632202D-06	5.100717D-06
5.039212D-06	4.382846D-06	4.711029D-06
4.728353D-06	4.021727D-06	4.375040D-06
4.254767D-06	3.753492D-06	4.004130D-06
3.756650D-06	3.241959D-06	3.499305D-06
3.180167D-06	2.781371D-06	2.980769D-06
2.512095D-06	2.286855D-06	2.399475D-06
2.067392D-06	1.843608D-06	1.955500D-06
1.980868D-06	1.588251D-06	1.784560D-06
1.969399D-06	1.599122D-06	1.784261D-06
1.670250D-06	1.547033D-06	1.608642D-06
1.519302D-06	1.290086D-06	1.404694D-06
1.565119D-06	1.201340D-06	1.383230D-06
1.407819D-06	1.208864D-06	1.308342D-06
1.075634D-06	1.006142D-06	1.040888D-06
7.671018D-07	8.213859D-07	7.942439D-07
7.322690D-07	6.812000D-07	7.067345D-07
8.426236D-07	5.727959D-07	7.077098D-07
7.209568D-07	6.622368D-07	6.915968D-07
4.142148D-07	5.614758D-07	4.878453D-07
3.938089D-07	3.843719D-07	3.840904D-07
3.610915D-07	2.988150D-07	3.299533D-07
3.204833D-07	3.184838D-07	1.761658D-07
3.094228D-07	2.432023D-07	2.763126D-07
3.214840D-07	2.234353D-07	5.449173D-07

BENCHMARK SOURCE SITUATION

Identification: 6
(To be filled in by Benchmark Committee)

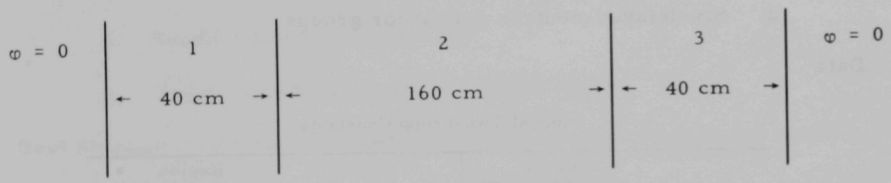
Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Adopted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Infinite Slab Reactor Model

Suggested Functions: Test 1D Neutron Kinetics Solutions

Configuration



BENCHMARK PROBLEM

Identification: 6-A1 Source Situation ID.6

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Subcritical Transient, 1D 2-group Neutron Diffusion Problem in Thermal Reactor

Reduction of Source Situation

1. Two-group diffusion theory

2. Six delayed-neutron precursor groups

Data

Initial Two-group Constants

	Region			Region	
	1 and 3	2		1 and 3	2
D^1 (cm)	1.50	1.00	$\Sigma^{1\to 2}$ (cm^{-1})	.015	.010
D^2 (cm)	.50	.50	χ^1	1.00	1.00
Σ_a^1 (cm^{-1})[a]	.026	.02	χ^2	0	0
Σ_a^2 (cm^{-1})[a]	.18	.08	v^1 (cm/sec)	1.0×10^7	1.0×10^7
$\nu\Sigma_f^1$ (cm^{-1})	.010	.005	v^2 (cm/sec)	3.0×10^5	3.0×10^5
$\nu\Sigma_f^2$ (cm^{-1})	.200	.099			

[a]Total removal cross section, including Σ_c, Σ_f, and $\Sigma^{1\to 2}$.

Additional Data

Delayed Neutron Parameters

Type	Effective Delay Fraction	Decay Constant (sec^{-1})	Type	Effective Delay Fraction	Decay Constant (sec^{-1})
1	.00025	.0124	4	.00296	.3010
2	.00164	.0305	5	.00086	1.1400
3	.00147	.1110	6	.00032	3.0100

Initiating Perturbation

Σ_a^2 in region 1 is linearly increased by 3% in 1.0 sec. (The initial configuration is made critical by dividing the fission cross sections by k_{eff}, and the initial precursor concentrations are in equilibrium with the initial critical flux distribution.)

Expected Primary Results

1. Total power vs time

2. Regional power vs time

Possible Additional Results

1. Comparison of time-difference algorithms

2. Sensitivity to time-step size

3. Time-dependent group flux distributions

Best Solution Available: RAUMZEIT II solution described in the solution 6-A1-1.

Solutions

1. Direct Finite Differences: 6-A1-1, 6-A1-2

2. Flux Factorization Method: 6-A1-3

BENCHMARK PROBLEM SOLUTION

Identification: 6-A1-1 Benchmark Problem ID.6-A1

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite-difference Solution with RAUMZEIT

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The time dependence of neutron and precursor equations is approximated by the time-integrated (TI), average (AV), or backward difference (BD) algorithm.[1]

Pertinent Features of Solution Methods

Differenced (discrete) equations are formulated as a matrix source problem at each time step; solution is obtained by matrix inversion.

Theory

The one-dimensional multigroup diffusion equations solved by RAUMZEIT are

$$\underline{v}^{-1}\dot{\underline{\varphi}}(r,t) = \underline{L}(r,t)\underline{\varphi}(r,t) + (1-\beta)\,\underline{\chi}\underline{F}^{T}(r,t)\underline{\varphi}(r,t) + \underline{\chi}\sum_{m=1}^{M}\lambda_m C^m(r,t) \tag{1}$$

where

$$\underline{L}(r,t) = \left\{\frac{1}{r^{\rho}}\frac{d}{dr}\left[r^{\rho}\underline{D}(r,t)\frac{d}{dr}\right] - \underline{H}(r,t)\right\} \tag{2}$$

and the delayed neutron precursor equations are

$$\dot{C}^m(r,t) = \beta_m\underline{F}^{T}(r,t)\underline{\varphi}(r,t) - \lambda_m C^m(r,t), \qquad m = 1, \ldots, M. \tag{3}$$

The column vectors $\underline{\chi}$, $\underline{F}$, and $\underline{\varphi}$ represent the fission spectra, the nu-fission cross section, and the flux, respectively. The matrices $\underline{D}$ and $\underline{H}$ represent the diffusion coefficient and the removal and scattering

cross sections, respectively, while $\underline{v}$ is a diagonal matrix of group-average neutron speeds. The integer ρ is 0, 1, or 2 for slab, cylindrical, and spherical geometry, respectively.

The precursor equations are approximated in RAUMZEIT by

$$C^m(r,t_i) = E_i^m C^m(r,t_{i-1}) + \frac{1}{\lambda_m} G_i^m \underline{F}^T(r,t_{i-1})\underline{\varphi}(r,t_{i-1})$$

$$+ \frac{1}{\lambda_m} GG_i^m \underline{F}^T(r,t_i)\underline{\varphi}(r,t_i), \tag{4}$$

where

	Time Integrated	Average
$E_i^m =$	$e^{-\lambda_m \Delta_i}$	$\dfrac{2 - \lambda_m \Delta_i}{2 + \lambda_m \Delta_i}$
$G_i^m =$	$\dfrac{\beta_m}{\lambda_m \Delta_i}\left[1 - (1 + \lambda_m \Delta_i)\, e^{-\lambda_m \Delta_i}\right]$	$\dfrac{\beta_m \lambda_m \Delta_i}{2 + \lambda_m \Delta_i}$
$GG_i^m =$	$\dfrac{\beta_m}{\lambda_m \Delta_i}\left[e^{-\lambda_m \Delta_i} - (1 - \lambda_m \Delta_i)\right]$	$\dfrac{\beta_m \lambda_m \Delta_i}{2 + \lambda_m \Delta_i}$

with $\Delta_i \equiv t_i - t_{i-1}$.

The equations evaluated for the neutron flux in RAUMZEIT are

$$[\underline{L}(r,t_i) + \underline{A}(r,t_i)]\,\underline{\Phi}(r,t_i) = \underline{S}(r,t_{i-1}) \tag{5}$$

where $\underline{L}$ is given by Equation (2), and $\underline{A}$ and $\underline{S}$ are given by:

1. Time-integrated:

$$\underline{A}(r,t_i) = \underline{\chi}\left[1 - \sum_{m=1}^{M} \frac{2}{\lambda_m \Delta_i} GG_i^m\right]\underline{F}^T(r,t_i) - \frac{2}{\Delta_i}\underline{v}^{-1},$$

$$\underline{S}(r,t_{i-1}) = \left\{-\underline{L}(r,t_{i-1}) - \frac{2}{\Delta_i}\underline{v}^{-1} - \underline{\chi}\left[1 - \sum_{m=1}^{M} \frac{2}{\lambda_m \Delta_i} G_i^m\right]\underline{F}^T(r,t_{i-1})\right\}\underline{\Phi}(r,t_{i-1})$$

$$- \underline{\chi} \sum_{m=1}^{M} \frac{2}{\Delta_i}(1 - E_i^m)\, C^m(r,t_{i-1}).$$

2. Backward Difference:

$$\underline{A}(r,t_i) = \underline{\chi}\left[1 - \beta + \sum_{m=1}^{M} GG_i^m\right]\underline{F}^T(r,t_i) - \frac{1}{\Delta_i}\underline{v}^{-1},$$

$$\underline{S}(r,t_{i-1}) = \left\{-\frac{1}{\Delta_i}\underline{v}^{-1} - \underline{\chi}\sum_{m=1}^{M} G_i^m \underline{F}^T(r,t_{i-1})\right\}\underline{\Phi}(r,t_{i-1}) - \underline{\chi}\sum_{m=1}^{M} E_i^m \lambda_m C^m(r,t_{i-1}).$$

3. Average:*

$$\underline{A}(r,t_i) = \underline{\chi}\left[1 - \beta + \sum_{m=1}^{M} GG_i^m\right]\underline{F}^T(r,t_i) - \frac{2}{\Delta_i}\underline{v}^{-1},$$

$$\underline{S}(r,t_{i-1}) = \left\{-\underline{L}(r,t_{i-1}) - \frac{2}{\Delta_i}\underline{v}^{-1} - \underline{\chi}\left[1 - \beta + \sum_{m=1}^{M} G_i^m\right]\underline{F}^T(r,t_{i-1})\right\}\underline{\Phi}(r,t_{i-1})$$

$$- \underline{\chi}\sum_{m=1}^{M}(1 + E_i^m)\lambda_m C^m(r,t_{i-1}).$$

Equation (5), which may be written

$$-\frac{1}{r^\rho}\frac{d}{dr}\left[r^\rho \underline{D}(r)\frac{d}{dr}\underline{\Phi}(r)\right] + \underline{B}(r)\underline{\Phi}(r) = \underline{S}(r),$$

is represented in RAUMZEIT by the spatial difference approximation

$$-\frac{\frac{2}{h_j}\underline{D}_j}{1 + \frac{h_j\rho}{2r_j}}\underline{\Phi}_{j-1} + \left[\frac{h_j\underline{B}_j + \frac{2}{h_j}\underline{D}_j}{1 + \frac{h_j\rho}{2r_j}} + \frac{h_{j+1}\underline{B}_{j+1} + \frac{2}{h_{j+1}}\underline{D}_{j+1}}{1 - \frac{h_{j+1}\rho}{2r_j}}\right]\underline{\Phi}_j - \frac{\frac{2}{h_{j+1}}\underline{D}_{j+1}}{1 - \frac{h_{j+1}\rho}{2r_j}}\underline{\Phi}_{j+1} =$$

$$h_j\underline{S}_j + h_{j+1}\underline{S}_{j+1}, \tag{6}$$

where $h_j \equiv r_j - r_{j-1}$, and the subscript j associates the term with the value taken on in h_j.

*The average algorithms for the neutron and precursor equations correspond to the method used in the WIGLE code (WAPD-TM-416) when $\theta = \theta_d = 1/2$.

Computer: CDC-6600 Date Solved: May 1969

at: KAPL

Program: RAUMZEIT

Reference

1. C. H. Adams and W. M. Stacey, Jr., RAUMZEIT - A Program to Solve Coupled Time-Dependent Neutron Diffusion Equations in One Space Dimension, Knolls Atomic Power Laboratory Report KAPL-M-6728 (1967).

Results

Uniform mesh with Δx = 2 cm; 120 intervals

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1	0.2790
Region 2	0.4421
Region 3	0.2790
	1.0001

EXHIBIT A Thermal (Group 2) flux distribution at 0.0, 1.0, and 2.0 seconds.

EXHIBIT B Total power (relative to initial value) vs time, as a function of difference algorithm and time-step size.

EXHIBIT A

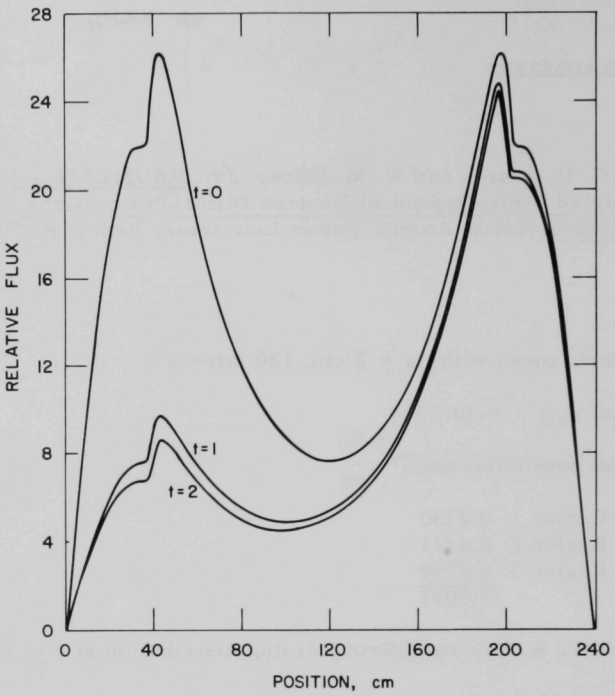


EXHIBIT B

NDA[a]	TI	TI	TI	BD	BD	AV
PDA[b]	TI	TI	TI	TI	AV	AV
Δt (sec)	10^{-3}	10^{-2}	10^{-1}	10^{-1}	10^{-1}	10^{-1}
Time (sec)						
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	.9299	.9299	.9321	.9300	.9301	.9318
0.2	.8733	.8733	.8718	.8735	.8735	.8719
0.5	.7597	.7597	.7608	.7598	.7598	.7607
1.0	.6588	.6588	.6584	.6589	.6589	.6584
1.5	.6432	.6432	.6435	.6433	.6433	.6435
2.0	.6307	.6307	.6305	.6307	.6307	.6305

[a]Neutron Difference Algorithm.
[b]Precursor Difference Algorithm.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A1-2 Benchmark Problem ID.6-A1

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite Difference Solution with WIGLE[1]

Mathematical Model

Symmetric 3-point difference approximations are used for the leakage operator of the neutron diffusion equations. The time dependence is approximated by central differences (Crank-Nicholson).

Computer: CDC-3600 Date Solved: January 1971

at: ANL

Program: WIGLE

Reference

1. W. R. Cadwell, A. F. Henry, and A. J. Vigilotti, WIGLE - A Program for the Solution of the Two-Group Space-Time Diffusion Equations in Slab Geometry, Westinghouse Electric Corporation, Bettis Atomic Power Laboratory Report WAPD-TM-416 (1964).

Results

Uniform mesh with Δx = 2 cm; 120 intervals, time-step size = 10^{-3} sec.

Initial k_{eff} = 0.9015507 (Value taken from problem solution 6-A1-1 to establish initial conditions.)

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power
0.0	1.0000
0.1	.9298
0.2	.8732
0.5	.7596
1.0	.6588
1.5	.6432
2.0	.6306

EXHIBIT B

	Relative Regional Power		
Time (sec)	Region 1	Region 2	Region 3
0.0	1.0000	1.0000	1.0000
0.1	.8621	.9339	.9910
0.2	.7520	.8804	.9830
0.5	.5336	.7724	.9655
1.0	.3452	.6753	.9461
1.5	.3235	.6587	.9381
2.0	.3066	.6455	.9311

BENCHMARK PROBLEM SOLUTION

Identification: 6-A1-3 Benchmark Problem ID.6-A1

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Flux Factorization Solution with QX1[2]

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The improved quasistatic method as described in Ref. 1 is used to solve the time-dependent problem.

Pertinent Features of Solution Methods

The method consists of factoring the total flux into the product of a space-energy-time-dependent shape function and a purely time-dependent amplitude function. The coupled set of equations which results is solved iteratively. (See Ref. 2 for details.) The code includes automatic time-step selection procedures, so that the solution is normally obtained on an uneven time mesh. Solutions were forced at particular time points for the purpose of comparison.

Computer: CDC-3600 Date Solved: January 1971

at: ANL

Program: QX1

References

1. K. O. Ott and D. A. Meneley, Accuracy of the Quasistatic Treatment of Spatial Reactor Kinetics, Nucl. Sci. Eng. 36, 402-411 (1969).

2. D. A. Meneley, K. O. Ott, and E. S. Wiener, Fast-reactor Kinetics--The QX1 Code, Argonne National Laboratory Report ANL-7769 (March 1971).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) and flux shape function count vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power	Flux Shape Function No.[a]
0.0	1.0000	1
0.1	.9298	7
0.2	.8733	11
0.5	.7597	21
1.0	.6588	36
1.5	.6433	41
2.0	.6307	44

[a]Cumulative up to the given time.

EXHIBIT B

	Relative Regional Power		
Time (sec)	Region 1	Region 2	Region 3
0.0	1.0000	1.0000	1.0000
0.1	.8621	.9340	.9910
0.2	.7521	.8805	.9831
0.5	.5336	.7724	.9655
1.0	.3452	.6753	.9463
1.5	.3235	.6588	.9383
2.0	.3066	.6455	.9312

BENCHMARK PROBLEM

Identification: 6-A2 Source Situation ID.6

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Delayed Super-critical Transient, 1D 2-Group Neutron Diffusion Problem in Thermal Reactor

Reduction of Source Situation

1. Two-group diffusion theory

2. Six delayed-neutron precursor groups

Data

Same as for problem 6-A1, except for:

Initiating Perturbation

Σ_a^2 in region 1 is linearly decreased by 1% in 1.0 sec. (The initial configuration is made critical by dividing the fission cross sections by k_{eff}, and the initial precursor concentrations are in equilibrium with the initial critical flux distribution.)

Expected Results

1. Total power vs time

2. Regional power vs time

Best Solution Available: RAUMZEIT II solution described in the solution 6-A2-1.

Solutions

1. Direct Finite Differences: 6-A2-1, 6-A2-2.

2. Flux Factorization Method: 6-A2-3.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A2-1 Benchmark Problem ID.6-A2

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite-difference Solution with RAUMZEIT

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The time dependence of neutron and precursor equations is approximated by the time-integrated (TI), average (AV), or backward difference (BD) algorithm.[1]

Pertinent Features of Solution Methods

Differenced (discrete) equations are formulated as a matrix source problem at each time-step; solution is obtained by matrix inversion.

Theory

Same as for solution 6-A1-1.

Computer: CDC-6600 Date Solved: May 1969

at: KAPL

Program: RAUMZEIT

Reference

1. C. H. Adams and W. M. Stacey, Jr., RAUMZEIT - A Program to Solve Coupled Time-Dependent Neutron Diffusion Equations in One Space Dimension, Knolls Atomic Power Laboratory Report KAPL-M-6728 (1967).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.2790
Region 2 0.4421
Region 3 0.2790

EXHIBIT A Thermal (Group 2) flux distribution at 0.0, 1.0, and 4.0 seconds.

EXHIBIT B Total power (relative to initial value) vs time, as a function of difference algorithm and time-step size.

EXHIBIT C Regional power (relative to initial value) vs time as a function of difference algorithm and time-step size.

EXHIBIT A

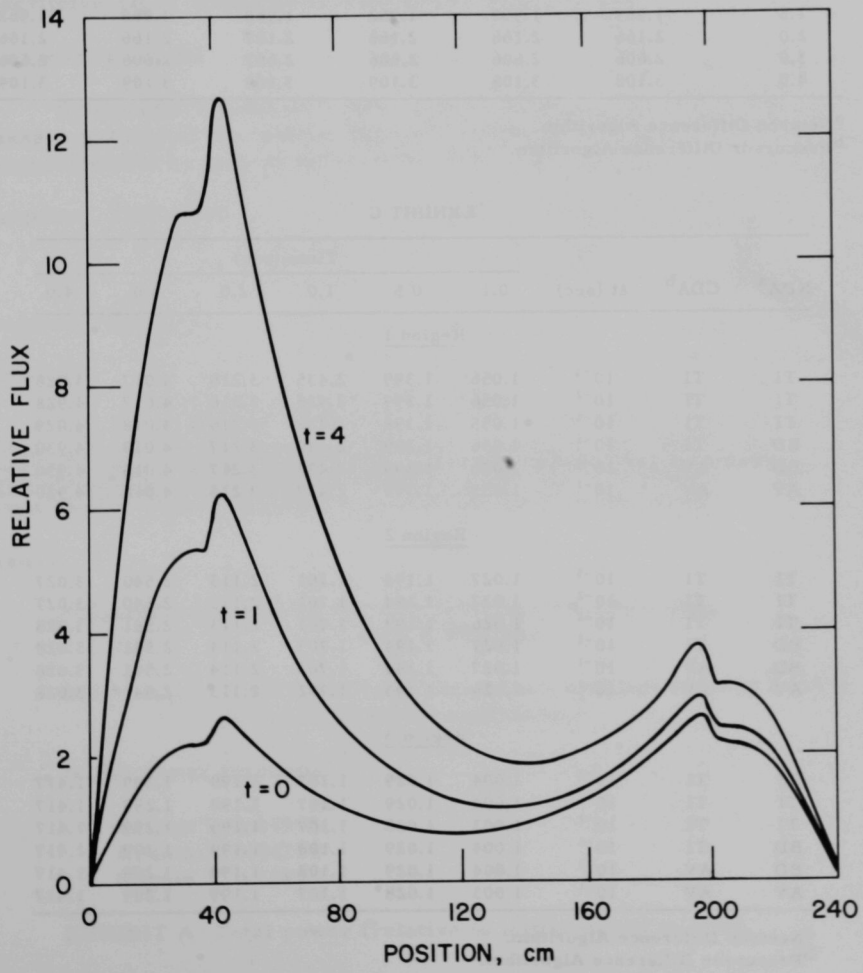


EXHIBIT B

NDA[a]	TI	TI	TI	BD	BD	AV
PDA[b]	TI	TI	TI	TI	AV	AV
Δt (sec)	10^{-3}	10^{-2}	10^{-1}	10^{-1}	10^{-1}	10^{-1}
Time (sec)						
0.0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.028	1.028	1.028	1.029	1.029	1.028
0.2	1.063	1.063	1.063	1.063	1.063	1.063
0.5	1.205	1.205	1.204	1.205	1.205	1.204
1.0	1.740	1.740	1.740	1.743	1.742	1.741
1.5	1.959	1.959	1.962	1.960	1.960	1.962
2.0	2.166	2.166	2.166	2.167	2.166	2.166
3.0	2.606	2.606	2.606	2.607	2.606	2.606
4.0	3.108	3.108	3.109	3.109	3.109	3.109

[a]Neutron Difference Algorithm.
[b]Precursor Difference Algorithm.

EXHIBIT C

			Time (sec)					
NDA[a]	CDA[b]	Δt (sec)	0.1	0.5	1.0	2.0	3.0	4.0
			Region 1					
TI	TI	10^{-3}	1.056	1.399	2.435	3.216	4.017	4.928
TI	TI	10^{-2}	1.056	1.399	2.435	3.216	4.017	4.928
TI	TI	10^{-1}	1.055	1.398	2.436	3.216	4.018	4.929
BD	TI	10^{-1}	1.056	1.399	2.440	3.217	4.019	4.930
BD	AV	10^{-1}	1.056	1.399	2.439	3.217	4.018	4.930
AV	AV	10^{-1}	1.055	1.398	2.437	3.215	4.018	4.930
			Region 2					
TI	TI	10^{-3}	1.027	1.194	1.701	2.113	2.540	3.027
TI	TI	10^{-2}	1.027	1.194	1.701	2.113	2.540	3.027
TI	TI	10^{-1}	1.026	1.193	1.701	2.113	2.541	3.028
BD	TI	10^{-1}	1.027	1.194	1.703	2.114	2.541	3.028
BD	AV	10^{-1}	1.027	1.194	1.703	2.114	2.541	3.028
AV	AV	10^{-1}	1.026	1.193	1.702	2.113	2.541	3.028
			Region 3					
TI	TI	10^{-3}	1.004	1.029	1.107	1.199	1.299	1.417
TI	TI	10^{-2}	1.004	1.029	1.107	1.199	1.299	1.417
TI	TI	10^{-1}	1.003	1.028	1.107	1.199	1.299	1.417
BD	TI	10^{-1}	1.004	1.029	1.108	1.199	1.299	1.417
BD	AV	10^{-1}	1.004	1.029	1.108	1.199	1.299	1.417
AV	AV	10^{-1}	1.003	1.028	1.107	1.199	1.299	1.417

[a]Neutron Difference Algorithm.
[b]Precursor Difference Algorithm.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A2-2 Benchmark Problem ID.6-A2

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite Difference Solution with WIGLE[1]

Mathematical Model

Symmetric 3-point difference approximations are used for the leakage operator of the neutron diffusion equations. The time dependence is approximated by central differences (Crank-Nicholson).

Computer: CDC-3600 Date Solved: January 1971

at: ANL

Program: WIGLE

Reference

1. W. R. Cadwell, A. F. Henry and A. J. Vigilotti, WIGLE - A Program for the Solution of the Two-Group Space-Time Diffusion Equations in Slab Geometry, Westinghouse Electric Corporation, Bettis Atomic Power Laboratory Report WAPD-TM-416 (1964).

Results

Uniform mesh with Δx = 2 cm; 120 intervals, time-step size = 10^{-3} sec.

Initial k_{eff} = 0.9015507 (Value taken from problem solution 6-A1-1 to establish initial conditions.)

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power
0.0	1.000
0.1	1.028
0.2	1.062
0.5	1.205
1.0	1.740
1.5	1.959
2.0	2.165
3.0	2.605
4.0	3.107

EXHIBIT B

	Relative Regional Power		
Time (sec)	Region 1	Region 2	Region 3
0.0	1.000	1.000	1.000
0.1	1.056	1.027	1.004
0.5	1.399	1.193	1.028
1.0	2.435	1.701	1.107
2.0	3.215	2.113	1.119
3.0	4.016	2.539	1.298
4.0	4.927	3.026	1.416

BENCHMARK PROBLEM SOLUTION

Identification: 6-A2-3 Benchmark Problem ID.6-A2

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Flux Factorization Solution with $QX1^2$

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The improved quasistatic method as described in Ref. 1 is used to solve the time-dependent problem.

Pertinent Features of Solution Methods

The method consists of factoring the total flux into the product of a space-energy-time-dependent shape function and a purely time-dependent amplitude function. The coupled set of equations which results is solved iteratively. (See Ref. 2 for details.) The code includes automatic time-step selection procedures, so that the solution is normally obtained on an uneven time mesh. Solutions were forced at particular time points for the purpose of comparison.

Computer: CDC-3600 Date Solved: January 1971

at: ANL

Program: QX1

References

1. K. O. Ott and D. A. Meneley, Accuracy of the Quasistatic Treatment of Spatial Reactor Kinetics, Nucl. Sci. Eng. 36, 402-411 (1969).

2. D. A. Meneley, K. O. Ott and E. S. Wiener, Fast Reactor Kinetics--The QX1 Code, Argonne National Laboratory Report ANL-7769 (March 1971).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) and flux shape function count vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power	Flux Shape Function No.[a]
0.0	1.000	1
0.1	1.028	5
0.2	1.063	7
0.5	1.205	12
1.0	1.740	26
1.5	1.959	33
2.0	2.166	37
3.0	2.606	43
4.0	3.108	48

[a]Cumulative up to the given time.

EXHIBIT B

	Relative Regional Power		
Time (sec)	Region 1	Region 2	Region 3
0.0	1.000	1.000	1.000
0.1	1.056	1.027	1.004
0.5	1.398	1.193	1.029
1.0	2.435	1.701	1.701
2.0	3.216	2.113	1.199
3.0	4.017	2.540	1.298
4.0	4.928	3.027	1.416

BENCHMARK PROBLEM

Identification: 6-A3 Source Situation ID.6

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Prompt Super-critical Transient, 1D 2-group Neutron Diffusion Problem in Thermal Reactor

Reduction of Source Situation

1. Two-group diffusion theory

2. Six delayed-neutron precursor groups

Data

Same as for problem 6-A1, except for:

Initiating Perturbation

Σ_a^2 in region 1 is linearly decreased by 5% in 0.01 sec. (The initial configuration is made critical by dividing the fission cross sections by k_{eff}. The initial precursor concentrations are in equilibrium with the initial critical flux distribution.)

Expected Results

1. Total power vs time

2. Regional power vs time

Best Solution Available: RAUMZEIT II solution described in the solution 6-A3-1.

Solutions

1. Direct Finite Differences: 6-A3-1, 6-A3-2

2. Flux Factorization Method: 6-A3-3

BENCHMARK PROBLEM SOLUTION

Identification: 6-A3-1 Benchmark Problem ID.6-A3

Date Submitted: July 1969 By: W. M. Stacey, Jr. (KAPL)
(Name and Organization)

Date Accepted: November 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite-difference Solution with RAUMZEIT

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The time dependence of neutron and precursor equations is approximated by the time-integrated (TI), average (AV), or backward difference (BD) algorithm.[1]

Pertinent Features of Solution Methods

Differenced (discrete) equations are formulated as a matrix source problem at each time step; solution is obtained by matrix inversion.

Theory

Same as for solution 6-A1-1.

Computer: CDC-6600 Date Solved: May 1969

at: KAPL

Program: RAUMZEIT

Reference

1. C. H. Adams and W. M. Stacey, Jr., RAUMZEIT - A Program to Solve Coupled Time-Dependent Neutron Diffusion Equations in One Space Dimension, Knolls Atomic Power Laboratory Report KAPL-M-6728 (1967).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.2790
Region 2 0.4421
Region 3 0.2790

EXHIBIT A Thermal (Group 2) flux distribution at 0.00, 0.01, and 0.02 seconds.

EXHIBIT B Total power (relative to initial value) vs time, as a function of difference algorithm and time-step size.

EXHIBIT C Regional power (relative to initial value) vs time, as a function of difference algorithm and time-step size.

EXHIBIT A

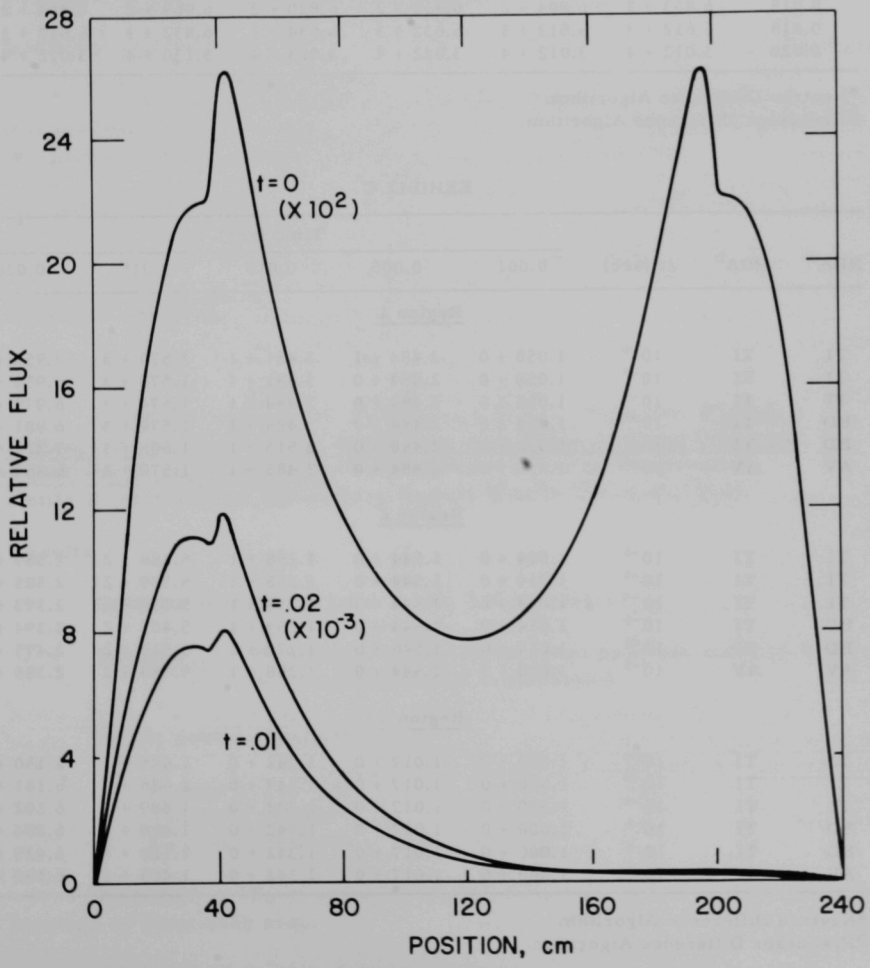


EXHIBIT B

NDA[a]	TI	TI	TI	BD	BD	AV
PDA[b]	TI	TI	TI	TI	TI	AV
Δt (sec)	10^{-6}	10^{-5}	10^{-4}	10^{-6}	10^{-5}	10^{-5}
Time (sec)						
0.000	1.000 + 0	1.000 + 0	1.000 + 0	1.000 + 0	1.000 + 0	1.000 + 0
0.001	1.022 + 0	1.022 + 0	1.022 + 0	1.022 + 0	1.022 + 0	1.022 + 0
0.005	1.659 + 0	1.659 + 0	1.659 + 0	1.659 + 0	1.661 + 1	1.659 + 0
0.010	1.565 + 1	1.565 + 1	1.566 + 1	1.566 + 1	1.580 + 1	1.565 + 1
0.012	7.019 + 1	7.019 + 1	7.028 + 1	7.030 + 1	7.128 + 1	7.019 + 1
0.015	6.803 + 2	6.804 + 2	6.818 + 2	6.820 + 2	6.969 + 2	6.804 + 2
0.018	6.612 + 3	6.613 + 3	6.632 + 3	6.634 + 3	6.832 + 3	6.613 + 3
0.020	3.012 + 4	3.012 + 4	3.022 + 4	3.023 + 4	3.130 + 4	3.012 + 4

[a]Neutron Difference Algorithm.
[b]Precursor Difference Algorithm.

EXHIBIT C

NDA[a]	PDA[b]	Δt (sec)	Time (sec) 0.001	0.005	0.010	0.015	0.020
			Region 1				
TI	TI	10^{-6}	1.058 + 0	2.484 + 1	3.481 + 1	1.570 + 3	6.955 + 4
TI	TI	10^{-5}	1.058 + 0	2.484 + 0	3.481 + 1	1.570 + 3	6.956 + 4
TI	TI	10^{-4}	1.058 + 0	2.484 + 0	3.484 + 1	1.574 + 3	6.979 + 4
BD	TI	10^{-6}	1.058 + 0	2.484 + 0	3.484 + 1	1.574 + 3	6.981 + 4
BD	TI	10^{-5}	1.058 + 0	2.488 + 0	3.515 + 1	1.608 + 3	7.228 + 4
AV	AV	10^{-5}	1.058 + 0	2.484 + 0	3.481 + 1	1.570 + 3	6.954 + 4
			Region 2				
TI	TI	10^{-6}	1.014 + 0	1.544 + 0	1.258 + 1	5.388 + 2	2.385 + 4
TI	TI	10^{-5}	1.014 + 0	1.544 + 0	1.258 + 1	5.389 + 2	2.385 + 4
TI	TI	10^{-4}	1.014 + 0	1.544 + 0	1.259 + 1	5.399 + 2	2.393 + 4
BD	TI	10^{-6}	1.014 + 0	1.544 + 0	1.259 + 1	5.401 + 2	2.394 + 4
BD	TI	10^{-5}	1.014 + 0	1.546 + 0	1.270 + 1	5.519 + 2	2.479 + 4
AV	AV	10^{-5}	1.014 + 0	1.544 + 0	1.258 + 1	5.388 + 2	2.385 + 4
			Region 3				
TI	TI	10^{-6}	1.000 + 0	1.017 + 0	1.342 + 0	1.485 + 1	6.180 + 2
TI	TI	10^{-5}	1.000 + 0	1.017 + 0	1.342 + 0	1.486 + 1	6.181 + 2
TI	TI	10^{-4}	1.000 + 0	1.017 + 0	1.342 + 0	1.489 + 1	6.202 + 2
BD	TI	10^{-6}	1.000 + 0	1.017 + 0	1.342 + 0	1.489 + 1	6.204 + 2
BD	TI	10^{-5}	1.000 + 0	1.017 + 0	1.342 + 0	1.520 + 1	6.423 + 2
AV	AV	10^{-5}	1.000 + 0	1.017 + 0	1.342 + 0	1.485 + 1	6.180 + 2

[a]Neutron Difference Algorithm.
[b]Precursor Difference Algorithm.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A3-2 Benchmark Problem ID.6-A2

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite Difference Solution with WIGLE[1]

Mathematical Model

Symmetric 3-point difference approximations are used for the leakage operator of the neutron diffusion equations. The time dependence is approximated by central differences (Crank-Nicholson).

Computer: CDC-3600 Date Solved: January 1971

at: ANL

Program: WIGLE

Reference

1. W. R. Cadwell, A. F. Henry, and A. J. Vigilotti, WIGLE - A Program for the Solution of the Two-Group Space-Time Diffusion Equations in Slab Geometry, Westinghouse Electric Corporation, Bettis Atomic Power Laboratory Report WAPD-TM-416 (1964).

Results

Uniform mesh with $\Delta x = 2$ cm; 120 intervals

Initial $k_{eff} = 0.9015507$ (Value taken from problem solution 6-A1-1 to establish initial conditions.)

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) vs time, as a function of time-step size.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power			
	$\Delta t = 2 \times 10^{-6}$	$\Delta t = 10^{-5}$	$\Delta t = 10^{-4}$	$\Delta t = 10^{-3}$
0.000	1.000	1.000	1.000	1.000
.001	1.022	1.022	1.022	1.022
.005	1.659	1.659	1.659	1.672
.010	1.565 + 1	1.565 + 1	1.565 + 1	1.802 + 1
.012	7.018 + 1	7.019 + 1	7.033 + 1	8.768 + 1
.015	6.802 + 2	6.803 + 2	6.824 + 2	9.582 + 2
.018	6.611 + 3	6.611 + 3	6.639 + 3	1.049 + 4
.020	3.011 + 4	3.011 + 4	3.026 + 4	5.176 + 4

EXHIBIT B

Time (sec)	Relative Regional Power[a]		
	Region 1	Region 2	Region 3
0.000	1.000	1.000	1.000
0.001	1.058	1.014	1.000
0.005	2.484	1.544	1.017
0.010	3.481 + 1	1.258 + 1	1.342
0.015	1.570 + 3	5.388 + 2	1.485 + 1
0.020	6.954 + 4	2.385 + 4	6.179 + 2

[a]Time-step size = 10^{-5} sec.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A3-3 Benchmark Problem ID.6-A3

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Flux Factorization Solution with $QX1^2$

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The improved quasistatic method as described in Ref. 1 is used to solve the time-dependent problem.

Pertinent Features of Solution Methods

The method consists of factoring the total flux into the product of a space-energy-time-dependent shape function and a purely time-dependent amplitude function. The coupled set of equations which results is solved iteratively. (See Ref. 2 for details.) The code includes automatic time-step selection procedures, so that the solution is normally obtained on an uneven time mesh. Solutions were forced at particular time points for the purpose of comparison.

Computer: IBM 360/75 Date Solved: January 1971

at: ANL

Program: QX1

References

1. K. O. Ott and D. A. Meneley, Accuracy of the Quasistatic Treatment of Spatial Reactor Kinetics, Nucl. Sci. Eng. 36, 402-411 (1969).

2. D. A. Meneley, K. O. Ott, and E. S. Wiener, Fast Reactor Kinetics--The QX1 Code, Argonne National Laboratory Report ANL-7769 (March 1971).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) and flux shape function count as a function of number of time-steps vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

Time (sec)	Total Power[a]				Flux Shape Function Number[b]			
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
0.000	1.000	1.000	1.000	1.000	1	1	1	1
0.001	1.022	1.022	1.022	1.022	20	4	4	4
0.005	1.660	1.666	1.673	1.673	266	40	17	8
0.010	15.65	15.73	15.83	15.71	577	140	57	13
0.012	70.18	70.55	70.92	69.93	626	188	78	16
0.015	680.0	683.6	686.8	674.0	653	215	91	19
0.018	6608.	6642.	6674.	6544.	659	221	95	22
0.020	30100.	30250.	30400.	29800.	661	223	97	24

[a]Case 2 shows the results obtained with the default time-step selection criteria in the QXI code; Case 1 has a tight criterion (.01 β) on the asymmetric reactivity insertion. Case 3 has a loose criterion (.05) on the fractional shape function change in any region, and Case 4 has a .001 sec fixed-step length (following the initial steps) plus one step at .0101 sec to obtain power definition of the end of the ramp at .01 sec.
[b]Cumulative up to the given time.

EXHIBIT B[a]

Time (sec)	Relative Regional Power		
	Region 1	Region 2	Region 3
0.000	1.000	1.000	1.000
0.001	1.061	1.014	0.999
0.005	2.486	1.545	1.016
0.010	34.82	12.59	14.93
0.015	1569.	538.6	14.93
0.020	69500.	23830.	618.2

[a]For Case 1.

BENCHMARK PROBLEM

Identification: 6-A4 Source Situation ID.6

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Prompt Super-critical Transient, 1D 2-group Neutron Diffusion Problem in Thermal Reactor with Modified Neutron Velocities

Reduction of Source Situation

1. Two-group diffusion theory

2. Six delayed-neutron precursor groups

Data

Same as for problem 6-A1, except for:

Region	1 and 3	2
v^1 (cm/sec)	1.00 + 9	1.00 + 9
v^2 (cm/sec)	3.00 + 7	3.00 + 7

Initiating Perturbation

Σ_a^2 in region 1 is linearly decreased by 5% in 0.01 sec. (The initial configuration is made critical by dividing the fission cross sections by k_{eff}. The initial precursor concentrations are in equilibrium with the initial critical flux distribution.)

Expected Results

1. Total power vs time

2. Regional power vs time

Best Solution Available: QX1 solution described in the solution 6-A4-2.

Solutions

1. Direct Finite Differences: 6-A4-1

2. [illegible]ization Method: 6-A4-2

BENCHMARK PROBLEM SOLUTION

Identification: 6-A4-1 Benchmark Problem ID.6-A4

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Finite-difference Solution with RAUMZEIT

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The time dependence of neutron and precursor equations is approximated by the time-integrated (TI), average (AV), or backward difference (BD) algorithm.[1]

Pertinent Features of Solution Methods

Differenced (discrete) equations are formulated as a matrix inversion.

Theory

Same as for solution 6-A1-1.

Computer: IBM 360/75 Date Solved: June 1971

at: ANL

Program: RAUMZEIT

Reference

1. C. H. Adams and W. M. Stacey, Jr., RAUMZEIT - A Program to Solve Coupled Time-Dependent Neutron Diffusion Equations in One Space Dimension, Knolls Atomic Power Laboratory Report KAPL-M-6728 (1967).

Results

Note: These results were obtained using the time-integrated (TI) algorithm for both the neutron and precursor equations.

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) vs time, as a function of time-step size.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

	Total Power			
Time (sec)	$\Delta t = 10^{-6}$	$\Delta t = 10^{-5}$	$\Delta t = 5 \times 10^{-5}$	$\Delta t = 10^{-4}$
0.000	1.000	1.000	1.000	1.000
0.001	1.178	1.178	1.178	1.178
0.002	1.558	1.558	1.558	1.559
0.003	2.797	2.797	2.797	2.798
0.0035	5.284	5.284	5.287	5.297
0.0040	20.72	20.73	20.85	21.25
0.0045	472.0	473.0	498.4	596.6
0.0050	153700.	155300.	203700.	632700.

EXHIBIT B

	Relative Regional Power[a]		
Time (sec)	Region 1	Region 2	Region 3
0.000	1.000	1.000	1.000
0.001	1.351	1.167	1.022
0.002	2.101	1.525	1.070
0.003	4.547	2.686	1.222
0.004	40.36	19.37	3.224
0.005	317100.	139500.	12710.

[a] $\Delta t = 10^{-6}$ sec.

BENCHMARK PROBLEM SOLUTION

Identification: 6-A4-2 Benchmark Problem ID.6-A4

Date Submitted: November 1971 By: E. L. Fuller (ANL)
(Name and Organization)

Date Accepted: December 1971 By: D. A. Meneley (ANL)
(Name and Organization)

Descriptive Title: Flux Factorization Solution with $QX1^2$

Mathematical Model

Symmetric 3-point difference approximations at mesh interval boundaries are used for the leakage operator of the neutron diffusion equations. The improved quasistatic method as described in Ref. 1 is used to solve the time-dependent problem.

Pertinent Features of Solution Methods

The method consists of factoring the total flux into the product of a space-energy-time-dependent shape function. The couples set of equations which results is solved iteratively. (See Ref. 2 for details.) The code includes automatic time-step selection procedures, so that the solution is normally obtained on an uneven time mesh. Solutions were forced at particular time points for the purpose of comparison.

Computer: IBM 360/75 Date Solved: June 1971

at: ANL

Program: QX1

References

1. K. O. Ott and D. A. Meneley, Accuracy of the Quasistatic Treatment of Spatial Reactor Kinetics, Nucl. Sci. Eng. 36, 402-411 (1969).

2. D. A. Meneley, K. O. Ott and E. S. Wiener, Fast Reactor Kinetics--The QX1 Code, Argonne National Laboratory Report ANL-7769 (March 1971).

Results

Initial k_{eff} = 0.9015507

Initial power fractions

Region 1 0.27895
Region 2 0.44209
Region 3 0.27895

EXHIBIT A Total power (relative to initial value) and flux shape function count as a function of number of time-steps vs time.

EXHIBIT B Regional power (relative to initial value) vs time.

EXHIBIT A

	Total Power[a]					Flux Shape Function Number[b]				
Time (sec)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
0.000	1.000	1.000	1.000	1.000	1.000	1	1	1	1	1
0.001	1.178	1.178	1.178	1.178	1.178	10	7	7	7	6
0.002	1.558	1.558	1.558	1.558	1.558	21	9	9	9	7
0.003	2.797	2.797	2.787	2.787	2.759	42	14	11	11	8
0.0035	5.284	5.284	5.245	5.227		64	18	13	12	
0.004	20.72	20.72	20.66	20.06	18.32	212	48	28	13	9
0.0045	471.6	471.1	470.0	431.7		248	57	34	14	
0.005	153500.	153200.	152700.	140700.	118900.	258	60	35	15	11

[a]Case 1: Default time-step selection criteria; Case 2: Shape-function criterion 0.1; asymmetric reactivity and amplitude change tests off; Case 3: Shape-function criterion 0.2; asymmetric reactivity and amplitude change tests off; Case 4: Shape-function, asymmetric reactivity, and amplitude change tests off; force time steps at 5×10^{-4} sec intervals; and Case 5: Same as Case 4, except time steps forced at 1×10^{-3} sec intervals.
[b]Cumulative up to the given time.

EXHIBIT B[a]

	Relative Regional Power		
Time (sec)	Region 1	Region 2	Region 3
0.000	1.000	1.000	1.000
0.001	1.351	1.167	1.022
0.002	2.101	1.525	1.070
0.003	4.547	2.687	1.222
0.004	40.36	19.37	3.224
0.005	316800.	139400.	12700.

[a]For Case 1, as defined in EXHIBIT A.

BENCHMARK SOURCE SITUATION

Identification: 7
(To be filled in by Benchmark Committee)

Date Submitted: November 1970 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: July 1972 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Monoenergetic Point Reactor Model

Suggested Function: Test point dynamics solution methods

Configuration

Temporal behavior of the reactor is described by the following equations:

$$\frac{dP}{dt} = \left[\frac{\rho(t,\omega) - \beta}{\Lambda}\right] P + \sum_i \lambda_i \eta_i$$

$$\frac{d\eta_i}{dt} = \frac{\beta_i}{\Lambda} P - \lambda_i \eta_i (1 \le i \le 1)$$

$$\rho(t,\omega) = f(t) + \sum_j b_j[\omega_j(t), \omega_j(0)]$$

$$\frac{d\omega_j}{dt} = g_j[P(t), P(0)](1 \le j \le J)$$

Initial conditions are defined consistent with

$$\frac{dP}{dt} = \frac{d\eta_i}{dt} = 0 \text{ for } t = (0 - \varepsilon).$$

Functions f, b_j, and g_j are piecewise continuous functions of time.

Symbols are defined as follows:

Name	Physical Meaning	Dimensions
P	power density	w/cm^3
t	time	sec
$\rho(t,\omega)$	reactivity	-
β	total delayed-neutron fraction	-
Λ	prompt-neutron generation time	sec
λ_i	delayed-neutron-precursor decay constant	$(sec)^{-1}$
η_i	delayed-neutron-precursor density	$(cm^3)^{-1}$
β_i	delayed-neutron fraction $\sum_i \beta_i = \beta$	-

BENCHMARK PROBLEM

Identification: 7-A1 Source Situation ID.7

Date Submitted: November 1970 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: July 1972 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Point Reactor, Ramp Reactivity Insertion, and Linear Temperature Feedback

Reduction of Source Situation

1. No delayed neutrons;

2. Linear reactivity insertion; $f(t) = At$

3. Linear temperature feedback; $J = 1$

and

$$b_j[\omega_j(t),\omega(0)] = B[T(t) - T(0)]$$

with

$$\frac{dT}{dt} = \frac{1}{C} P(t)$$

Data

Initial power density $P(0) = 10\ w/cm^3$

Initial temperature $T(0) = 0°C$

Generation time $\Lambda = 5 \times 10^{-4}$ sec

Input reactivity ramp rate $A = 5 \times 10^{-2}\ sec^{-1}$

Temperature coefficient $B = -1 \times 10^{-4}\ (°C)^{-1}$

Heat capacity $C = 2 \frac{w \cdot sec}{cm^3 \cdot °C}$

Expected Primary Results

1. Period of power density oscillation

2. Maximum power density

3. Temperature rise per period.

Possible Additional Results

1. Plot of temperature vs time

2. Plot of power density vs time

3. Sensitivity of results to time-step length.

4. Tabular temperatures and power densities at selected time points.

Best Solution Available: Analytic result.

BENCHMARK PROBLEM SOLUTION

Identification: 7-A1-1 Benchmark Problem ID.7-A1

Date Submitted: June 1972 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: January 1973 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Analytic Solution of Prompt Point Reactor Equation with Linear Temperature Feedback

Mathematical Model

The equations given in ID.7-A1 are written in dimensionless form and solved exactly by the methods described in Refs. 1 and 2, using the definitions:

$$\alpha = \frac{A}{\Lambda}, \quad \gamma = \frac{B}{C\Lambda}$$

$$V(t) = \frac{\gamma}{\alpha} P(t), \quad V(0) = \frac{\gamma}{\alpha} P(0),$$

$$X = \sqrt{\alpha}\, t.$$

The power density P(t) and the temperature T(t) are given by:

$$\left.\begin{aligned} P(t) &= \frac{\alpha}{\gamma} V(X) \\ T(t) &= T(0) + \frac{\sqrt{\alpha}}{\gamma C} S(X) \end{aligned}\right\} \quad 0 \le \sqrt{\alpha} \le \frac{P_x}{z},$$

where P_x is the period of the power oscillation.

Introducing the definition $w = \ln [V(X)]$, the functions X(w) and S(w) are calculated from:

$$X(w) = \int_{w(0)}^{w} \frac{du}{\sqrt{2\left[u - e^{u} - w(0) + e^{w(0)}\right]}},$$

$$S(w) = \int_{w(0)}^{w} \frac{e^{u}\, du}{\sqrt{2\left[u - e^{u} - w(0) + e^{w(0)}\right]}},$$

with $w(0) < 0$. The period P_x is determined by integration of the first of these equations to w_1, where w_1 is the solution $\neq w(0)$ of the equation

$$e^w - w = e^{w(0)} - w(0).$$

Pertinent Features of Solution Methods

The solution for w_1 was found by iteration, and the W(w) and S(w) equations were integrated by Simpson's rule with an analytical treatment in small intervals around $w(0)$ and w_1.

Date Solved: November 1970

at: GGA

References

1. R. Froehlich and S. R. Johnson, Reactor Excursions with Ramp Reactivity Insertion and Linear Temperature Feedback, General Atomic Report GA-8676 (1968).

2. R. Froehlich and S. R. Johnson, Exact Solution of the Non-Linear Prompt Kinetics Equations, A Benchmark Problem in Reactor Kinetics, Nukleonik, 12, 93 (1969).

Results

Period of power density oscillation = 0.8198 sec

Maximum power density = 6484.60 w/cm^3

Temperature rise per period = 409.9°C

EXHIBIT A: Tabular values of X, V, and S for $v(0) = 1 \times 10^{-2}$

EXHIBIT B: Tabular values of X, V, and S for $v(0) = 9 \times 10^{-3}$

EXHIBIT C: Tabular values of maximum power density, period of power density oscillation, and time of maximum reactivity as a function of V(0).

EXHIBIT A

Relation between V, S and $X = \sqrt{\alpha} \cdot t$

V(0) = 1.000-02

X	V	S
0.000	1.000-02	0.000
6.822-01	1.259-02	7.385-03
9.650-01	1.585-02	1.136-02
1.182+00	1.995-02	1.522-02
1.365+00	2.512-02	1.932-02
1.527+00	3.162-02	2.387-02
1.673+00	3.981-02	2.907-02
1.806+00	5.012-02	3.509-02
1.934+00	6.310-02	4.216-02
2.052+00	7.943-02	5.055-02
2.164+00	1.000-01	6.056-02
2.271+00	1.259-01	7.258-02
2.374+00	1.585-01	8.710-02
2.473+00	1.995-01	1.047-01
2.569+00	2.512-01	1.262-01
2.662+00	3.162-01	1.524-01
2.752+00	3.981-01	1.847-01
2.841+00	5.012-01	2.244-01
2.925+00	6.310-01	2.736-01
3.015+00	7.943-01	3.348-01
3.100+00	1.000+00	4.114-01
3.135+00	1.098+00	4.479-01
3.170+00	1.206+00	4.879-01
3.205+00	1.324+00	5.320-01
3.240+00	1.453+00	5.807-01
3.275+00	1.596+00	6.344-01
3.311+00	1.752+00	6.939-01
3.347+00	1.924+00	7.599-01
3.383+00	2.112+00	8.332-01
3.420+00	2.319+00	9.151-01
3.458+00	2.546+00	1.007+00
3.496+00	2.796+00	1.110+00
3.536+00	3.070+00	1.227+00
3.577+00	3.371+00	1.359+00
3.621+00	3.701+00	1.512+00
3.666+00	4.064+00	1.690+00
3.716+00	4.462+00	1.900+00
3.770+00	4.899+00	2.154+00
3.833+00	5.379+00	2.477+00
3.912+00	5.906+00	2.927+00
4.099+00	6.485+00	4.099+00

EXHIBIT B

Relation between V, S and $X = \sqrt{\alpha} \cdot t$

V(0) = 9.000-03

X	V	S
0.000	9.000-03	0.000
6.896-01	1.139-02	6.731-03
9.754-01	1.442-02	1.038-02
1.195+00	1.824-02	1.393-02
1.380+00	2.309-02	1.773-02
1.543+00	2.922-02	2.197-02
1.691+00	3.698-02	2.684-02
1.827+00	4.680-02	3.251-02
1.954+00	5.923-02	3.920-02
2.074+00	7.496-02	4.717-02
2.187+00	9.487-02	5.674-02
2.295+00	1.201-01	6.829-02
2.399+00	1.519-01	8.231-02
2.499+00	1.923-01	9.941-02
2.595+00	2.434-01	1.203-01
2.689+00	3.080-01	1.461-01
2.781+00	3.898-01	1.779-01
2.870+00	4.933-01	2.172-01
2.958+00	6.243-01	2.662-01
3.046+00	7.902-01	3.275-01
3.132+00	1.000+00	4.045-01
3.167+00	1.099+00	4.408-01
3.201+00	1.208+00	4.808-01
3.236+00	1.327+00	5.248-01
3.271+00	1.459+00	5.734-01
3.306+00	1.603+00	6.271-01
3.341+00	1.762+00	6.868-01
3.377+00	1.937+00	7.527-01
3.413+00	2.128+00	8.263-01
3.450+00	2.339+00	9.085-01
3.488+00	2.571+00	1.001+00
3.526+00	2.825+00	1.104+00
3.566+00	3.105+00	1.222+00
3.607+00	3.412+00	1.355+00
3.650+00	3.750+00	1.509+00
3.695+00	4.121+00	1.688+00
3.744+00	4.529+00	1.900+00
3.798+00	4.978+00	2.157+00
3.861+00	5.471+00	2.484+00
3.940+00	6.012+00	2.939+00
4.125+00	6.608+00	4.125+00

EXHIBIT C

Dimensionless Quantities v_1, P_x, and x^* as Functions of v_0

v_1 dimensionless maximum power density
P_x dimensionless period of power density oscillation
x^* dimensionless time of maximum reactivity
v_0 dimensionless initial power density

v_0	v_1	P_x	x^*
9.000-01	1.107+00	6.286+00	1.606+00
8.000-01	1.231+00	6.295+00	1.646+00
7.000-01	1.375+00	6.313+00	1.691+00
6.000-01	1.547+00	6.341+00	1.743+00
5.000-01	1.756+00	6.385+00	1.806+00
4.000-01	2.019+00	6.450+00	1.882+00
3.000-01	2.365+00	6.550+00	1.982+00
2.000-01	2.860+00	6.713+00	2.122+00
1.000-01	3.715+00	7.031+00	2.359+00
9.000-02	3.845+00	7.083+00	2.395+00
8.000-02	3.989+00	7.141+00	2.435+00
7.000-02	4.153+00	7.207+00	2.480+00
6.000-02	4.342+00	7.284+00	2.531+00
5.000-02	4.564+00	7.376+00	2.592+00
4.000-02	4.835+00	7.490+00	2.665+00
3.000-02	5.182+00	7.637+00	2.758+00
2.000-02	5.657+00	7.845+00	2.887+00
1.000-02	6.485+00	8.198+00	3.100+00
9.000-03	6.608+00	8.251+00	3.132+00
8.000-03	6.745+00	8.310+00	3.167+00
7.000-03	6.900+00	8.376+00	3.207+00
6.000-03	7.079+00	8.453+00	3.252+00
5.000-03	7.290+00	8.543+00	3.305+00
4.000-03	7.547+00	8.652+00	3.368+00
3.000-03	7.876+00	8.792+00	3.449+00
2.000-03	8.337+00	8.985+00	3.561+00
1.000-03	9.119+00	9.308+00	3.746+00
9.000-04	9.237+00	9.356+00	3.773+00
8.000-04	9.369+00	9.410+00	3.803+00
7.000-04	9.518+00	9.470+00	3.838+00
6.000-04	9.690+00	9.539+00	3.877+00
5.000-04	9.893+00	9.621+00	3.923+00
4.000-04	1.014+01	9.719+00	3.978+00
3.000-04	1.046+01	9.845+00	4.049+00
2.000-04	1.091+01	1.002+01	4.146+00
1.000-04	1.167+01	1.031+01	4.308+00
9.000-05	1.178+01	1.035+01	4.333+00
8.000-05	1.191+01	1.040+01	4.359+00

EXHIBIT C (Contd.)

v_0	v_1	P_x	x^*
7.000-05	1.206+01	1.046+01	4.390+00
6.000-05	1.222+01	1.052+01	4.424+00
5.000-05	1.242+01	1.059+01	4.465+00
4.000-05	1.267+01	1.068+01	4.514+00
3.000-05	1.298+01	1.080+01	4.577+00
2.000-05	1.342+01	1.095+01	4.664+00
1.000-05	1.416+01	1.122+01	4.810+00

BENCHMARK PROBLEM SOLUTION

Identification: 7-A1-2 Benchmark Problem ID.7-A1

Date Submitted: June 1972 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: January 1973 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: GAKIT[1] Finite-difference Solution

Mathematical Model: See Ref. 1.

Pertinent Features of Solution Methods

1. GAKIT was run with one group and reflecting boundary conditions.

2. The GAKIT flux equation:

$$\frac{\partial \Phi}{\partial t} = v[\nu(T)\Sigma_f - \Sigma_a(t)]\,\Phi$$

was adjusted to match the point reactor equation:

$$\frac{dP}{dt} = \left\{ \frac{At + B[T(t) + T(0)]}{\Lambda} \right\} P$$

by choosing the parameters listed below:

Parameter	Value
Σ_f	1.6×10^{-3} cm^{-1}
v	5×10^{5} cm sec^{-1}
$\Sigma_a(0)$	2.4×10^{-3} cm^{-1}
$\frac{d\Sigma_a}{dt}$	-2.0×10^{-4} cm^{-1} sec^{-1}
$\nu(0)$	2.5
$\frac{d\nu}{dT}$	-2.5×10^{-4} °C^{-1}

3. The heat-transfer coefficient to the coolant was set to a very small value to simulate the temperature equation in the problem statement.

Computer: UNIVAC 1108 Date Solved: November 1970

at: GGA

Program: GAKIT

Reference

1. R. Froehlich, S. R. Johnson, and M. H. Merrill, GAKIT--A One-Dimensional Multigroup Kinetics Code with Temperature Feedback, General Atomic Report GA-8576 (1968).

Results

Period of power density oscillation = 0.820 sec
Maximum power density = 6502 w/cm^3
Temperature rise per period = 410.1°C
EXHIBIT A Power density vs time compared with analytical result.
EXHIBIT B Temperature vs time compared with analytical result.
EXHIBIT C Power density and temperature vs time for different GAKIT time-step sizes.

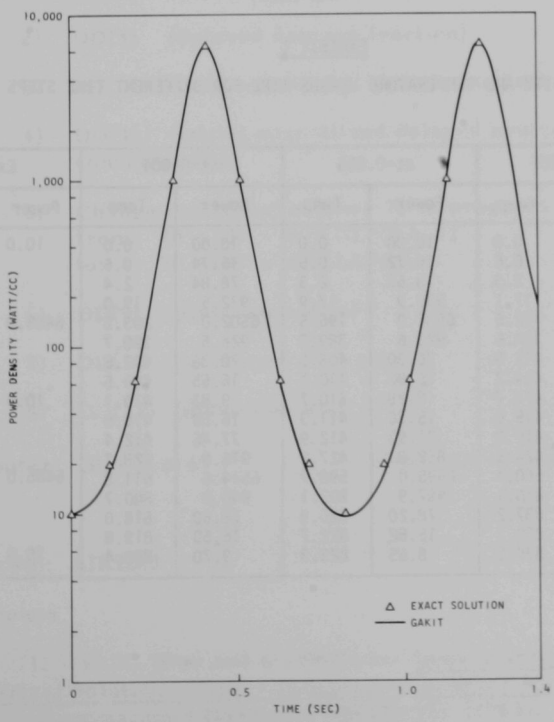


EXHIBIT A

EXHIBIT B

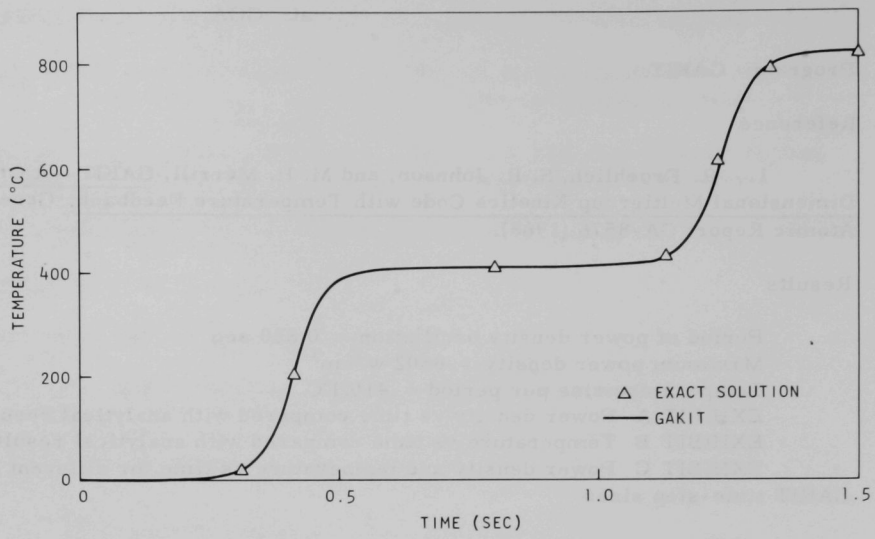


EXHIBIT C

POWER DENSITY AND TEMPERATURE VERSUS TIME FOR DIFFERENT TIME STEPS

Time	Δt=0.025		Δt=0.005		Δt=0.001		Exact	
	Power	Temp.	Power	Temp.	Power	Temp.	Power	Temp.
0.0	10.00	0.0	10.00	0.0	10.00	0.0	10.0	0.0
0.1025	16.71	0.6	16.72	0.5	16.74	0.6		
0.2050	78.06	2.3	78.63	2.3	78.84	2.4		
0.3075	908.7	17.1	930.9	17.9	932.5	19.0		
0.4100	6591.3	192.6	6535.0	196.5	6502.0	203.2	6485.0	204.9
0.5125	939.8	394.6	923.6	389.3	926.5	390.7		
0.6150	70.00	412.5	76.30	408.2	78.30	407.6		
0.7175	13.05	414.2	15.90	410.1	16.55	409.5		
0.8200	6.89	414.7	8.28	410.7	9.83	410.1	10.0	409.9
0.9225	10.48	415.0	15.36	411.3	16.50	410.6		
1.0250	45.80	416.0	71.93	412.9	77.46	412.4		
1.1275	528.1	424.5	852.8	427.2	916.0	428.7		
1.2300	6501.3	550.1	6595.0	598.7	6514.6	611.8	6485.0	614.9
1.3325	1482.5	810.1	987.9	802.1	940.0	800.7		
1.4350	87.40	837.2	78.20	820.9	78.60	818.0		
1.5375	12.21	839.1	15.52	822.7	16.50	819.8		
1.6400	4.79	839.5	8.65	823.3	9.70	820.4	10.0	819.8

BENCHMARK PROBLEM SOLUTION

Identification: 7-A1-3 Benchmark Problem ID.7-A1

Date Submitted: May 1971 By: W. H. Köhler and R. T. Perry (Texas A&M) (Name and Organization)

Date Accepted: January 1973 By: D. A. Meneley (Ontario Hydro) (Name and Organization)

Descriptive Title: AIREK3[1] Finite-difference Solution

Mathematical Model: See Ref. 1.

Pertinent Features of Solution Methods

The following input parameters and code change were necessary to adapt AIREK3 to the benchmark problem. Remaining input was as given in the problem statement.

1) DD(1) (number of delayed neutron groups) = 1

2) DD(3) (delayed neutron fraction) = 1

3) DD(31) (delayed neutron decay constant) = 0

4) DD(46) (initial normalized delayed neutron concentration) = 0

5) Change statement number 70 in program
from: 70 ALN = B/ELL*(RA-1)
to: 70 ALN = B/ELL*RA

6) DD(9) (time-step size) = 3.28×10^{-3}

7) DD(10) (minimum step size) = 3.28×10^{-3}

8) DD(11) (maximum step size) = 3.28×10^{-3}

Computer: IBM 360/65 Date Solved: May 1971

at: Texas A&M

Program: AIREK3

Reference

1. L. R. Blue and M. Hoffman, Generalized Program for the Numerical Solution of Space Independent Reactor Kinetics Equations, [illegible] Document AMTD-131 (1963).

Results

Period of power density oscillation = 0.8200 sec

Maximum power density = 6484.49 w/cm^3

Temperature rise per period = 409.86°C.

BENCHMARK PROBLEM SOLUTION

Identification: 7-A1-4 Benchmark Problem ID.7-A1

Date Submitted: July 1971 By: J. J. Kaganove (ANL)
(Name and Organization)

Date Accepted: January 1973 By: H. Greenspan (ANL)
(Name and Organization)

Descriptive Title: R123JUMP - Reactor Kinetics

Mathematical Model: Collocation Solution[1,2]

Pertinent Features of Solution Methods

The following input parameters were used in R123 to solve the benchmark problem.

1) Number of delayed neutron groups = 1

2) Delayed neutron fraction = 0.0

3) Relative accuracy error control = 10^{-5}

Computer: CDC 3600 Date Solved: June 7, 1971

Program: R123JUMP

References

1. J. J. Kaganove, Numerical Solution of the One-group, Space-independent Reactor Kinetics Equations for Neutron Density Given the Excess Reactivity, Argonne National Laboratory Report ANL-6132 (Feb 1960).

2. H. Greenspan, C. N. Kelber, and D. Okrent (Eds.), "Computing Methods in Reactor Physics," Gordon and Breach Science Publishers, New York (1968).

Results

Period of power density oscillation = 0.81975 sec

Maximum power density = 6484.601 w/cm^3

Temperature rise per period = 409.8741°C.

BENCHMARK PROBLEM

Identification: 7-A2 Source Situation ID.7

Date Submitted: June 1972 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: July 1972 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Point Reactor, Ramp Reactivity Insertion, and Linear Temperature Feedback with Constant Heat Removal

Reduction of Source Situation

1. No delayed neutrons;

2. Linear reactivity insertion; $f(t) = At$

3. Linear temperature feedback; $J = 1$

and

$$b_j[\omega_j(t), \omega(0)] = B[T(t) - T(0)]$$

with

$$\frac{dT}{dt} = \frac{1}{C}[P(t) - P(0)].$$

Data

Initial power density $P(0) = 10\ w/cm^3$

Initial temperature $T(0) = 0°C$

Generation time $\Lambda = 5 \times 10^{-4}$ sec

Input reactivity ramp rate $A = 5 \times 10^{-2}\ sec^{-1}$

Temperature coefficient $B = -1 \times 10^{-4}\ (°C)^{-1}$

Heat capacity $C = 2\ \frac{w \cdot sec}{cm^3 \cdot °C}$

Expected Primary Results

1. Period of power density oscillation

2. Maximum power density

3. Temperature rise per period.

Possible Additional Results

1. Plot of temperature vs time

2. Plot of power density vs time

3. Sensitivity of results to time-step length

4. Tabular results at selected time points.

Best Solution Available: Analytic result.

BENCHMARK PROBLEM SOLUTION

Identification: 7-A2-1 Benchmark Problem ID.7-A2

Date Submitted: June 1972 By: R. H. Brogli (GGA)
(Name and Organization)

Date Accepted: January 1973 By: D. A. Meneley (Ontario Hydro)
(Name and Organization)

Descriptive Title: Analytic Solution of Prompt Point Reactor Problem with Linear Temperature Feedback and Constant Heat Removal

Mathematical Model: Same as that described in ID.7-A1-1, except for the definitions

$$\alpha = \frac{A}{\Lambda} + \frac{BP(0)}{C\Lambda'}$$

and

$$T(t) = T(0) + \frac{\sqrt{\alpha}}{\gamma C} S(x) - \frac{BP(0)}{C} t$$

Pertinent Features of Solution Methods: See Solution ID.7-A1-1

Date Solved: June 1972

at: GGA

References: See ID.7-A1-1

Results

Period of power density oscillation = 0.8162 sec

Maximum power density = 6562.0 w/cm^3

Temperature rise per period = 408.11°C.

BENCHMARK PROBLEM SOLUTION

Identification: 7-A2-2 Benchmark Problem ID.7-A2

Date Submitted: May 1971 By: W. H. Köhler and R. T. Perry (Texas A&M) (Name and Organization)

Date Accepted: January 1973 By: D. A. Meneley (Ontario Hydro) (Name and Organization)

Descriptive Title: AIREK3[1] Finite-difference Solution with Constant Heat Removal

Mathematical Model: See Ref. 1.

Pertinent Features of Solution Methods: See Solution ID.7-A1-3

Computer: IBM 360/65 Date Solved: May 1971

at: Texas A&M

Program: AIREK3

Reference

1. L. R. Blue and M. Hoffman, Generalized Program for the Numerical Solution of Space Independent Reactor Kinetics Equations, Atomics International Document AMTD-131 (1963).

Results

Period of power density oscillation = 0.817 sec

Maximum power density = 6558 w/cm^3

Temperature rise per period = 408.07°C

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