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STUDY ON FLOW INSTABILITIES IN TWO-PHASE MIXTURES

by

M. Ishii

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M. Ishii

Reactor Analysis and Safety Division

March 1976

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NOMENCLATURE

A_c	Flow area per channel
C_k	Kinematic wave velocity, Eq. 30
C_m	Two-phase friction-factor coefficient ($f_m = C_m f_s$)
D	Hydraulic diameter
f_0	Friction factor in inlet section
f_m	Two-phase friction factor
f_s	Single-phase friction factor at saturation
g	Gravity
i	Enthalpy
i_1	Subcooled inlet enthalpy
j	Mixture volumetric flux, Eq. 30
k	Orifice coefficient
ℓ	Length of heated channel
P	Pressure
P_s	System pressure
Q	Characteristic function
q''_w	Wall heat flux
S	Perturbation parameter, $S = a + \sqrt{-1}\,\omega$
t	Time
v	Velocity
V_{gj}	Vapor drift velocity ($= v_g - j$)
X	Vapor quality
z	Axial coordinate
α	Vapor void fraction
Γ_g	Mass generation for vapor phase
$\Delta\rho$	Density difference
ΔP_{ex}	Total pressure drop of channel
Δi_{fg}	Latent heat
Δi_s	Maximum possible subcooling
δX	Perturbation of X
ϵ	Perturbation magnitude ($\ll \bar{v}_{fi}$)
λ	Heated length
ξ	Heated perimeter
ρ	Density
Ω	Reaction (phase-change) frequency
ω	Angular frequency
τ_{12}	Time delay in heated-liquid region
τ_{23}	Time delay in mixture region

Subscripts

e	Exit
f	Liquid
g	Vapor
i	Inlet
m	Mixture
s	Saturation

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ABSTRACT

Thermally induced flow instabilities can result in operational and safety problems to various components of interest to power, heat-transfer, and process systems. The appearance of these instabilities may not only degrade the performance of the systems, but can also result in premature burnout and control problems which can be destructive.

The various mechanisms that can induce these flow instabilities in two-phase flow systems are reviewed and their relative importance discussed. In view of its practical importance, the density-wave instabilities have been analyzed in detail based on the one-dimensional two-phase flow formulation.

The dynamic response of the system to the inlet flow perturbations has been derived from the model; thus the characteristic equation that predicts the onset of instabilities has been obtained. The effects of various system parameters, such as the heat flux, subcooling, pressure, inlet velocity, inlet orificing, and exit orificing on the stability boundary have been analyzed. In addition to numerical solutions, some simple stability criteria under particular conditions have been obtained. Both results have been compared with various experimental data, and a satisfactory agreement has been demonstrated.

I. INTRODUCTION

A. Relevance of the Problem

Two-phase flow instabilities can introduce operational and safety problems to systems and components of interest to power-generating utilities, chemical-process industries, and aerospace industries. Examples of such systems and components include nuclear reactors, liquid rocket engines, heat exchangers, cryogenic equipment, boilers, evaporators, and various chemical-process units.

The existence of thermohydraulic instabilities in two-phase mixtures has been known for some time, and the appearance of these instabilities at either subcritical or supercritical pressures is undesirable. They may not only degrade the performance of the system, but can also result in premature burnout and control problems that can become destructive.

Therefore, it is desirable to accomplish following objectives:

1. Derive from the appropriate mathematical model both stability maps and stability criteria that can be used to predict the onset of these two-phase flow instabilities.

2. Obtain correct and important similarity groups that can reduce the number of governing parameters.

3. Present the results in a parametric plane that is useful to researchers and designers.

4. Obtain from the analysis simple stability criteria that can be used for design purposes.

5. Corroborate the predicted results with experimental data.

B. Mechanism of Two-phase Flow Instabilities

Two-phase flow instabilities can be divided into two main categories: static instabilities and dynamic (or oscillatory) instabilities.

The static instabilities are related to discontinuous changes in steady-state operational conditions. In other words, a steady-state flow becomes unstable under certain conditions, and it translates to another quite different operational condition. The causes of the static flow instabilities can be interfacial instabilities, some relation between flow and pressure drop, or changes in heat-transfer mechanisms. The most commonly observed static instabilities are:

1. Flow-regime transitions.

2. Flow excursion.

3. Dryout and quenching.

The flow-regime transitions[1-3] can be caused by the various mechanisms mentioned previously. Bubbly to slug flow, slug to churn flow, churn to annular flow, annular to drop-annular flow, and countercurrent to concurrent flow transitions are important examples of the flow-regime changes mainly due to interfacial instabilities. The last two flow-regime transitions are of particular interest to various chemical-engineering fields, and are known as the droplet-entrainment inception and flooding, respectively.

Excursive instabilities were first analyzed successfully by Ledinegg in 1938.[4] Under certain conditions, the curve of steady-state system pressure drop versus flow has a negative slope; hence, since the flow rate is not a single-valued function of the pressure drop, a flow excursion may occur. In his analysis, Ledinegg assumed that the heat flux was uniform, but later other investigators[5,6] extended this criterion to more general cases. It is also known that the flow-excursion instability can induce cyclic oscillations called pressure-drop oscillations.[7,8]

Dryout and quenching can be considered as flow-regime transitions due to changes in heat-transfer mechanisms. When a hot surface is no longer in direct contact with the liquid phase, the heat transfer is very much reduced from that of the nucleate boiling or liquid-film evaporation. Dryout[1,2,9] in heat-generating systems such as nuclear reactors is accompanied by a sudden rise of wall temperature, which can be destructive. Quenching or rewetting of a hot dry surface is essential to emergency core-cooling systems in light-water nuclear reactors in order to protect the cladding surface.[10-15] When the temperature of the heating surface exceeds the so-called Leidenfrost temperature,[16] the vapor generated by the change of phase forms a continuous film between the liquid and the solid. Unless the hot-wall temperature is reduced, rewetting cannot occur. Therefore the dryout and rewetting front are the flow-regime transition points that are governed by heat-transfer mechanisms.

The oscillatory-type instabilities are rather complicated dynamic phenomena, which may be divided into four different mechanisms:

1. Instabilities due to kinematic-wave (density-wave) propagation.
2. Instabilities due to pressure-wave propagation.
3. Instabilities due to thermodynamic nonequilibrium.
4. Instabilities due to flow-regime change.

The most common oscillations[17-27] encountered in heated channels are low-frequency, i.e., density-wave, oscillations. There is considerable evidence that some relationship exists between the residence time of the particle and the period of the oscillations. Therefore, several analyses have been formulated and carried out by considering the propagation of density waves and the attendant time-lag effects. Inlet flow perturbations in a heated channel result in delayed mixture-density changes throughout the channel. These disturbances in the mixture density affect the local mixture velocity and the total pressure drop in a channel. Under certain conditions, the inlet flow perturbations and the internal pressure-drop perturbations satisfy a self-exciting relation such that sustained oscillations with considerable amplitudes appear in the system. This instability is caused by finite time necessary (1) for the enthalpy wave to propagate in the subcooled-liquid region and (2) for the density or the void-fraction wave to propagate in the mixture region. These finite

propagation times induce the time-lag effects and phase shift in the channel-pressure-drop response, leading to the oscillatory instabilities.

It was found both experimentally[25] and analytically[28,29] that there could be several modes of the density-wave instabilities. The most commonly encountered mode is the lowest-order instability (lowest frequency); however, higher-order instabilities or compound instabilities, which produce heat effects, are possible. In view of its great importance in practical applications, the density-wave instability will be discussed in more detail in the subsequent sections. The essence of the stability analysis based on the drift-flux formulation is also given there.

Although high-frequency oscillations, which are associated with the propagation of pressure waves, have been observed in some experiments,[30] they may be of less importance for practical applications than the low-frequency oscillations. The period of the oscillations is of the same order of magnitude as the residence time of the acoustic waves. These high-frequency oscillations have been also encountered in subcooled boiling;[31] however, they are much more common in combustion processes. The high-frequency instabilities are called "screaming" in the field of rocket-engine-combustion instabilities and have been studied quite extensively.[32]

The instabilities observed by Jeglic and Grace[33] in experiments conducted with water at low pressures flowing through a smooth pipe were apparently due to thermodynamic nonequilibrium. Under these conditions and because of poor nucleation, the liquid can become highly superheated. However, once a bubble is nucleated, it grows explosively (because of the high liquid superheat), ejecting the liquid from the duct while interrupting the inlet flow. After the liquid is ejected from the duct, the pressure decreases, new liquid enters and becomes superheated, and the process repeats itself.

Chugging instabilities can occur for liquid metals at low pressures associated with the reentry of liquid into voided channels.[34,35] In this case the instabilities are caused by a large ratio of liquid to vapor density and the rapid evaporation from thin liquid films at the lower end of the fuel pins. When the liquid enters the voided channel, the high evaporation leads to a higher vapor velocity and eventual pressurization. Thus the level of the liquid at the bottom is pushed down, reducing the evaporation rate and the vapor velocity. This will depressurize the channel, the liquid reenters from the bottom again, and the process repeats.

Geysering instabilities occur mainly in a vertical channel of a natural-circulation loop[36] or in a closed-end tube. The main cause of the instabilities is the reduced total pressure drop for two-phase flow due to smaller hydrostatic head. As soon as sufficient vapor is generated in the channel such that the pressure drop is smaller than the liquid head, an expulsion of the mixture and the reentry of liquid follow.

The disturbances created by flow-regime changes can also produce oscillatory behavior. Wallis and Heasley[37] analyzed the slug flow in a long, large-diameter riser and concluded that cyclic variation of vapor content could produce periodic fluctuations of loop flow rate.

So-called thermal oscillations[8] are associated with the instability of the liquid film and accompanied by large fluctuations in wall temperatures in a constant-heat-flux system. As the heat-transfer coefficient of the two-phase regime oscillates between the wet- and dry-wall conditions, the wall superheat fluctuates accordingly to accommodate the constant heat generation in the wall. In general, the thermal oscillations are triggered by other hydrodynamic instabilities, such as the density-wave oscillations.

Further references on various instabilities appear in a review by Bouré et al.[38]

II. STATE OF THE ART OF DENSITY-WAVE-INSTABILITY ANALYSIS

Numerous experiments are concerned with density-wave instabilities.[17-27] It was found that the period of the oscillations is closely related to the residence time of the kinematic wave (density wave), so that the product of the frequency and the residence time remains approximately constant. From the above observation, this type of instability is called a "density-wave instability." This density refers to the mixture density, and the wave propagation of concern is that of the void fraction and not that of the compressible pressure wave. In a two-phase flow system the mixture density can change, even though each constituent phase is incompressible, because the mixture density is also a function of the void concentration.

It was also observed experimentally that the density-wave instabilities are strongly related to the response characteristics of the total channel pressure drop and are therefore not caused by local phenomena. Consequently, several important parametric observations have been made on the effects of the inlet and exit flow restrictions, single- and two-phase frictional pressure drop, heat flux, subcooling, inlet flow, and system pressure. In general, any increase in the frictional pressure drop in the liquid region has a stabilizing effect, whereas the increase in two-phase region has a destabilizing effect. When the channel geometry is fixed, an increase in the inlet velocity has a significant stabilizing effect in terms of the heat flux. On the other hand, increased heat flux always results in a smaller stability margin or flow instabilities. Furthermore, an increase in the system pressure is a stabilizing effect in terms of the quality, since at higher pressure the density change due to phase change is less significant.

The effect of parallel channels is generally stabilizing compared with that of an identical single channel.[39] This may be due to the dumping effect of one channel with respect to others unless they are oscillating completely in phase. In other words, the parallel channels have a tendency to equalize the pressure drop or the pressure gradient (if they are interconnected) by adjusting the ratio of the mass flows or by interchannel mixing.

Numerous analytical studies have been directed at obtaining a better understanding of thermally induced flow oscillations, determining their mechanism, and deriving stability criteria. Although most of the investigations have been done in the last decade, we cannot omit two early studies.[33,40]

The initial analysis concerned with the transient operation of boiler channels was done in 1953 by Teletov and Serov,[40] who were able to obtain transfer functions for a distributed parameter system that gave the response of the fluid enthalpy and density to perturbations of the heat flux. The derivation of this transfer function was based on a simple but important transformation which is discussed in subsequent sections. Their analysis was limited, however, to a homogeneous flow (in which the effect of the relative velocity between phases is neglected) and a thermodynamic-equilibrium condition.

The excellent studies by Crocco and Cheng, which are summarized in their book,[32] are not directly related to the two-phase flow oscillations. They were concerned with instabilities in rocket-engine combustion; however, their analysis has offered valuable information on the instability mechanisms of heated channels due to time-lag effects. Furthermore, by comparing theoretical results with experimental data, Crocco and Cheng demonstrated the applicability and reliability of the linearized (small-disturbance) theory to analyses and studies of combustion instabilities.

Besides these early studies by Teletov and Serov[40] and by Crocco and Cheng,[32] numerous analytical investigations have been conducted in this field.

In general, two approaches have been followed. The first is based on phenomenological models obtained from the assumed similarity with a simple mechanical system or an electrical circuit having excitations. The second approach is to formulate the problem from the conservation laws for the mixture.

Because of simplicity, many of the early studies belong to the first group; however, the applicability of their results is severely restricted by the nature of the model. Needless to say, one must supply several experimental coefficients or correlation functions into these formulations, since, strictly speaking, the assumed methods are not based on conservation laws.

Shortcomings of such analyses are evidenced by the fact that the values of the coefficients or the functions change with operating conditions and design configurations. We may point out two reasons for this deficiency:

1. Inaccuracy of the assumed models.
2. The lack of knowledge of similarity groups which characterize phenomena.

This latter aspect constitutes one of the most important problems in the general analyses of two-phase flow systems. We shall close our discussion of the phenomenological approach without referring to individual models, because the models seem to be of limited value. Furthermore, the phenomenological approach can be looked upon as a transitional method of analysis that leads to the second approach based on the conservation laws.

In general, a theoretical approach must be based on the conservation equations, appropriate constitutive equations, and imposed boundary conditions, together with correct physical approximations. In particular, if the system has complicated characteristics such as existence of interfaces, boiling heat transfer, change of two-phase flow regimes, turbulent flow, and time-dependent variables, the formulation of the model should be manageable, and, at the same time, the model should state the basic physical nature of the phenomenon. From this point of view, two models (a homogeneous-flow and a slip-flow model) have

been used in various analyses. The first model neglects the relative velocity between the two phases, whereas the slip-flow model takes this important characteristic of two-phase flow systems into account.

Besides this classification into homogeneous- and slip-flow models, generally two distinct methods have been applied to obtain a solution of the problem. In the first, the system of partial differential equations is linearized by assuming small disturbances about a steady state. The response of the system to various perturbations, as well as stability criteria, is then obtained by using standard techniques. The second method is based on a numerical solution whereby the set of partial differential equations is solved by numerical methods.

As previously mentioned, Teletov and Serov[40] were the first to formulate the dynamic problem of two-phase flow systems, although the analyses of similar dynamic problems of instabilities in rocket-engine combustion had already been studied in detail by Summerfield,[41] Tsien,[42] and Crocco and Cheng,[32] among others. The study of Teletov and Serov[40] is limited to low-frequency oscillations and deals with the transient responses of enthalpy and density to a heat-flux disturbance. It also takes into account the effect of the wall heat capacity. By considering only low-frequency oscillations, Teletov and Serov were apparently the first to realize that the density could be considered as a function of enthalpy only and not of both enthalpy and pressure. Note, however, that, Teletov and Serov did not integrate the momentum equation; consequently, the characteristic equation was not derived. Furthermore, their analysis was limited to homogeneous flow and thermodynamic equilibrium.

The importance of Serov's first analysis[40] rests on:

1. Decoupling the momentum equation from the energy and continuity equations.

2. Deriving the transfer function for a distributed parameter system.

3. Finding a simple transformation that relates the divergence of the velocity to the heat flux:

$$\frac{\partial v_m}{\partial z} = \Omega \equiv \frac{\Delta\rho}{\rho_g \rho_f} \frac{1}{\Delta i_{fg}} \frac{q_w'' \xi}{A_c}, \tag{1}$$

where v_m is a velocity of the mixture and Ω is a reaction frequency of the phase change, which can be expressed by the densities of each phase ρ_g and ρ_f, the density difference $\Delta\rho$, the latent heat Δi_{fg}, and the heat input $q_w'' \xi / A_c$.

An analysis similar to that by Serov but including the response of the pressure was given by Terano.[43] The system was divided into three parts: liquid, mixture, and superheated vapor region. However, he used a simple capacitance model for the pressure response to reduce the generality of the solution for the same reasons discussed in connection with the phenomenological models.

Wallis and Heasley[37] used a model similar to that of Serov. Using Lagrangian coordinates, they integrated the energy and continuity equations for a disturbed inlet flow. However, in contrast to the analysis of Serov, they neglected the effect of a variable heat-transfer coefficient. They obtained the characteristic equation from the momentum balance and used the Nyquist criterion to discuss the stability. They also used $\rho_m = \rho_m(i_m)$ and rederived Eq. 1. As in the analysis of Serov, their analysis is limited to the thermodynamic equilibrium, homogeneous flow, and low-frequency oscillations.

About 10 yr after his original paper, Serov[44,45] integrated the momentum equation and obtained the characteristic equation for a distributed parameter system. His analysis takes into account the variation of inlet flow and heat-transfer coefficient, but neglects the displacement of the boiling boundary. The characteristic equation derived is a fifth-order exponential polynomial with two time delays. It has been solved for the stability boundaries in a simplified form by the D-partition method. Some qualitative success was shown in the ζ plane (inlet orifice).

Bouré[46,47] used a model similar to that of Serov. Consequently, his analysis is applicable to thermodynamic equilibrium, homogeneous flow, and low-frequency oscillations. Bouré, like Serov, assumed that $\rho_m = \rho_m(i_m)$, expressed the continuity equation in terms of Lagrange's differential equation, and independently rederived Eq. 1. Bouré also integrated the momentum equation and thus obtained a characteristic equation for a distributed parameter system. The characteristic equation given in Ref. 46 is a fifth-order exponential polynomial with two time delays. The coefficients of polynomials differ from those of Serov[44,45] because the analysis[47] accounts for the variation of inlet flow and the displacement of a boiling boundary (which Serov neglected), but neglects the wall heat capacity (which Serov included). Satisfactory agreement with experimental data is reported in Ref. 48, where the comparison is shown in the dimensionless subcooling and inlet velocity plane. A parametric study is also given in Ref. 48.

The method of Bouré[48] was used by Zuber[49] to analyze flow instabilities in the near- and super-critical thermodynamic region. However, Zuber's analysis differs in two aspects from that in Ref. 48: (1) The constitutive equations are different, and (2) the resulting characteristic equations are different. Under some conditions he was able to obtain a simple algebraic criterion that was useful as a design criterion for the friction-dominated flow at high pressure.

So far we have discussed the analyses based on the homogeneous-flow model and a condition of thermodynamic equilibrium. Such conditions can be attained with high flow rates at high reduced pressures. However, at lower pressures the effects of relative velocity and of thermodynamic nonequilibrium become important and may not be neglected.

As is mentioned in Refs. 50 and 51, the traditional "slip"-flow models were not formulated in terms of the center of mass of the mixture; therefore

these models use several different expressions for the mixture density or for the slip ratio. In the formulation of a dynamical model, properly averaged two-phase flow equations based on the conservation law should be used.[52,53]

Besides this basic shortcoming of traditional formulations, almost all "slip"-flow models have been solved directly by computers.[54-57] Meyer and Rose[54] were early users of this method of applying a momentum integral and a finite-difference method. The works by Jones[55] and by Carver,[19] among others, are notable in their detailed treatments of various effects. These direct methods require expensive computer time and great care in the programming in order to avoid numerical instabilities. We may quote Carver (Ref. 19, p. 4) in this connection: "Such approaches require considerable expensive computer time, the expense increasing with the degree of sophistication. Again they depend heavily on void, pressure drop, and heat transfer correlations used, and under certain conditions the validity of these is questionable."

In the analysis, the set of simultaneous partial differential equations was solved step by step in the time domain with one parameter disturbed, but all others were held fixed. This approach, in addition to being expensive for parametric studies, does not provide an insight into the physical aspects of the problem. Consequently, it is not very helpful in advancing the understanding of the phenomenon. However, these computer codes can be useful as a design tool for complex systems such as nuclear reactors. Furthermore, with computers a nonlinear analysis is also possible. This has an advantage over a linear theory when ultimate nonlinear responses in an unstable region or responses to large changes in steady-state operational conditions are required.

Zuber[50] formulated the problem in terms of the drift-flux model in order to take into account the effect of slip; then he used a small-perturbation analysis to obtain a characteristic equation. This characteristic equation was solved in the stability plane, i.e., nondimensional subcooling number N_{sub} versus phase-change number N_{pch}.[28] The model has been extended to non-uniformly heated systems[29] and a parametric study concerning the system pressure, heat flux, inlet subcooling, mass flow, inlet and exit flow restrictions, and various friction factor models has been carried out. A very simple algebraic stability criterion was also developed.[29] This simple criterion has proved to be almost as good as the exact solution, except for very small inlet subcooling. The model[28,50] has also been extended to the case of thermal nonequilibrium by Saha.[26] The above various stability criteria based on the drift model were compared to the experimental data from a Freon-113 loop.[26,27] The agreement of the equilibrium theory[28,29] with the experimental data was shown to be satisfactory. It was also found that subcooled boiling has an important stabilizing effect at small inlet subcooling. This has been predicted by the nonequilibrium model.[26]

III. DRIFT-FLUX FORMULATION APPLIED TO DENSITY-WAVE INSTABILITY

A. Thermodynamic Process

To understand the mechanism of the thermally induced flow oscillations and to formulate the mathematical model that describes the real physical system, we must examine the thermodynamic processes and the flow characteristics of the system. The typical components of the system of interest are shown in Fig. 1. They consist basically of four different regions:

Upstream unheated region (A)
Heated liquid region (B) } Single-phase

Heated mixture region (C)
Downstream unheated region (D) } Two-phase

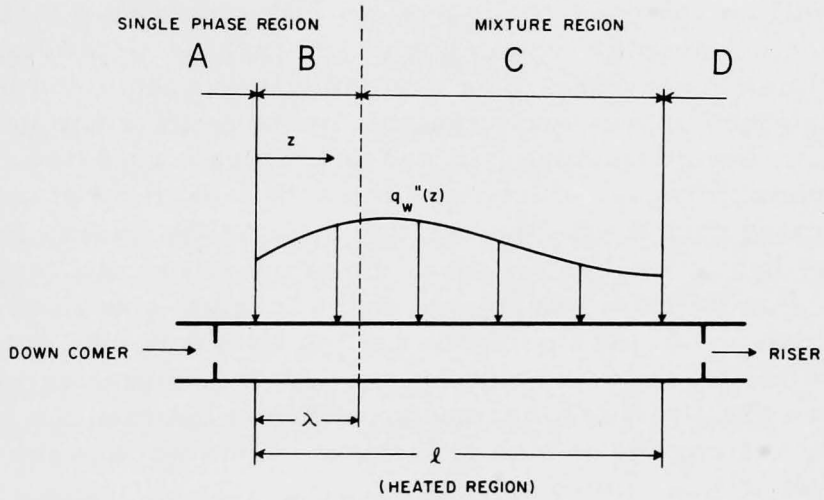


Fig. 1. System Used for Analysis of Density-wave Instability

The system of interest extends to the components where the thermally induced flow instabilities can be affected in a systematic manner. If the system consists of a single heated channel without any bypass (see Fig. 2), a large number of the components in the loop should be considered. This is because any disturbance propagating in the loop has a definite functional relation in terms of time and space lag to the heated section and, therefore, can influence the stability of the system. On the other hand, if the heated section consists of multiple channels which coverage to one channel at the riser and downcomer with sufficiently large volume capacitance (see Fig. 3), it is sufficient to consider the system between two volume reservoirs. In this case, the systematic form of disturbance propagation is effectively insulated from the remaining portion of the system, i.e., from the pump and the turbine, etc.

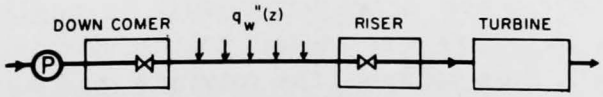


Fig. 2

Single Heated-channel System

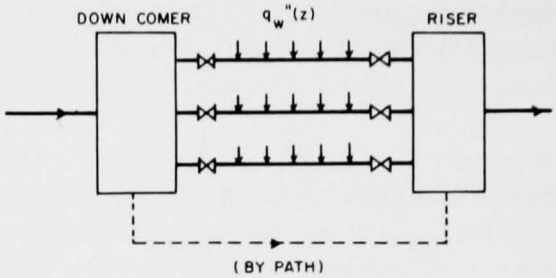


Fig. 3

Multiple Heated-channel System

The case of the single-heated-channel system is, therefore, far more complicated than the case of a multichannel system and requires information on the dynamic responses of the pump, turbine, etc. However, in view of practical applications, such as nuclear reactors and conventional power plants with boiling, it is sufficient to consider the second case.

For generality, we imagine two volume capacitances that can insulate any systematic propagation of disturbances. Then the system between these capacitances will be analyzed. This system may or may not include such accessories as the downcomer, riser, pump, and turbine, depending on the positions of the volume reservoirs. The system will be subdivided into the four previously mentioned approximate regions, namely, upstream unheated (A), heated liquid (B), heated mixture (C), and downstream unheated (D) regions. The thermodynamic process starts with the subcooled fluid of enthalpy i_1, entering the heated duct B with the velocity v_{fi}. As the energy is being transferred from the heated wall to the fluid, the temperature and the enthalpy i_f will increase. Due to the developments of the thermal boundary layer or the superheat capacitance of the liquid, the boiling boundary may not coincide with the point where the bulk-liquid enthalpy reaches the saturation value i_{fs}. In our analysis, we shall assume thermal equilibrium between the phases, which is a reasonable assumption at high pressures. Thus we take the boundary between B and C at $z = \lambda$, where $i_f = i_{fs}$.

In region C, the phase change takes place, and the mixture enthalpy i_m and void fraction α increase toward the end of C as more heat is added to the fluid. For a number of systems of practical interest, particularly for systems at high reduced pressures, it is reasonable to assume that this process takes place at an approximately constant pressure, since the pressure drop is relatively small compared with the absolute pressure of the system.

The processes in both A and D, i.e., the unheated regions, can be considered as isenthalpic. Furthermore, the complete liquid phase occupies the upstream unheated region A. Thus, the assumption of incompressibility is appropriate. On the other hand, in the downstream unheated region D, the mixture of vapor and liquid enters either into the steam separator or into the volume capacitance directly. From the previous discussion, either of the system components can be considered as the end of the system of interest. Therefore, from this point any systematic wave propagation will be neglected. Furthermore, we assume that the pressure-drop effect on the mixture properties in region D is negligibly small; thus we treat the mixture in D as isochoric.

The assumption that the effect of pressure variation on the thermodynamic properties of each phase can be neglected is implied by the assumption that the density is a function of enthalpy only and not of both enthalpy and pressure. This condition will be valid if the two-phase Eckert number is smaller than unity. From this limitation we can see that the assumption holds only if the rate of propagation of disturbance is much slower than the velocity of the pressure wave; otherwise these two waves interact with each other. Thus, under this assumption, only low-frequency oscillation can be analyzed.

B. Transport Process

The transport processes of mass, momentum, and energy are best understood from the field equations with appropriate constitutive relations.[52,53] In our analysis the drift-flux formulation will be used, and some further simplifications of this one-dimensional model will be made. Now, let us examine the transport process in the four different regions separately.

In the upstream unheated region A, the liquid flows incompressibly and isenthalpically; hence, only the momentum transport is important. The kinematics of the fluid (velocity field) can be obtained directly from the continuity equation. From a knowledge of the velocity field, we can obtain the acceleration, gravity, frictional, valve, and orifice pressure drops.

Since in the liquid heated region B the fluid is still considered to be incompressible, the velocity field can be immediately obtained from the mass-conservation law. The boundary between B and C will be calculated from the energy equation under thermal-equilibrium conditions and by neglecting any dissipation effect on the liquid enthalpy in comparison with the heat input. The pressure drops can be calculated by the same method as used in A.

In the heated-mixture region C, the transport processes are complicated. Following the discussion of the thermodynamic process, we assume that the density is a function only of the enthalpy and not of both the enthalpy and pressure. This important assumption, first used by Teletov and Serov,[40] permits us to decouple the momentum equation from the continuity and energy equations. Thus, again we can proceed with the integration of the latter two equations and obtain the kinematics of the fluid independent of the dynamic conditions imposed by the momentum equation. By knowing the density and velocity of the mixture, we can calculate the various pressure drops. For simplicity and because of the lack of knowledge on the constitutive equation, we also assume that the capillary body force is negligible.

From the previous discussion, we assume that the mixture in the downstream unheated region D is isochoric and isenthalpic. This does not imply constant density, since the mixture entering region D may have different densities depending on the upstream disturbances. The kinematics of the fluid can be obtained from the mass-conservation law, whereas the pressure drop will be calculated in a way similar to that used in the boiling region. Furthermore, the pressure drop at the orifice or valve will be calculated by using the orifice coefficients.

C. Time Lag and Space Lag

It is useful to consider the process from the Eulerian and Lagrangian points of view. If we follow a fluid particle entering section A at time τ_0 and study the change of its properties from the mass center of that particle in the time coordinate, our observation is from the Lagrangian or particle-coordinate point of view (see Fig. 4). The center of mass enters in regions B, C, and D at times τ_1, τ_2, and τ_3, respectively, and leaves region D at τ_4. The transition from B to C, i.e., the inception of bulk boiling, occurs when the liquid enthalpy increases due to the heat input and reaches the saturation liquid enthalpy. In region C, the enthalpy of the mixture continually increases due to the evaporation. The residence time of the particle in region B is denoted by τ_{12}, and the total residence time in the heated section by τ_{13}. In a similar manner, we can also define τ_{01} and τ_{34}. Since each residence time expresses the time necessary to bring about specific changes in the position or properties, it will be called the time lag. Except when the inlet enthalpy is disturbed, the time lag τ_{01} has no significant physical meaning other than that it represents the residence time, since the disturbances of the velocity, pressure, etc., propagate with infinite velocities. On the other hand, the time lags τ_{12}, τ_{13}, and τ_{34} are significantly related to the propagation of disturbances and hence are important for the stability analysis.

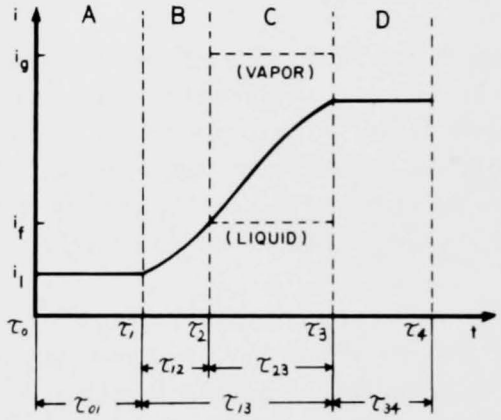


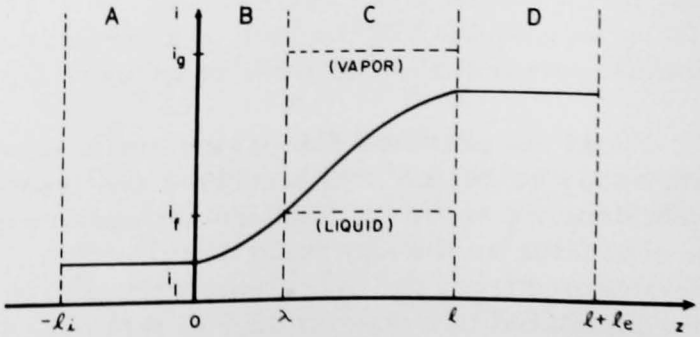
Fig. 4

Lagrangian Description of Enthalpy

It is of interest also to consider the spatial or Eulerian description of the process (see Fig. 5). In this case the time lags are replaced by the space lags and hence indicate the boundaries of the various operational regions.

Fig. 5

Eulerian Description of Enthalpy



Among the indicated four space lags, that corresponding to the time lag τ_{12} and denoted by λ is particularly important, since it defines the boundary between the liquid and the mixture regions, and it is only obtainable by solving the conservation equations. Other space lags are given by the geometrical description of the system.

In a dynamic analysis, the fluctuations of space lag λ are important, since they can be considered as the source of a fluid undergoing a phase change. Changing the position of this source causes the mixture enthalpy, density, and pressure drop to fluctuate; thus we have a generation and propagation of waves created at λ due to the fluctuation of λ.

D. Governing Equations for the Upstream Unheated Region (A)

Following the above discussion, we assume that the liquid density is a function only of the system pressure P_s, which can be considered to be uniform throughout the system. Thus we have $\rho_f = \rho_f(P_s)$. Without loss of generality we can assume that region A consists of a constant-area duct and a pressure-drop device such as an orifice. In this case, from the continuity equation we have $v_{f0} = (A_c/A_0)v_{fi}$, where A_c and A_0 denote the flow area for the heated section and upstream unheated region A.

The equation of motion for the duct is

$$-\frac{\partial P}{\partial z} = \rho_f\left(\frac{\partial v_{f0}}{\partial t} + v_{f0}\frac{\partial v_{f0}}{\partial z} + g_0 + \frac{f_0}{2D_0}v_{f0}^2\right). \tag{2}$$

Assuming that the position of the orifice or valve is at the entrance to the heated region, we have

$$\Delta P_i = k_i \rho_f v_{fi}^2. \tag{3}$$

The friction factor f_0 can be obtained from the Reynolds number and the roughness parameters of the duct. Therefore, $f_0 = f_0(N_{Re}, \epsilon_0)$. In addition to the above equations, we impose the isenthalpic condition; thus,

$$\frac{Di_{f0}}{Dt} = 0. \tag{4}$$

E. Governing Equations for the Heated Liquid Region (B)

By neglecting the velocity and enthalpy covariant terms, axial conduction, normal stress, pressure, and the dissipation effects on the enthalpy, we obtain the following three conservation equations:

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial \rho_f v_f}{\partial z} = 0, \tag{5}$$

$$\frac{\partial i_f}{\partial t} + v_f \frac{\partial i_f}{\partial z} = \frac{q''_w \xi}{\rho_f A_c}, \tag{6}$$

and

$$-\frac{\partial P}{\partial z} = \rho_f \left(\frac{\partial v_f}{\partial t} + v_f \frac{\partial v_f}{\partial z} + g + \frac{f_s}{2D} v_f^2 \right). \tag{7}$$

The constitutive equation of state is given by $\rho_f = \rho_f(P_s)$. These above equations specify the four variables P, ρ_f, v_f, and i_f in the heated liquid region B. The friction factor f_s should be given by the constitutive relation, $f_s = f_s(N_{Re_s}, \epsilon)$. These four equations can be solved if the heat flux q''_w is a known function of independent variables z and t. Although the wall has some heat capacity, which can influence the heat input to the fluid, we assume that it is small compared to the heat capacity of the fluid. This enables us to neglect the entire wall effect on the heat transfer if the heat source generates a steady flux. Thus we take

$$q''_w = q''_w(z). \tag{8}$$

Assuming that P_s, ξ/A_c, ϵ, and D are known and constant, we have six dependent variables--ρ_f, v_f, i_f, q''_w, P, and f --whereas the equations describing the system are three field equations (Eqs. 5-7) and three constitutive equations. Thus the total number of unknowns is the same as the number of equations. Consequently, the formulation is mathematically consistent.

F. Governing Equations for the Heated Mixture Region (C)

To take into account the relative motions of each phase, it is useful to formulate the problem in terms of mixture field equations with a vapor-continuity equation (drift-flux formulation). If the velocity and enthalpy covariant terms, axial conduction, normal stress, capillary force, pressure, and dissipation effects on the enthalpy are neglected, the drift-flux formulation reduces to the following forms:

The continuity equation of the mixture:

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m v_m}{\partial z} = 0. \tag{9}$$

The continuity equation for the vapor:

$$\frac{\partial \alpha \rho_g}{\partial t} + \frac{\partial}{\partial z}(\alpha \rho_g v_m) = \Gamma_g - \frac{\partial}{\partial z}\left(\frac{\alpha \rho_g \rho_f}{\rho_m} V_{gj} \right). \tag{10}$$

The energy equation for the mixture:

$$\rho_m\left(\frac{\partial i_m}{\partial t} + v_m\frac{\partial i_m}{\partial z}\right) = \frac{q_w''\xi}{A_c} - \frac{\partial}{\partial z}\left(\frac{\alpha\rho_g\rho_f}{\rho_m}V_{gj}\Delta i_{fg}\right). \tag{11}$$

The momentum equation for the mixture:

$$-\frac{\partial P}{\partial z} = \rho_m\left(\frac{\partial v_m}{\partial t} + v_m\frac{\partial v_m}{\partial z}\right) + g\rho_m + \frac{f_m}{2D}\rho_m v_m^2 + \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g}\frac{\rho_g\rho_f}{\rho_m}V_{gj}^2\right). \tag{12}$$

In addition to the above four field equations, we should specify the constitutive relations. Following the discussion of the thermodynamic process, we assume thermal equilibrium between two phases. Furthermore, the fluid properties of both vapor and liquid phases can be assumed to be a function only of the system pressure P_s, which is uniform throughout the system. This implies that enthalpy and density of each phase are constant once P_s is specified. Hence, the thermal and calorical equation of state can be described as

$$\left.\begin{aligned} &\rho_f = \rho_f(P_s), \quad \rho_g = \rho_g(P_s); \\ &\rho_m = \rho_m(P_s, \alpha) = \alpha\rho_g + (1 - \alpha)\rho_f; \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} &i_f = i_f(P_s), \quad i_g = i_g(P_s); \\ &i_m = i_m(P_s, \alpha) = \frac{\alpha\rho_g i_g + (1 - \alpha)\rho_f i_f}{\rho_m}. \end{aligned}\right\} \tag{14}$$

In addition to the above relations, we need:

a. The constitutive relation for the relative motion,[58] which specifies the vapor drift velocity V_{gj}.

b. The rheological constitutive equation, which supplies f_m.

c. The boundary condition for heat flux, which gives $q_w'' = q_w''(z)$.

If we assume that P_s, ξ/A_c, D, and g are known, the dependent variables are ρ_m, v_m, α, P, f_m, ρ_g, Γ_g, ρ_f, V_{gj}, i_m, q_w'', i_{gs}, and i_{fs}. The equations describing the system are the four field equations and the nine constitutive equations. Thus, the total number of unknowns is the same as the number of equations.

Note that velocities of each phase, v_g and v_f, do not appear in this formulation explicitly. However, they are related to the mixture velocity and the vapor drift velocity by definitions.

G. Governing Equations for the Downstream Unheated Region (D)

In this region the mixture is treated as an isochoric fluid. Furthermore, without losing generality we can assume that region D consists of a constant-area duct and a pressure-drop device such as an orifice. The thermal equation of state is given by

$$\frac{D\rho_{me}}{DT} = 0, \tag{15}$$

so that the continuity equation becomes $v_{me} = (A_c/A_e)v_m(\ell, t)$. Here A_e is the flow area of the duct in region D.

The equation of motion for the duct is given by

$$-\frac{dP}{dz} = \rho_{me}\left(\frac{\partial v_{me}}{\partial t} + v_{me}\frac{\partial v_{me}}{\partial z}\right) + g_e\rho_{me} + \frac{f_{me}}{2D_e}\rho_{me}v_{me}^2$$

$$+ \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_{me}}{\rho_{me} - \rho_g}\frac{\rho_f\rho_g}{\rho_{me}}V_{gj}^2\right). \tag{16}$$

For the orifice, assuming that it is on the exit of the heated region, we have

$$\Delta P_e = k_e\rho_m(\ell, t)v_m^2(\ell, t). \tag{17}$$

The friction factor f_{me} can be given by the function similar to the one used in region C, and the vapor drift velocity should be supplied by a constitutive equation.

In addition to the above equation, we have an isenthalpic condition, so that

$$\frac{Di_{me}}{Dt} = 0. \tag{18}$$

H. Method of Solution

The dynamic problem of our system of interest has been formulated by considering four different regions, i.e., the upstream unheated, heated liquid, heated mixture, and downstream unheated regions. To obtain solutions for the system, boundary and/or initial conditions should be given. Several methods can be used to obtain the transient response and the stability criteria of the system. For example, linear and nonlinear theories are available and could be used to obtain the solution. Furthermore, we have a choice of the disturbance to impose on the system. As we can see from our formulation, the

governing differential equations are nonlinear; thus, it will be expected that for unstable operation the nonlinearity becomes important for large departures from the equilibrium state. Due to the presently limited knowledge about nonlinear partial differential equations, the general solution for our system by analytical means is almost inaccessible.

However, in most practical cases, the information on the stability boundary is much more important than the ultimate unstable dynamical response beyond the stability boundary. According to the Liapunov theorem,[59] the stability of the linearized system corresponds to the stability of the nonlinear system that operates under quasi-equilibrium conditions. By taking advantage of this theorem, we can use the linear theory. In our analysis, the disturbance will be given in the form of an inlet-velocity perturbation. Consequently, we impose the following boundary and initial conditions on the density, pressure, enthalpy, and velocity. By taking the origin of the z coordinate at the boundary between A and B, the inlet of the heated region, we have

$$\left.\begin{aligned}
\rho_f &= \rho_f(P_s) && \text{at } z = 0,\ t \geq 0;\\
P &= P_s = \text{const.} && \text{at } z = 0,\ t \geq 0;\\
i_f &= i_1 = \text{const.} && \text{at } z = 0,\ t \geq 0;\\
v_f &= v_{fi}(t) = \bar{v}_{fi} + \delta v(t) && \text{at } z = 0,\ t \geq 0;\\
i_f &= i_s(P_s) && \text{at } z = \lambda(t).
\end{aligned}\right\} \tag{19}$$

Hence, the density ρ_f and the saturation liquid enthalpy i_{fs}, which are functions of only P_s, can be treated as constants. The steady-state inlet velocity is denoted by $\bar{v}_{fi}$, and the perturbation of the velocity is given by $\delta v(t)$. In our analysis, we use a frequency-response method; therefore, the perturbation $\delta v(t)$ can be given by an exponential function:

$$\left.\begin{aligned}
\delta v(t) &= \epsilon e^{St};\\
S &= a + j\omega \qquad (j = \sqrt{-1}).
\end{aligned}\right\} \tag{20}$$

Thus, S is a complex number; the real part gives the amplification coefficient of the particular oscillation mode, and the imaginary part represents the angular frequency ω. For the linear-perturbation analysis, we assume that $\epsilon/\bar{v}_{fi}$ is much smaller than unity; therefore, ϵ is infinitesimal compared with finite $\bar{v}_{fi}$. In the following analysis, we shall retain the first-order terms in ϵ and neglect second- and higher-order terms.

The schematic procedure for the analytical solution is given in Fig. 6. This diagram indicates the dramatic effects of decoupling the momentum equation from the continuity and energy equations. The kinematics of the fluid, i.e.,

the velocity field and the density variation, can be solved independently of the dynamics of the system. Recalling single-phase potential-flow theory, we find an analogy in attacking the two problems. As will be clarified in what follows, the only difference is that, in the potential-flow analysis, the divergence of the velocity of the center of mass is zero, whereas in a two-phase boiling system, the divergence of the center of volume equals the volume source due to evaporation.

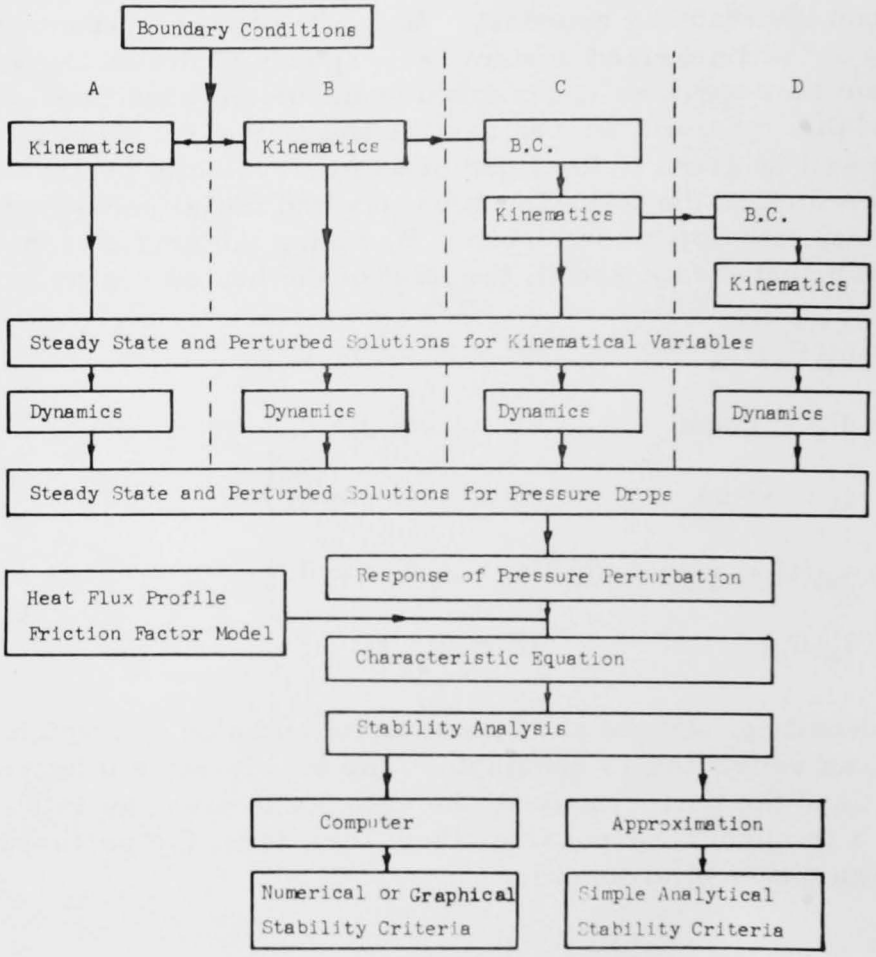


Fig. 6. Schematic Procedure for Solution

IV. KINEMATICS OF FLUID IN THE HEATED SECTION

A. Liquid Region

From the continuity equation with constant density,

$$v_f(t) = \bar{v}_{fi} + \epsilon e^{St}. \tag{21}$$

The velocity in region B is therefore a function of time only.

Knowing the velocity field, the energy equation becomes

$$\frac{\partial i_f}{\partial t} + v_f(t)\frac{\partial i_f}{\partial z} = \frac{q''_w \xi}{\rho_f A_c}. \tag{22}$$

This is a first-order partial differential equation whose solution can be obtained by the method of characteristics. For simplicity let us take a case with a constant q''_w. This implies that the system is uniformly heated along the channel. The detailed calculation with a nonuniform heat flux appears in Ref. 29.

Now Eq. 22 can be transformed to the Lagrange form:

$$dt = \frac{dz}{v_f(t)} = \frac{di_f}{\dfrac{q''_w \xi}{A_c \rho_f}}, \tag{23}$$

with the initial and boundary conditions

$$\left.\begin{aligned} &\text{at } t = \tau_1 \text{ and } z = 0, \qquad i_f = i_1; \\ &\text{at } t = \tau_2 \text{ and } z = \lambda(t), \qquad i_f = i_{fs}(P_s). \end{aligned}\right\} \tag{24}$$

By use of the small-perturbation analysis the steady-state and the first-order solutions are given by

$$i_f - i_1 = \frac{q''_w \xi}{A_c \rho_f}\left[\frac{z}{\bar{v}_{fi}} - \frac{\epsilon e^{St}}{\bar{v}_{fi}} \frac{1 - \exp^{(-Sz/\bar{v}_{fi})}}{s}\right], \tag{25}$$

where

$$\tau_{12} \equiv \tau_2 - \tau_1 = \frac{\Delta i_{12} A_c \rho_f}{q''_w \xi} \tag{26}$$

and

$$\lambda(t) = \bar{\lambda} + \delta\lambda = \tau_{12}\bar{v}_{fi} + \epsilon e^{St} \frac{1 - \exp^{(-S\tau_{12})}}{S}. \tag{27}$$

B. Mixture Region

The set of governing equations for heated mixture region C has been given in the previous section. We recall the condition of constant properties for the liquid and vapor phases in the mixture region; this condition is valid for the relatively high-pressure system assuming thermal equilibrium. Furthermore, for simplicity, we assume that the vapor drift velocity can be treated as a constant. This is supported by experimental data[58] and is valid for several flow regimes, i.e., bubbly, slug, churn turbulent, etc. Under these conditions, the mixture and the vapor continuity equations can be transformed to the volumetric flux equation and the density propagation equation; thus,

$$\frac{\partial j}{\partial z} = \Gamma_g \frac{\Delta\rho}{\rho_g \rho_f} \tag{28}$$

and

$$\frac{\partial \rho_m}{\partial t} + C_k \frac{\partial \rho_m}{\partial z} = -\Gamma_g \frac{\Delta\rho \rho_m}{\rho_g \rho_f}, \tag{29}$$

where

$$\left.\begin{aligned} C_k &\simeq j + V_{gj} \\ \text{and} \quad & \\ j &= v_m + \frac{\alpha \Delta\rho}{\rho_m} V_{gj} \end{aligned}\right\}. \tag{30}$$

On the other hand, the thermal-equilibrium condition gives

$$\Gamma_g = \frac{q_w'' \xi}{A_c \Delta i_{fg}}, \tag{31}$$

which relates the vapor generation Γ_g to the heat input and the latent heat. The detailed analysis of the thermal-nonequilibrium case is given in Ref. 26.

The characteristic frequency Ω of the phase change is defined by

$$\Omega \equiv \frac{\Gamma_g \Delta\rho}{\rho_g \rho_f} = \frac{q_w'' \xi}{A_c \Delta i_{fg}} \frac{\Delta\rho}{\rho_g \rho_f}. \tag{32}$$

Then we have

$$\frac{\partial j}{\partial z} = \Omega, \tag{33}$$

and the solution for the total volumetric flux is given by

$$j(z,t) = v_{fi}(t) + \Omega(z - \lambda). \tag{34}$$

When the above equation is substituted into Eq. 30, the kinematic wave velocity becomes

$$C_k(z,t) = \bar{C}_k(z) + \delta C_k(t) = [\bar{v}_{fi} + V_{gj} + \Omega(z - \bar{\lambda})] + (\delta v - \Omega\delta\lambda). \tag{35}$$

The solution for the mixture density becomes

$$\frac{\rho_m(z)}{\rho_f} = \frac{\bar{C}_k(\bar{\lambda})}{\bar{C}_k(z)} \tag{36}$$

and

$$\frac{1}{\rho_f}\frac{\delta\rho_m}{\delta v} = \left[\frac{\bar{C}_k(\bar{\lambda})}{\bar{C}_k(z)}\right]^2 \frac{1}{\bar{C}_k(\bar{\lambda})}\frac{\Omega}{S - \Omega}\frac{\delta C_k}{\delta v}$$

$$+ \left[\frac{C_k(\bar{\lambda})}{C_k(z)}\right]^{(S/\Omega)+1} \frac{1}{C_k(\bar{\lambda})}\left(1 - \frac{S}{S - \Omega}\frac{\delta C_k}{\delta v}\right). \tag{37}$$

Similarly, the solution for the mixture velocity can be obtained from Eq. 30:

$$\frac{v_m(z,t)}{\bar{v}_{fi}} = \frac{\bar{C}_k(z)}{\bar{C}_k(\bar{\lambda})} + \left\{\frac{\delta C_k(t)}{\bar{v}_{fi}} + \left[\frac{\bar{C}_k(z)}{\bar{C}_k(\bar{\lambda})}\right]^2 \frac{V_{gj}}{\bar{v}_{fi}}\frac{\delta\rho_m}{\rho_f}\right\}. \tag{38}$$

V. DYNAMICS OF THE SYSTEM

In the previous section, we have solved for the kinematics of the fluid. If the velocity field and the density variation are known, the pressure drop can be calculated by integrating the momentum equation. By considering the heated section with the inlet and exit flow restrictions, the total channel pressure drop becomes

$$\Delta P_{ex} = k_i \rho_f v_{fi}^2 + k_e \rho_m(\ell, t) v_m^2(\ell, t)$$

$$+ \int_0^{\lambda} \rho_f \left(\frac{\partial v_f}{\partial t} + v_f \frac{\partial v_f}{\partial z} + g + \frac{f_s}{2D} v_f^2 \right) dz$$

$$+ \int_{\lambda}^{\ell} \left[\rho_m \left(\frac{\partial v_m}{\partial t} + v_m \frac{\partial v_m}{\partial z} + g + \frac{f_m}{2D} v_m^2 \right) \right.$$

$$\left. + \frac{\partial}{\partial z} \left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_g \rho_f}{\rho_m} V_{gj}^2 \right) \right] dz. \tag{39}$$

Therefore, the inertia, gravity, and friction terms in the inlet and exit unheated regions are dropped for simplicity, though calculating these terms is straightforward without any difficulties.

The steady-state part of the solution is given by

$$\overline{\Delta P}_{ex} = k_i \rho_f \bar{v}_{fi}^2 + \rho_f \bar{v}_{fi}^2 (C_r^* - 1) + \rho_f g \left[\bar{\lambda} + \int_{\bar{\lambda}}^{\ell} \frac{\bar{C}_k(\bar{\lambda})}{\bar{C}_k(z)} dz \right]$$

$$+ \rho_f \bar{v}_{fi}^2 \left[\frac{f_s \bar{\lambda}}{2D} + \frac{1}{2D} \int_{\bar{\lambda}}^{\ell} \bar{f}_m \frac{\bar{C}_k(z)}{\bar{C}_k(\bar{\lambda})} dz \right] + \rho_g C_r^* (C_r^* - 1) V_{gj}^2 + k_e \rho_f \bar{v}_{fi}^2 C_r^*, \tag{40}$$

where

$$C_r^* \equiv \bar{C}_k(\ell) / \bar{C}_k(\bar{\lambda}).$$

Here we made a simplifying assumption that $\rho_m - \rho_g \simeq \rho_m$ in the drift-pressure-drop term. The two integrals appearing in Eq. 40 can be easily carried out by knowing the constitutive equation for V_{gj} and f_m. The most simple two-phase friction-factor model is

$$f_m = C_m f_s, \tag{41}$$

where $C_m = 2$ can be a good approximation for high-pressure systems and gives a slightly higher pressure drop than the Martinelli-Nelson correlation. The effects of various other two-phase friction-factor correlations have been discussed in detail in Ref. 29.

On the other hand, the first-order solution is given as a form of a functional relation between the perturbation of the system pressure drop and of the incoming fluid velocity δv. Thus, by carrying out the integration in Eq. 39 we obtain

$$\delta \Delta P_{ex}(s, t) = Q(s) \delta v(s, t). \tag{42}$$

Here the exact form of the function Q(s) is determined by the integration of Eq. 39 and appears in Ref. 29. To examine the stability of the system, we must specify the generalized input force and the output displacement. Although in our analysis we assumed the initial flow perturbation δv and obtained the response of ΔP_{ex} to it, the physical process occurring in the system is exactly the opposite.

In other words, the input force imposed on the system is the pressure-drop perturbation $\delta \Delta P_{ex}$, and it induces the change in the flow field. Hence, δv is the generalized displacement. Here $1/Q(s)$ is called the system transfer function, and the dynamic response can be represented by the transformation.

$$\delta v = \frac{1}{Q(s)} \delta \Delta P_{ex}, \tag{43}$$

where Q(s) is the characteristic function. According to control theory, the asymptotic stability of the system can be determined by nature of the roots of the characteristic equation

$$Q(s) = 0. \tag{44}$$

With Eq. 44, the formulation is now essentially complete, since the initial problem of determining the dynamic stability of the physical system is reduced to the mathematical problem of the complex functional analysis. More specifically, our problem becomes: to examine the nature of the roots in the complex plane for the characteristic equation given by Eq. 44.

Here it can be mentioned briefly that, if the characteristic equation has all its roots in the left half of the S plane, every component of small disturbance tends to zero as $t \to \infty$. Thus, this is the necessary condition for asymptotic stability. Furthermore, if the characteristic equation has a root with a positive real part, the disturbance grows with time and hence the system is unstable.

On the other hand, when the excursive instability is considered, it is useful to transform Eq. 42 into

$$\frac{\delta \Delta P_{ex}}{\delta v} = Q(s). \tag{45}$$

In the limit $S \rightarrow 0$ the perturbation becomes constant. Hence the excursive stability condition can be given by

$$\lim_{S \rightarrow 0} \frac{\delta \Delta P_{ex}}{\delta v} > 0. \tag{46}$$

Since ϵ, the magnitude of the velocity perturbation, is an arbitrarily small constant, Inequality 46 reduces to the criteria obtained by Leginegg,[4] namely,

$$\frac{\partial \Delta P_{ex}}{\partial v} > 0. \tag{47}$$

It follows that

$$\lim_{S \rightarrow 0} Q(s) > 0 \tag{48}$$

is the condition for the excursive stability. In view of Eq. 44, this corresponds to a singularity in the dynamic stability analysis. Therefore, the examination of the characteristic equation 44 and of the nature of its roots in the complex S-plane is sufficient for both the dynamic and excursive stability analyses. We note here that the excursive stability criterion based on the above discussion has been obtained in Ref. 29.

VI. SIMILARITY GROUPS GOVERNING THE DYNAMICS OF THE SYSTEM

In the following analysis, we shall nondimensionalize the characteristic equation for the uniformly heated system. We choose the length scale as ℓ, the heated channel length, and the time scale as $1/\Omega$, the reaction (phase-change) time in the mixture region given by Eq. 32. Based on the above reference scales, we obtain following nondimensional parameters:

$$\left.\begin{array}{ll}
\text{Geometrical groups:} & k_i,\ k_e,\ D^*; \\[2ex]
\text{Froude number:} & N_{Fr} \equiv v_{fi}^2/g\ell; \\[2ex]
\text{Reynolds number:} & N_{Re_s} \equiv \rho_f v_{fi} D/\mu_f; \\[2ex]
\text{Subcooling number:} & N_{sub} \equiv \tau_{12}\Omega = \dfrac{\Delta i_{12}}{\Delta i_{fg}}\dfrac{\Delta\rho}{\rho_g}; \\[2ex]
\text{Phase-change number:} & N_{pch} \equiv \dfrac{\Omega\ell}{\bar{v}_{fi}} = \dfrac{q_w''\xi\ell}{A_c\bar{v}_{fi}\Delta i_{fg}}\dfrac{\Delta\rho}{\rho_g\rho_f}; \\[2ex]
\text{Drift number:} & N_d \equiv V_{gj}/\bar{v}_{fi}; \\[2ex]
\text{Density ratio:} & N_\rho \equiv \rho_g/\rho_f; \\[2ex]
\text{Perturbation parameter:} & S^* \equiv a/\Omega + j\omega/\Omega = a^* + j\omega^*.
\end{array}\right\} \qquad (49)$$

The above groups are independent of each other and are the basic parameters governing the dynamics of the system. For a smooth pipe, the single phase-friction factor f_s can be correlated to the Reynolds number N_{Re_s}. The two-phase friction factor f_m is given by $C_m f_s$, where C_m is supposed to be a constant. The geometrical parameters, the Froude number, and the Reynolds number have their standard significances. On the other hand, the subcooling, phase-change, drift, and density numbers are associated with the two-phase flow systems.

The subcooling number takes into account the time-lag effects in the liquid region due to the subcooling of the fluid entering the heated duct. Thus it is one of the important parameters for the stability analysis.

The phase-change number corresponds to Damköehler's Group I in chemical kinetics,[60] and it scales the change of phase due to the heat transfer to the system. Since N_{pch} is the inverse of the nondimensional inlet velocity, it can be seen that N_{pch} is one of the decisive parameters for the kinematical similarity. Both N_{sub} and N_{pch} are significant not only for stability analyses, but also for the description of the steady-state operational conditions.

The drift number takes account of the drift effects due to the relative motion of the fluids and thus plays a role in two-phase flow similar to that of Damköehler's Group II in diffusion processes.[60] Since the vapor drift velocity V_{gj} depends on the flow regime,[58] this group characterizes the flow pattern. We note here that two-phase drift processes are not due to the molecular random motions, but to the macroscopic geometrical orientation of each phase.

The drift number is important for the kinematical similarity of mixture. If $N_d >> N_{pch}$, then the change of the density and velocity are controlled by the drift, i.e., redistribution of phases.

The density number explicitly appears only in the drift stress term of the characteristic equation. Depending on the constitutive equation for C_m, this group N_ρ may also appear in the two-phase frictional-pressure-drop terms, in which case C_m is a function of N_ρ. The density number actually scales the dynamic effect of the system pressure P_s in contrast to the kinematical effects of P_s, which are taken into account for by the term $\Delta\rho/\rho_g$ in N_{sub} and N_{pch}. This is quite important in connection with the above statements on the drift and frictional pressure-drop terms.

For example, if the condition $N_\rho N_d^2 << 1$ is satisfied, then the pressure drop due to drift stress is small. Furthermore, if we can assume that, for a wide range of system pressures $\partial C_m/\partial N_\rho \simeq 0$, then the influence of the system pressure can be effectively taken into account by the groups N_{sub} and N_{pch}. In this case, N_ρ can be eliminated.

Following the above similarity analysis, the characteristic equation can be rewritten in general as

$$Q^*(k_i, k_e, D^*, N_{Fr}, N_{Re_s}, N_{sub}, N_{pch}, N_d, N_\rho, S^*) = 0. \tag{50}$$

Equation 50 shows the complete parametric dependence of the characteristic equation. Under usual circumstances D^* cannot be considered as an operational parameter, but rather as a design parameter. Thus, once the system is given, the dynamic behavior can be represented in eight-dimensional space with S^* as a parameter.

VII. STABILITY ANALYSIS

The characteristic equation governing the dynamic response of the system can be expressed by the combination of the rational functions and the exponential functions in terms of the perturbation parameter S^*. The asymptotic stability of the system can be determined by the nature of the characteristic equation given by Eq. 50.

The stability theorem applicable to our characteristic equation can be classified in two groups, the stability test criterion and the stability boundary criterion. In the following, we shall discuss the above groups separately.

A. Graphical Method Based on Encirclement Theorem

The stability criterion from the encirclement theorem was first developed for the linear system with no time delays by Mikhailov[61] and by Leonhard,[62] and slightly modified by Nyquist[63] for the closed-loop control system. The theorem is well known and not restricted to the rational algebraic functions. The extension of the criterion to the time-delay system was studied by Sokolov, Miasnikov, and Satche among others. A detailed review of these works has been done by Popov[64] or by Porter.[59] Briefly, the criterion was obtained from the application of conformal mapping and the residue theorem. Basically, the stability test criterion states that the system is unstable if the characteristic equation has any roots in the right-hand half-plane of S^*. The number of zeros can be found by mapping S^* on the contour C, which covers the right-hand half-plane into the Q^* plane, and then counting the clockwise encirclement of the origin of Q^* plane.

The above criterion is extensively based on graphical means, and a plotting of the Nyquist or Mikhailov diagram is required. The algebraic criteria of stability for the linear-time-delay system were developed by Pontriagin;[65] however, the application of the criteria to the practical problem usually becomes quite complicated. Therefore, it is not recommended here.

B. Stability-boundary Criterion (D-partition Method)

The criterion discussed in the foregoing section is best suited for the examination of the stability when all operational parameters are known constants. On the other hand, if a parametric study for the system is required, the D-partition method is preferable. It was first studied by Neimark,[66] and a detailed discussion on the application for the time-delay system appears in Refs. 59 and 64. By taking the characteristic function Q^*, we have

$$Q^*(S^*, N_1, N_2, \ldots, N_m) = 0, \tag{51}$$

where N's are the nondimensional groups appearing in the argument of Q^* in Eq. 50.

Now consider harmonic oscillations. We set $S^* = j\omega^*$, where $j = \sqrt{-1}$. By separating the real and the imaginary parts, we obtain

$$Q^*_{Re}(\omega^*, N_1, N_2, \ldots, N_m) = 0 \tag{52}$$

and

$$Q^*_{Im}(\omega^*, N_1, N_2, \ldots, N_m) = 0. \tag{53}$$

Equations 52 and 53 give the harmonic-frequency surfaces in an m-dimensional space; i.e., $N_1, \ldots, N_m$ are coordinates, with ω^* as an auxiliary parameter.

Since the complex roots are always conjugate for a function with real coefficients, it is sufficient to consider the domain $0 < \omega^* < \infty$ with two singular surfaces:

$$\lim_{\omega^* \to 0} Q^* = 0 \quad \text{and} \quad \lim_{\omega^* \to \infty} Q^* = 0. \tag{54}$$

Now the m-dimensional space is divided into regions bounded by the harmonic-frequency surfaces and two singular surfaces.

The theorem states that the number of roots lying in the right half of the S^* plane for each region divided by the surfaces do not change within a subdivision. Therefore, the stability of each region can be decided by testing the stability for any point in that region. For this purpose, the criterion derived in the previous section is useful. We note here that, since the characteristic equation is a rather complicated function of the similarity parameters and S^*, it is necessary to use a computer to obtain these boundaries. Such a computer code has been developed for the thermal equilibrium as well as non-equilibrium cases.[26,29]

C. Stability Plane

The parametric study of the stability of the system can be performed by using the D-partition method discussed above. The governing parameters obtained from dimensional analysis are k_i, k_e, D^*, N_{Fr}, N_{Re_s}, N_{sub}, N_{pch}, N_d, and N_ρ, whereas the harmonic frequency ω^* is an auxiliary variable at the stability boundaries. As has been shown in a previous section, the neutral stability surfaces in a multidimensional space are given by setting $S^* = j\omega^*$ in the characteristic equation. However, to present these stability boundaries in a two-dimensional plane, we must select two representative parameters for the coordinates of such a stability plane. Since, for constant system pressure and inlet velocity with fixed geometry, the parameters k_i, k_e, D^*, N_{Fr}, N_{Re_s}, N_d, and N_ρ are fixed, the subcooling and phase-change numbers are best suited for the coordinate of such a plane. For example, it was found that the operational domain in the stability plane was bounded by the physical

restrictions on the subcooling and the heat flux. From the condition that the subcooling is a positive entry and has an upper bound given by Δi_s, corresponding to the operational limit on subcooling or the freezing point, we have

$$0 \leq N_{sub} \leq \frac{\Delta i_s}{\Delta i_{fg}} \frac{\Delta \rho}{\rho_g}. \tag{55}$$

On the other hand, from the condition that boiling takes place in the channel and that superheating the vapor does not occur, we obtain

$$N_{pch} - \frac{\Delta \rho}{\rho_g} < N_{sub} < N_{pch}. \tag{56}$$

In addition to the stability boundaries, some important operational characteristics can be represented in simple form on the stability plane. For example, the constant exit-quality line is given by

$$N_{sub} = N_{pch} - X_e \frac{\Delta \rho}{\rho_g}, \tag{57}$$

whereas the length of the nonboiling region is given by

$$\lambda^* \equiv \frac{\lambda}{\ell} = N_{sub}/N_{pch}. \tag{58}$$

The basic characteristics of this stability plane are shown in Fig. 7.

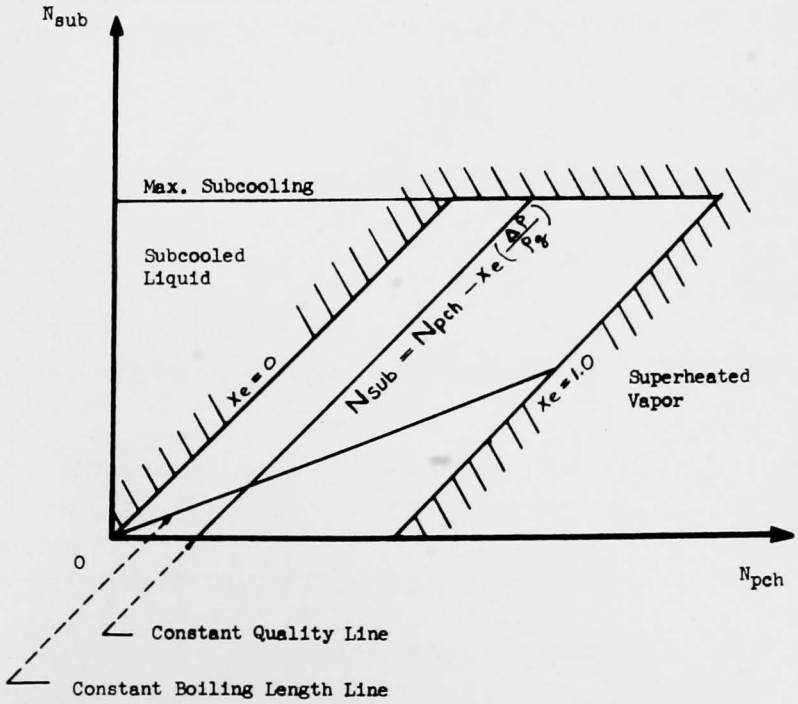


Fig. 7. Stability Plane

VIII. SIMPLE ALGEBRAIC CRITERIA

The exact neutral-stability boundaries of Eqs. 52 and 53 can also be given in the form

$$Q^*(j\omega^*) = \frac{1}{(\omega^{*3} - 5\omega^*)^2 + (4\omega^{*2} - 2)^2}\left(\frac{\mathrm{Re}}{\omega^*} + j\frac{\mathrm{Im}}{\omega^*}\right) = 0. \tag{59}$$

Consequently, we obtain parametric equations $\mathrm{Re}/\omega^* = 0$ and $\mathrm{Im}/\omega^* = 0$ for the neutral-stability surfaces. Here Re and Im are complicated trigonometric polynomial functions in ω^*; however, by using the asymptotic condition $\omega^* \ll 1$, we can simplify them to a great extent. Since the first crossover of the real axis in the Mikhailov diagram happens with relatively small ω^*, the exponential terms have usually more significance than the polynomial parts for the determination of the first crossover. Thus, by considering the asymptotic case of $\omega^* \ll 1$, we can neglect the higher-order terms of ω^*.

By canceling ω^* between the simplified form of Re and Im, we obtain an approximate algebraic criterion[29] given by

$$X_e\frac{\Delta\rho}{\rho_g} = N_{pch} - N_{sub} < \frac{2\left(k_i + \frac{f_s C_m}{2D^*} + k_e\right)}{1 + \frac{1}{2}\left(\frac{f_s C_m}{2D^*} + 2k_e\right)}. \tag{60}$$

Here we have neglected the effects of the relative velocity and gravity. The algebraic criterion including these effects and a more detailed derivation are given in Ref. 29. The above stability criterion is applicable in the range of

$$0 < \omega^* N_{sub} < \pi. \tag{61}$$

However, for a system with a sufficiently large excursive stability margin, we have

$$\omega^* N_{sub} \lesssim \pi. \tag{62}$$

In view of the asymptotic condition $\omega^* < 1$, the limit of the criterion, Eq. 60, becomes

$$N_{sub} \gtrsim \pi. \tag{63}$$

IX. RESULTS OF THE STABILITY ANALYSIS

In this section, we shall study the effects of various operational variables on the boundary of the stability of the system by using the results from the computer runs. Typical stability maps obtained from the calculations are given in Figs. 8-15.

It has been found that more than one neutral frequency curve exists and ω^* appears quasi-periodically. However, the most important curve is the first neutral-stability curve, since it is the stability boundary.

A. Effects of N_{sub} (Subcooling)

In view of Figs. 8-15, it can be seen that there is a characterized $(N_{sub})_c$ such that increasing N_{sub} is stabilizing when $N_{sub} > (N_{sub})_c$ and destabilizing when $N_{sub} < (N_{sub})_c$.

For the range of $N_{sub} > (N_{sub})_c$, the stability-boundary curve is almost a straight line nearly parallel to the constant exit-quality line. This result and the form of the simple criterion given by Eq. 60 suggest that this criterion can be applied for $N_{sub} > (N_{sub})_c$.

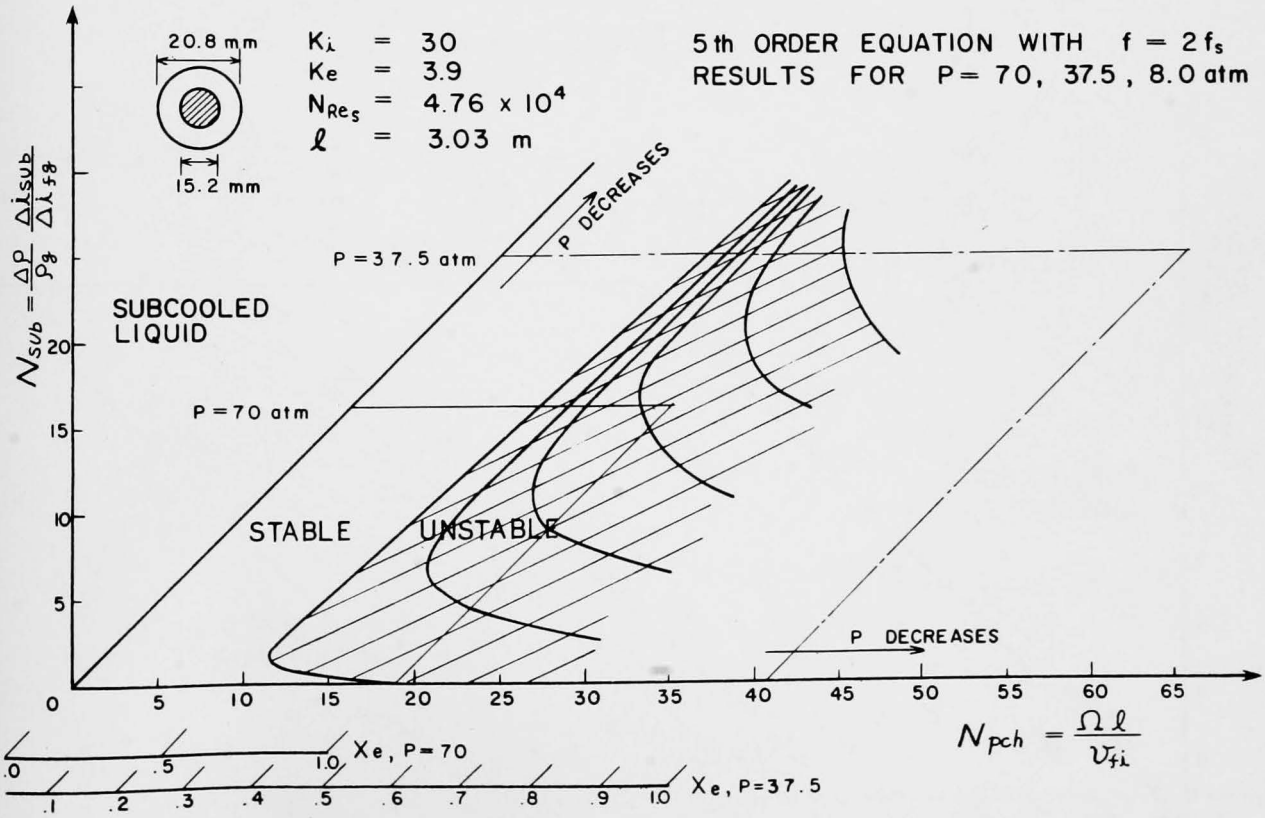


Fig. 8. Effect of System Pressure

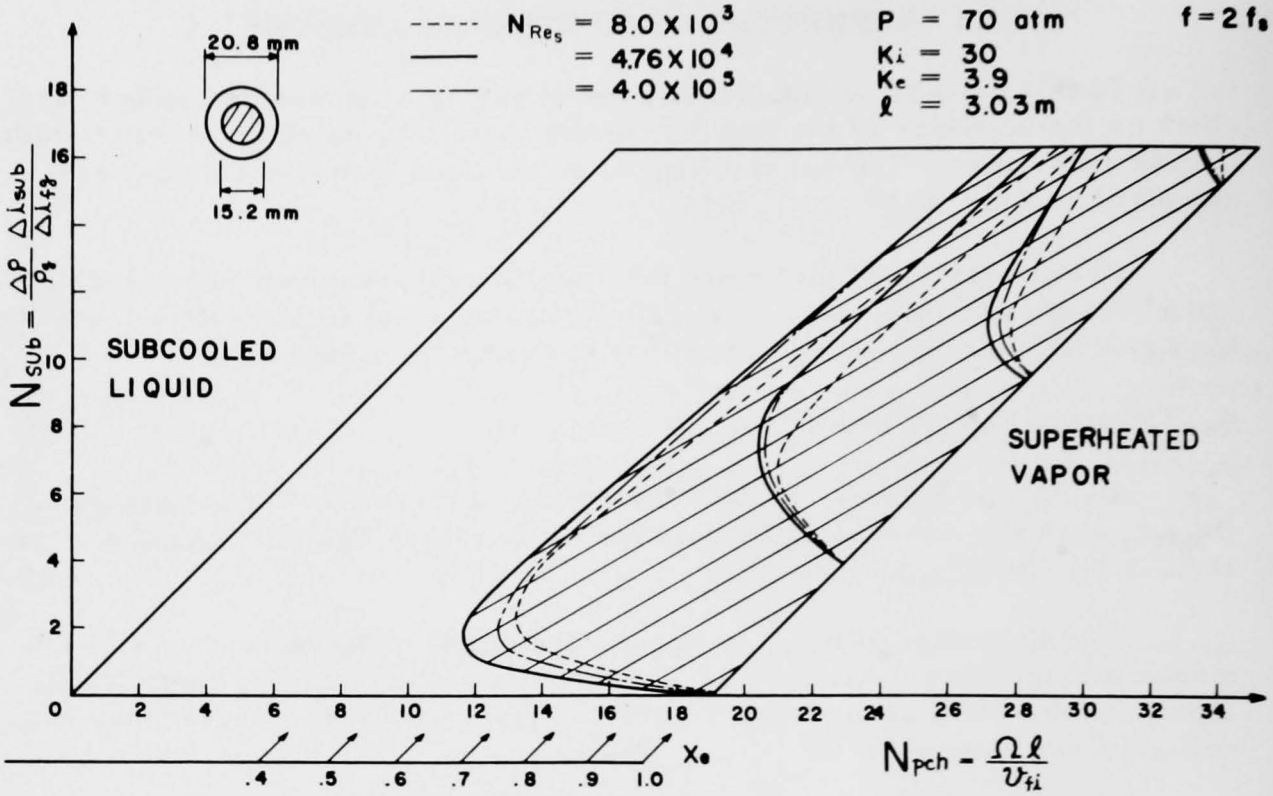


Fig. 9. Effect of Reynolds Number

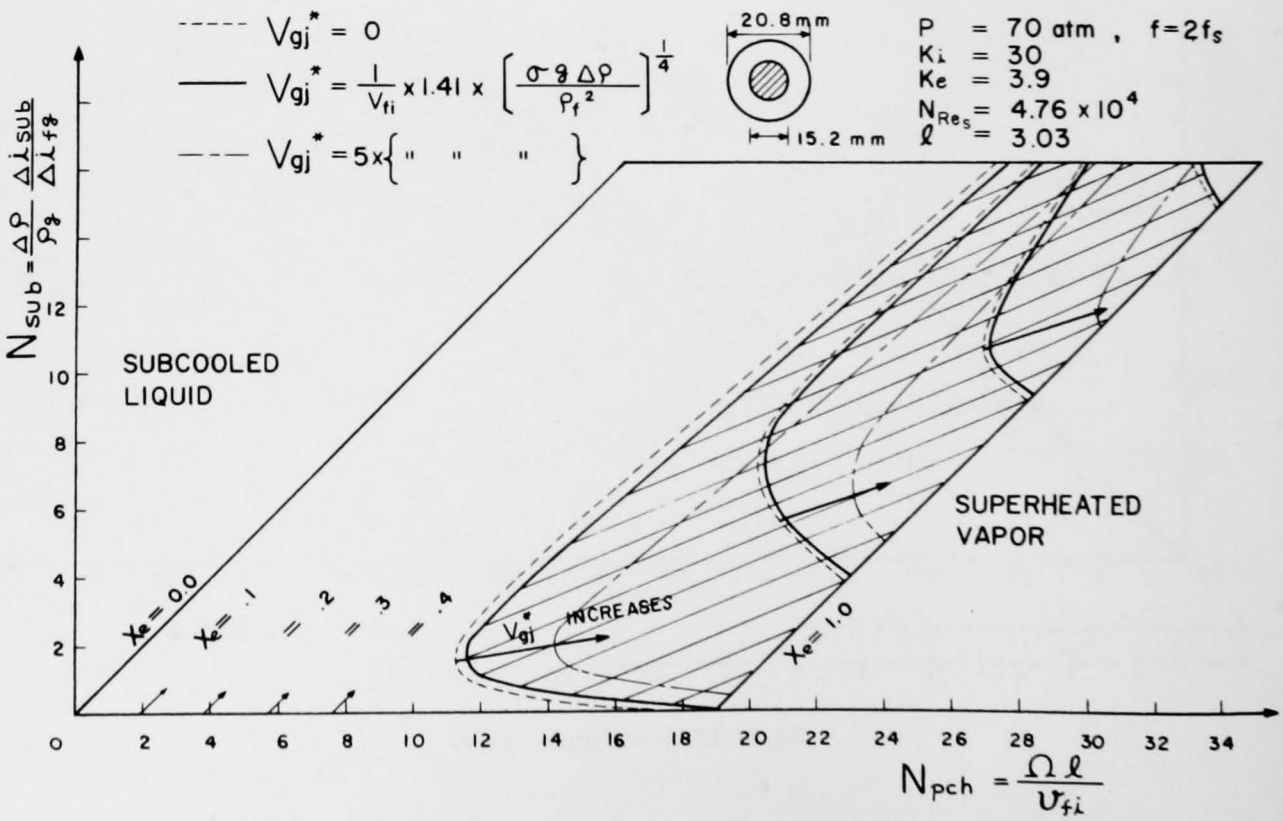


Fig. 10. Effect of V^*_{gj} (= N_d)

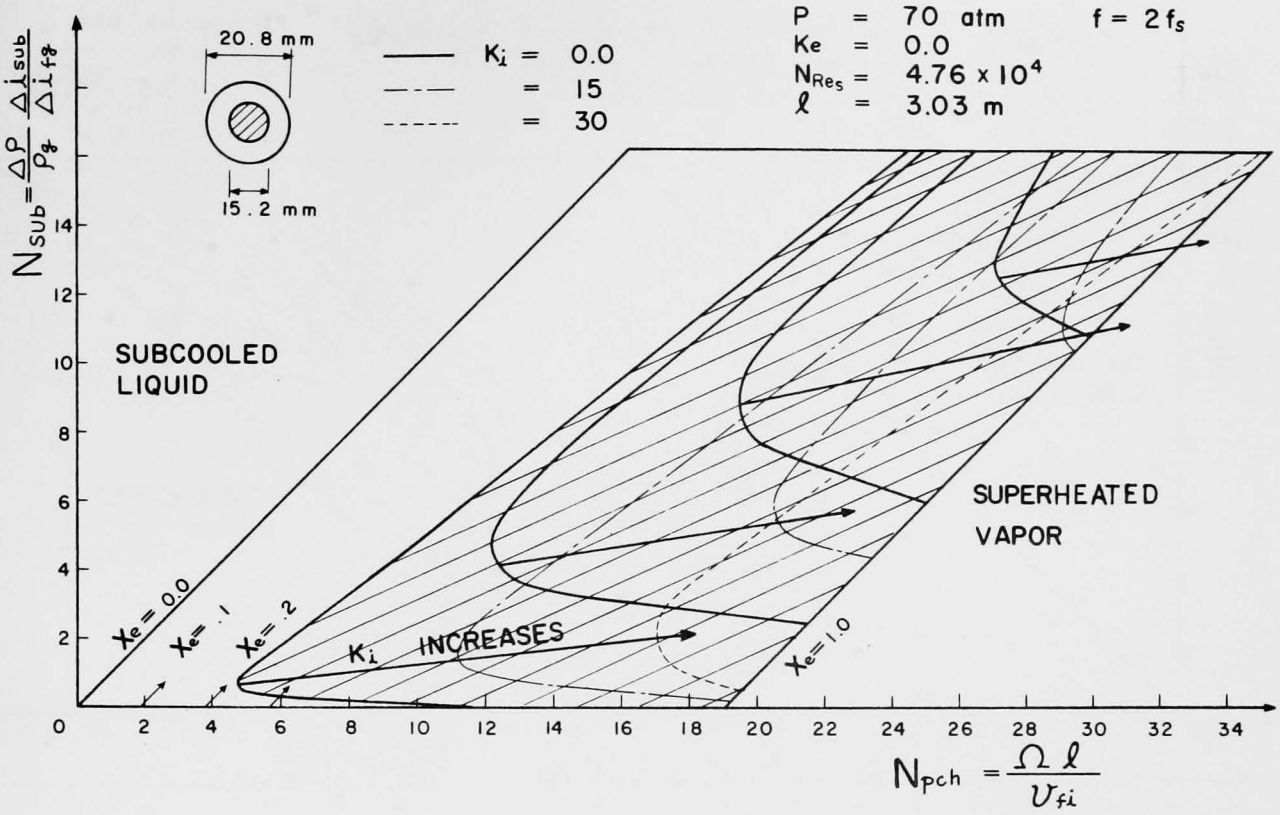


Fig. 11. Effect of k_i

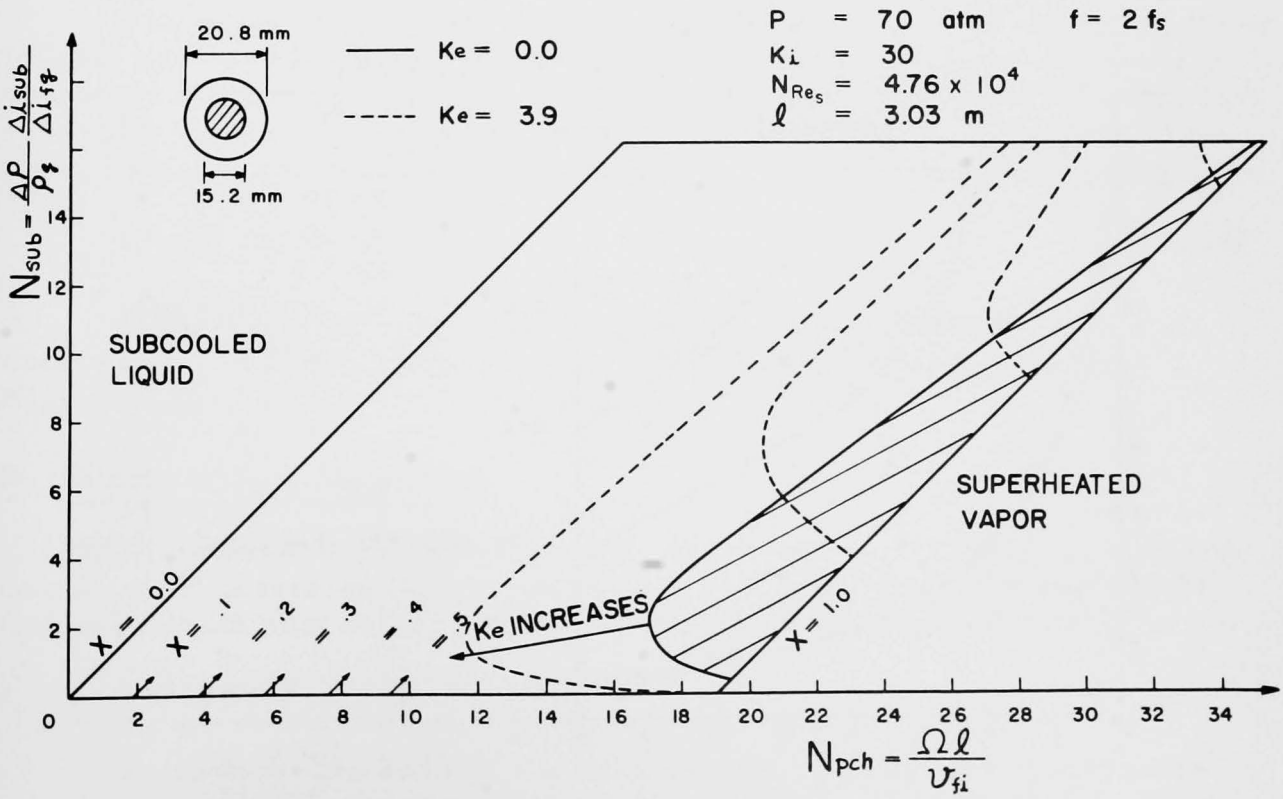


Fig. 12. Effect of k_e

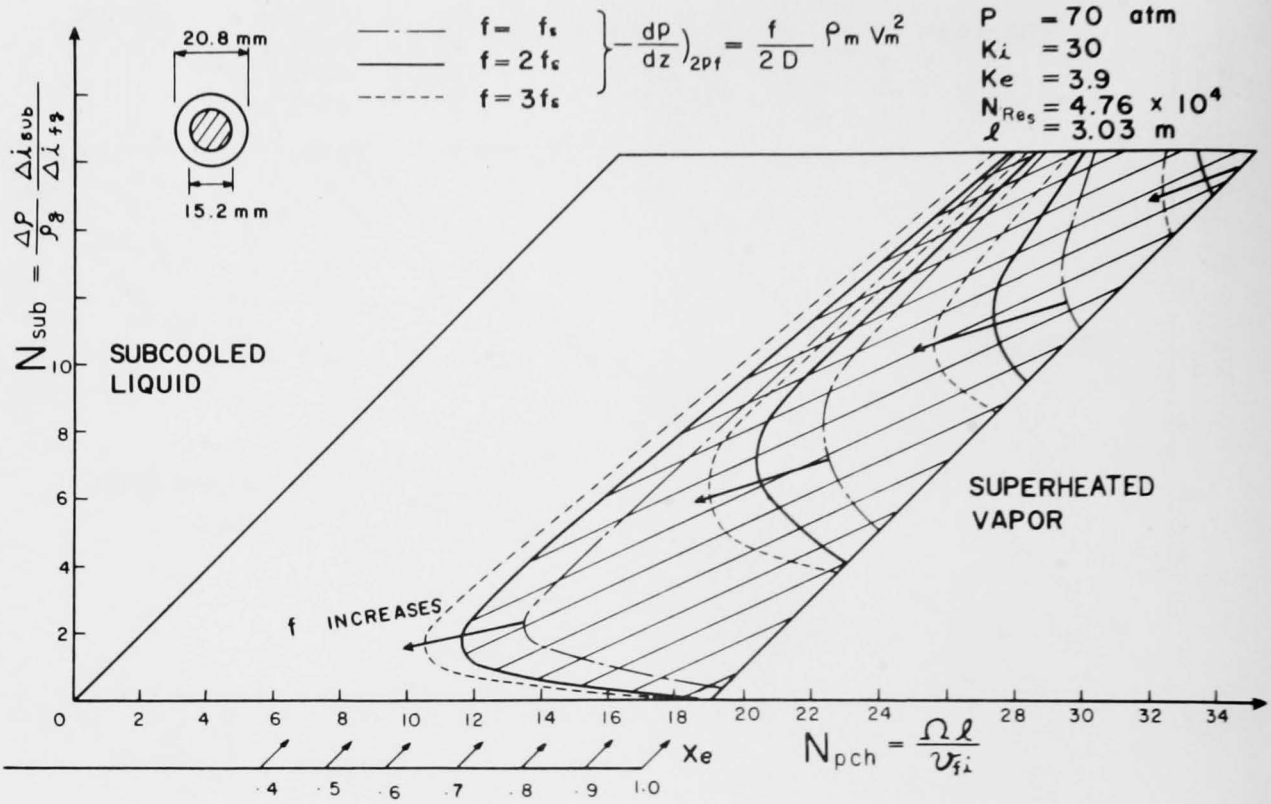


Fig. 13. Effect of Magnitude of Friction Factor

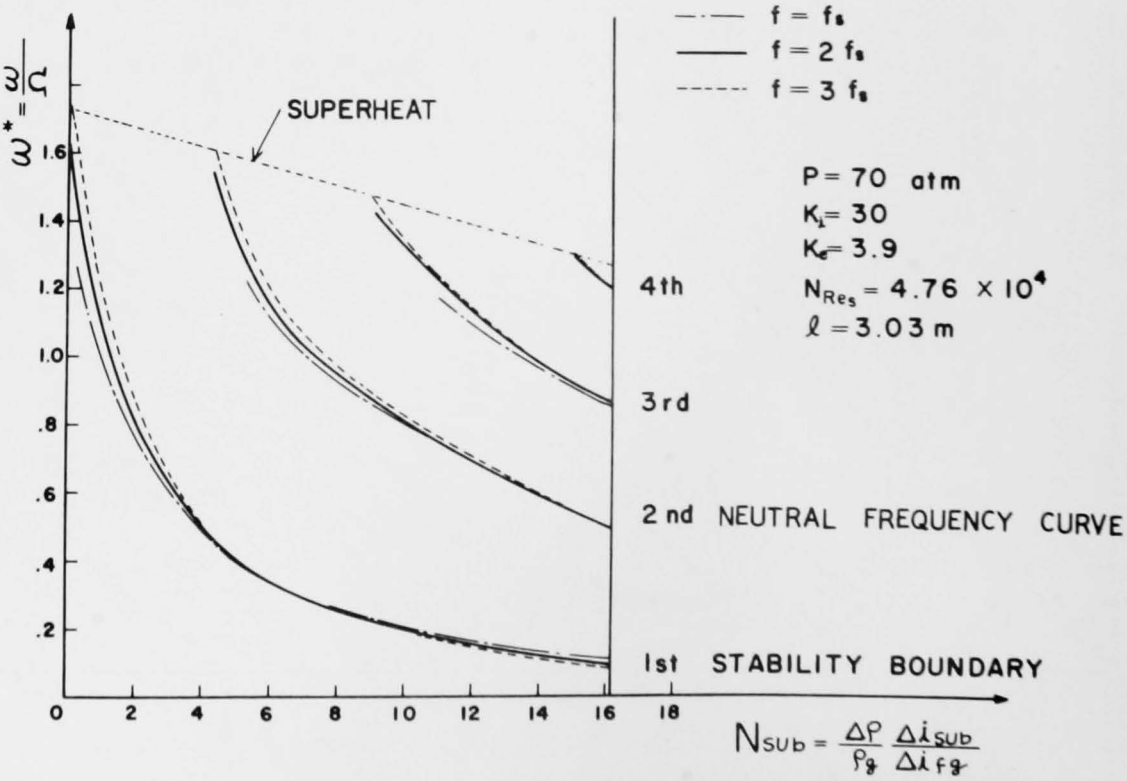


Fig. 14. Effect of Friction Factor on Frequency

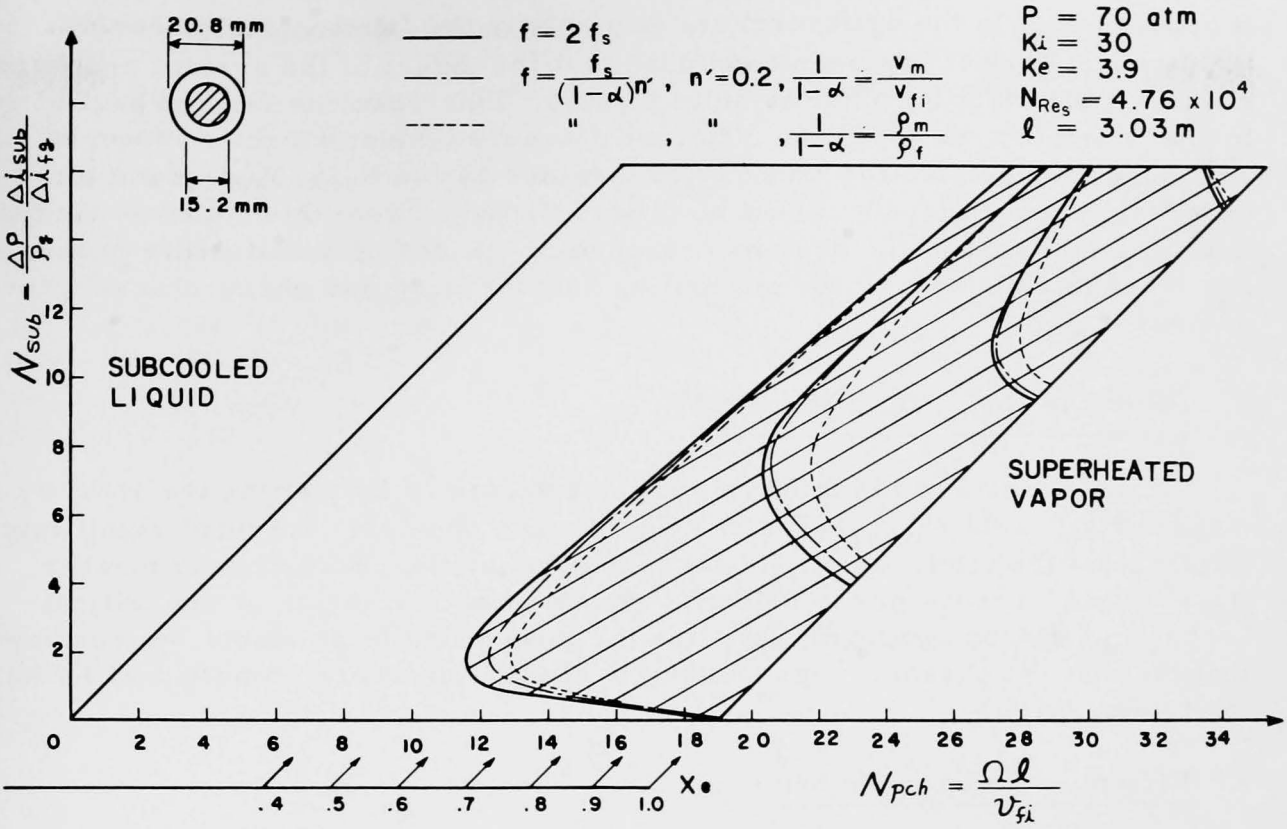


Fig. 15. Effect of Different Friction-factor Models

The large change of the stability-boundary curve at the neighborhood of $(N_{sub})_c$ can be explained by the fact that the frequency ω^* increases as N_{sub} decreases along the curve; thus the higher-order polynomial term in the characteristic equation becomes important as ω^* approaches 1. From this argument, it can be said that $(N_{sub})_c$ happens at $\omega^* \approx 1$. Thus from Eq. 62 we have

$$(N_{sub})_c \lesssim \pi. \tag{64}$$

In terms of the subcooling at a given N_{pch}, we conclude that the increase of N_{sub} is stabilizing for $N_{sub} > (N_{sub})_c$ and destabilizing for $N_{sub} < (N_{sub})_c$.

B. Effects of N_{pch} (Heat Flux)

From Figs. 8-15 as well as from the simple criterion (Eq. 62), it can be seen that increasing N_{pch} is always destabilizing. Thus increasing heat flux at constant inlet velocity shifts the system to the unstable direction.

C. Effects of System Pressure

As explained in Sec. VI, the dynamic effects of the system pressure are characterized by the density number N_ρ. In the characteristic equation,

it appears only in the drift term and possibly in the friction-factor coefficients C_m. Thus it has been concluded that the extent of the system pressure effects in the stability plane is quite limited. This result is further backed up by the computer run for three different pressure levels, which is shown in Fig. 8, where the stability boundaries obtained on the N_{sub}-N_{pch} plane for different pressure levels cannot be differentiated. From this we conclude that the important part of the system-pressure influences on the stability boundaries are taken into account by the subcooling number N_{sub} and phase-change number N_{pch}.

D. Effects of N_{Re_s} (Inlet Velocity)

The effects of the inlet velocity are examined by plotting the stability maps for different N_{Re_s} in Fig. 9. The results show that the most significant influence of the inlet velocity is accounted for by the phase-change number N_{pch}. Thus, increasing the velocity is stabilizing. In terms of the critical heat flux, i.e., the maximum heat flux for the system to be stable, we can say that the critical phase-change number is almost inversely proportional to the inlet velocity.

E. Effects of Relative Velocity

By changing the drift number N_d, we have examined the effects of relative velocity. The result, shown in Fig. 10, agrees with the analytical conclusion that the relative velocity has a stabilizing effect.

F. Effects of the Inlet Restriction k_i

Figure 11 shows the drastic influence of k_i on the stability boundary. As has been concluded analytically, increasing k_i is a strong stabilizing factor.

G. Effects of the Exit Restriction k_e

Figure 12 shows the important influence of k_e on the stability of the system. The effect of increasing k_e is shown to be strongly destabilizing, as has been expected.

H. Effects of Static Friction Factor

The effect of the static magnitude of the friction factor is examined by changing the value of the friction-factor coefficient C_m. As can be seen from Fig. 13, the increase in two-phase frictional pressure drop is destabilizing. The quantitative effect of C_m on the stability boundary largely depends on the values of k_i and k_e. Thus, it can be said that the friction factor becomes more important as the values of k_i and k_e decrease.

I. Effects of Dynamic Friction Factor

The dynamic effects of the two-phase frictional pressure drop have been examined for three different models. The results are shown in Figs. 13-15. By comparing these three stability maps, we can see some differences among them; however, if proper values are taken for C_m and n', the differences are limited. Thus we may conclude that, for a system with sufficiently large k_i and/or k_e, the difference in the friction-factor model is not important for operations at high system pressure. However, it also should be noted that introducing the complicated dynamic friction factor into the characteristic equation may generate a numerical instability due to the complication in the transfer function for the frictional pressure drop.

X. COMPARISON WITH THE EXPERIMENTAL DATA AND DISCUSSIONS

The analyses developed in previous chapters have been compared to the experimental data of Refs. 19, 21, 26, and 27. The geometry of the three independent experiments includes both circular and annular tubes at different diameters. The working fluids were water[19,21] and Freon-113.[26,27] The experimental data in Refs. 19 and 21 were well-suited to examine the thermal-equilibrium model,[28,29] since the data were for a high-pressure water system. On the other hand, the data of Refs. 26 and 27 are expected to show some effects of the thermal nonequilibrium. Therefore, the later data were also compared to the nonequilibrium model.[26]

The theory is compared to the experimental data in Figs. 16-21. The quantitative agreements of the stability boundary as well as of the frequency of the oscillation are excellent. As predicted by the theory, the effect of the system pressure is absorbed by the nondimensional groups, namely, N_{pch} and N_{sub}. Therefore the data for three different pressures are scattered around one single stability boundary, as seen in Fig. 20. However, the exit quality corresponding to a particular value of N_{pch} increases with increasing pressure. The equilibrium theory and the simple stability criterion of Eq. 60 are in good agreement with the experimental data, except for low subcooling numbers. For Freon-113, the nonequilibrium effects are pronounced, and the nonequilibrium theory of Saha[26] well predicts this trend (see Fig. 20). Further discussions on the comparison of the theories to experimental data are given in Refs. 26, 27, and 29.

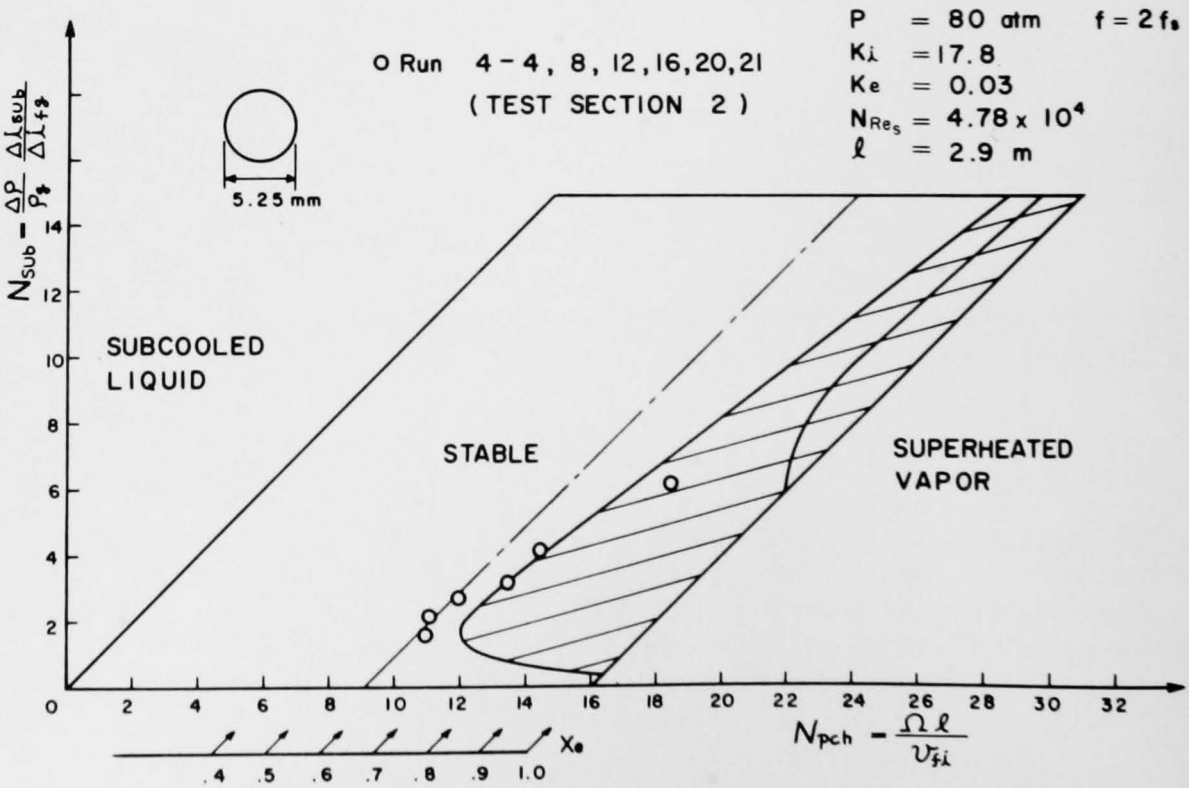


Fig. 16. Comparison with Solberg's Experiments. The solid lines indicate the computer solution and the dashed line the simple criterion.

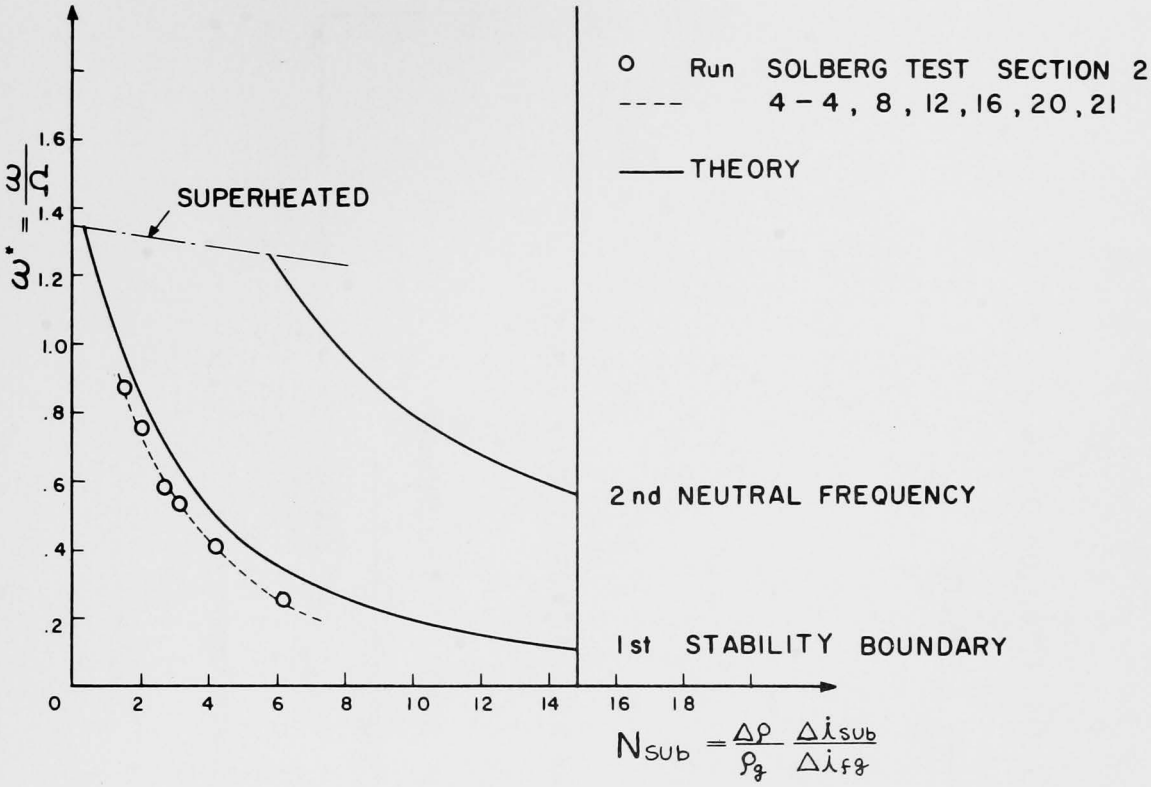


Fig. 17. Comparison of Predicted with Measured Frequency

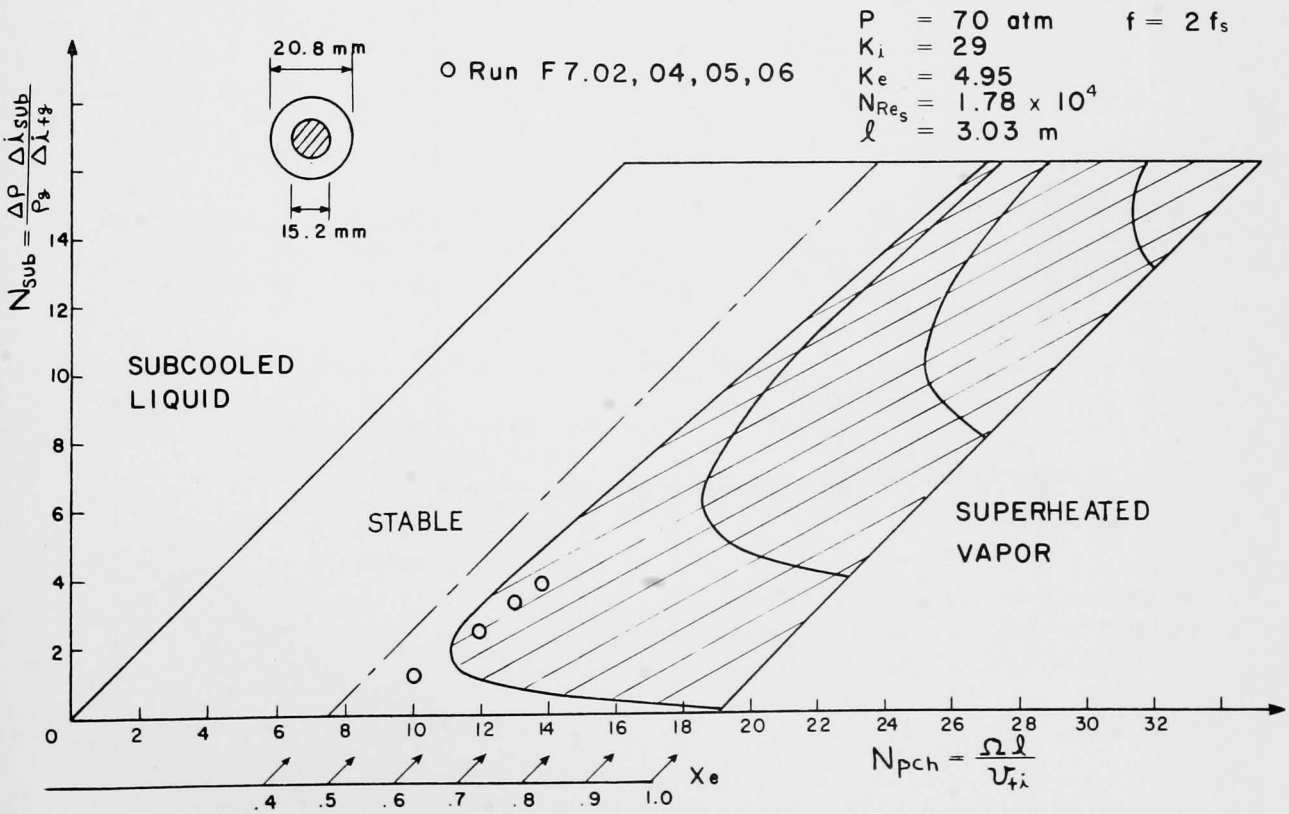


Fig. 18. Comparison with FLARE Experiments. The solid lines indicate the computer solution and the dashed line the simple criterion.

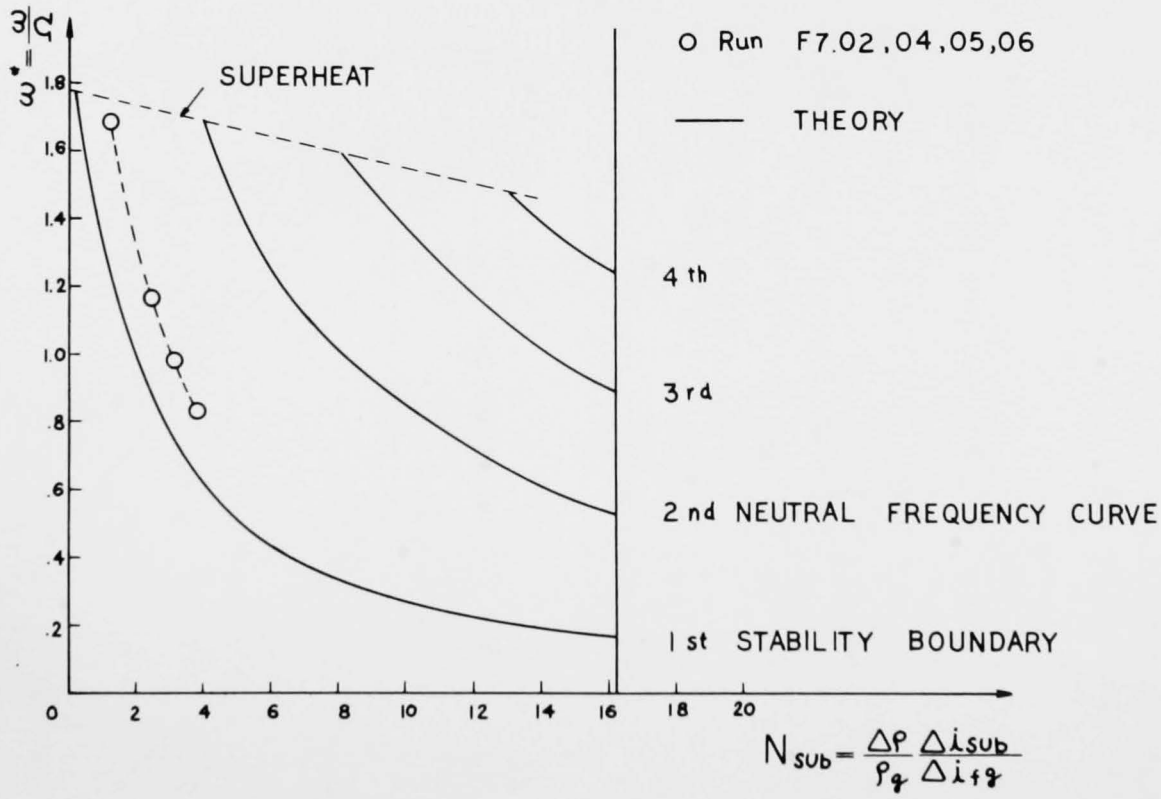


Fig. 19. Comparison of Predicted with Measured Frequency

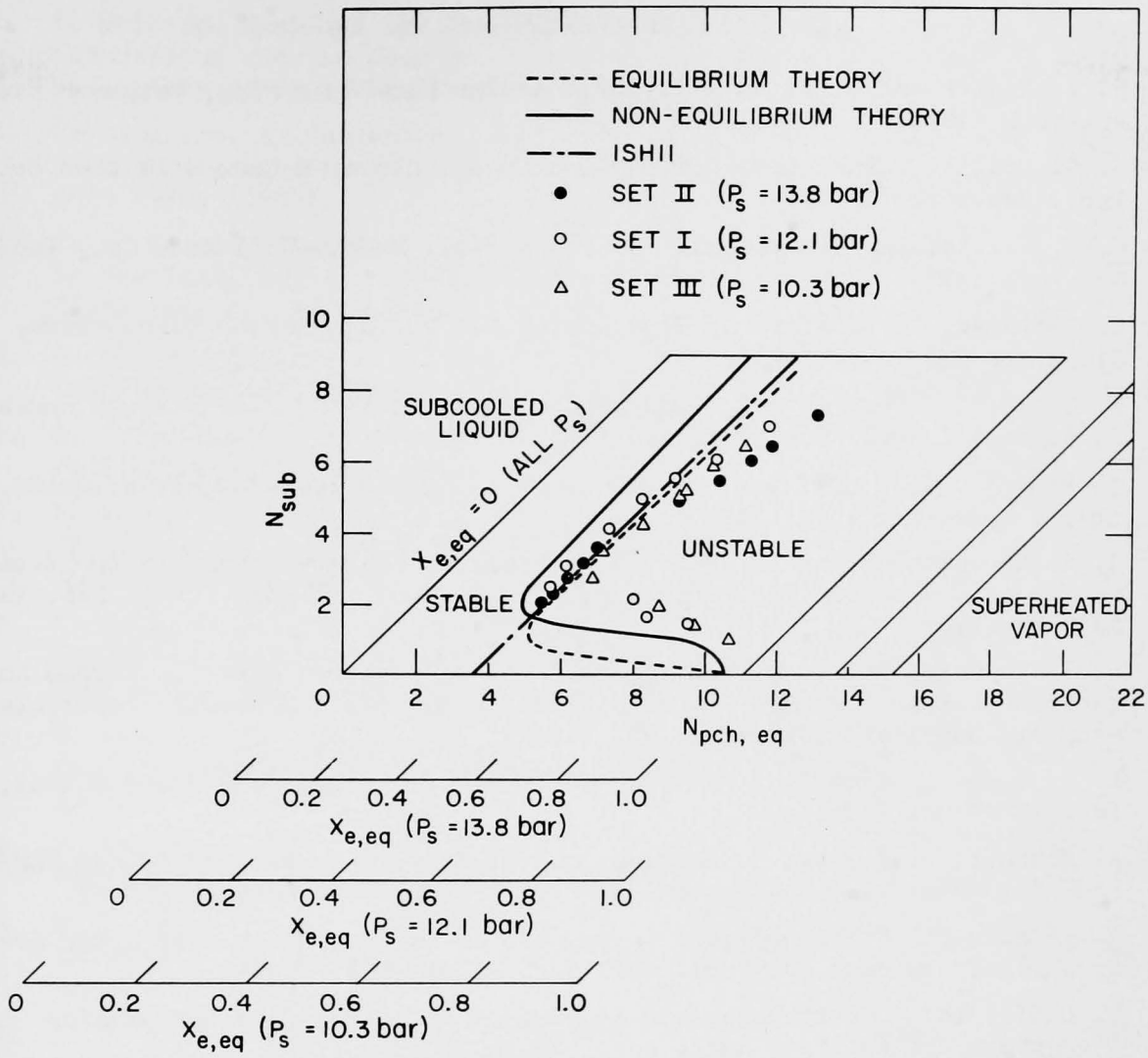


Fig. 20. Comparison with Freon-113 Experiments[26] at Various Pressure Levels. The test section was of 10-mm ID and 274.3-cm long, with k_i = 2.85, k_e = 2.03, and N_{Res} = 5.9 x 10^4. The dashed line indicates the simple criterion. ANL Neg. No. 900-5353.

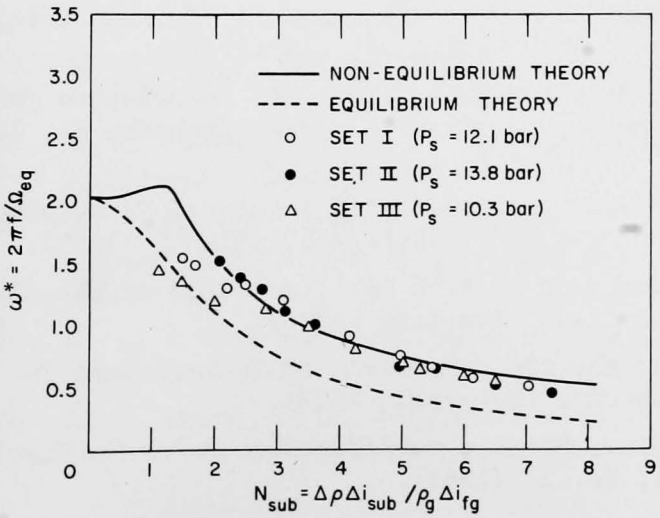


Fig. 21

Comparison of Predicted with Measured Frequency. The data were identical to those in Fig. 20. ANL Neg. No. 900-5360 Rev. 1.

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