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A METHOD FOR RECONSTRUCTING A THERMAL-EXPANSION GRAPH FROM TWO VALUES OF THE MEAN EXPANSION COEFFICIENT

by

Frank L. Yaggee and Frank G. Foote

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Materials Science Division

December 1969

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A METHOD FOR RECONSTRUCTING A THERMAL-EXPANSION GRAPH FROM TWO VALUES OF THE MEAN EXPANSION COEFFICIENT

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ABSTRACT

A simplified method is presented for reconstructing a linear thermal-expansion graph from two known values of the mean expansion coefficient ($\bar{\alpha}$). This method assumes that the expansion curve is a monotonic function of parabolic form, and is adequately defined by a quadratic polynomial. The method provides a means of calculating values of the instantaneous expansion coefficient (α) over any range of temperature between 0 and 1000°C, and of determining the polynomial coefficients A and B, which can be used to reconstruct the entire expansion curve. The simple computations can be done either graphically or on a desk calculator, and the results compare well within 1% of the values obtained by performing the dilatation experiment. The validity of this method is demonstrated for metals, alloys, ceramics, and cermets that have body-centered-cubic, face-centered-cubic, close-packed-hexagonal, and monoclinic crystal structures, and mean expansion coefficients with a fivefold difference in magnitude (5×10^{-6} to 23×10^{-6} units/unit-°C) for the temperature range from 0 to 1000°C.

INTRODUCTION

In one form or another, linear thermal-expansion information is basic to the design and construction of all engineering structures that must operate in an environment of varying temperatures. This is especially true for engineering power-producing systems such as boilers, turbines, steam generators, and nuclear power reactors that normally operate at elevated temperatures (above 500°C) and cycle over a wide temperature range. Since the expansions and contractions of the system components cannot be physically restricted, they must be adequately accommodated in the physical design.

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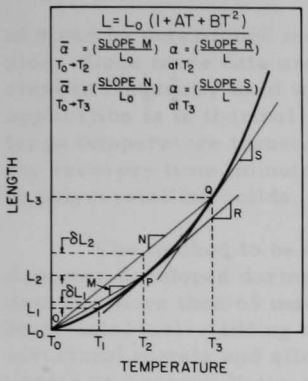

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Fig. 1. Relationship of Both the Mean Expansion Coefficient ($\bar{\alpha}$) and the Instantaneous Expansion Coefficient (α) to the Thermal-expansion Curve

Linear thermal-expansion data are usually presented either as a curve of length or fractional length versus temperature, as shown in Fig. 1, or as a coefficient of expansion for a finite temperature range. The expansion curve graphically illustrates the relationship between length and temperature. If the fractional expansion is plotted as the ordinate, the expansion curve provides the multiplying factor needed to determine the expanded length at any temperature. The information contained in an expansion curve, such as that shown in Fig. 1, cannot be used effectively in computer codes designed for numerical analyses of physical problems unless a mathematical expression is fitted to the experimental data.

The most familiar form for presenting linear thermal-expansion data is the mean expansion coefficient ($\bar{\alpha}$) for a stated temperature range. This coefficient is easily determined by measuring expansion over any temperature range of interest, which eliminates the necessity to determine the entire expansion curve. The relationship of $\bar{\alpha}$ to the expansion curve is shown in Fig. 1. In the temperature intervals T_0 to T_2 and T_0 to T_3, $\bar{\alpha}$ is related to the chords OP and OQ with slopes M and N, respectively. The value of $\bar{\alpha}$ between T_0 to T_3 is in error at T_2 by an amount δL_2. However, the error at T_2 in $\bar{\alpha}$ between T_0 and T_2 is considerably less, and is, in fact, the experimental error involved in making the expansion measurement. The error δL_1 at T_1 in $\bar{\alpha}$ for the temperature interval T_0 to T_2 is also very much less than δL_2. As is evident from Fig. 1, the error in $\bar{\alpha}$ increases as the curvature (degree of concavity) of the expansion curve increases, and for a curve of a given and constant curvature, the error in $\bar{\alpha}$ increases as the temperature interval increases. For this reason, the value of $\bar{\alpha}$ for a given situation must be selected judiciously. This is especially true when either the initial (reference) length of the structural member is very large or the temperature interval of heating is large.

The instantaneous expansion coefficient (α) is related to the slope (dL/dT) of the expansion curve at a particular temperature (as shown in Fig. 1). At temperatures T_2 and T_3, the values of α are related, respectively, to the slopes R and S at the points P and Q. The value of α will always be greater than the value of $\bar{\alpha}$ at corresponding temperatures for materials that exhibit a normal expansion curve (concave upward). If the experimental expansion data are plotted as the total specimen length versus temperature (see Fig. 1), the value of α can be determined by graphical construction. If, however, the experimentally determined length versus temperature data are fitted to a polynominal expression, the value

of α can be determined mathematically by manipulating the resulting expression. Since more data are needed to determine α than $\bar{\alpha}$, the latter coefficient is frequently used when α would be more appropriate. One such application is in thermal-stress calculations, where the time interval for large temperature transients is very small (microseconds) compared with the recovery time (minutes to hours) necessary to relieve thermal stresses in polycrystalline solids.

The method to be described for reconstructing thermal-expansion data was developed during the computer analysis of experimental expansion data for more than 65 materials, which include potential nuclear fuel materials, fuel-cladding alloys, and some of the more common engineering structural metals and alloys. The experimental expansion data were generated at Argonne National Laboratory in an automated fused-quartz dilatometer[1] at temperatures between 25 and 1000°C. Quadratic and cubic polynomial expressions were fitted to these data by the nonlinear regression-analysis method. With a single exception (chromium), the quadratic polynomial fit was as good as or better than the cubic fit and was preferred because of its simpler form.

The validity of this method is demonstrated for four general classes of materials, namely, unalloyed metals (M_1,* M_2, ...), binary and ternary alloys (M_1-M_2, M_1-M_2-M_3), ceramics (M_1X,** M_1X-M_2X), and cermets (M_1-M_2X). The method assumes that the expansion curve is parabolic and adequately defined by a quadratic polynomial and is also a monotonic function of temperature. Only two reliable values of $\bar{\alpha}$ are required, and they may be determined for any temperature interval within the temperature range of interest. The only necessity is that the values of $\bar{\alpha}$ be spaced at a sufficient interval to permit a reliable linear extrapolation.

DESCRIPTION OF THE METHOD

Since a wide variety of materials exhibit parabolic expansion curves between 25 and 1000°C, the expanded length at any temperature is adequately defined by the expression

$$L = L_0(1 + aT + bT^2), \tag{1}$$

where

L = expanded length at temperature,

L_0 = length at the reference temperature T_0,

a, b = polynomial coefficients,

---

*M_1 and M_2 are polycrystalline metals such as iron, copper, molybdenum, rhenium, tungsten, and vanadium.
**X represents an oxide, nitride, or carbide.

of α can be determined mathematically by manipulating the resulting expressions. Since more data are needed to determine α than $\bar{\alpha}$, the latter coefficient is frequently used when α would be more appropriate. One such application is in the thermal-stress calculations, where the time interval for large temperature transients is very small (microseconds) compared with the recovery time (minutes to hours) necessary to relieve the real stresses in polycrystalline solids.

The method to be described for reconstructing thermal-expansion data was developed during the computer analysis of experimental expansion data for more than 65 materials, which include potential nuclear fuel materials, fuel-cladding alloys, and some of the more common engineering structural metals and alloys. The experimental expansion data were generated at Argonne National Laboratory in an automated fused-quartz dilatometer[1] at temperatures between 25 and 1000°C. Quadratic and cubic polynomial expressions were fitted to these data by the nonlinear regression analysis method. With a single exception (chromium), the quadratic polynomial fit was as good as or better than the cubic fit and was preferred because of its simpler form.

The validity of this method is demonstrated for four general classes of materials, namely, unalloyed metals [M_1,* M_2, ...], binary and ternary alloys [M_1-M_2, M_1-M_2-M_3], ceramics [M_1X,** M_2X-M_3X], and cermets [M_1-M_2X]. The method assumes that the expansion curve is parabolic and adequately defined by a quadratic polynomial and is also a monotonic function of temperature. Only two reliable values of $\bar{\alpha}$ are required, and they may be determined for any temperature interval within the temperature range of interest. The only necessity is that the values of $\bar{\alpha}$ be spaced at a sufficient interval to permit a reliable linear extrapolation.

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*M_1 and M_2 are polycrystalline metals such as iron, copper, molybdenum, titanium, copper, and vanadium.

**X represents an oxide, nitride, or carbide.

and

T = temperature in °C.

By definition, $\bar{\alpha}$ between any two temperatures T_0 and T is given by

$$\bar{\alpha} = \frac{L - L_0}{L_0} \frac{1}{T - T_0}. \tag{2}$$

Equation 2 can be rewritten in terms of the polynomial coefficients a and b as

$$\bar{\alpha} = \frac{a(T - T_0) + b(T - T_0)^2}{T - T_0}. \tag{3}$$

If the reference temperature is taken as 0°C (T_0 = 0°C), then Eq. 3 reduces to

$$\bar{\alpha} = A + BT. \tag{4}$$

The translation of the reference temperature from 25 to 0°C can be made without difficulty, since the expansion curve for most materials will probably not exhibit an inflection within this narrow temperature range. Such a translation will only affect the values of the polynomial coefficients a and b in Eq. 1, which now become A and B.

Similarly, α at any temperature is given by

$$\alpha = \frac{1}{L} \frac{dL}{dT}, \tag{5}$$

which can be rewritten in terms of the polynomial coefficients A and B as

$$\alpha = \frac{A + 2BT}{1 + AT + BT^2}. \tag{6}$$

As T approaches 0°C, $\bar{\alpha}$ in Eq. 4 approaches α in Eq. 6, and both approach the limiting value A, which is arbitrarily called α_0 (the value of $\bar{\alpha}$ at T = 0°C):

$$A = \alpha_0. \tag{7}$$

Substituting Eq. 7 in Eq. 4 and solving for B yields

$$B = \frac{\bar{\alpha} - \alpha_0}{T}. \tag{8}$$

and

T = temperature in °C.

By definition, $\bar{\alpha}$ between any two temperatures T_0 and T is given by

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Substituting Eq. 7 in Eq. 4 and solving for B yields

$$B = \frac{\bar{\alpha} - \alpha_0}{T}. \tag{8}$$

Similarly, substituting Eqs. 7 and 8 in Eq. 6 and simplifying the results yields an expression for α in terms of $\bar{\alpha}$, the constant α_0, and the temperature T, namely,

$$\alpha = \frac{2\bar{\alpha} - \alpha_0}{1 + \bar{\alpha}T}. \tag{9}$$

Equation 9 is used to calculate values of α from known or extrapolated values of $\bar{\alpha}$.

Since Eqs. 6 and 9 are equivalent expressions and their numerators and denominators are equal, Eq. 6 can be rewritten as

$$\alpha = \frac{A + 2BT}{1 + \bar{\alpha}T}. \tag{10}$$

Multiplying through by the factor $(1 + \bar{\alpha}T)$ yields

$$K = A + BN, \tag{11}$$

where

$$K = \alpha(1 + \bar{\alpha}T)$$

and

$$N = 2T.$$

Equation 11 has the form of a straight line with slope B and intercept A when values of K are plotted against values of N. The numerical values of A and B in Eq. 11 can be determined either by linear-regression analysis of by graphical means. These values are then substituted in Eq. 1, or variations thereof (for example, L/L_0 versus T and $(L/L_0 - 1) \times 100$ versus T), to reconstruct the desired form of the thermal-expansion curve.

RESULTS

The nine materials used to demonstrate the method for reconstructing a thermal-expansion curve are vanadium, copper (OFHC*), V-20 wt % Ti, V-15 wt % Cr-5 wt % Ti, AISI Type 304 stainless steel, UO_2, UO_2-10 mole % Gd_2O_3, W-20 vol % UO_2, and W-80 vol % UO_2. It is assumed that three values of $\bar{\alpha}$ are known for each material for the temperature ranges 0-100, 0-200, and 0-400°C. The three known values of $\bar{\alpha}$ are plotted as a function of temperature, as shown in Fig. 2, and the values

*Oxygen-free, high-conductivity copper.

Similarly, substituting Eqs. 1 and 4 in Eq. 5 and simplifying the results yields an expression for σ in terms of $\bar{\sigma}$, the constant σ_0, and the temperature T, namely,

$$\sigma = \frac{\bar{\sigma}T - \sigma_0}{1 + \bar{\sigma}T} \tag{9}$$

Equation 9 is used to calculate values of $\bar{\sigma}$ from known or extrapolated values of σ.

Since Eqs. 6 and 9 are equivalent expressions and their numerators and denominators are equal, Eq. 6 can be rewritten as

$$A = \frac{A + \bar{\sigma}T}{1 + \bar{\sigma}T} \tag{10}$$

Multiplying through by the factor $(1+\bar{\sigma}T)$ yields

$$K = A + \bar{\sigma}M \tag{11}$$

where

$$K = A(1 + \bar{\sigma}T)$$

and

$$M = \bar{\sigma}T$$

Equation 11 has the form of a straight line with slope $\bar{\sigma}$ and intercept A when values of K are plotted against values of M. The numerical values of A and $\bar{\sigma}$ in Eq. 11 can be determined either by linear-regression analysis or by graphical means. These values are then substituted in Eq. 1 or variations thereof (for example, $\ln T_0$ versus T and $(\ln \sigma_0 - \ln \sigma)$ versus T_0) to reconstruct the desired form of the thermal-expansion curve.

RESULTS

The data materials used to demonstrate the method for reconstructing a thermal-expansion curve are vanadium, copper (OFHC*), V-20 wt % Ti, V-15 wt % Cr-5 wt % Ti, AISI Type 316 stainless steel, UO_2, UO_2-10 mole % Gd_2O_3, W-20 vol % UO_2, and W-40 vol % UO_2. It is assumed that three values of $\bar{\sigma}$ are known, for each material for the temperature ranges 0-100°, 0-200°, and 0-400°C. The three known values of $\bar{\sigma}$ are plotted as a function of temperature, as shown in Fig. 2, and the values

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*Oxygen-free, high-conductivity copper.

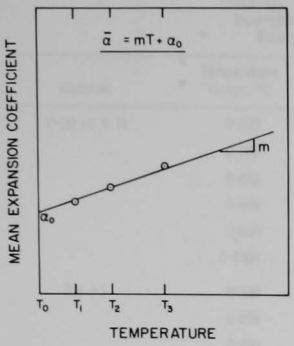

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Fig. 2. Graphical Extrapolation of the Mean Thermal-expansion Coefficient from Three Known Values

of α_0 and slope m are determined. Extrapolated values of $\bar{\alpha}$ between 0 and 1000°C can be read directly from Fig. 2, or estimated by using the values of α_0 and m determined from Fig. 2. Table I lists the three known values of $\bar{\alpha}$ and the values of m and α_0 determined from Fig. 2 for these materials. Extrapolated values of $\bar{\alpha}$ are tabulated in column 3 of Tables II-V. The tables also compare values of α calculated from experimental data by Eq. 6 with corresponding values of α calculated from the three known values of $\bar{\alpha}$ by Eq. 9. In the former case, a quadratic polynomial of the type shown in Eq. 1 is computer fitted to the experimental thermal-expansion data by nonlinear regression analysis with a 50- to 60-point input.[2] The computer-determined values of A and B are used in Eq. 6. Table VI compares the computer-determined polynomial coefficients A and B determined from an analysis of the experimental data with corresponding coefficients determined from Eq. 11.

TABLE I. Values of the Mean Expansion Coefficient Used to Reconstruct the Thermal-expansion Curve

Material	Known Values of the Mean-expansion Coefficient, $\bar{\alpha}$, 10^{-6}/°C			$\bar{\alpha} = mT + \alpha_0$	
	0-100°C	0-200°C	0-400°C	m, 10^{-6}/°C	α_0, 10^{-6}
Vanadium	9.1900	9.3400	9.6500	0.00154	9.035
Copper (OFHC)	19.8100	19.9700	20.3100	0.00167	19.6400
V-20 wt % Ti	8.9700	9.1600	9.5300	0.00186	8.7850
304 SS	16.3300	16.5300	16.7200	0.0039	16.1367
V-15 wt % Cr-5 wt % Ti	9.4800	9.7000	10.1500	0.00223	9.2550
UO_2	9.9289	10.0434	10.2724	0.00114	9.8144
UO_2-10 mole % Gd_2O_3	9.9480	10.1380	10.5170	0.00189	9.7585
W-20 vol % UO_2	5.1720	5.2300	5.3461	0.00058	5.1140
W-80 vol % UO_2	8.0600	8.1770	8.4050	0.0015	7.946

TABLE II. Comparison of the Calculated Value of the Instantaneous Expansion Coefficient with the Value Obtained by Experiment for Materials of Type M_1, M_2, ...

Material	Temperature Range, °C	$\bar{\alpha}$,[a] 10^{-6}/°C	Temperature, °C	α Experiment,[b] 10^{-6}/°C	α Calculated,[c] 10^{-6}/°C	Percent Difference
Vanadium	0-100	9.1886	100	9.3336	9.3342	0.006
	0-200	9.3421	200	9.6312	9.6366	0.056
	0-400	9.6493	400	10.2241	10.2387	0.143
	0-600	9.9564	600	10.8132	10.8370	0.220
	0-800	10.2636	800	11.3994	11.4314	0.281
	0-1000	10.5707	1000	11.9804	12.0217	0.345
Copper (OFHC)	0-100	19.8071	100	19.9345	19.9352	0.003
	0-200	19.9743	200	20.2264	20.2272	0.004
	0-400	20.3086	400	20.8033	20.8090	0.027
	0-600	20.6429	600	21.3727	21.3812	0.040
	0-800	20.9711	800	21.9344	21.9457	0.051
	0-1000	21.3114	1000	22.4883	22.5025	0.063

(a) $\bar{\alpha} = mT + \alpha_0$. (b) $\alpha = \dfrac{A + 2BT}{1 + AT + BT^2}$. (c) $\alpha = \dfrac{2\bar{\alpha} - \alpha_0}{1 + \bar{\alpha}T}$.

TABLE III. Comparison of the Calculated Value of the Instantaneous Expansion Coefficient with the Value Obtained by Experiment for Binary and Ternary Alloys of Type M_1-M_2 and M_1-M_2-M_3

Material	Temperature Range, °C	$\bar{\alpha}$,[a] 10^{-6}/°C	Temperature, °C	α Experiment,[b] 10^{-6}/°C	α Calculated,[c] 10^{-6}/°C	Percent Difference
V-20 wt % Ti	0-100	8.9714	100	9.1519	9.1496	0.025
	0-200	9.1579	200	9.5168	9.5134	0.035
	0-400	9.5307	400	10.2434	10.2374	0.059
	0-600	9.9036	600	10.9656	10.9571	0.078
	0-800	10.2764	800	11.6829	11.6711	0.101
	0-1000	10.6493	1000	12.3952	12.3812	0.113
304 SS	0-100	16.5283	100	16.8871	16.8898	0.016
	0-200	16.9183	200	17.6335	17.6380	0.026
	0-400	17.6983	400	19.1223	19.1142	0.042
	0-600	18.4783	600	20.5777	20.5894	0.057
	0-800	19.2583	800	22.0230	22.0382	0.069
	0-1000	20.0383	1000	23.4488	23.4675	0.080
V-15 wt % Cr-5 wt % Ti	0-100	9.4786	100	9.6963	9.6930	0.034
	0-200	9.7021	200	10.1345	10.1295	0.049
	0-400	10.1493	400	11.007	10.9984	0.028
	0-600	10.5964	600	11.8736	11.8616	0.101
	0-800	11.0436	800	12.7341	12.7206	0.106
	0-1000	11.4907	1000	13.5879	13.5711	0.123

(a) $\bar{\alpha} = mT + \alpha_0$. (b) $\alpha = \dfrac{A + 2BT}{1 + AT + BT^2}$. (c) $\alpha = \dfrac{2\bar{\alpha} - \alpha_0}{1 + \bar{\alpha}T}$.

TABLE IV. Comparison of the Calculated Value of the Instantaneous Expansion Coefficient with the Value Obtained by Experiment for Ceramic Materials of Type M_1X and M_1X-M_2X

Material	Temperature Range, °C	$\bar{\alpha}$,[a] 10^{-6}/°C	Temperature, °C	α Experiment,[b] 10^{-6}/°C	α Calculated,[c] 10^{-6}/°C	Percent Difference
UO_2	0-100	9.9289	100	10.0034	10.0335	<0.001
	0-200	10.0434	200	10.2518	10.2518	0
	0-400	10.2724	400	10.6866	10.6866	0
	0-600	10.5014	600	11.1183	11.1184	<0.001
	0-800	10.7304	800	11.5474	11.5474	0
	0-1000	10.9594	1000	11.9730	11.9733	<0.001
UO_2-10 mole % Gd_2O_3	0-100	9.9481	100	10.1276	10.1280	0.004
	0-200	10.0430	200	10.4959	10.4962	0.003
	0-400	10.5171	400	11.2281	11.2283	0.002
	0-600	10.8964	600	11.9554	11.9553	0.001
	0-800	11.2756	800	12.6774	12.6791	0.013
	0-1000	11.6549	1000	13.3939	13.3954	0.011

(a) $\bar{\alpha} = T + \alpha_0$. (b) $\alpha = \dfrac{A + 2BT}{1 + AT + BT^2}$. (c) $\alpha = \dfrac{2\bar{\alpha} - \alpha_0}{1 + \bar{\alpha}T}$.

TABLE V. Comparison of the Calculated Value of the Instantaneous Expansion Coefficient with the Value Obtained by Experiment for Cermets of Type M_1-M_2X

Material	Temperature Range, °C	$\bar{\alpha}$,[a] 10^{-6}/°C	Temperature, °C	α Experiment,[b] 10^{-6}/°C	α Calculated,[c] 10^{-6}/°C	Percent Difference
W-20 vol % UO_2	0-100	5.1721	100	5.2274	5.2274	0
	0-200	5.2301	200	5.3405	5.3408	<0.001
	0-400	5.3461	400	5.5662	5.5664	<0.001
	0-600	5.4621	600	5.7911	5.7910	<0.001
	0-800	5.5781	800	6.0153	6.0154	<0.001
	0-1000	5.6941	1000	6.2380	6.2386	<0.001
W-80 vol % UO_2	0-100	8.0609	100	8.1692	8.1707	0.018
	0-200	8.1757	200	8.3917	8.3922	0.006
	0-400	8.4054	400	8.8334	8.8351	0.019
	0-600	8.6351	600	9.2723	9.2761	0.041
	0-800	8.8649	800	9.7087	9.7149	0.064
	0-1000	9.0946	1000	10.1509	10.1509	0.082

(a) $\bar{\alpha} = mT + \alpha_0$. (b) $\alpha = \dfrac{A + 2BT}{1 + AT + BT^2}$. (c) $\alpha = \dfrac{2\bar{\alpha} - \alpha_0}{1 + \bar{\alpha}T}$.

TABLE VI. Comparison of the Calculated Polynomial Coefficients A And B with Those Obtained by Experiment

Material	Temperature Range, °C	Calculated[a]		Experiment[b]		Percent Difference	
		A, 10^{-6}/°C	B, 10^{-9}/°C	A, 10^{-6}/°C	B, 10^{-9}/°C	A/A	B/B
Vanadium	0-1000	9.035	1.535	9.031	1.559	0.047	1.55
Copper (OFHC)	0-1000	19.640	1.671	19.642	1.662	0.012	0.519
V-20 wt % Ti	0-1000	8.785	1.864	8.786	1.871	0.009	0.353
304 SS	0-950	16.092	3.891	16.137	3.891	0.276	0.001
V-15 wt % Cr-5 wt % Ti	0-1000	9.2546	2.236	9.2567	2.244	0.023	0.339
UO_2	0-1000	9.8144	1.145	9.8144	1.145	0	0
UO_2-10 mole % Gd_2O_3	0-1000	9.7583	1.896	9.7589	1.896	0.006	0
W-20 vol % UO_2	0-1000	5.1141	0.5806	5.1140	0.5804	<0.001	<0.001
W-80 vol % UO_2	0-1000	7.9460	1.148	7.9487	1.143	0.034	0.484

(a) $K = A + BN$.
(b) $L = L_0(1 + AT + BT^2)$.

DISCUSSION

The proposed method for reconstructing thermal-expansion data permits the calculation of reliable values of α from a minimum of two reliable values of $\bar{\alpha}$, and also provides a shortcut for determining the polynomial coefficients A and B so that the entire expansion curve can be reconstructed. This method assumes the expansion curve is a monotonic function of temperature, has a parabolic form and, therefore, can be

adequately defined by a quadratic polynomial, and uses the fact that $\bar{\alpha}$ is not a sensitive function of temperature. Equation 9 also satisfies the necessary condition that α be greater than $\bar{\alpha}$ at all temperatures for materials exhibiting the normal, concave-upward, thermal-expansion curve.

The nine materials used to demonstrate this technique represent four crystal classes, namely, body-centered cubic, face-centered cubic, close-packed hexagonal, and monoclinic, and have mean expansion coefficients from 5×10^{-6} to 23×10^{-6} units/unit-°C. This technique has also been applied successfully to Armco iron (body-centered cubic), a variety of refractory metals and alloys such as tungsten, rhenium, molybdenum, W-25 wt % Re, and W-30 at. % Re-30 at. % Mo (body-centered cubic and close-packed hexagonal), titanium (close-packed hexagonal), and the U-Pu-Ti- and U-Pu-Zr-type potential fuel alloys. Possibly the same technique would apply equally well to fiber-reinforced composites, provided coherence is maintained between the reinforcing fibers and the matrix.

The simplicity of the technique provides a means of determining the change in expansion behavior exhibited by a material after exposure to an integrated fast-neutron flux on the order of 10^{22} to 10^{23} n/cm^2 with a simple dial-gage dilatometer. Many potential cladding materials for nuclear fuels are highly radioactive after such neutron exposures and therefore must be remotely handled to manipulators in a hot-cell facility. Installation of sophisticated equipment in a hot-cell facility for remote operation can be very costly and can present operational problems. Since the technique described requires only two values of the mean expansion coefficient, these values can be obtained easily with an inexpensive dial-gage dilatometer that can be operated remotely by manipulators. This approach to the problem of obtaining postirradiation thermal-expansion data on potential nuclear fuel and fuel-cladding alloys is being implemented at Argonne National Laboratory.

Although many of the computations indicated in Eqs. 1-11 were carried out on a time-sharing computer, they can be solved either graphically or by a desk calculator. The results would be equally as acceptable as those presented in this report.

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