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# SENSITIVITY STUDIES OF THE EFFECT OF UNCERTAINTY IN THE $^{238}U(n,\gamma)$ AND IN THE $^{239}Pu(n,f)$ AND $(n,\gamma)$ CROSS SECTIONS

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# SENSITIVITY STUDIES OF THE EFFECT OF UNCERTAINTY IN THE $^{238}U(n,\gamma)$ AND IN THE $^{239}Pu(n,f)$ AND $(n,\gamma)$ CROSS SECTIONS

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## ABSTRACT

The effects of current uncertainties in the above cross sections below 1 MeV, believed to be the most serious for fast reactors, were studied for a spherical model of a large LMFBR. Total variations found for a probably pessimistic assumption for $^{238}U$ $\sigma(n,\gamma)$ uncertainty were about 3% in k, 0.10 in breeding ratio, 5% in sodium-void effect (out of a total effect of +2.7% k), and 20% in $^{238}U$ Doppler effect. For $^{239}Pu$ $\sigma(n,f)$ and $\sigma(n,\gamma)$ below 30 keV, disagreements among recent measurements (which are usually within error bars) correspond to a variation of about 1% in k, 0.05 in breeding ratio, 15% in sodium-void effect, and 12% in $^{238}U$ Doppler effect. Uncertainty in the $^{239}Pu$ fission cross section above 30 keV corresponds to a variation of several percent in k, about a ±5% uncertainty in the sodium-void effect, and minor changes in the other quantities. The most serious of these uncertainties are those in k and in breeding ratio. Improvement in knowledge of low energy $^{239}Pu$ cross sections has significantly reduced the uncertainty in reactivity coefficients.

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*NOTE:* This report is an expanded version of the paper of the same title presented at the *Third Conference on Neutron Cross Sections and Technology, University of Tennessee, Knoxville, March 15-17, 1971.*

## INTRODUCTION

It is generally recognized that nuclear data uncertainties are the principal cause of unreliability in fast reactor physics calculations. The most important data uncertainties are commonly considered to be in the fission and capture cross sections of $^{239}$Pu and in the capture cross section of $^{238}$U. Because there have been a number of recent measurements and evaluations of these cross sections, it seemed to be of interest to assess the current uncertainty in the most important fast reactor characteristics associated with uncertainty in these cross sections. The reactor properties selected for study were reactivity, sodium-void effect, Doppler effect of $^{238}$U, and breeding ratio (B.R.).

## NUCLEAR DATA SELECTION

### $^{239}$Pu Fission and Capture Below 30 keV

A number of authors[1-7] have recently presented results for these cross sections in a form that facilitates comparison: a tabulation for common energy intervals ranging from 0.1 keV at low energies to 5 keV at higher energies. Results in most cases now agree within error bars; these error bars are sometimes rather large, however, particularly for the capture-to-fission ratio, α. It seemed that simply computing the reactor properties corresponding to the various reported cross section values would give a reasonable estimate of the uncertainty from this source. The discrepancies obtained in this way are probably smaller than those that would correspond to the uncertainty in individual measurements because of the cancellation of positive and negative effects. The reference cross sections from which variations were made were those presented by Pitterle, et al.[8] The energy-averaged values for ENDF/B-II[(9)] were taken from Ref. 5.

The fission and capture cross sections were used directly as tabulated in the various papers, with self-shielding factors calculated by Kikuchi[10] applied to variations in the cross sections. These factors were assumed to be independent of the data, which should be good enough for the present purpose.

The base cross sections as calculated by MC$^2$ (11) were effective cross sections in that flux correction factors differing for different isotopes to take account of accidental resonance overlap were used.[12,13] The flux correction factors calculated by Kikuchi[10] were also based on this method for a $\sigma_p$ (scattering cross section per atom) consistent with the reactor composition being studied.

## $^{239}$Pu Fission and Capture Above 30 keV

In this case the variation made from the cross sections of Pitterle, et al was to lower fission and capture by up to 16% between 40 keV and 1 MeV, corresponding to the difference between the White[14] and Poenitz[15] (preliminary) $^{235}$U fission cross sections used as a standard. This was considered to be a representative uncertainty. The effect of increasing alpha by 20% from 30 to 800 keV was also determined; this is the uncertainty estimated by Greebler, et al.[16]

## $^{238}$U Capture

The ENDF/B Version I cross section[17] was used as a standard in this case. Upper and lower curves were constructed (Fig. 1) which were intended to represent extreme limits for this cross section, based on available experiments and evaluations. Below 25 keV the curves were calculated from the unresolved resonance parameters given in Table 1. The parameters for the upper curve are those of Schmidt[18] except that $D_{J=3/2}$ is 10.4 eV instead of 11.4. This curve agrees well with the 1966 evaluation of Schmidt below 100 keV.

The upper curve also agrees rather well with the data of Macklin, Gibbons and Pasma,[19] as renormalized by Davey,[20] extending up to 55 keV. Davey[20] included these measurements in his Category A of best available measurements, "Good Absolute Data." The upper curve lies from 6 to 10% above Davey's "Best Values" over most of the energy range below 100 keV. At higher energies up to 1 MeV, the upper curve agrees well with the measurements of Barry, Bunce and White,[21] and therefore with Davey's evaluated results,[20] which were based on these measurements.

The lower curve up to 100 keV agrees well with the data of Moxon,[22] except that it is at about the lower limit of the data between 10 and 30 keV. Several

recent evaluations[20,23,24] have chosen to renormalize Moxon's data upwards by factors of 1.09 to 1.15. This was justified by a desire for consistency with certain other measurements rather than by identification of any specific deficiency in Moxon's technique, however. The shapes of both upper and lower curves below 100 keV are similar to that of Moxon's data. At higher energy the lower curve agrees with the measurements of Menlove and Poenitz,[25] and above 140 keV also agrees closely with measurements of Fricke, et al.[26] As pointed out by Davey,[20] the discrepancy between his evaluated values and the Menlove and Poenitz values is essentially the same as the difference between the White[14] and Poenitz[15] $^{235}U$ fission cross sections and probably results from discrepancy in neutron flux monitoring.

If ratio measurements among $^{235}U$ fission, $^{239}Pu$ fission, and $^{238}U$ capture available above 100 keV[(27)] are used to obtain $^{238}U$ capture from the $^{239}Pu$ based on the White $^{235}U$ fission data, values close to the upper curve are obtained. If the preliminary Poenitz $^{235}U$ fission cross sections are used as a standard instead, values close to the lower curve are obtained up to about 600 keV. However, the error bars in the ratio measurements would allow a reduction of at least 5% from the upper curve for $^{238}U$ capture using the White $^{235}U$ fission cross section as a standard.

If the shapes of the Moxon and of the Menlove and Poenitz measurements are accepted, acceptance of the Barry, Bunce and White data requires high values of $^{238}U$ capture at low energy close to those of Macklin, Gibbons and Pasma. The measurements of Fricke, et al, however, which differ in shape from those of Moxon, indicate the possibility of cross sections substantially below those of Macklin, et al, below 30 keV even with agreement with the values of Barry, et al, at high energies.

Davey's best values for $^{238}U$ capture[20] are very close to ENDF/B-I values up to 30 keV and are within a few percent of them plus or minus up to 1 MeV. The ENDF/B Version II evaluated results prepared by Pitterle[23] are about 5% below Version I over most of the range below 100 keV, the discrepancy increasing to 15% over a small range around 80 keV. Between 100 keV and 1 MeV the two versions are in close agreement.

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The evaluated values by Konshin[24] are within several percent of the Version I values up to 1 MeV. Recent estimates of the uncertainty in the $^{238}U$ capture cross section are (1) ±5-10% below 2 keV, ±10% between 2 and 150 keV, ±5% between 0.15 and 2 MeV;[23] and (2) ±10%.[16] Since the difference between the upper and lower curves used here, which were meant to represent pessimistic limits, is more like ±15% above their average over most of the range below 100 keV and ±10% up to 1 MeV, the variation in reactor characteristics obtained depending on which curve is used should be divided by about 1.5 to be consistent with these estimates of error limits. The most reasonable reduction of the uncertainty assumed here is in the lowering of the upper limit between 1 and 30 keV, as there are no recent measurements to support values this high.

In calculating the effect of resonance self-shielding on the altered cross sections, only the effect on the numerator of the effective cross section was taken into account; the effect on the flux correction factor was neglected. This causes some error in calculation of the variation in the Doppler effect, but this error is not believed to be large enough to be important for the present purpose.

## METHOD OF CALCULATION

The cross section variations were made for a spherical model of a 1000-MWe oxide-fueled fast reactor used for parametric studies of LMFBRs.[28] The 5500-liter core contained two enrichment zones of equal volume, and had 40 vol-% fuel ($\rho$ = 0.85), 40% sodium, and 20% stainless steel. The isotopic plutonium composition was 66% $^{239}Pu$, 28% $^{240}Pu$, 4% $^{241}Pu$, and 20% $^{242}Pu$. Fission products corresponding to 5 heavy at-% burnup were present, and 0.5% homogeneously distributed tantalum in the outer zone simulated shim control effects. The core was surrounded by a 25-cm thick blanket containing 55 vol-% depleted $UO_2$, 30% sodium, and 15% stainless steel, and a reflector 15-cm thick containing 80% stainless steel and 20% sodium. The cross sections were ENDF/B Version I except for use of the Pitterle $^{239}Pu$ data and of lowered values for $^{238}U$ inelastic scattering.

The reference calculations were carried out with a 26-group set of cross sections with 0.5 lethargy unit width, generated by ultra-fine-group $MC^2$

calculations for a base temperature of 1300°K and also at 2500°K for Doppler-effect calculations. Doppler and sodium-void effect calculations were carried out by first-order perturbation calculations, assuming a uniform temperature rise in the core in the former case and uniform core voiding in the latter.

For study of the effect of the variation of $^{239}$Pu fission and capture below 30 keV, derivatives of the effect of variation of $\sigma(n,f)$ at constant $\alpha$ and of $\alpha$ at constant $\sigma(n,f)$ were obtained by direct k-calculations, with additional derivatives obtained for the adjustment to critical by enrichment search. The derivatives for the energy intervals of the tabulated cross sections were obtained from those in calculation groups by linear interpolation in energy. The effects of other cross section variations were obtained by direction enrichment search.

## RESULTS OF CALCULATIONS

The importance of the variation of reactor characteristics given in Tables 2 and 3 can be judged by comparing them with what are believed to be reasonable goals for the next few years for acceptable errors from all cross section uncertainties (see, for example, Ref. 16): reactivity, ±1% k; sodium-void effect, ±0.3% k; Doppler coefficient, ±5%; and breeding ratio, ±0.02.

### Variations in $^{239}$Pu $\sigma_f$ and $\sigma_\gamma$

Results obtained for variations of $^{239}$Pu $\sigma(n,f)$ and $\alpha$ are displayed in Table 2. Of the results of variations in $\sigma(n,f)$ at constant $\alpha$ below 30 keV, only the divergence in reactivity of about ±0.5% k seems unacceptably high. This deviation is mainly due to $\sigma(n,f)$ variation above 10 keV. Perhaps the most noteworthy result is the relatively small variation in sodium-void effect, amounting to ±0.1% k. A variation of this size is not very significant for safety, considering other uncertainties in dealing with accidents involving sodium voiding. This conclusion is in contrast with what was found from comparison calculations made in 1965, when differences amounting to ±1.0 to 1.5% k were found for total core voiding.[29] In that case, however, variations by factors as large as two occurred in choices of $\sigma(n,f)$ of $^{239}$Pu in the energy region below 30 keV made by various organizations, and this is believed to be the main source of such large discrepancies in the sodium-void effect.[30] The recent data as averaged over common energy intervals agree for the most part within 10 to 20%.

The self-shielding factors applied to σ(n,f) were about 0.7 at 0.1 keV, 0.8 at 0.3 keV, 0.9 at 0.7 keV, 0.93 at 1 keV, and 1.00 at 5 keV and above. No factor was applied to α since the factors for σ(n,f) and σ(n,γ) were nearly the same.

The energy region in which the sodium-void effect is most sensitive to a given percentage change in the fission cross section at constant α is from about 100 eV up to about 2 keV as can be seen in Table 4, in which the derivatives of reactor characteristics with respect to effective cross section changes are given as a function of energy. Since strong fluctuations in the fission cross section occur in this energy range, significant errors in reactor calculations are possible if the data are not properly averaged. It is desirable to have the data given in as much detail as possible as a function of energy so that the reactor physicist can perform his own averaging, taking into account the group energy structure he wishes to use, the attenuation of the neutron flux over such groups, and the perturbing effect of wide scattering resonances. A representation of the fluctuations to the extent possible in terms of resonance parameters is, of course, highly desirable for accurate resonance self-shielding and overlap and $^{239}$Pu Doppler-effect calculations. The latter were not attempted here because of the nonavailability of any resonance parameter representation for the various sets of data. Experimentally, the $^{239}$Pu Doppler effect appears to be small so that this deficiency does not seem serious. The $^{239}$Pu Doppler effect is defined here in the effective cross section sense,[12,13] which is the customary definition.

The effects of a variation in the low energy α at constant σ(n,f) are somewhat more significant for reactivity coefficients and breeding ratio, fairly marked differences between results from the ORNL-RPI data[5] and the data of Schomberg, et al,[1] being evident. The former data are generally higher than the latter, the difference exceeding error bars in some energy regions. It was found that for variation from ENDF/B-I[(31)] to II a loss in k of 0.5% occurs both for the σ(n,f) variation at constant α and the α variation at constant σ(n,f). The low α values of ENDF/B-I give variations of -0.16% k for the sodium-void effect and -0.03% for the Doppler effect relative to the base values. Since it is rather certain that the I values are too low, this difference represents a narrowing of the uncertainty range for these quantities.

The self-shielding factors applied to $\sigma(n,f)$ were about 0.7 at 0.1 keV, 0.8 at 0.2 keV, 0.9 at 0.7 keV, 0.93 at 1 keV, and 1.00 at 5 keV and above. No factor was applied to $\alpha$ since the factors for $\sigma(n,f)$ and $\sigma(n,\gamma)$ were nearly the same.

The energy region in which the sodium-void effect is most sensitive to a given percentage change in the fission cross section at constant $\alpha$ is from about 100 eV up to about 2 keV as can be seen in Table 4, in which the derivatives of reactor characteristics with respect to effective cross section changes are given as a function of energy. Since strong fluctuations in the fission cross section occur in this energy range, significant errors in reactor calculations are possible if the data are not properly averaged. It is desirable to have the data given in as much detail as possible as a function of energy so that the reactor physicist can perform his own averaging, taking into account the group energy structure he wishes to use, the attenuation of the neutron flux over such groups, and the perturbing effect of wide scattering resonances. A representation of the fluctuations to the extent possible in terms of resonance parameters is, of course, highly desirable for accurate resonance self-shielding and overlap and $^{239}Pu$ Doppler-effect calculations. The latter were not attempted here because of the nonavailability of any resonance parameter representation for the various sets of data. Experimentally, the $^{239}Pu$ Doppler effect appears to be small so that this deficiency does not seem serious. The $^{239}Pu$ Doppler effect is defined here in the effective cross section sense,[12,13] which is the customary definition.

The effects of a variation in the low energy $\alpha$ at constant $\sigma(n,f)$ are somewhat more significant for reactivity coefficients and breeding ratio, fairly marked differences between results from the ORNL-RPI data[5] and the data of Schomberg, et al,[1] being evident. The former data are generally higher than the latter, the difference exceeding error bars in some energy regions. It was found that for variation from ENDF/B-I[(31)] to II a loss in k of 0.5% occurs both for the $\sigma(n,f)$ variation at constant $\alpha$ and the $\alpha$ variation at constant $\sigma(n,f)$. The low $\alpha$ values of ENDF/B-I give variations of −0.16% k for the sodium-void effect and −0.03% for the Doppler effect relative to the base values. Since it is rather certain that the I values are too low, this difference represents a narrowing of the uncertainty range for these quantities.

In Table 4 it is seen that the derivatives are depressed in Group 17, which contains most of the large sodium resonance at 2.85 keV. At higher energies the derivatives of reactivity coefficients with respect to cross section changes decrease markedly even on a per group rather than per unit energy basis. Also, relative uncertainties in cross sections are smaller at high energy so that the contribution of the region above several keV to uncertainty in reactivity coefficients is relatively unimportant. To the derivatives in Table 4 there must still be added the effect of an enrichment search to critical, as given in Footnote "a" in Table 2.

Of the variations above 30 keV, the reactivity change is the most important. The corresponding decreases in k calculated for critical assemblies are unacceptably large in comparison with experiment.

Variations in $^{238}U\ \sigma_{\gamma}$

The "Unmod. $\delta\sigma$" results for $^{238}U$ Doppler effect given in Table 3 correspond to neglect of the change in effective $^{238}U$ capture cross section with temperature. The effect of this change is smaller for the lower curve because of the weaker p-wave self-shielding with the smaller strength function. The indicated Doppler-effect variation corresponds to an uncertainty considerably less than the deviations between experiment and calculation of the order of 30% that have been observed.[32] Although other parameter uncertainties affect Doppler effect calculations also, the ones considered here are probably the most important. Large uncertainties in Doppler-effect calculations because of uncertainty in the cross sections considered here seem unlikely.

Use of the ENDF/B-II values would produce variations from the base values about a third of those obtained with the lower curve.

The uncertainties indicated for reactivity and breeding ratio are much too large even after reduction to allow for an overly pessimistic choice of error limits.

## ALTERNATE METHODS OF ADJUSTING FOR CROSS SECTION CHANGES

The results given in Tables 2 and 3 are, as noted before, based on an enrichment search to critical, in which the fissile content of both core regions

is changed by the same ratio. Other means of adjustment which might be more realistic in an actual case are the variation of the relative sizes of the two zones, holding the total core volume constant, and a uniform change in the core size, maintaining the ratio of the two zones constant. The final reactor properties obtained with a given initial composition and given initial zone sizes for the various ways of achieving criticality are shown in Table 5.

It is noted in Table 5 that there is essentially no change in any of the components of the sodium-void effect on adjusting enrichment to critical at constant core volume. There is still not much change when varying the ratio of zone volumes but holding the radius of the outer zone constant. A larger but still not very important change occurs when criticality is achieved by a uniform change in core size at constant enrichment. This indicates that the most important effect on the scattering component is that of the variation of effective geometrical $B^2$ through core size changes on the energy dependence of the adjoint function. This effect is considerably diminished when only the inner zone radius is varied, and would be smaller still if there were more than two enrichment zones with the outer core dimension kept constant. In effect, this represents an approach to the case in which enrichment is varied by a given ratio throughout the core. There is also, of course, a change in the leakage component when core size is altered, which is considerably less when the outer core radius is kept constant. The spherical model used is not the best possible one for studying the leakage component effect, but conclusions should be qualitatively valid even in this case.

Comparison of the scattering component for the enrichment search to critical at constant core dimensions and for the search by altering both core radii shows the change in sodium-void effect when core size and enrichment are altered simultaneously. The change in scattering component with fertile-to-fissile ratio under these circumstances is comparable to that observed in earlier studies with a fundamental mode flux with $B^2$ being adjusted for criticality.[13,33]

The situation is different for the Doppler effect; here the effect of varying enrichment on the amount of low-energy flux is evident, while a change in core size at constant enrichment has a smaller effect. The change in Doppler effect for a critical reactor, balancing size against core enrichment, is of the same order as observed in fundamental mode studies.[13,34]

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In the case of the total breeding ratio, there is a significant change in adjusting the system to criticality, as this effectively amounts to a change in $\bar{\nu}$. There is little change for a critical system in balancing change in enrichment against change in core dimensions, however.

The error in critical mass corresponding to a 1% error in k is 1.9% for adjustment of relative zone size at constant total core volume comapred to 1.7% for a uniform enrichment search. If a uniform core size change is made, the error in mass per per cent k is much larger, 14%. The former type of adjustment is more likely to be made in practice than the latter, however.

## CONCLUSIONS

While a change in the method of adjusting to critical would change the results in Tables 2 and 3 slightly, the conclusions drawn from them would not be changed in any significant way. Usually, of course, changes will be made in more than one cross section at a time with compensating effects on reactivity so that the problem of adjustment to criticality is even less than is implied here.

Recent improvement in knowledge of $^{239}$Pu fission and capture cross sections below 30 keV has considerably reduced the uncertainty in reactivity coefficient calculations, particularly for the sodium-void effect. Uncertainty in breeding ratio and k calculations is still much too large.

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The error in critical mass corresponding to a 1% error in k is 1.98 for adjustment of relative zone size at constant total core volume compared to 1.74 for a uniform enrichment search. If a uniform core size change is made, the error in mass per per cent k is much larger, 14%. The former type of adjustment is more likely to be made in practice than the latter, however.

CONCLUSIONS

While a change in the method of adjusting to critical would change the results in Tables 2 and 3 slightly, the conclusions drawn from them would not be changed in any significant way. Usually, of course, changes will be made in more than one cross section at a time with compensating effects on reactivity so that the problem of adjustment to criticality is even less than is implied here.

Recent improvement in knowledge of $^{239}$Pu fission and capture cross sections below 20 keV has considerably reduced the uncertainty in reactivity coefficient calculations, particularly for the sodium-void effect. Uncertainty in breeding ratio and K calculations is still much too large.

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Table 1. UNRESOLVED RESONANCE PARAMETERS USED TO CALCULATE $\sigma_\gamma$ OF $^{238}U$ BELOW 25 keV

| | Upper Curve | Lower Curve | ENDF/B-I |
|---|---|---|---|
| $\Gamma_\gamma$, mV | 24.8 | 24.8 | 24.6 |
| $S_0 \times 10^4$ | 0.90 | 0.90 | 0.94 |
| $S_1 \times 10^4$ | 2.5 | 1.0 | 1.58 |
| $R \times 10^{13}$, cm | 9.18 | 9.18 | 8.74 |
| $D_{J=1/2}$ | 20.8 | 20.8 | 18.5 |
| $D_{J=3/2}$ | 10.4 | 10.4 | 9.25 |

Table 2. RESULTS OF $^{239}Pu$ $\sigma(n,f)$ AND $\alpha$ VARIATIONS

| | Variation with σ(n,f) at Constant α | | | | Variation with α at Constant σ(n,f) | | | | Total Variation | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sodium Void[a] (% k) | $^{238}U$ Doppler[ab] (% k) | Total[a] B.R. | Reac.[c] (% k) | Sodium Void (% k) | $^{238}U$ Doppler (% k) | Total B.R. | Reac. (% k) | Sodium Void (% k) | $^{238}U$ Doppler (% k) | Total B.R. | Reac. (% k) |
| Base-Pitterle σ(n,f) and α[(8)] | 2.683 | -0.485 | 1.277 | 100. | 2.683 | -0.485 | 1.277 | 100. | 2.683 | -0.485 | 1.277 | 100. |
| | | | | Variations Below 30 keV | | | | | | | | |
| ENDF/B-II[(9)] | -0.112 | 0 | -0.003 | 0.018 | -0.092 | -0.015 | 0.005 | 0.096 | -0.204 | -0.015 | 0.002 | 0.114 |
| ORNL-RPI[d(5)] | | | | | | | | | | | | |
| 11-g Foil | -0.004 | 0 | 0.001 | -0.196 | 0.132 | 0.035 | -0.031 | -0.444 | 0.126 | 0.035 | -0.030 | -0.640 |
| Ioniz. Chamber | -0.036 | 0.002 | -0.001 | -0.118 | 0.202 | 0.045 | -0.040 | -0.569 | 0.166 | 0.047 | -0.041 | -0.687 |
| Saclay[(2)] | -0.120 | -0.001 | -0.003 | 0.359 | | | | | | | | |
| Petrel[e(6)] | 0.061 | 0.001 | 0.001 | 0.396 | | | | | | | | |
| James[e(3)] | -0.061 | -0.005 | 0 | -0.182 | | | | | | | | |
| Dubna[e(7)] | -0.003 | 0.004 | 0.001 | -0.665 | 0.185 | 0.028 | -0.012 | -0.154 | +0.182 | 0.032 | -0.011 | -0.819 |
| Schomberg et al[(1)] | -0.065 | 0.008 | 0 | -0.509 | -0.096 | -0.015 | 0.012 | 0.193 | -0.161 | -0.007 | 0.012 | -0.316 |
| Czirr & Lindsey[(2)] | | | | | 0.030 | 0.011 | -0.001 | -0.067 | | | | |
| | | | | Variations Above 30 keV | | | | | | | | |
| Poenitz σ(n,f)[(12)] | -0.256 | -0.001 | -0.001 | -3.130 | | | | | | | | |
| Increase of 20% in α | | | | | -0.019 | 0.004 | -0.022 | -0.281 | | | | |

[a] For enrichment search to critical, $\delta k_{Doppler}/\delta k_{Reactivity} = -0.0088$, where $\delta k_{Reactivity}$ is k after cross section adjustment, $\delta k_{B.R.}/\delta k_{Reactivity} = +2.77$, $\delta k_{Na\ Void}/\delta k_{Reactivity} \approx 0$.

[b] Doppler temperature change 1300°K to 2500°K.

[c] An increase of 1% k corresponds to a decrease of 1.6% in fissile inventory for an enrichment search to critical.

[d] Gaps in one set of ORNL data were filled with values from the other set. Fission data extend only to 25 keV.

[e] Data extend only to 20 keV.

Table 3. RESULTS OF $^{238}U$ $\sigma(n,\gamma)$ VARIATIONS[a]

| | Base Value - ENDF/B-I $\sigma(n,\gamma)$ | High Curve | | Low Curve | |
|---|---|---|---|---|---|
| | | Above 67 keV | Above 1 keV | Above 67 keV | Above 1 keV |
| % $\delta k$[b] | —— | -0.11 | -0.56 | 1.10 | 2.90 |
| Enrichment | | | | | |
| Total Fissile | | | | | |
| Reg. 1 | 0.1093 | 0.1095 | 0.1121 | 0.1074 | 0.1042 |
| Reg. 2 | 0.1628 | 0.1631 | 0.1669 | 0.1599 | 0.1552 |
| $\delta$ Sodium Void (% k) | 2.683 | -0.015 | 0.112 | 0.107 | 0.035 |
| $\delta$ $^{238}U$ Doppler (% k)[c] | | | | | |
| Unmod. $\delta\sigma$ | -0.485 | 0.001 | 0.052 | -0.015 | -0.074 |
| Mod. $\delta\sigma$ | -0.485 | 0.001 | 0.030 | -0.014 | -0.063 |
| $\delta$ (B.R.) | | | | | |
| Core | 0.896 | 0.001 | 0.029 | -0.015 | -0.049 |
| Total | 1.277 | 0.001 | 0.039 | -0.017 | -0.063 |

[a]Enrichment adjustment to critical.

[b]Before enrichment adjustment.

[c]Doppler temperature change 1300°K to 2500°K.

Table 4. DERIVATIVES OF REACTOR PROPERTIES WITH RESPECT TO CROSS SECTION VARIATIONS PER UNIT ENERGY[a]

| Group | E, keV | ΔE, keV | σ(n,f) at Constant α | | | | α at Constant σ(n,f) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\frac{\%\ \delta k_{Na}}{(\delta\sigma_f/\sigma_f)\Delta E}$ | $\frac{\%\ \delta k_{Doppler}}{(\delta\sigma_f/\sigma_f)\Delta E}$ | $\frac{\%\ \delta k_{React.}}{(\delta\sigma_f/\sigma_f)\Delta E}$ | $\frac{\delta(B.R.)}{(\delta\sigma_f/\sigma_f)\Delta E}$ | $\frac{\%\ \delta k_{Na}}{\delta\alpha\ \Delta E}$ | $\frac{\%\ \delta k_{Doppler}}{\delta\alpha\ \Delta E}$ | $\frac{\%\ \delta k_{React.}}{\delta\alpha\ \Delta E}$ | $\frac{\delta(B.R.)}{\delta\alpha\ \Delta E}$ |
| 11 | 40.9-67.4 | 26.5 | 0.005 | 0.001 | 0.146 | -0.003 | -0.001 | 0.0004 | -0.045 | -0.0027 |
| 12 | 24.8-40.9 | 16.1 | 0.009 | 0.002 | 0.188 | -0.005 | -0.002 | 0.0007 | -0.057 | -0.0035 |
| 13 | 15.0-24.8 | 9.8 | -0.025 | 0.003 | 0.270 | -0.007 | -0.003 | 0.0014 | -0.080 | -0.0050 |
| 14 | 9.12-15.0 | 5.9 | -0.021 | 0.005 | 0.392 | -0.011 | 0.001 | 0.0027 | -0.112 | -0.0071 |
| 15 | 5.53-9.12 | 3.59 | -0.021 | 0.006 | 0.468 | -0.013 | 0.014 | 0.0043 | -0.143 | -0.0081 |
| 16 | 3.35-5.53 | 2.18 | 0.089 | 0.008 | 0.417 | -0.013 | -0.023 | 0.0055 | -0.139 | -0.0073 |
| 17 | 2.03-3.35 | 1.32 | 0.005 | 0.008 | 0.265 | -0.010 | 0.005 | 0.0048 | -0.114 | -0.0048 |
| 18 | 1.23-2.03 | 0.80 | -0.660 | 0.026 | 1.50 | -0.063 | 0.294 | 0.0364 | -0.640 | -0.0314 |
| 19 | 0.748-1.23 | 0.48 | -1.24 | 0.008 | 2.19 | -0.083 | 0.582 | 0.0816 | -0.988 | -0.0434 |
| 20 | 0.454-0.748 | 0.294 | -1.50 | -0.034 | 2.24 | -0.085 | 0.850 | 0.139 | -1.30 | -0.0380 |
| 21 | 0.275-0.454 | 0.179 | -1.37 | -0.073 | 1.84 | -0.067 | 0.854 | 0.147 | -1.25 | -0.0380 |
| 22 | 0.167-0.275 | 0.108 | -1.22 | -0.127 | 2.04 | -0.046 | 1.080 | 0.234 | -1.30 | -0.0305 |
| 23 | 0.101-0.167 | 0.066 | -0.23 | -0.091 | 1.06 | -0.030 | 0.561 | 0.097 | -0.44 | -0.0167 |

[a]Energy in keV.

Table 5. EFFECT ON REACTOR CHARACTERISTICS OF VARIOUS WAYS OF ADJUSTING TO CRITICAL

| Case | Region 1 Enrichment / Fert/Fiss Ratio | Region 1 Outer Radius, cm | Region 2 Enrichment / Fert/Fiss Ratio | Region 2 Outer Radius, cm | k | Sodium-Void Effect, % δk: Zone | Scatt. | Capture | Leakage | Total | $^{238}U$ Doppler Effect, % k | B.R. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial | 0.11011 | 86.912 | 0.16397 | 109.501 | 1.00454 | 1 | 2.653 | 0.144 | -0.210 | 2.587 | -0.3861 | 1.2662 |
| | 8.081 | | 5.099 | | | 2 | 0.931 | 0.047 | -0.880 | 0.098 | -0.0941 | |
| | | | | | | Total | 3.584 | 0.191 | -1.090 | 2.685 | -0.4808 | |
| Enrichment search | 0.10929 | 86.912 | 0.16276 | 109.501 | 1.00000 | 1 | 2.652 | 0.144 | -0.211 | 2.586 | -0.3893 | 1.2765 |
| | 8.150 | | 5.144 | | | 2 | 0.931 | 0.047 | -0.880 | 0.098 | -0.0955 | |
| | | | | | | Total | 3.583 | 0.191 | -1.091 | 2.683 | -0.4848 | |
| Zone 1 radius altered | 0.11011 | 88.181 | 0.16397 | 109.501 | 1.00000 | 1 | 2.731 | 0.149 | -0.253 | 2.627 | -0.4104 | 1.2771 |
| | 8.081 | | 5.099 | | | 2 | 0.847 | 0.043 | -0.842 | 0.048 | -0.0873 | |
| | | | | | | Total | 3.578 | 0.192 | -0.1095 | 2.675 | -0.4887 | |
| Both zone radii altered | 0.11011 | 85.166 | 0.16397 | 107.301 | 1.00002 | 1 | 2.629 | 0.143 | -0.224 | 2.548 | -0.3844 | 1.2769 |
| | 8.081 | | 5.099 | | | 2 | 0.921 | 0.047 | -0.895 | 0.073 | -0.0944 | |
| | | | | | | Total | 3.550 | 0.190 | -1.119 | 2.621 | -0.4788 | |

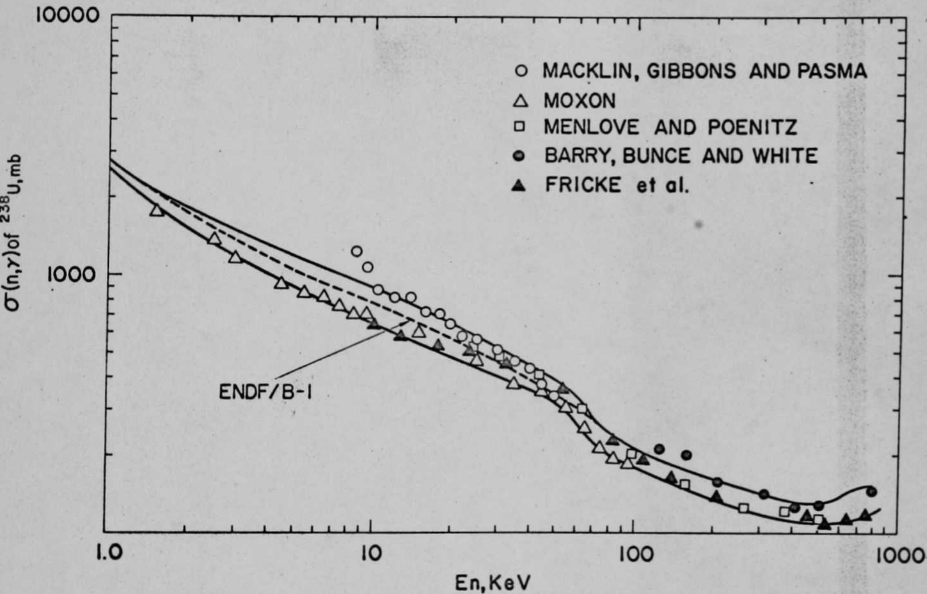


Fig. 1. Capture cross section of $^{238}U$. The upper and lower curves are estimated extreme uncertainty limits. (ANL Neg. 116-592)