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ANALYSIS OF AIR-TEMPERATURE MEASUREMENTS FROM THE THREE MILE ISLAND UNIT 2 REACTOR BUILDING

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ABSTRACT

The performance of the ambient air resistance temperature detectors (RTDs) just after the hydrogen burn in the TMI-2 Reactor Building is examined. The performance of the sensors is compared with physical models of the sensor/ambient air system. With one exception, the RTD data appear to be valid for the period examined. Based on the data, the hydrogen burn ended considerably before the first data points were recorded.

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ANALYSIS OF AIR TEMPERATURE MEASUREMENTS FROM THE THREE MILE ISLAND UNIT 2 REACTOR BUILDING

INTRODUCTION

This is the second in a series of reports on the resistance temperature detectors (RTDs) used to measure ambient air temperature in the TMI-2 Reactor Building. The first report detailed the current status of the RTDs and compiled data for the period of the TMI accident.[1] This report deals with the performance of the RTDs for the period just after the hydrogen burn in the TMI-2 Reactor Building. During this period, the ambient air sensors were subjected to extreme temperature changes, and some of them were sprayed with a water and sodium hydroxide solution.

The performance of these instruments during and after the hydrogen burn is important because of what the RTD data imply about the survivability of the instruments and about the hydrogen burn.

RTD DATA

Frequencey of Recording

There are sixteen Rosemount Series 78 ambient air RTDs in the TMI-2 Reactor Building.[2] The readings of these instruments were recorded in a round robin fashion on a strip chart recorder. Fifteen seconds elapsed between each recording, and no data were taken for 2 min between the first and last sensor. Each sensor reading was recorded once every 6 min. The slow frequency of recording precludes obtaining information about the first few seconds after hydrogen ignition from the RTD data. Longer term building heating and cooling information can be obtained, as will be shown.

Locations of Instruments

The locations of the sensors are given in Table 1.[1] Nine of the RTDs are at the 282-ft elevation of the Reactor Building, five between the 305- and 330-ft elevation, and two at the 353-ft elevation.

TABLE 1. RTD LOCATIONS

RTD	Location	Foot Elevation	RTD	Location	Foot Elevation
5010	Sump pump	282	5018	Primary shield	282
5011	Letdown cooler	282	5019	Primary shield	282
5012	RC drain tank	282	5020	Top ceiling	353
5013	Impinge barrier	282	5021	Top ceiling	353
5014	NR equipment hatch	305	5022	SE stairwell	330
5015	A/C plenum out	319	5023	WE stairwell	330
5016	Primary shield	282	5027	A/C plenum out	305
5017	Primary shield	282	5088	SE stairwell	310

Accuracy

Steady state accuracy of the RTDs is ±1°F at 200°F[2], and the strip chart recorder is accurate to ±1°F. The time calibration of the strip chart recorder was rather poor, and timing for data reduction and modeling was taken relative to the hydrogen burn. The time origin of the data is

uncertain to ±90 s because the burn occurred during a period when no data were recorded and the primary shield sensors showed no response to the burn.[3]

Response time of the RTDs is less than 5.5 s in 76°C water flowing at 3 ft/s.[2] The response time in air is not specified in the manufacturer's literature, so it was estimated as part of the modeling effort described in this report.[a] Appendix A contains plots of the RTD data recorded during the time of the TMI-2 accident. In all the plots (except for the RTDs in the primary shield) a jump in temperature is seen at the time of the hydrogen burn. The primary shield data are not analyzed further because they do not provide any information about the hydrogen burn. One of the RTDs on the 353-ft elevation shows a jump decrease in temperature at the time of the hydrogen burn. This is probably due to its being sprayed with water. The instrument is obviously suspect and is not analyzed further in this report. The data from the remaining eleven sensors were analyzed to estimate what temperatures the RTDs experienced and the RTD's time response characteristics.

---

a. The manufacturers claim that response time should be less than 40 s.

MODELS

After a preliminary examination of the RTD data, it appeared that they could be explained by postulationg a jump in ambient air temperature at the time of the hydrogen burn, followed by an exponential temperature decay. The RTD behavior was modeled with a first order heat transfer equation.

Sensor Model

The rate of change of RTD temperature was assumed proportional to the temperature difference between the RTD and the ambient air,

$$\frac{dT}{dt} = -\alpha (T - T_A) \tag{1}$$

where

t = time

T_A = ambient temperature

T = RTD temperature

α = an unknown parameter depending, in a complicated way, on the construction of the RTD, the ambient conditions, and T; α was assumed constant over the time interval of interest, i.e., the first hour after the burn.

Ambient Air Temperature Model

The model assumed for the ambient air temperature was a step followed by an exponential decay. Specifically,

$$T_A = \begin{cases} T_0 & t = t_s \\ T_1 + T_2\, e^{-\beta (t-t_s)} & t > t_s \end{cases} \tag{2}$$

where

T_0	=	initial temperature
T_1, T_2, and β	=	constant parameters
t_s	=	time of the burn.

Obviously, this model does not account for the details of the hyurogen burn, but is probably adequate on average for long-time intervals after the burn.

Sensor Response

Inserting the ambient temperature expression (2) in the RTD sensor response Equation (1), and solving the resulting differential equation, gives

$$T = e^{-\alpha(t-t_s)} \times T_0 + [1 - e^{-\alpha(t-t_s)}] \times T_1$$

$$+ [e^{-\beta(t-t_s)} - e^{-\alpha(t-t_s)}] \times \frac{T_2}{\alpha - \beta} \alpha \qquad (3)$$

for $\alpha \neq \beta$, which is the case.

ESTIMATION OF MODEL PARAMETERS

A good technique for estimating model paramenters based on repeated measurements is Kalman filtering. The filter produces minimum variance estimates of the model parameters. Several excellent references are available on Kalman filtering, so a theoretical development of the method will not be presented in this report. An iterated and extended Kalman filter was used[4] because of the nonlinear relationship between RTD temperature and model parameters.

Filter Formulation

The filter state vector is

$$X = \begin{bmatrix} T_1 \\ T_2 \\ \alpha \\ \beta \end{bmatrix} . \tag{4}$$

The measurements (Z) are the RTD data and

$$Z = h(X) \quad .$$

The measurement equation relating the states to the measurements is given by (3). The H matrix is defined as

$$H = \left[\frac{\partial h}{\partial T_1}, \ \frac{\partial h}{\partial T_2}, \ \frac{\partial h}{\partial a}, \ \frac{\partial h}{\partial \beta} \right] . \tag{5}$$

The initial state covariance matrix is

$$P_o = \begin{bmatrix} 2.5 \times 10^3 & 0 & 0 & 0 \\ 0 & 2.5 \times 10^5 & 0 & 0 \\ 0 & 0 & 1.5 \times 10^{-2} & 0 \\ 0 & 0 & 0 & 6.5 \times 10^{-5} \end{bmatrix} . \tag{6}$$

The filter algorithm is as follows:

1. Save current state estimate

   $xsave = x_k$,

   where

   k is the discrete time index

2. Calculate gain

   $$K_k = P_k H^T(x_k) \, [H(x_k) \, P_k H^T(x_k) + R_k]^{-1}$$

   where

   R_k = measurement noise matrix

   P_k = state estimate covariance matrix

3. Update state, using linearized measurement equation

   $$x_k = xsave + K_k \, [Z_k - h(x_k) - H(x_k)(xsave - s_k)]$$

4. Test for convergence of each element of state vector

5. If converged, read in next measurement, Z_{k+1}, and update the covariance matrix and state,

   $$P_{k+1} = [I - K_k H(x_k)] \, P_k$$

   $$x_{k+1} = x_k$$

   then go to 1

6. If not, go to 2.

The filter was run on the data for each sensor, covering the period of 1 h after the hydrogen burn. Estimates of the model parameters T_1, T_2, α, and β were obtained for each sensor. Covariances of these estimates were also obtained. The results are discussed in the next section.

Appendix B contains a list of filter computer programs designed to be executed on the INEL CYBER 176 system, and these programs use library routines available on the system.

Parameter Estimates

Table 2 shows the temperature values and time constant estimates for each RTD. The standard deviations of these estimates are included in the table.

The standard deviations are probably somewhat too small because the RTD and ambient air models were assumed correct in their calculation. The average value of the sensor time constant is 41 s, with a standard deviation of less than 24 s. This is not an extremely precise estimate of the time constant, but it limits the expected values to much shorter times than previously estimated.[5] Time constants of the magnitude calculated in this report are consistent with the plots of RTD shown in Appendix A; i.e., RTD readings are rapidly declining within 90 s after the hydrogen burn.

TABLE 2. PARAMETER ESTIMATES

Sensor	T_1	σ	T_2	σ	α	σ	β	σ
5010	112	6	20	4	0.031	0.015	0.0006	0.0005
5011	114	2	15	4	0.027	0.022	0.0009	0.0006
5012	124	3	31	4	0.025	0.015	0.0009	0.0004
5013	121	7	41	4	0.022	0.009	0.0007	0.0004
5014	122	2	32	3	--	--	0.0006	0.0001
5015	106	8	33	3	--	--	0.003	0.0004
5021	144	8	59	3	--	--	0.004	0.0004
5022	139	6	51	3	0.020	0.03	0.0009	0.0004
5023	123	2	51	3	--	--	0.0001	0.00002
5027	106	1	25	2	--	--	0.001	0.0003
5088	126	1	35	4	--	--	0.002	0.0003

The sensor time constants (α's) were not calculated for several of the RTDs because the filter failed to converse. Since the values of T_1, T_2, and β are not very sensitive to the α value, T_1, T_2 and β were estimated for these sensors with the value of α fixed at 0.02/s. A 3-state filter was used for these RTDs.

The maximum air temperature experienced at each RTD is the sum of T_1 and T_2. The maximum temperatures are given in Table 3. As would be expected, the higher elevation sensors got hotter than lower elevation sensors, with the exception of 5015 and 5027, which are located at air conditioning plenum outlets. These sensors experienced different cooling than the others, probably because of better heat exchange.

The maximum air temperatures are subject to considerable uncertainty, especially at higher elevations, because of the water spraying that occurred just after the burn. But the maximum temperature estimates are probably good upper bounds on the air temperatures a few minutes after burn. Most sensors are on the long-term cooling trend by the time the first data were recorded after the burn. Extremely high temperatures, of more than 1000°F, could not have lasted for more than a few minutes, according to the RTD data. This information will be useful in any consideration about fire resistant design of items in the Reactor Building.

TABLE 3. MAXIMUM AIR TEMPERATURES ACCORDING TO MODEL

Sensor	Temperature (°F)
5010	132
5011	129
5012	155
5013	162
5014	154
5015	139
5021	203
5022	190
5023	184
5027	131
5088	161

MODEL VALIDATION

Figures 1 through 11 show plots of model temperatures and RTD data. As can be seen from the plots, the model fits the data well in most cases. The air conditioning plenum outlet sensors and higher elevation sensors show the most disagreement between the model and the data. Again, this is probably the result of better heat transfer at the AC outlets and spraying at higher levels, respectively.

Table 4 shows the average difference, maximum difference, and standard deviation of the difference between the model and data for each sensor. The accuracy of the sensor/strip chart recorder is within ±2°F. The model and data agree to better than this at nearly all points for most RTDs.

TABLE 4. MODEL VALIDATION

Sensor	Average Difference (°F)	Standard Deviation (°F)	Maximum Difference (°F)
5010	0.13	0.17	-1.5
5011	-0.07	0.28	-1.7
5012	0.04	0.12	-0.87
5013	-0.19	0.10	-0.48
5014	-0.45	0.42	-3.1
5015	-0.36	1.3	-6.2
5021	0.02	0.83	4.1
5022	-0.21	0.30	-2.12
5023	-3.3	3.3	-29.7
5027	-1.6	1.4	-12
5088	-0.35	0.58	3.1

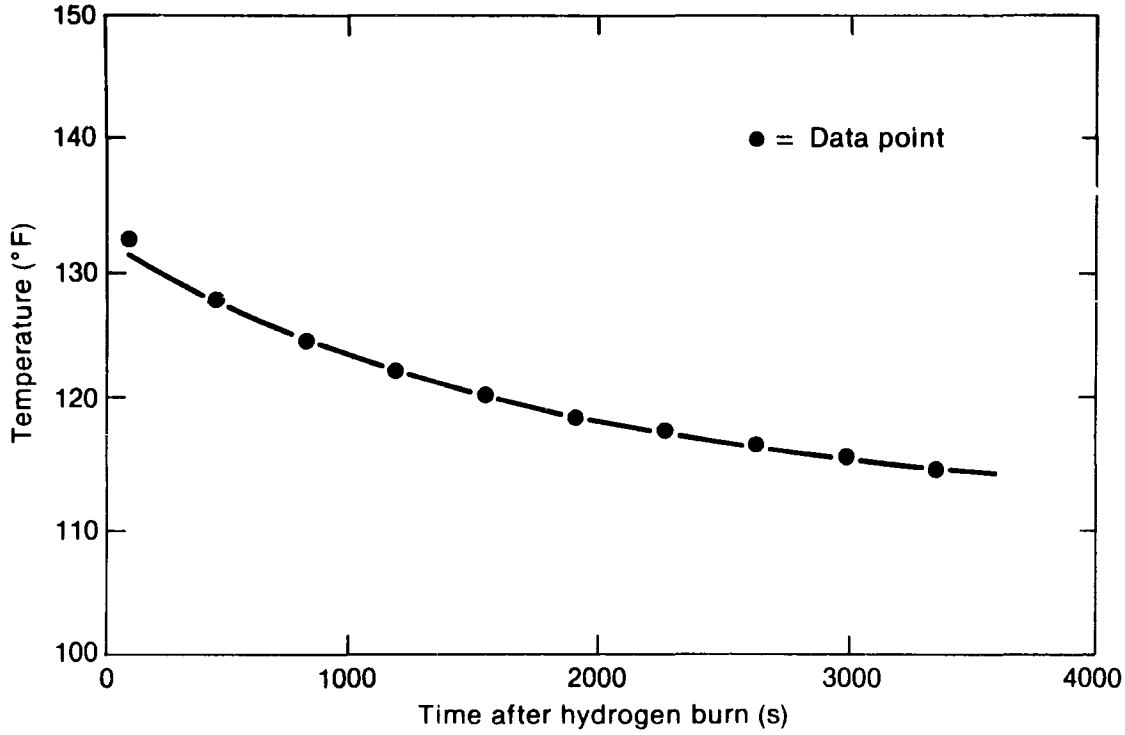

Figure 1. RTD 5010 data versus model.

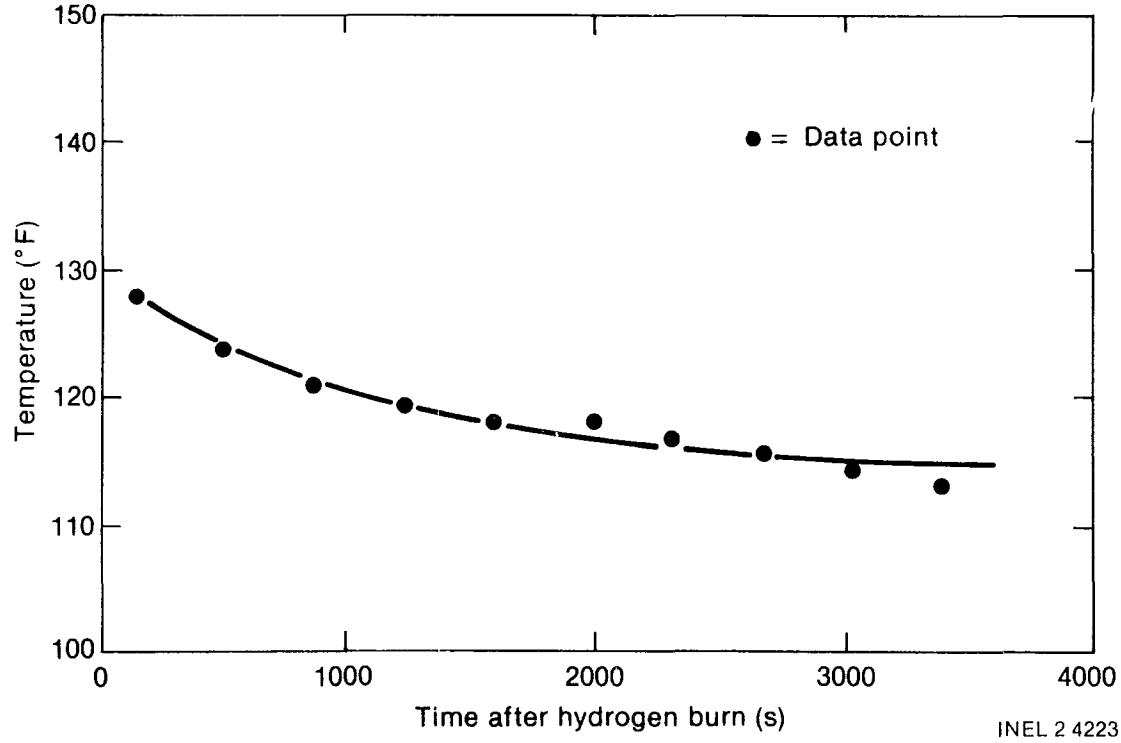

Figure 2. RTD 5011 data versus model.

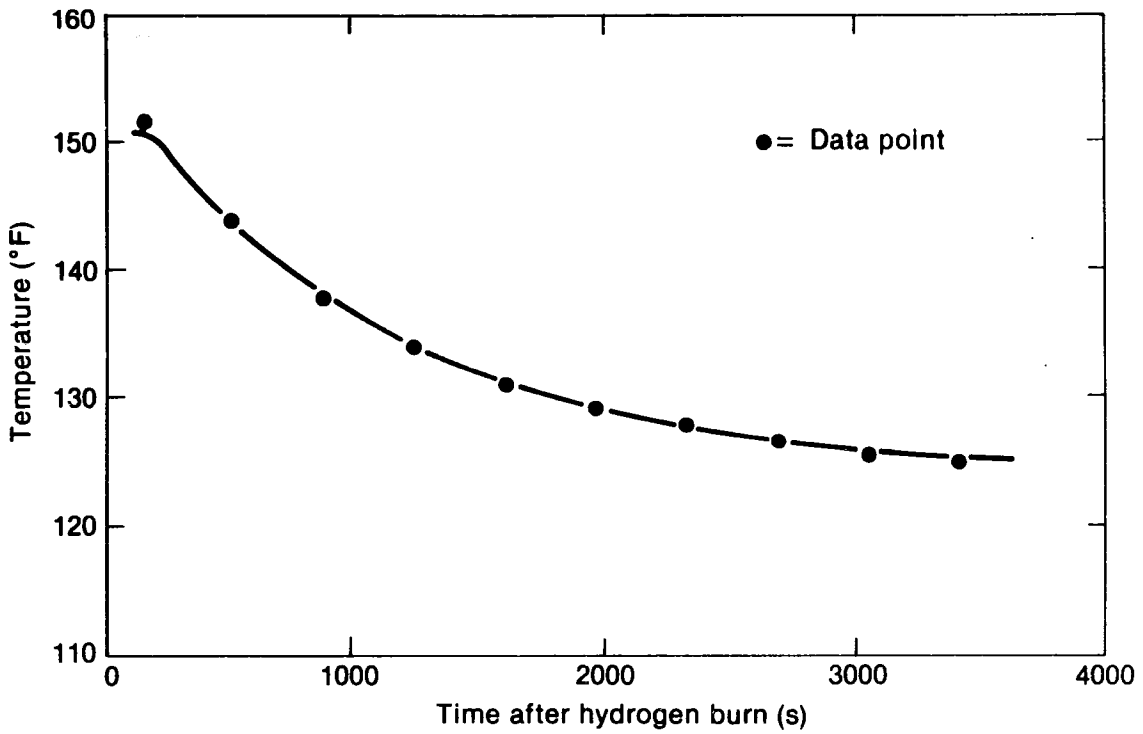

Figure 3. RTD 5012 data versus model.

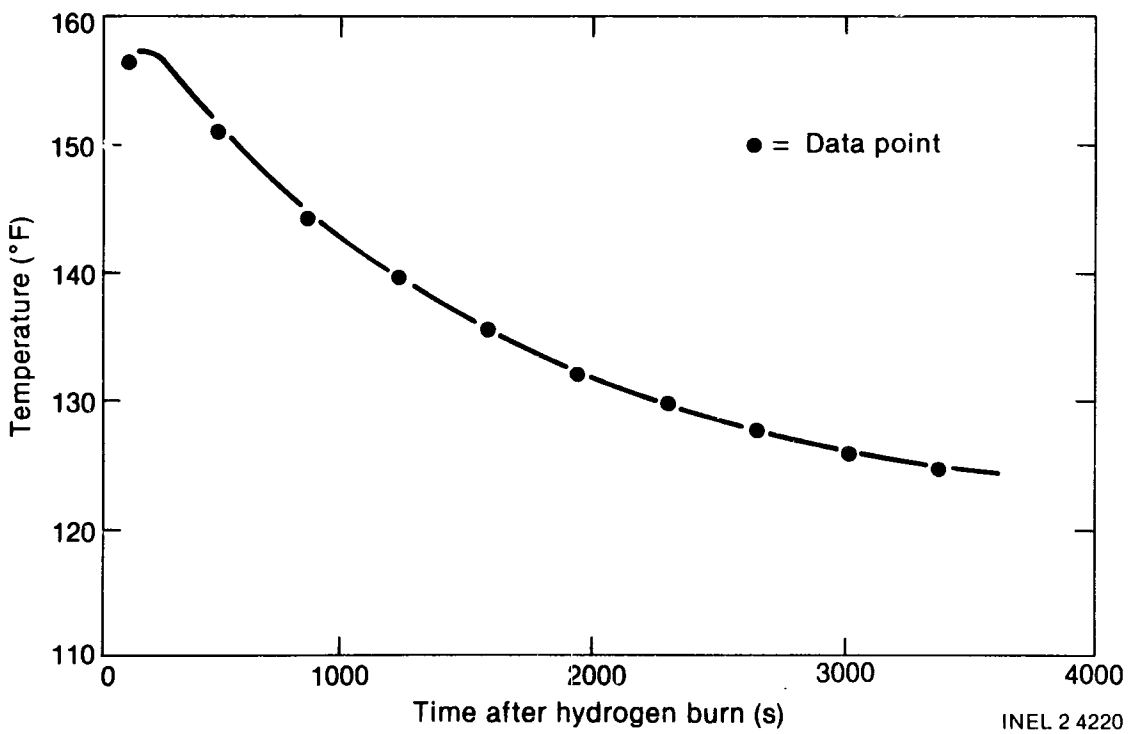

Figure 4. RTD 5013 data versus model.

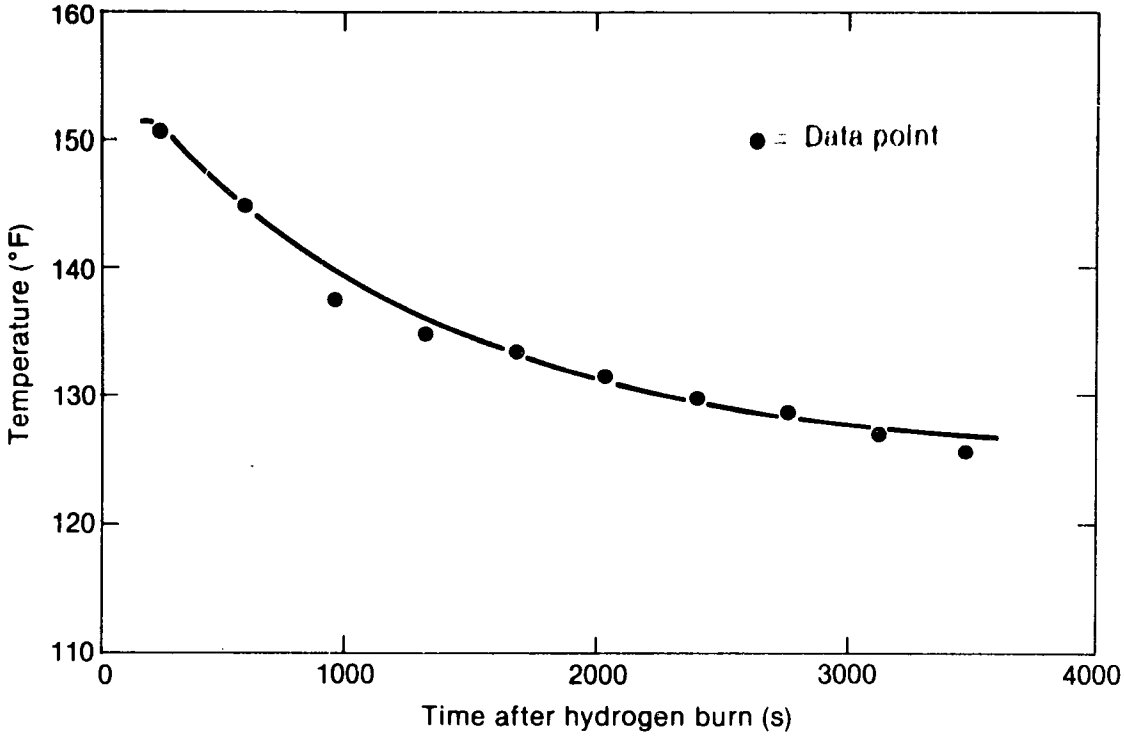

Figure 5. RTD 5014 data versus model.

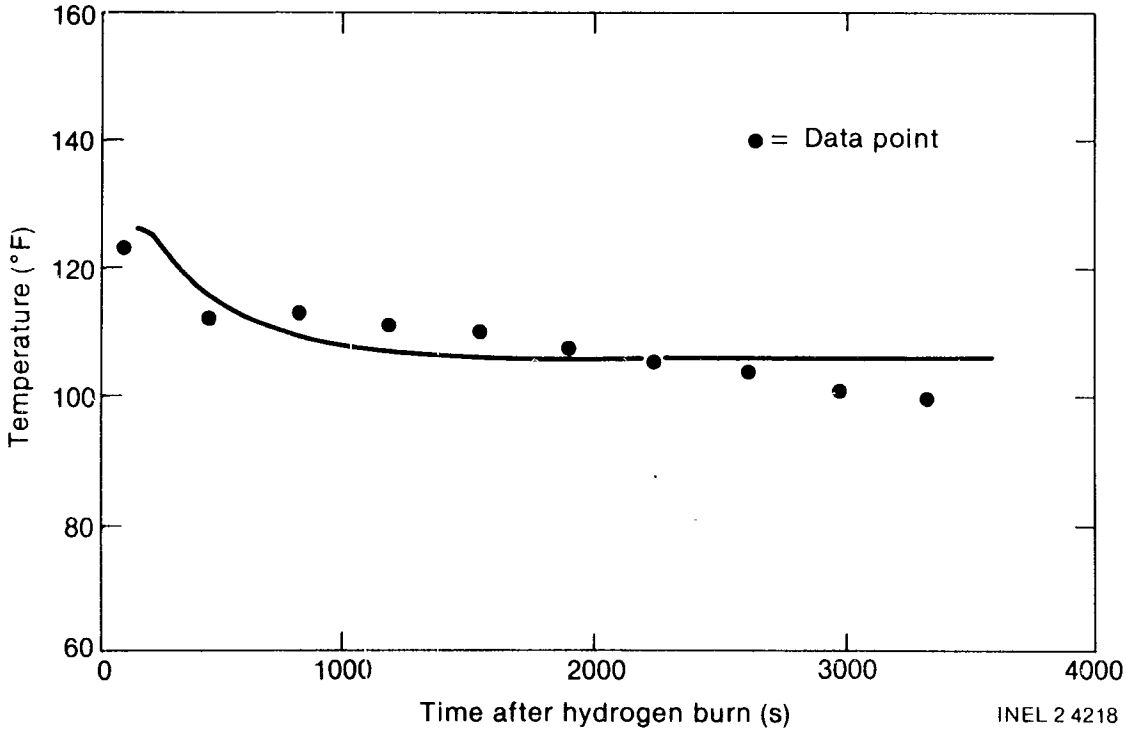

Figure 6. RTD 5015 data versus model.

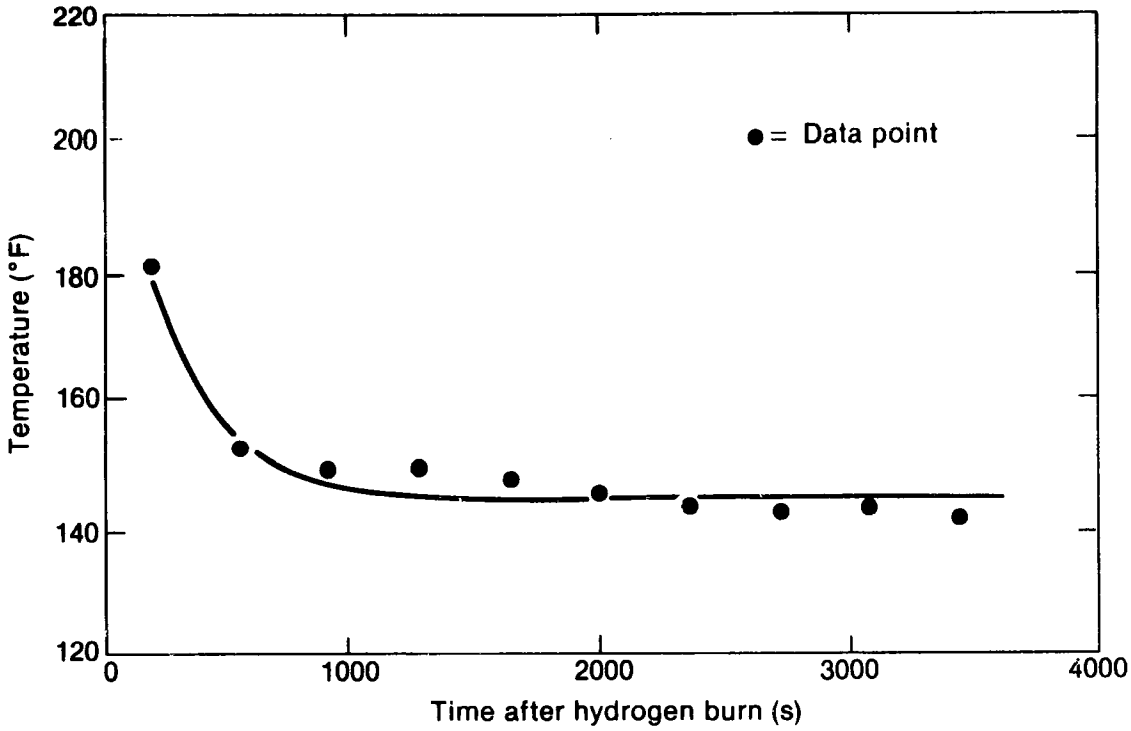

Figure 7. RTD 5021 data versus model.

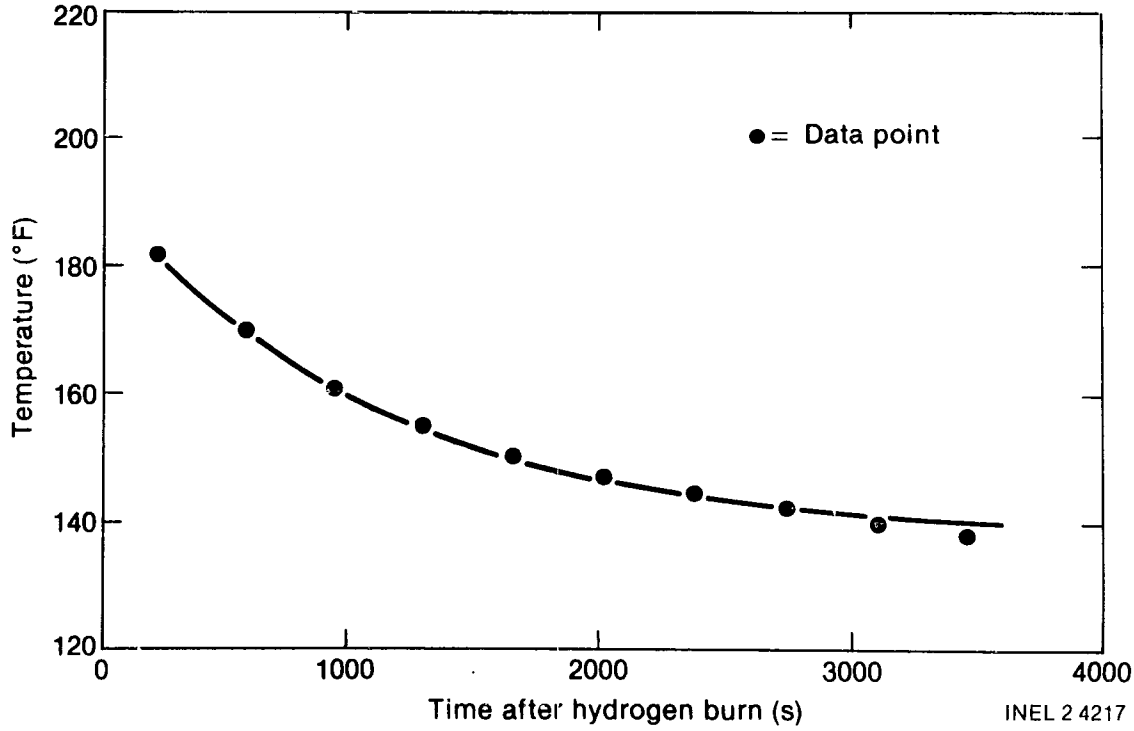

Figure 8. RTD 5022 data versus model.

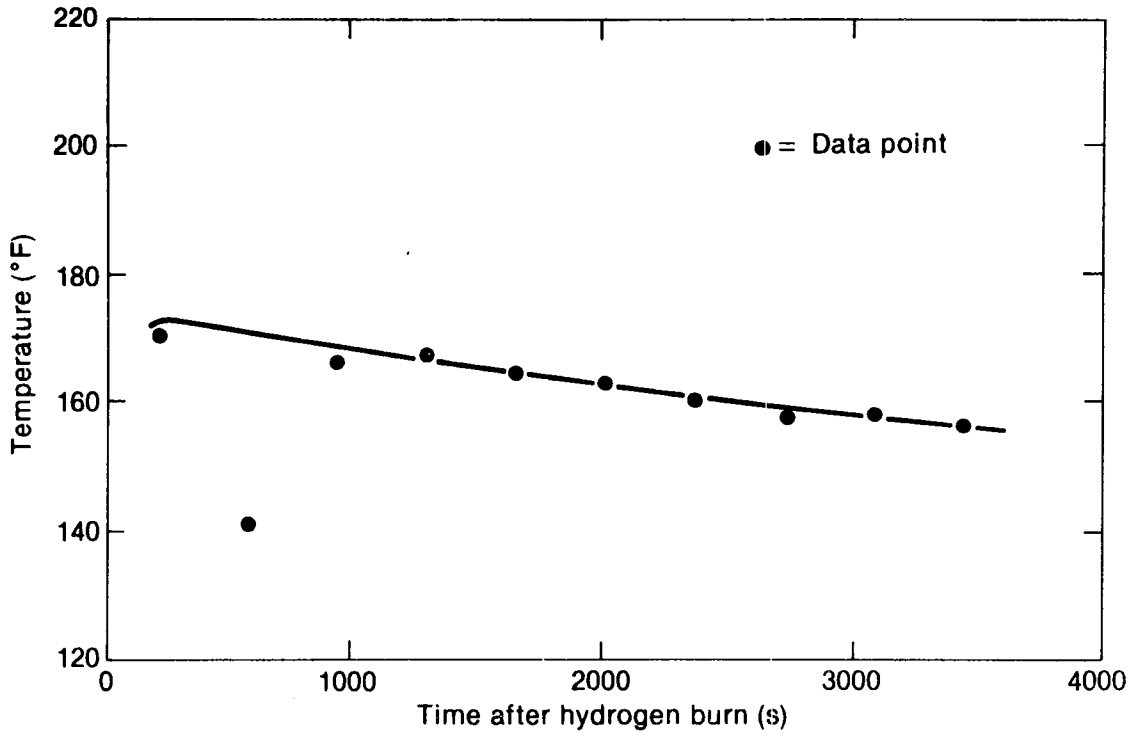

Figure 9. RTD 5023 data versus model.

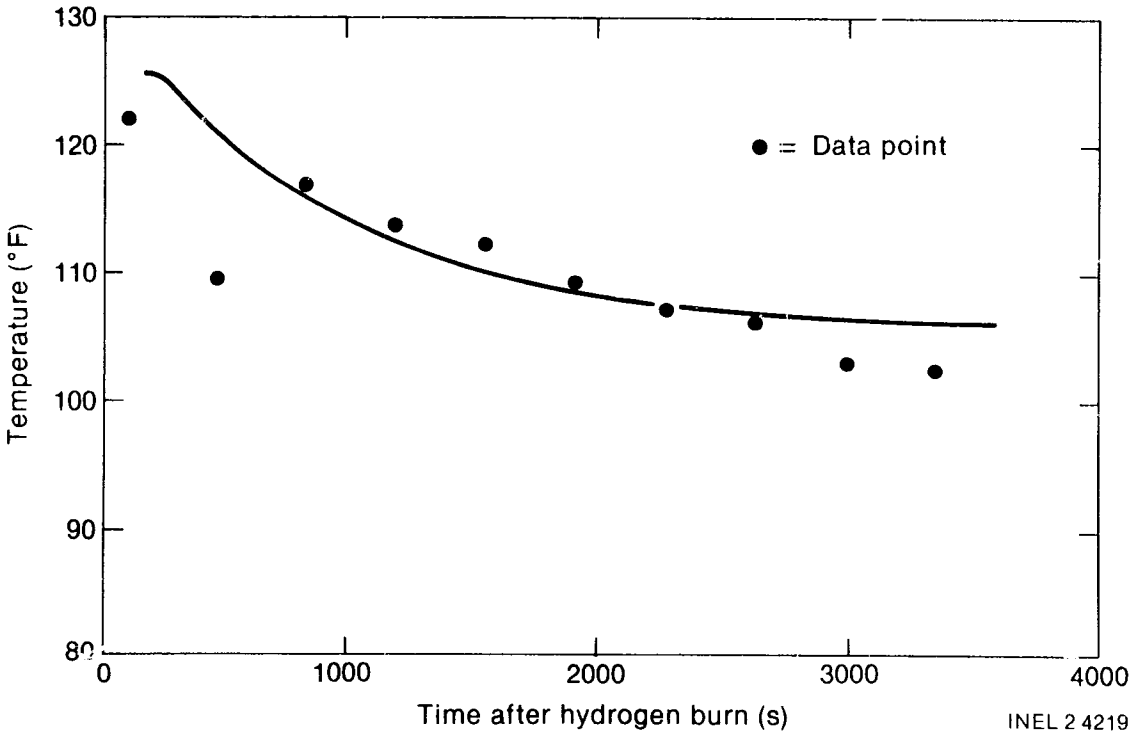

Figure 10. RTD 5027 data versus model.

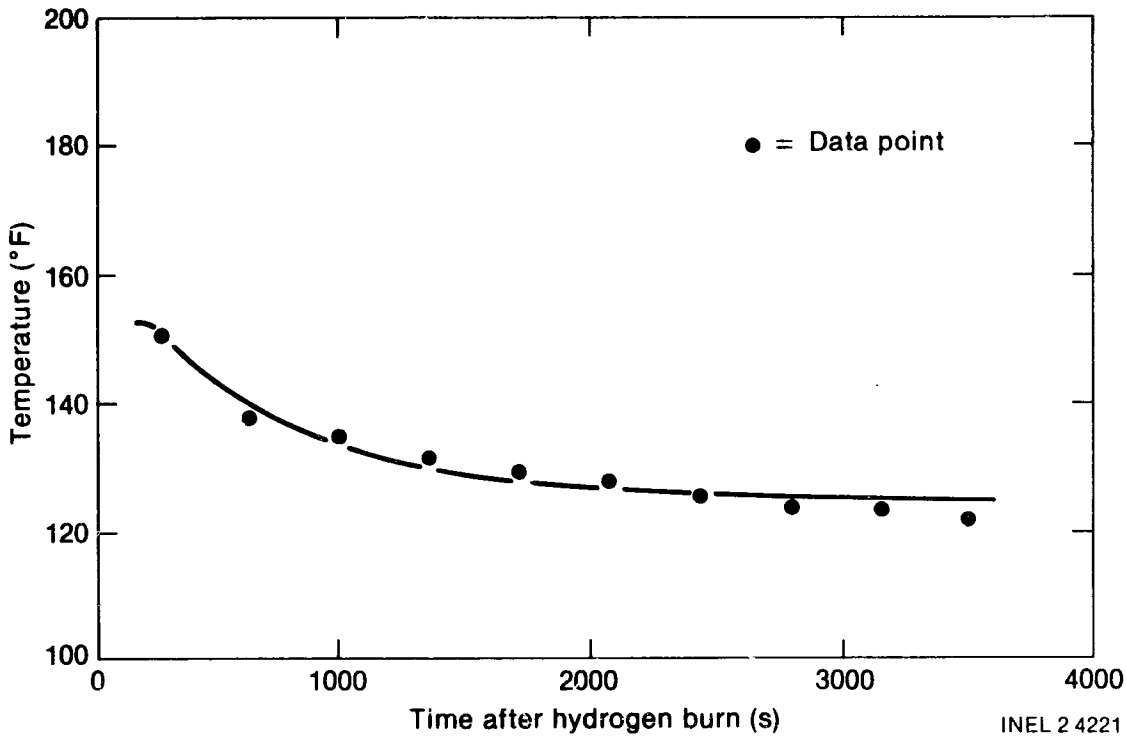

Figure 11. RTD 5088 data versus model.

DURATION OF HIGH TEMPERATURES

Information about the duration of the hydrogen burn can be obtained from the RTD data. An analysis was done for an RTD with average time response characteristics and average initial temperature to determine when the burning must have ended. Appendix C contains the mathematical details of the analysis. The results indicated that temperatures of greater than 1000°F must have persisted for less than 100 s after the start of the burn.

CONCLUSIONS

The RTDs behaved in a manner predictable from simple physical models. When these models were used to fit the data, good agreement was obtained in nearly all cases. It appears that the RTD data are valid for the period examined, with the exception of RTD 5020, which showed lower temperatures just after the hydrogen burn.

The high temperatures (>1000°F) indicative of hydrogen burning must have ended considerably before the first data points were recorded after the hydrogen burn.

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2. Series 78 Platinum Resistance Temperature Sensors, Product data sheet 2178, Rosemount, Inc.

3. Letter from H. F. Ring to D. L. Reeder (EG&G Idaho), TIO Control No. 007013099, June 25, 1982.

4. Arthur Gelb, Applied Optimal Estimation, Cambridge: M.I.T. Press, 1974.

5. H. R. Keltner, Potential Enhancement of TMI Containment Thermal Sensor Data, Sandia Laboratories, TIO Control No. 7-18212, July 2, 1982.

APPENDIX A

PLOTS OF RTD DATA

APPENDIX A
PLOTS OF RTD DATA

Figures A-1 through A-12 are RTD data plots of temperature excursions for the duration of the TMI-2 accident. The temperature excursion caused by the hydrogen burn is indicated on each plot. Other temperature excursions on the plots are closely correlated with operator actions, such as opening and closing the block valve.

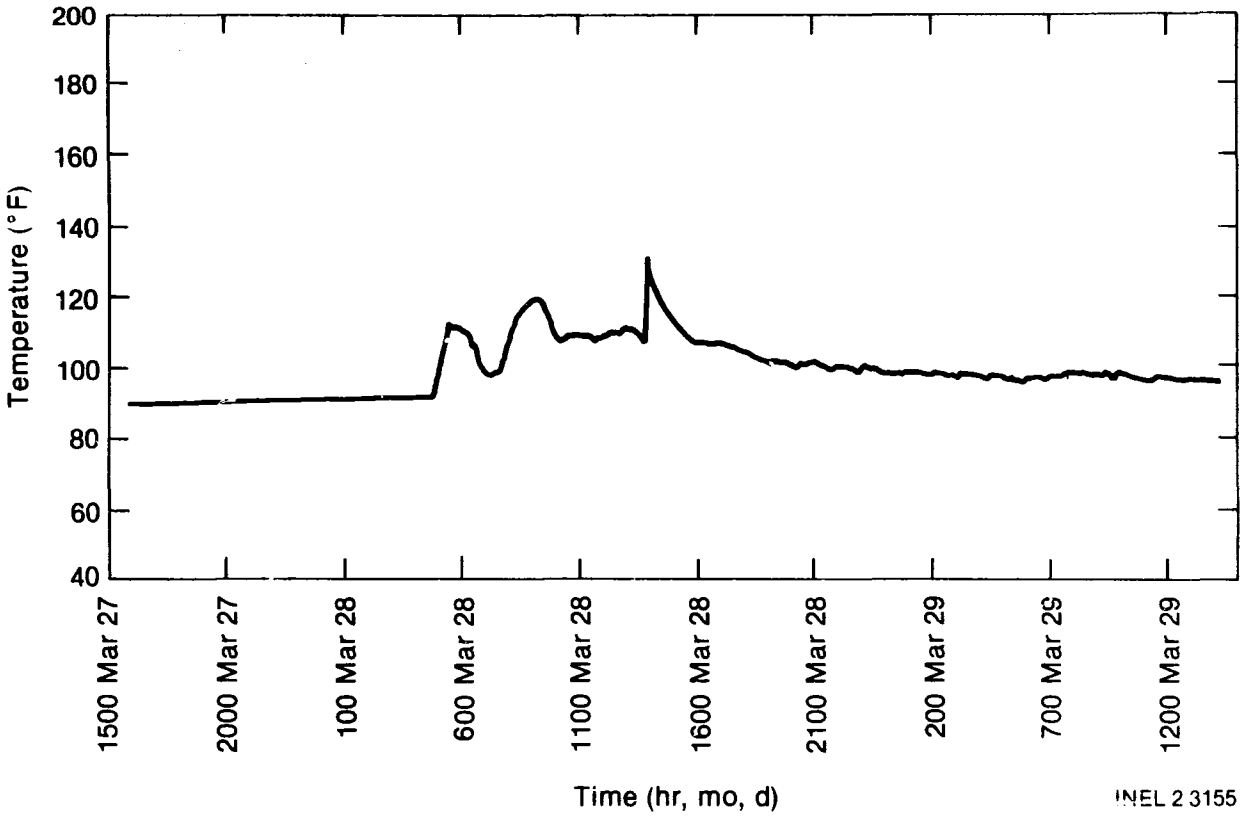

Figure A-1. RTD 5010 temperature excursions during the TMI-2 accident.

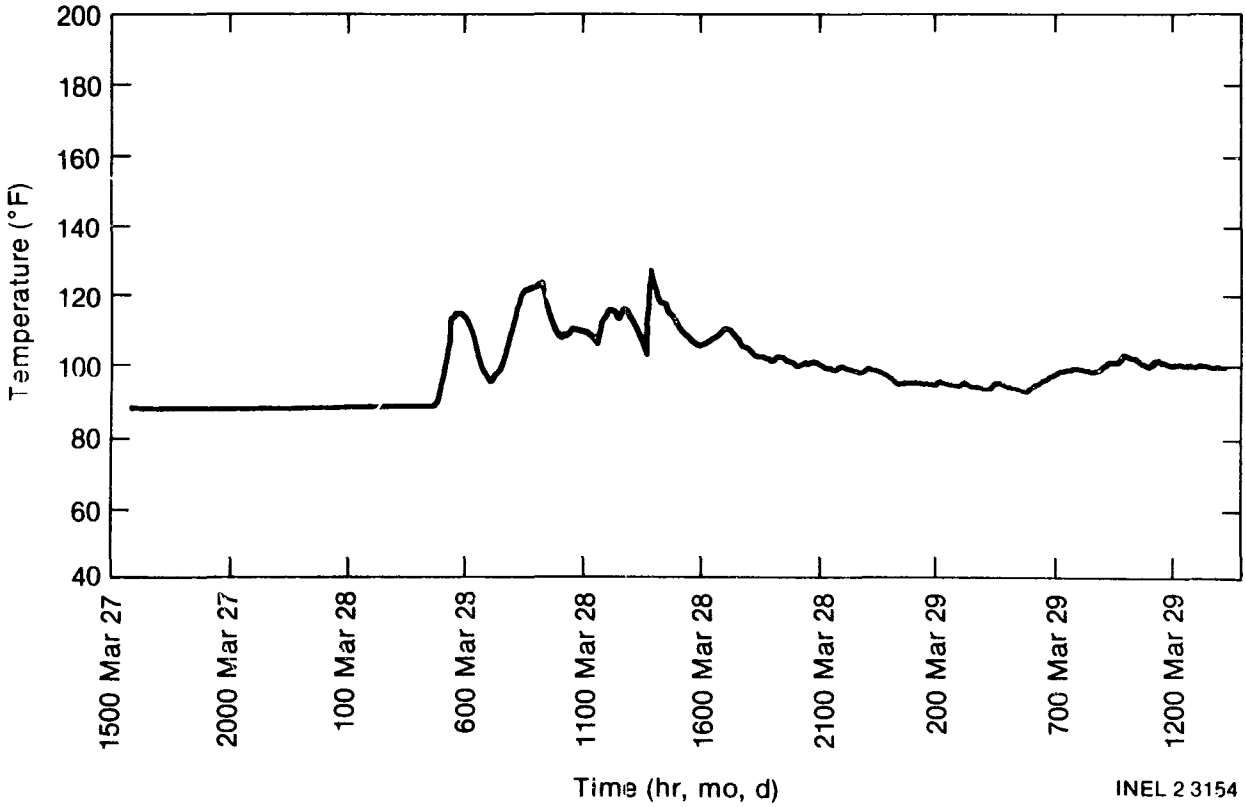

Figure A-2. RTD 5011 temperature excursions during the TMI-2 accident.

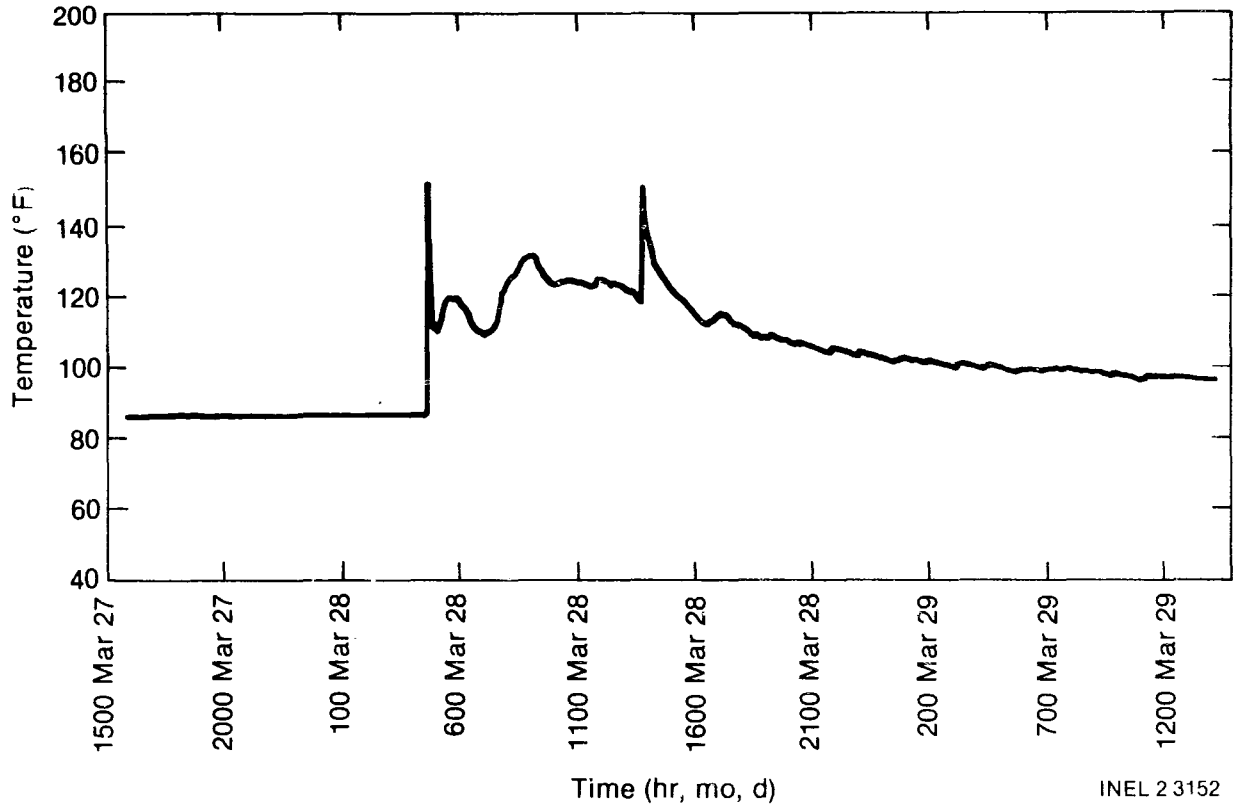

Figure A-3. RTD 5012 temperature excursions during the TMI-2 accident.

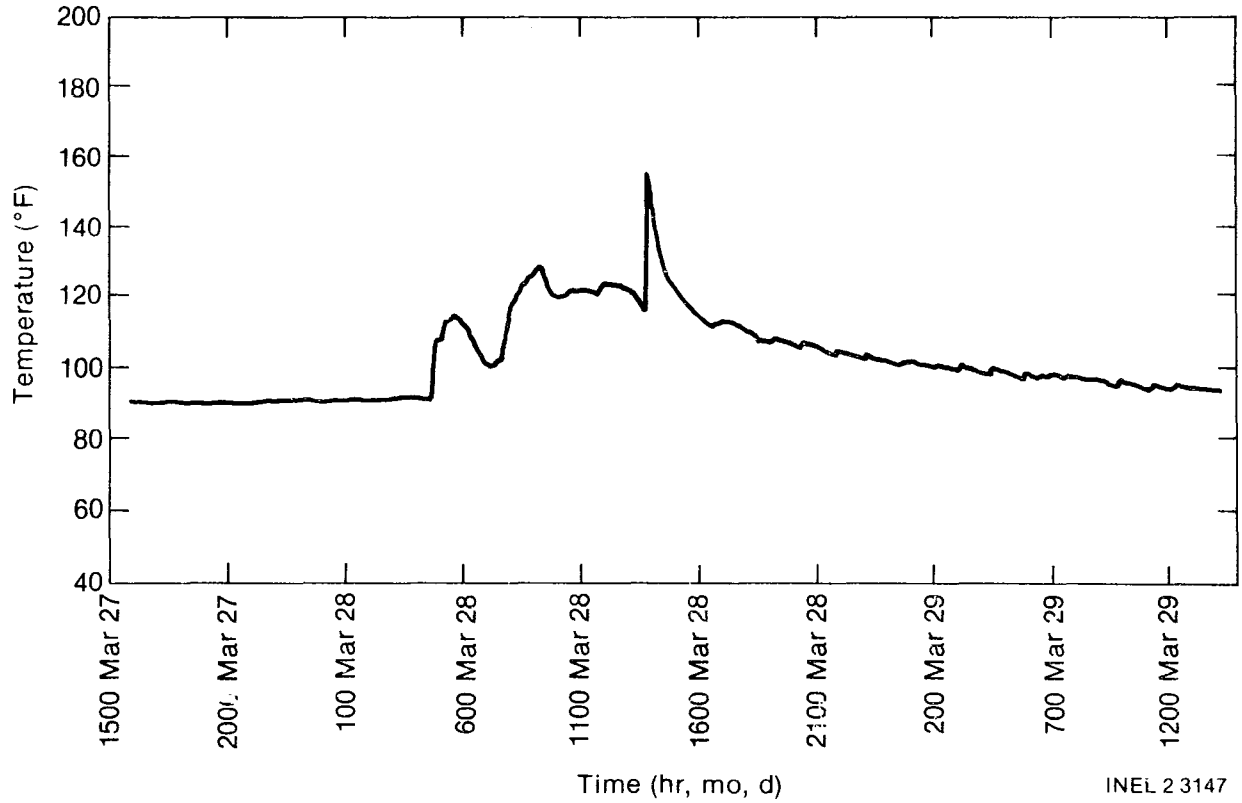

Figure A-4. RTD 5013 temperature excursions during the TMI-2 accident.

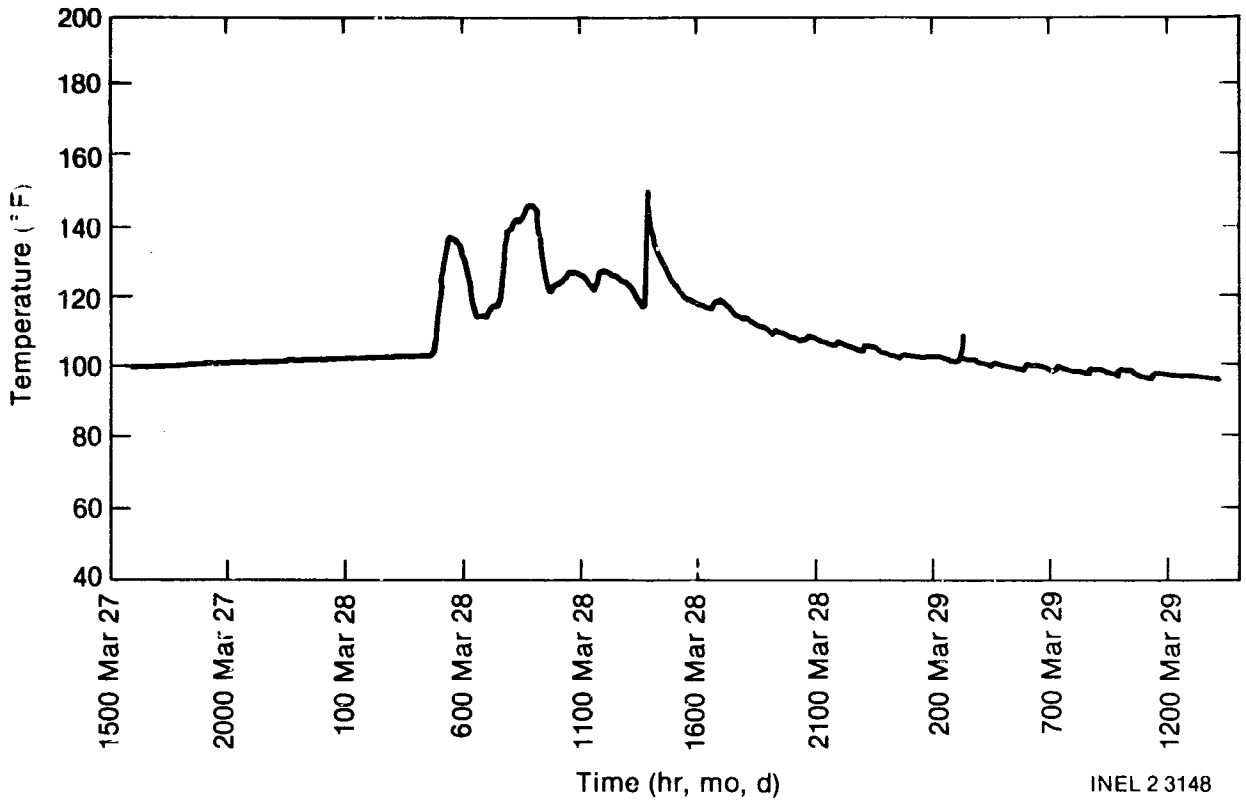

Figure A-5. RTD 5014 temperature excursions during the TMI-2 accident.

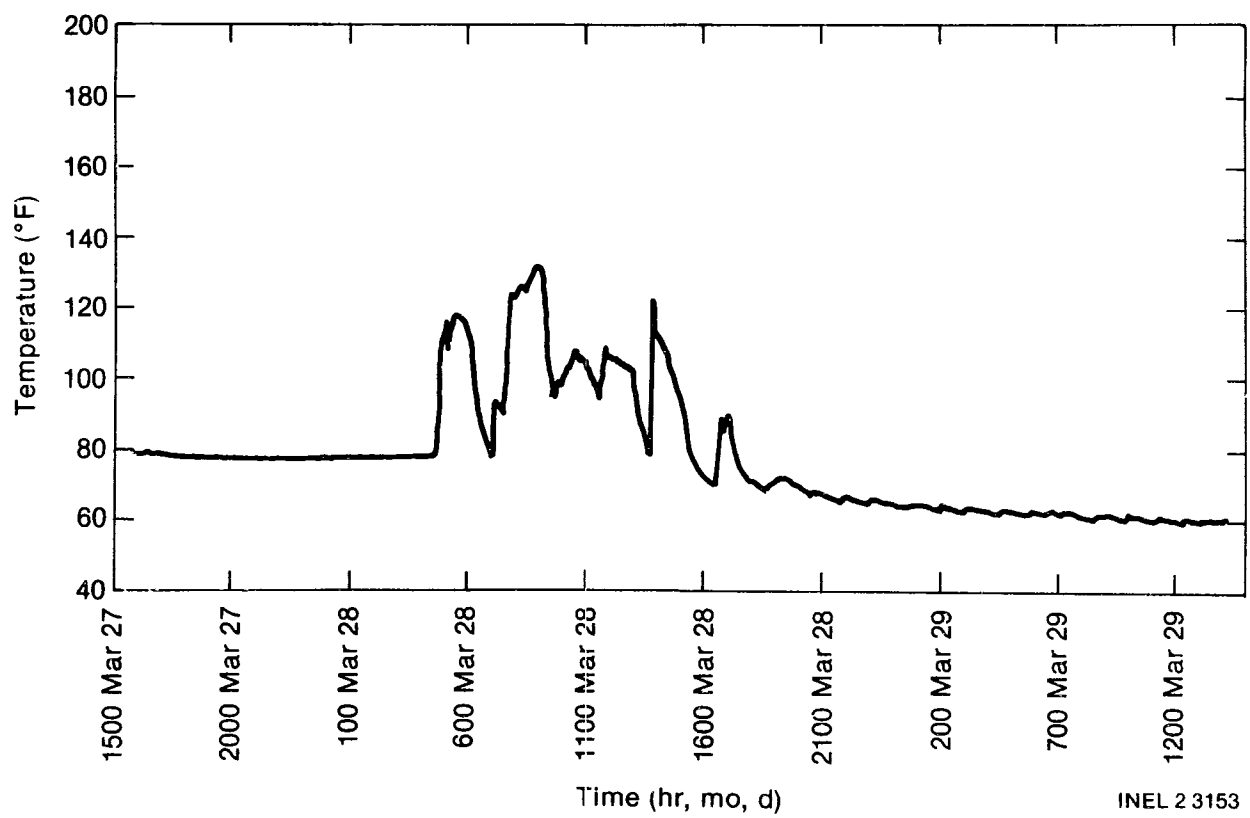

Figure A-6. RTD 5015 temperature excursions during the TMI-2 accident.

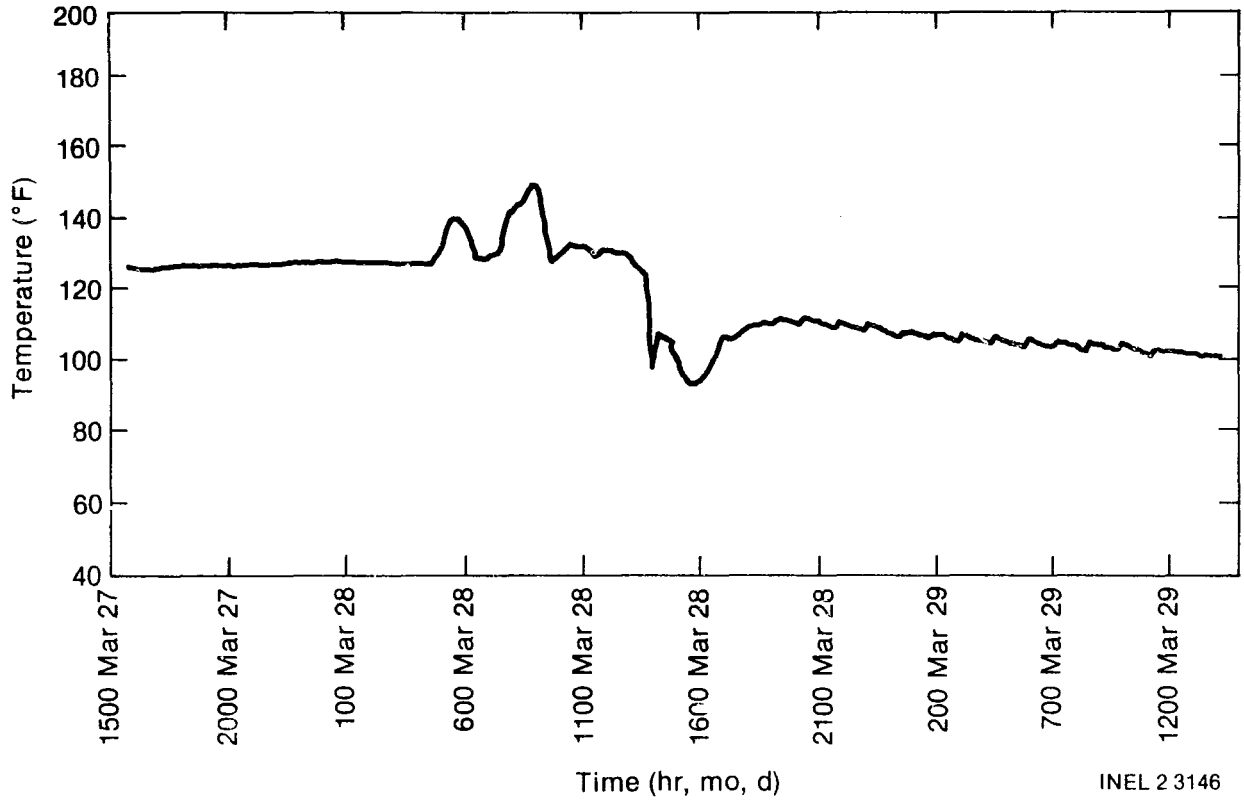

Figure A-7. RTD 5020 temperature excursions during the TMI-2 accident.

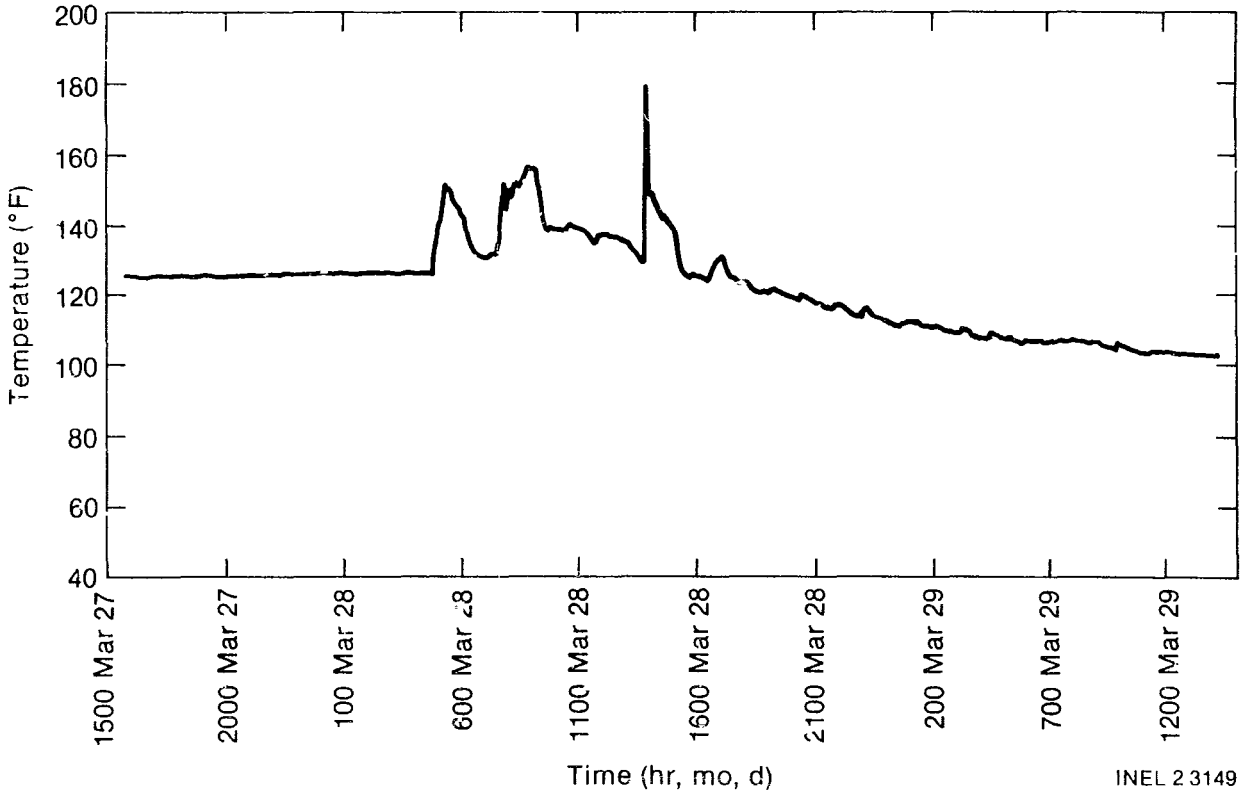

Figure A-8. RTD 5021 temperature excursions during the TMI-2 accident.

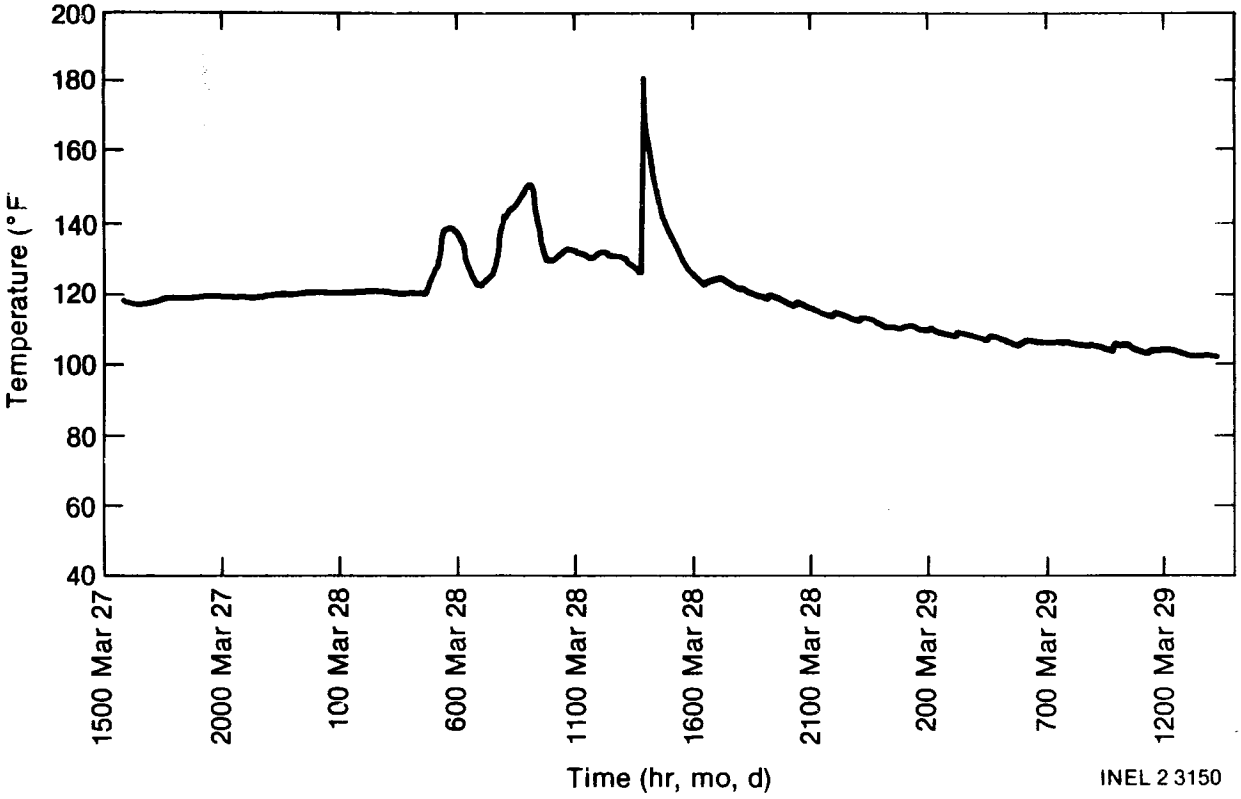

Figure A-9. RTD 5022 temperature excursions during the TMI-2 accident.

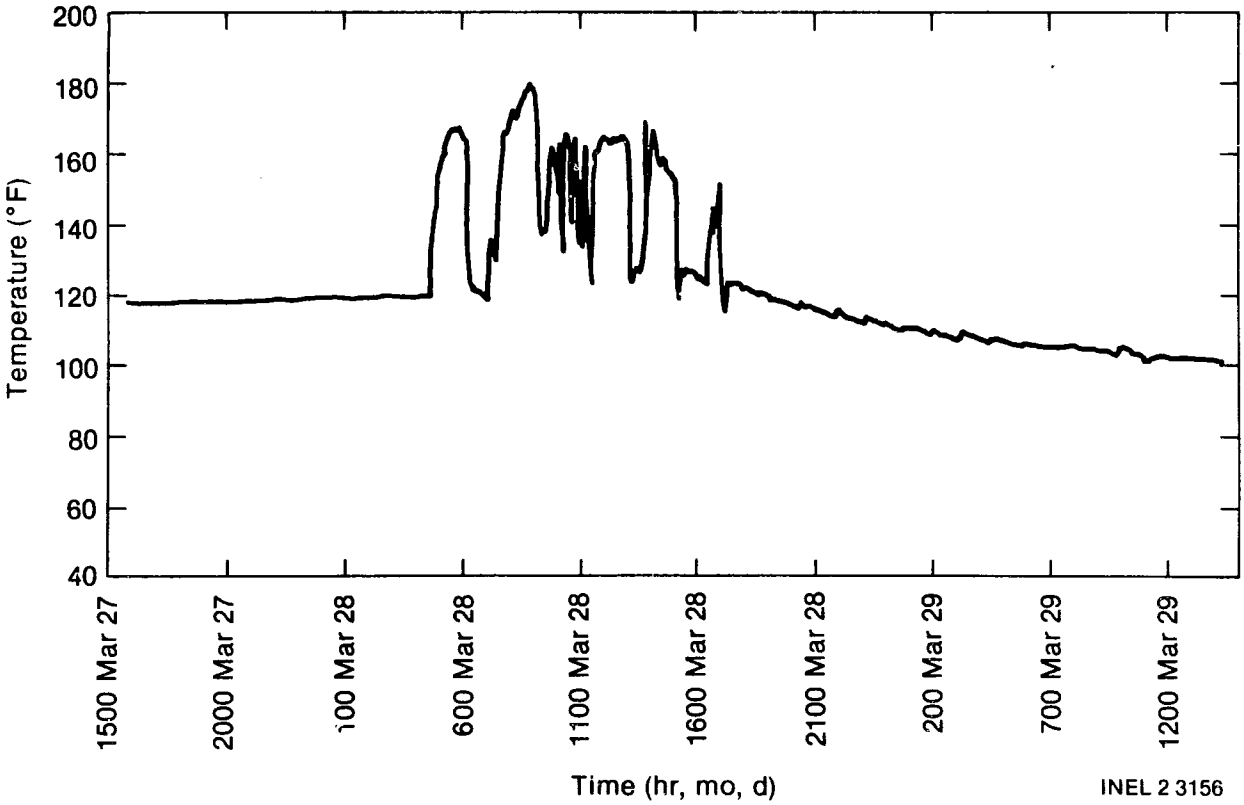

Figure A-10. RTD 5023 temperature excursions during the TMI-2 accident.

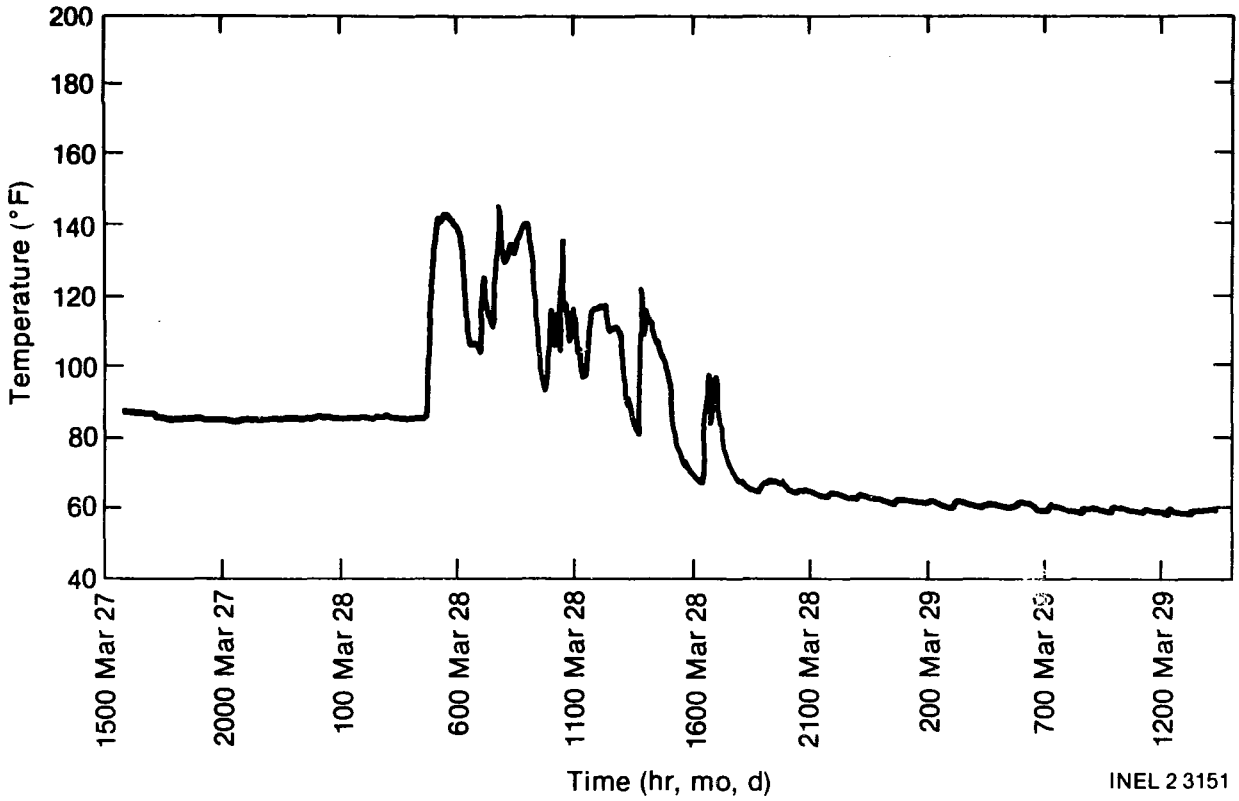

Figure A-11. RTD 5027 temperature excursions during the TMI-2 accident.

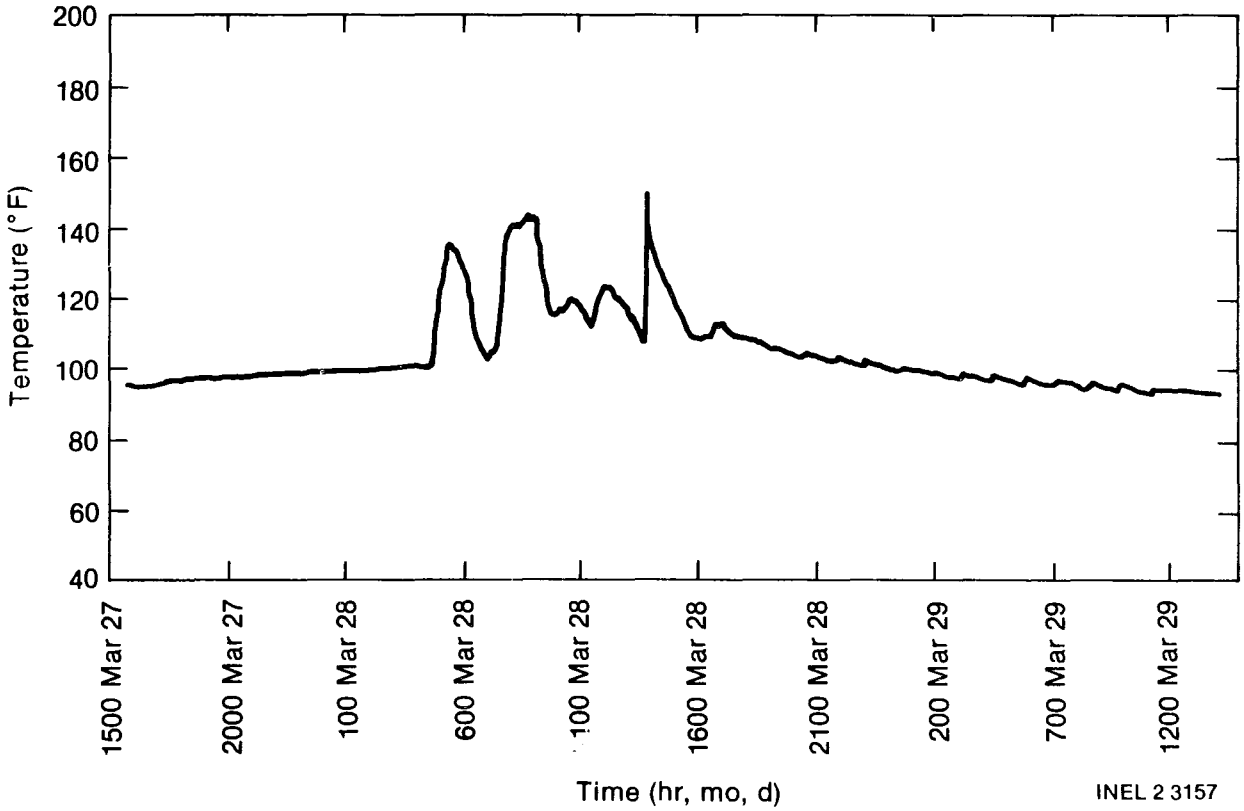

Figure A-12. RTD 5088 temperature excursions during the TMI-2 accident.

APPENDIX B
FILTER PROGRAMS

```
      PROGRAM RTD(TAPE1,TAPE2,TAPE3)
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      REAL MAX
      DIMENSION XISAVE(10)
      DIMENSION ERRMAG(10)
      DIMENSION AMEAS(10)
      DATA XI/10*0.0/
      DATA PMAT/100*0.0/,H/100*0.0/,RMAT/100*0.0/
      DATA HT/100*0.0/,GAIN/100*0.0/,HMAT/10*0.0/
      TBOOM=0.0
      PMAT(1,1)=2.5E+03
      PMAT(2,2)=2.5E+05
      PMAT(3,3)=.015
      PMAT(4,4)=6.5E-05
      RMAT(1,1)=4.24
      READ(1,*) NDATA,NMEAS,NSTATE
      READ(1,*) TSTART,(XI(J),J=1,NSTATE)
      DO 1000 IDUMMY=1,NDATA
      READ(1,*) (ZMEAS(I),I=1,NMEAS)
      AMEAS(IDUMMY)=ZMEAS(1)
      READ(1,*) TIME
      IF(IDUMMY.EQ.1) STIME=TIME
      DO 50 I=1,NSTATE
   50 X(I)=XI(I)
      DO 100 I=1,100
C---- 100 CHANCES TO CONVERGE
      CALL HVALU
      CALL KGAIN
      DO 60 J=1,NSTATE
60    XISAVE(J)=XI(J)
      CALL STATE(I)


      DO 75 J=1,NSTATE
   75 ERRMAG(J)=ABS(XI(J)-XISAVE(J))
      NUM=0
      DO 90 J=1,NSTATE
   90 IF(ERRMAG(J).LT.(.001*ABS(XI(J)))) NUM=NUM+1
      IF(NUM.EQ.NSTATE) GO TO 101
      IF(I.EQ.20) WRITE(2,' ( " FAILED TO CONVERGE ")')
  100 CONTINUE
  101 CONTINUE
      WRITE(2,105) (XI(I),I=1,NSTATE)
  105 FORMAT(' STATE ',E12.6)
      CALL PROPP
```

```
      WRITE(2,'(///, " COVARIANCE ",/)')                                000520
      DO 110 I=1,NSTATE                                                 000530
  110 WRITE(2,112) (PMAT(I,J),J=1,NSTATE)                               000540
  112 FORMAT(4(2X,E12.6))                                               000550
 1000 CONTINUE                                                          000560
      WRITE(3,'( " 1")')                                                000570
      TIME=0.0                                                          000590
C---- MAKE PLOT FILE OF MODEL FIT                                       000600
      DO 1500 II=1,360                                                  000610
      EX3=EXP(-XI(3)*TIME)                                              000620
      EX4=EXP(-XI(4)*TIME)                                              000630
      DATA=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*XI(3)/(XI(3)-XI(4)) 000640
      TIME=TIME+10.0                                                    000650
 1500 WRITE(3,*) TIME,DATA                                              000660
      WRITE(3,'( " END")')                                              000670
      AVE=0.0                                                           000672
      MAX=0.0                                                           000673
      SIG=0.0                                                           000674
      N=0                                                               000675
      TIME=STIME                                                        000680
      DO 2000 II=1,NDATA                                                000690
      EX3=EXP(-XI(3)*TIME)                                              000700
      EX4=EXP(-XI(4)*TIME)                                              000710
      DATA=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*XI(3)/(XI(3)-XI(4)) 000720
      A=AMEAS(II)-DATA                                                  000724
      TIME=TIME+360.0                                                   000730
      N=N+1                                                             000732
      IF(MAX.LT.ABS(A)) MAX=ABS(A)                                      000734

      AVE=(AMEAS(II)-DATA)/N+AVE*(N-1)/N                                000740
      SIG=SIG+(AMEAS(II)-DATA)**2                                       000750
 2000 CONTINUE                                                          000760
      SIG=SQRT(SIG)/(N-1)                                               000770

      WRITE(2,'(//," AVERAGE , MAXIMUM , SAMPLE VARIANCE OF ERROR")')   000780
      WRITE(2,*) AVE,MAX,SIG                                            000790
      STOP                                                              000800
      END                                                               000810
      SUBROUTINE HVALU                                                  000820
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)    000830
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE        000840
     +,TSTART,ZMEAS(10)                                                 000850
      DELTA=TIME-TBOOM                                                  000860
      EX3=EXP(-XI(3)*DELTA)                                             000870
      EX4=EXP(-XI(4)*DELTA)                                             000880
      ABA=XI(3)/(XI(3)-XI(4))                                           000890
      HMAT(1)=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*ABA              000900
```

```
      H(1,1)=1-EX3
      H(1,2)=(EX4-EX3)*ABA
      H(1,3)=-DELTA*EX3*(TSTART-XI(1)-XI(2)*ABA)
     + +(EX4-EX3)*ABA*XI(2)/XI(3)-(EX4-EX3)*(ABA**2)*XI(2)/XI(3)
      H(1,4)=-DELTA*EX4*XI(2)*ABA
     + +(EX4-EX3)*(ABA**2)*XI(2)/XI(3)
      RETURN
      END
      SUBROUTINE KGAIN
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      DIMENSION PH(10),HPH(10,10),TEMP(10,10),TEMPIV(10,10),WRK(1000)
      DO 100 I=1,NSTATE
      DO 100 J=1,NMEAS
  100 HT(I,J)=H(J,I)
      CALL VMULFF(PMAT,HT,NSTATE,NSTATE,NMEAS,10,10,PH,10,IER)
      CALL VMULFF(H,PH,NMEAS,NSTATE,NMEAS,10,10,HPH,10,IER)
      DO 200 I=1,NMEAS
      DO 200 J=1,NMEAS
  200 TEMP(I,J)=HPH(I,J)+RMAT(I,J)
      IDGT=0
      CALL LINV2F(TEMP,NMEAS,10,TEMPIV,IDGT,WRK,IER)

      CALL VMULFF(PH,TEMPIV,NSTATE,NMEAS,NMEAS,10,10,GAIN,10,IER)
      RETURN
      END
      SUBROUTINE STATE(NPASS)
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      DIMENSION AKR(10),RESID(10),DIFF(10),HX(10)
      DO 100 I=1,NSTATE
      DIFF(I)=X(I)-XI(I)

  100 CONTINUE
      N=1
      CALL VMULFF(H,DIFF,NMEAS,NSTATE,N,10,10,HX,10,IER)
      DO 200 I=1,NMEAS
  200 RESID(I)=ZMEAS(I)-HMAT(I)-HX(I)
      CALL VMULFF(GAIN,RESID,NSTATE,NMEAS,N,10,10,AKR,10,IER)
      DO 300 I=1,NSTATE
  300 XI(I)=X(I)+AKR(I)
      RETURN
      END
      SUBROUTINE PROPST
      RETURN
      END
```

```
      SUBROUTINE FROPP
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      REAL KH(10,10),TEMP(10,10),KH1(10,10)
      DATA KH/100*0.0/,KH1/100*0.0/,TEMP/100*0.0/
      CALL VMULFF(GAIN,H,NSTATE,NMEAS,NSTATE,10,10,KH,10,IER)
      DO 100 I=1,NSTATE
      DO 100 J=1,NSTATE
      KH1(I,J)=-KH(I,J)
  100 IF(I.EQ.J) KH1(I,J)=1+KH1(I,J)
      CALL VMULFF(KH1,PMAT,NSTATE,NSTATE,NSTATE,10,10,TEMP,10,IER)
      DO 200 I=1,NSTATE
      DO 200 J=1,NSTATE
      PMAT(I,J)=TEMP(I,J)
      IF(I.GT.J) PMAT(I,J)=TEMP(J,I)
  200 CONTINUE
      RETURN
      END
```

```
      PROGRAM RTD(TAPE1,TAPE2)
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      REAL MAX
      DIMENSION XISAVE(10)
      DIMENSION ERRMAG(10)
      DIMENSION AMEAS(10)
      DATA XI/10*0.0/
      DATA PMAT/100*0.0/,H/100*0.0/,RMAT/100*0.0/
      DATA HT/100*0.0/,GAIN/100*0.0/,HMAT/10*0.0/
      TBOOM=0.0
      PMAT(1,1)=2.5E+03
      PMAT(2,2)=2.5E+05
      PMAT(3,3)=6.5E-05
      RMAT(1,1)=4.24
      READ(1,*) NDATA,NMEAS,NSTATE
      READ(1,*) TSTART,(XI(J),J=1,NSTATE)
      DO 1000 IDUMMY=1,NDATA
      READ(1,*) (ZMEAS(I),I=1,NMEAS)
      AMEAS(IDUMMY)=ZMEAS(1)
      READ(1,*) TIME
      IF(IDUMMY.EQ.1) STIME=TIME
      DO 50 I=1,NSTATE
   50 X(I)=XI(I)
      DO 100 I=1,100
C---- 100 CHANCES TO CONVERGE
      CALL HVALU
      CALL KGAIN
      DO 60 J=1,NSTATE
60    XISAVE(J)=XI(J)
      CALL STATE(I)
      DO 75 J=1,NSTATE

   75 ERRMAG(J)=ABS(XI(J)-XISAVE(J))
      NUM=0
      DO 90 J=1,NSTATE
   90 IF(ERRMAG(J).LT.(.001*ABS(XI(J)))) NUM=NUM+1
      IF(NUM.EQ.NSTATE) GO TO 101
      IF(I.EQ.20) WRITE(2,' ( " FAILED TO CONVERGE ")')
  100 CONTINUE
  101 CONTINUE
```

```
  101 CONTINUE
      WRITE(2,105) (XI(I),I=1,NSTATE)
  105 FORMAT(' STATE ',E12.6)
      CALL PROPP
      CALL PROPST
      WRITE(2,'(///, " COVARIANCE ",/)')
      DO 110 I=1,NSTATE
  110 WRITE(2,112) (PMAT(I,J),J=1,NSTATE)
  112 FORMAT(4(2X,E12.6))
 1000 CONTINUE
      WRITE(3,'( " 1")')
      TIME=0.0
C---- MAKE PLOT FILE OF MODEL FIT
      DO 1500 II=1,360
      EX3=EXP(-.02*TIME)
      EX4=EXP(-XI(3)*TIME)
      DATA=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*.02/(.02-XI(3))
      TIME=TIME+10.0
 1500 WRITE(3,*) TIME,DATA
      WRITE(3,'( " END")')
      AVE=0.0
      MAX=0.0
      SIG=0.0
      N=0
      TIME=STIME
      DO 2000 II=1,NDATA
      EX3=EXP(-.02*TIME)
      EX4=EXP(-XI(3)*TIME)
      DATA=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*.02/(.02-XI(3))
      A=AMEAS(II)-DATA
      TIME=TIME+360.0
      N=N+1

      IF(MAX.LT.ABS(A)) MAX=ABS(A)
      AVE=(AMEAS(II)-DATA)/N+AVE*(N-1)/N
      SIG=SIG+(AMEAS(II)-DATA)**2
 2000 CONTINUE
      SIG=SQRT(SIG)/(N-1)
      WRITE(2,'(//," AVERAGE , MAXIMUM , SAMPLE VARIANCE OF ERROR")')
      WRITE(2,*) AVE,MAX,SIG
      STOP
      END
```

```
      STOP
      END
      SUBROUTINE HVALU
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      DELTA=TIME-TBOOM
      EX3=EXP(-.02*DELTA)
      EX4=EXP(-XI(3)*DELTA)
      ABA=.02/(.02-XI(3))
      HMAT(1)=EX3*TSTART+(1-EX3)*XI(1)+(EX4-EX3)*XI(2)*ABA
      H(1,1)=1-EX3
      H(1,2)=(EX4-EX3)*ABA
      H(1,3)=-DELTA*EX4*XI(2)*ABA
     + +(EX4-EX3)*(ABA**2)*XI(2)/.02
      RETURN
      END
      SUBROUTINE KGAIN
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      DIMENSION PH(10),HPH(10,10),TEMP(10,10),TEMPIV(10,10),WRK(1000)
      DO 100 I=1,NSTATE
      DO 100 J=1,NMEAS
  100 HT(I,J)=H(J,I)
      CALL VMULFF(PMAT,HT,NSTATE,NSTATE,NMEAS,10,10,PH,10,IER)
      CALL VMULFF(H,PH,NMEAS,NSTATE,NMEAS,10,10,HPH,10,IER)
      DO 200 I=1,NMEAS
      DO 200 J=1,NMEAS
  200 TEMP(I,J)=HPH(I,J)+RMAT(I,J)
      IDGT=0
      CALL LINV2F(TEMP,NMEAS,10,TEMPIV,IDGT,WRK,IER)
      CALL VMULFF(PH,TEMPIV,NSTATE,NMEAS,NMEAS,10,10,GAIN,10,IER)

      RETURN
      END
      SUBROUTINE STATE(NPASS)
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      DIMENSION AKR(10),RESID(10),DIFF(10),HX(10)
      DO 100 I=1,NSTATE
      DIFF(I)=X(I)-XI(I)
  100 CONTINUE
      N=1
      CALL VMULFF(H,DIFF,NMEAS,NSTATE,N,10,10,HX,10,IER)
      DO 200 I=1,NMEAS
  200 RESID(I)=ZMEAS(I)-HMAT(I)-HX(I)
```

```
      CALL VMULFF(H,DIFF,NMEAS,NSTATE,N,10,10,HX,10,IER)
      DO 200 I=1,NMEAS
  200 RESID(I)=ZMEAS(I)-HMAT(I)-HX(I)
      CALL VMULFF(GAIN,RESID,NSTATE,NMEAS,N,10,10,AKR,10,IER)
      DO 300 I=1,NSTATE
  300 XI(I)=X(I)+AKR(I)
      RETURN
      END
      SUBROUTINE PROPST
      RETURN
      END
      SUBROUTINE PROPP
      COMMON/FILTER/X(10),XI(10),GAIN(10,10),Z(10),HMAT(10),H(10,10)
     +,PMAT(10,10),HT(10,10),RMAT(10,10),TIME,TBOOM,NMEAS,NSTATE
     +,TSTART,ZMEAS(10)
      REAL KH(10,10),TEMP(10,10),KH1(10,10)
      DATA KH/100*0.0/,KH1/100*0.0/,TEMP/100*0.0/
      CALL VMULFF(GAIN,H,NSTATE,NMEAS,NSTATE,10,10,KH,10,IER)
      DO 100 I=1,NSTATE
      DO 100 J=1,NSTATE
      KH1(I,J)=-KH(I,J)
  100 IF(I.EQ.J) KH1(I,J)=1+KH1(I,J)
      CALL VMULFF(KH1,PMAT,NSTATE,NSTATE,NSTATE,10,10,TEMP,10,IER)
      DO 200 I=1,NSTATE
      DO 200 J=1,NSTATE
      PMAT(I,J)=TEMP(I,J)
      IF(I.GT.J) PMAT(I,J)=TEMP(J,I)
  200 CONTINUE
      RETURN
      END
```

APPENDIX C

ANALYSIS OF WORST-CASE AMBIENT TEMPERATURE PROFILE

APPENDIX C

ANALYSIS OF WORST-CASE AMBIENT TEMPERATURE PROFILE

Consider the worst-case ambient temperature profile shown in Figure C-1. This profile will give the longest duration of high temperatures without large differences with recorded data. A mathematical expression for the ambient temperature is

$$T_{AMB} = \begin{cases} T_0 & t<t_B \\ T_B & t_B<t<t_s \\ T_1 + T_2\, e^{-\beta(t-t_s)} & t>t_s \end{cases}$$

Given this, the RTD temperature is

$$T(t) = T_0\, e^{-\alpha(t-t_B)} + T_B\, [e^{-\alpha(t-t_s)} - e^{-\alpha(t-t_B)}]$$

$$+ T_1\, [1-e^{-\alpha(t-t_s)}] + \frac{\alpha T_2}{\alpha-\beta}\, [e^{-\beta(t-t_s)} - e^{-\alpha(t-t_s)}]$$

and the difference between the RTD temperature and the ambient at the first data point time t_D is

$$T_D - T_{AMB} = T_0\, e^{-\alpha(t_D-t_B)} + T_B\, [e^{-\alpha(t_D-t_s)} - e^{-\alpha(t_D-t_B)}]$$

$$- T_1\, e^{-\alpha(t_D-t_s)} + T_2 \left\{ \frac{\alpha}{\alpha-\beta}\, [e^{-\beta(t_D-t_S)}\right.$$

$$\left. - e^{-\alpha(t_D-t_S)}] - e^{-\beta(t_D-t_S)} \right\}$$

where $T_D = T(t_D)$. We want to find the maximum t_s so that the difference between the ambient and sensor temperature is small, say 10°F. Such a t_s can be found by solving the equation

$$10°F = T_D - T_{AMB}$$

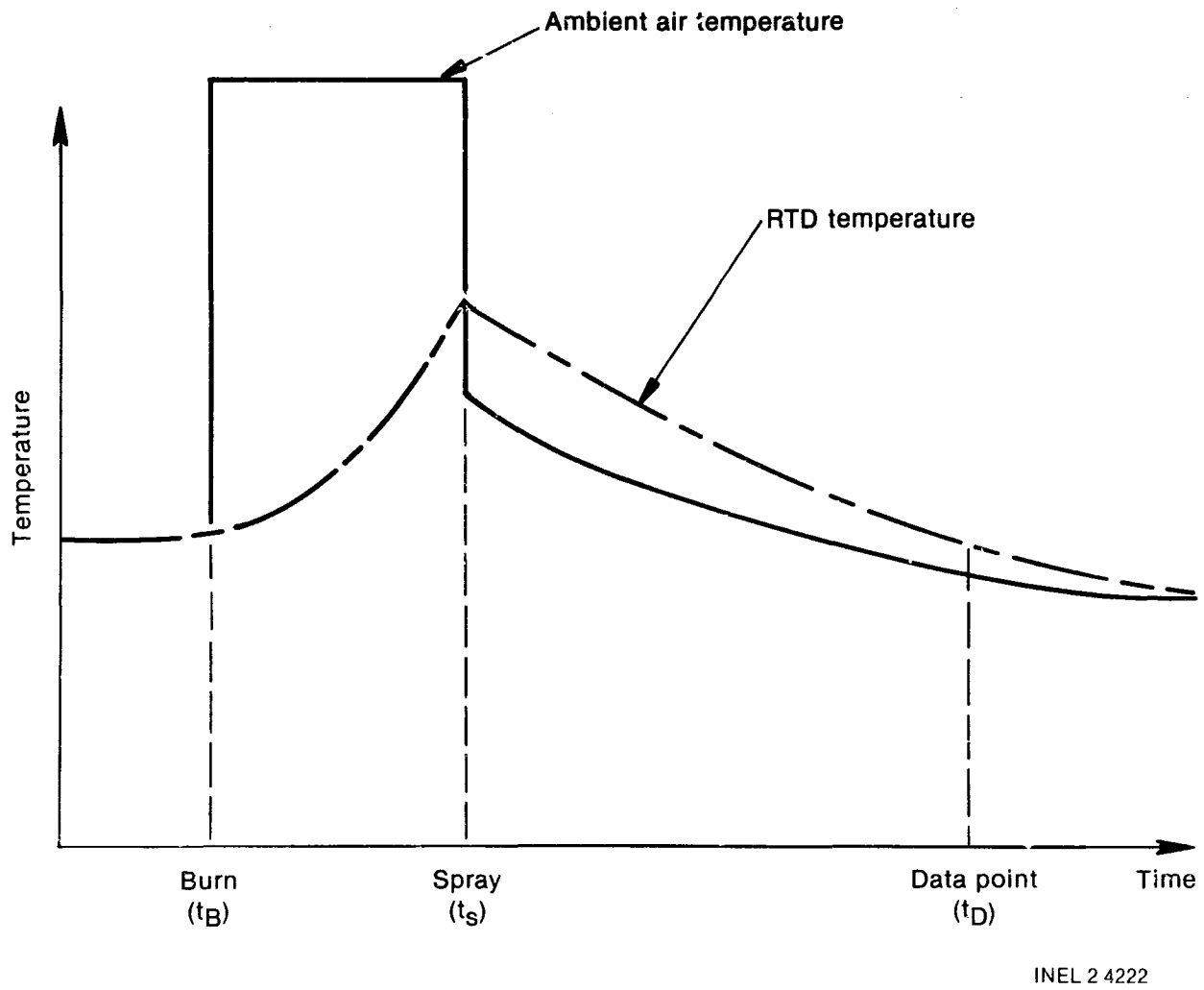

Figure C-1. Worst-case ambient temperature profile.

using the values

T_D = 110°F
T_1 = 120°F
T_2 = 30°F
α = 0.025/s
T_B = 1000°F

β = 0.0009/s
$t_D - t_B$ = 270 s

and solving the resulting equation using Newton's method we get $t_D - t_s$ = 170.5 s.