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UNDERSTANDING THE SEMI-PROBABILISTIC APPROACHES IN STRUCTURAL RELIABILITY USED TO SET DESIGN RELIABILITY TARGETS FOR GRAPHITE COMPONENTS USING ASME BPVC METHODS

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ABSTRACT

Graphite is a quasi-brittle material, resulting in random variability in tensile strength distributions. To account for the random variability in strength, HHA-3000 of the ASME BPVC provides two semi-probabilistic methods for qualifying nuclear graphite components in the design stage, the simplified and full assessments. The full and simplified assessments apply statistical methods to engineering-based design problems. This is often referred to as reliability-based design. Reliability-based design (RBD) is a method to develop reliable designs by accounting for uncertainties and result in small chances of failure when also considering safety factors. RBDs provide reliability targets using semi-probabilistic approaches. RBD is implemented in ASME BPVC HHA-3000 for nuclear graphite components, but is not specific to that application.

There has been much confusion around the methods implemented in ASME BPVC HHA-3000 for qualifying nuclear graphite components. To address the confusion, this paper takes a hierarchical approach. First, the general RBD framework is presented. Then, the semi-probabilistic methods and the underlying assumptions implemented in the assessments are presented. The semi-probabilistic methods are separated from the engineering modifications that have been made to the assessments. After building the framework and underlying assumptions, the specific methods in the full and simplified assessments are explained in three steps: inputs, methods, outputs. The methods are applied to an H-451 reflector block. Tensile strength properties for other graphite grades are provided.

Keywords: Graphite, reliability-based design, semi-probabilistic, ASME BPVC

NOMENCLATURE

ASME BPVC	American Society of Mechanical Engineers Boiler and Pressure Vessel Code
RBD	Reliability based design
POS	Probability of survival
POF	Probability of failure
PES	Peak equivalent stress
VHTR	Very high temperature reactor
FE	Finite element
SRC	Structural reliability class
ART	Advanced reactor technologies
BP	Baseline program
μ_S	Mean strength
μ_σ	Mean stress
Sf	Safety factor
σ_{TS}	Tensile stress
σ_{FS}	Flexural stress
μ_{FS}	Estimated average flexural strength
$\hat{\mu}_{TS}$	Estimated average tensile strength

1. INTRODUCTION

The American Society of Mechanical Engineers Boiler and Pressure Vessel Code (ASME BPVC) utilizes a reliability-based design (RBD) approach for setting design targets for graphite components used in high temperature nuclear reactors. This paper provides a description of the high-level concepts and definitions of reliability incorporated into HHA-3000 semi-probabilistic design applications. HHA-3000 provides two semi-probabilistic methods for determining reliability targets, the full and simplified assessments. To explain the methodologies incorporated in the full and simplified assessments, basic RBD concepts need to be understood. The purpose of this paper is to explain the basics of an RBD using reliable sources, and how those are incorporated into the semi-probabilistic methods for calculating reliability targets for graphite components using ASME BPVC. The reliability targets limit the likelihood of failure. Several definitions and concepts will be explained nearly verbatim from sources, to convey knowledge. The purpose of this paper is to provide a framework and foundation for understanding the RBD methods used for setting nuclear graphite components' design reliability targets. The methods are then applied for an illustrative case.

2. RELIABILITY ANALYSIS

Designers must design components that will work "theoretically and in practice" [6]. Reliability analysis attempts to quantify the probability of a component performing its function over a specified period of time and under specified operating conditions [6]. Reliability is not considered just in the design stage; it is also considered during development, production, quality control, shipping, operation, and maintenance [6]. The focus of this work is the reliability of graphite components at the design stage. There are electronic, mechanical, and structural systems that may use reliability analysis [6]. Structural systems and components primarily fail when extreme loads are encountered, such that the loads exceed the strengths of the components [6]. "A mechanical or structural component is considered to have failed when it ceases to function properly for its intended use" [6]. There are several types of failure, but here is a list of a few common failure modes for mechanical and structural components: static failure, fatigue failure, creep failure, corrosion failure, wear failure, and instability [6]. Reliability needs to be built into the component, to ensure enough confidence of component reliability during use, to limit the amount of component reliability that is left to chance. "Reliability is an inherent characteristic of the system, similar to the system's capacity or power rating" [6]. RBD's consider components, materials, processes and tolerances carefully at the decision stage, noting areas of uncertainty. "A mechanical or structural component is considered to be safe and reliable when the strength or resistance of the component exceeds the volume of the load acting on it. Thus, the reliability of the component requires knowledge of the random nature of the strength and the load." [6]. Rao, 1992 [6] explains ways to categorize systems:

"Systems with multiple members can be classified into two categories, weakest link systems and fail-safe system. A system is called a weakest link system if it fails whenever any of its elements fails. A fail-safe system (parallel system) is where the failure of a single element will not always result in the failure of the total system, because the remaining elements may be able to sustain the applied loads. If an element is perfectly brittle, it will lose its load-carrying capacity after its failure. On the other hand, if the element is ductile, it will not carry any additional load after failure, but it will continue to carry the load that it was carrying at the time of failure. Statistical models may be considered for modeling strength of brittle material. An ideal brittle material is one that fails whenever a single particle fails. Thus, the strength of the material is governed by the smallest strength of a particle [6]. The material is considered as a chain composed of several (n) elements in series. The strengths of the individual elements of a series chain are assumed statistically to be independent and follow a common distribution. The volume of the solid V is divided into several elementary volumes V0 such as the volume of a standard test specimen."

Choi, et al. [2] also define reliability as the probability that a system or component will perform its function over a specified period of time and under specified service conditions. Reliability analysis is defined as the probability a component has a strength which, if exceeded, will fail the component [6]. Simply put, reliability is the probability the material strength exceeds the component's applied stress, $P(Strength > Stress)$. Reliability analyses are important when there are strict performance requirements, narrow margins of safety, high liability, and to beat market competition [2]. The following paragraph is copied from Choi, et al. [2] to and describes how reliability interacts with structure limit states well:

"If, when a structure (or part of a structure) exceeds a specific limit, the structure (or part of the structure) is unable to perform as required, then the specific limit is called a ***limit-state****. The structure will be considered unreliable if the failure probability of the structure limit-state exceeds the required value. For most structures, the limit-state can be divided into two categories:*

- ***Ultimate limit-states*** *are related to a structural collapse of part or all of the structure. Examples of the most common ultimate limit-states are corrosion, fatigue, deterioration, fire, plastic mechanism, progressive collapse, fracture, etc. Such a limit-state should have a very low probability of occurrence, since it may risk the loss of life and major financial losses.*
- ***Serviceability limit-states*** *are related to disruption of the normal use of the structures. Examples of serviceability limit-states are excessive deflection, excessive vibration, drainage, leakage, local damage, etc. Since there is less danger than in the case of ultimate limit-states, a higher probability of occurrence may be tolerated in such limit-states."*

The classical (deterministic) approach to design is to define the mean strength (μ_S), the mean stress (μ_σ), and the safety factor (Sf) and ensure that $\mu_S \geq \mu_\sigma * Sf$ [5]. However, the deterministic approach to design does not ensure any level of reliability when there is randomness and variability in the component's applied loading and the material strength. Probabilistic designs are designs where there is variability in the strength and the stress distributions, and sometimes other variables, such as the component engineering aspects, such as boundaries, stiffness, mass properties, etc. [2] and component manufacturing processes [6]. There are two categories of variability, aleatoric and epistemic uncertainty. Choi [2] defines aleatoric uncertainty as the irreducible, inherent uncertainty and epistemic uncertainty as the reducible uncertainty. Epistemic uncertainty stems from lack of knowledge of the data, reduced as more information is gathered based on past experience or expert judgement [2]. The design is considered satisfactory when the reliability targets are met with an acceptable degree of certainty [2].

Reliability corresponds to specific failure modes. Dhillon [3] lists several types of failure modes: shear loading, instability, bending, compressive, tensile yield, fatigue, ultimate tensile, creep/rupture, stress concentration, material flaw (fatigue, small cracks/pores, poor quality assurance), metallurgical, bearing failures. For depth, failure modes described by Rao [6] are also listed: static failure, fatigue failure, creep failure, corrosion failure, wear failure, and instability. Static and fatigue failures both commonly result in fracture, creep often results in distortion or rupture, and instability often results in buckling of columns or plates [6]. "Each type of failure requires a different analysis, inspection, and test procedure" [6]. Dhillon [3] defines ultimate tensile strength failure as, "the applied stress exceeds the ultimate tensile strength and leads to a complete failure of the structure at a cross-sectional point." Technical analyses can be done to determine failure modes and effect and criticality analyses including determining the failure-governing stress and strength distributions for all critical failure modes, for example, see military standard MIL-STD-1629A [14]. This reference may be a useful consideration for designers and Code developers of maintenance and inspection guidelines.

3. ASME BPVC GRAPHITE DESGIN

ASME BPVC Section III, Construction of Nuclear Facility Components, Division 5, High Temperature Reactors, Subsection HH Subarticle A, Graphite provides two semi-probabilistic methodologies for setting reliability targets for nuclear graphite components in the ASME BPVC, the full and simplified assessments. The scope is described under section HHA-1100 as covering graphite core components and assemblies, for the reactor pressure vessel of a high temperature, graphite moderated fission reactor. HHA-1300 states that the rules of HHA apply to, "(a) Graphite Core Components utilized in a high temperature, graphite moderated fission reactor. Graphite core components include fuel blocks, reflector blocks, shielding blocks, and any keys or dowels used to interconnect them. (b) The rules shall also apply to the arrangement of Graphite Core Components that form the Graphite Core Assembly. (c) The rules shall not apply to fuel compacts, bushings, bearings, seals, blanket materials, instrumentation, or components internal to the reactor other than those defined above."

The full and simplified assessments are semi-probabilistic because only variability in the strength is considered in the design reliability analysis. Variability in load and other variables is not considered. Load uncertainty is aleatoric uncertainty and strength uncertainty is epistemic uncertainty. Although the assessments account for factors such as oxidation and irradiation effects in component stress distributions, the assessments do not account for oxidation and irradiation effects on the material reliability curves. The assessments provide design targets and may not accurately predict the cracking rate. The designer separately is instructed to evaluate the effects of cracking and ensure the assembly is damage tolerant. Methods outlined in [14] may be a good basis for evaluation.

The design targets are a function of the material reliability curve, the component stress distribution, and the component structural reliability class (SRC), which determines the allowable probability of failure (POF). That is, $POF_{allowable}(SRC_{comonent}) = P\big(Strength_{material} > Stress_{component}\big)$. The ASME BPVC graphite design articles seem to have largely been borrowed from the German KTA-3232 standard [13], with modifications by Dr. Hindley [9], presented in his 2015 doctoral thesis and related journal articles. Work by Ho [10], Strizak [11], and Schmdit [12] also seem to have ties to the ASME BPVC graphite design articles.

The 2- and 3- parameter Weibull distributions are used as material reliability curves in the simplified and full assessments, respectively. The sampling requirements to estimate the material reliability curves are outlined in HHA-III-4100, which through reference to Mandatory Appendix HHA-I, references ASTM guidelines in D7219-08 and D7301-08. Each graphite grade needs 24 spatially located specimens from each of four billets from each of three lots, totaling 288 tensile specimens.

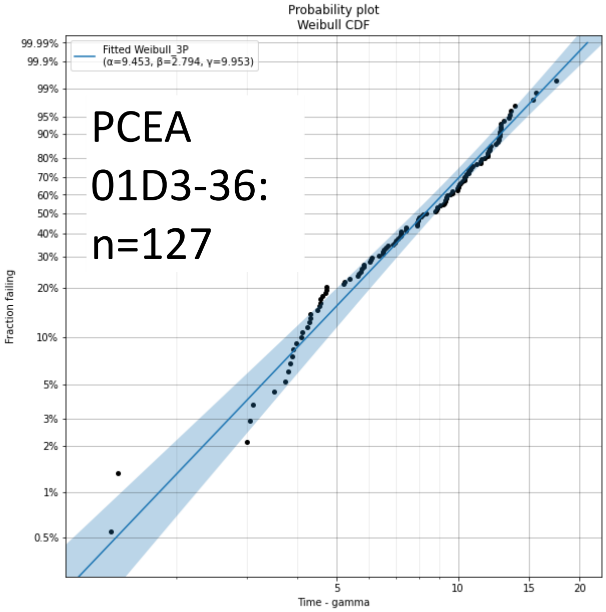
The finite element (FE) model is developed for the component of interest. The elemental principal stresses are converted to equivalent tensile stresses, see HHA-3213. Again, a fully probabilistic approach to design would account for uncertainty in the load creating the stress distribution. The graphite RBD is only semi-probabilistic, in that it assumes the load is fixed and does not vary with time/environmental conditions. The failure mode considered in the assessments is tensile fracture, or, as referred to in the Code, cracking.

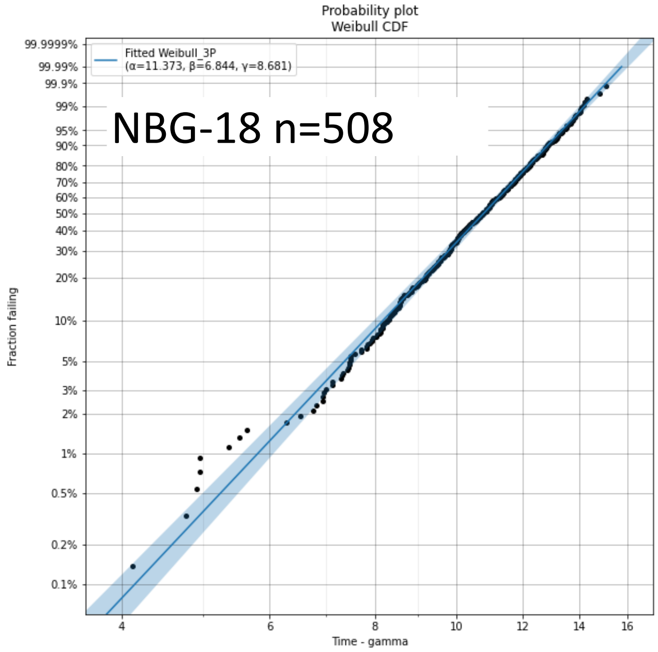
The simplified assessment utilizes Choi, et al.'s [2] definition of reliability by applying two checks. The checks are described for a component critical to safety, SRC-1, which limits the probability of failure to 10^{-4}: (1) the allowable tensile stress is σ_{TS}, such that $P(\sigma \leq \sigma_{TS}) = 10^{-4}$. If membrane stresses exceed σ_{TS}, the component fails to meet the design targets (2) the allowable flexural stress is found by multiplying σ_{TS} by the ratio of the average flexural to the average tensile strength, $\frac{\hat{\mu}_{FS}}{\hat{\mu}_{TS}}$, such that $\sigma_{FS} = \sigma_{TS} * \frac{\hat{\mu}_{FS}}{\hat{\mu}_{TS}}$. If the peak equivalent stress (PES) exceeds σ_{FS}, the component fails to meet the simplified assessment design targets. Again, these methods are not deterministic, because the uncertainty in the material strength is accounted for, but they are not fully probabilistic because the uncertainty in the loads, for example, are not considered.

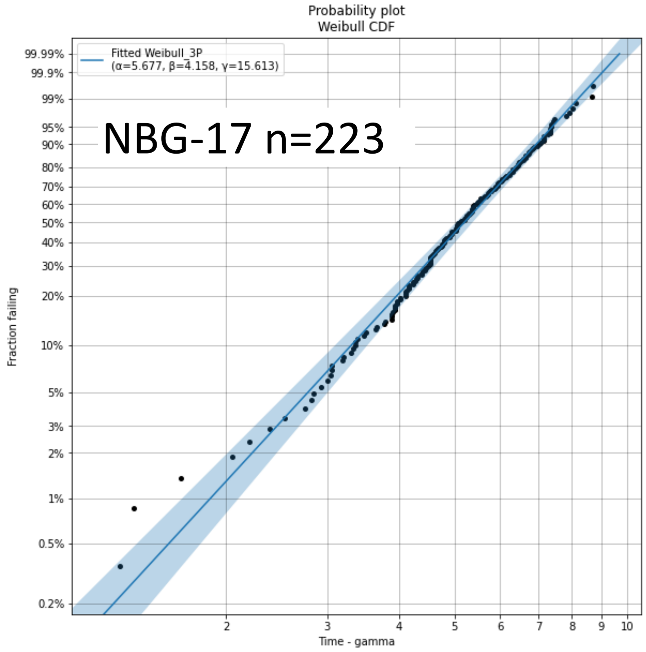
The full assessment utilizes Weibull's weakest link theory [7] for brittle materials, also described by Rao [6], but modified for graphite material properties. Assumptions of weakest link theory are: (1) that the strength of the entire volume is limited by the strength of the weakest volume and (2) the size effect assumption is that the strength decreases as a larger volume of material is tested. The Weibull threshold parameter is reduced if the stress distribution is too far from the strength distribution. Elements from the FE output are grouped according to volume and relative stress gradients. The Weibull reliability curve is used to determine the survival probabilities (POS_i) for each group within the component. The component failure probability is; $POF = 1 - \prod_i POS_i$.

4. APPLICATION

The Advanced Reactor Technologies (ART) baseline program (BP) collected tensile data, with specimens machined according to ASTM C749, for five grades of graphite. All billets of IG-110, 2114, NBG-17, and NBG-18 from the BP were used in the analyses in the paper. Only billet 01D336 of PCEA was used in the analysis of this paper, because all other billets of PCEA currently available through the BP are thought to have wriggler porosity. Therefore, PCEA-36 is used to emphasize that material properties are only representative of PCEA billet 01D336. While the sampling of these graphite grades did not follow the requirements of D7219, the total sample sizes are large enough to obtain estimated reliability curves. The 3-parameter Weibull distributions were estimated using maximum likelihood estimation in Python [8]. This is the recommended estimation method for the full assessment in HHA-II-3000. Figure 1 displays the linearized tensile reliability curve for the five graphite grades used in the BP. Tables 1 and 2 provide the 2- and 3- parameter Weibull estimates. Note that lower bounds are only reported for the shape and characteristic strength parameters and not for the threshold. This is aligned with the notation in the Code, which indicates such.







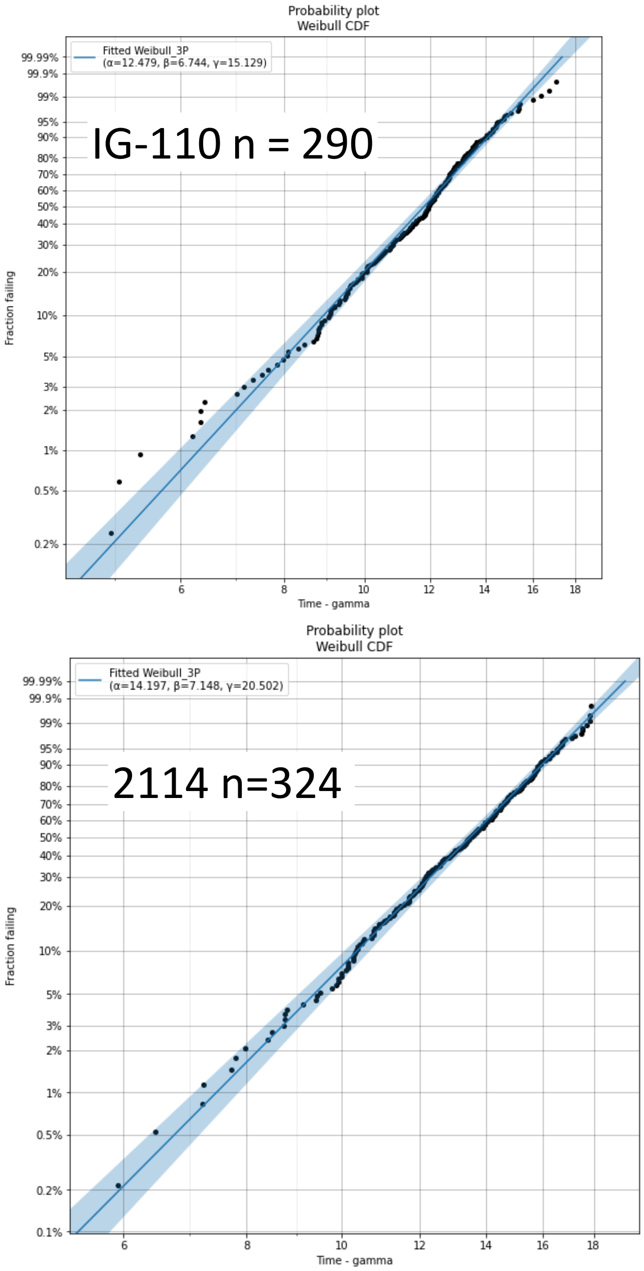


FIGURE 1: LINEARIZED MLE 3-PARAMETER WEIBULL DISTRIBUTIONS

		2 Parameter Weibull LSR		**2 Parameter Weibull LSR LB**	
Material	**N**	**Char. Strength**	**Shape**	**Char. Strength**	**Shape**
PCEA-36	**127**	19.69265	6.492431	19.27309	5.74336
NBG18	**508**	20.09379	12.88491	19.97917	12.10394
NBG17	**223**	21.26848	17.04267	21.13391	15.56016
IG-110	**290**	27.68002	16.03679	27.51515	14.80432
2114	**324**	34.76101	18.77677	34.59296	17.40437

TABLE 1: LEAST SQUARES REGRESSION ESTIMATES AND LOWER BOUNDS, 2-PARAMETER WEIBULL DISTRIBUTIONS

		3 Parameter Weibull MLE			**3 Parameter Weibull MLE LB**	
Material	**N**	**Thres.**	**Char. Strength**	**Shape**	**Char. Strength**	**Shape**
PCEA-36	**127**	9.95	19. 4053	2.794	18.8997	2.481
NBG18	**508**	15.613	21.2905	4.15761	21.1245	3.79762
NBG17	**223**	15.6131	21.2905	4.15761	21.1245	3.79762
IG-110	**290**	15.129	27.975	6.74376	27.608	6.2645
2114	**324**	20.5019	34.6989	7.14811	34.5091	6.654

TABLE 2: MLE AND LOWER BOUND, 3-PARAMETER WEIBULL DISTRIBUTIONS

The simplified assessment tensile and flexural allowable stresses are plotted in Figures 2 and 3, assuming SRC-1 components. The allowable tensile stress limits are found by using eq. 23 of HHA-II-3300. The allowable flexural stress limits are found by multiplying the allowable tensile stress limits by the ratio of the average flexural stress to the average tensile stress for each graphite grade.

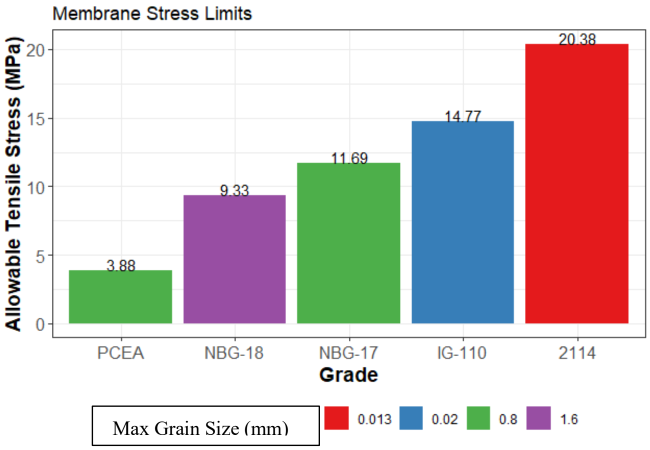


FIGURE 2: ALLOWABLE TENSILE STRESS LIMITS, APPLIED TO MEMBRANE STRESSES, EQ. 23 OF HHA-II-3300, ASSUMING SRC-1 COMPONENT

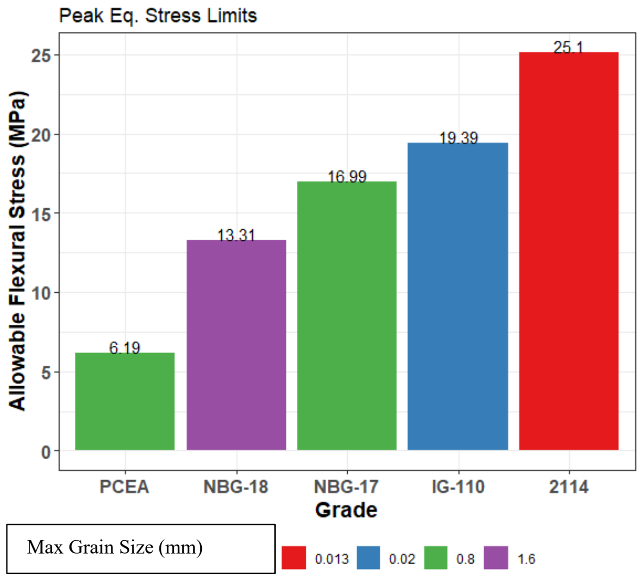


FIGURE 3: ALLOWABLE FLEXURAL STRESS LIMITS, APPLIED TO PEAK EQUIVALENT STRESSES, EQ. 23 OF HHA-II-3300, ASSUMING SRC-1 COMPONENT

The full assessment was applied to a reflector brick from the historical Fort St. Vrain H-451 reactor. Conditions from the model are described in detail in [15]. Figure 4 displays the reflector block's temperature and fluence profiles after 8 years. The temperature and fast neutron flux applied is constant over time, and along the length of the brick. The irradiation dose and temperature applied are based upon operating conditions of the Fort St. Vrain reactor. [16]

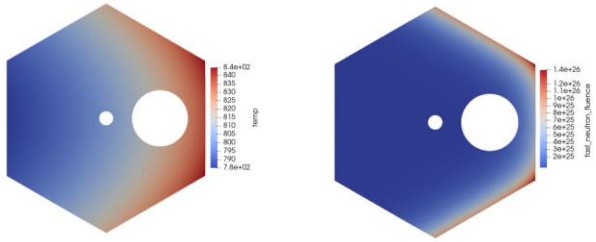
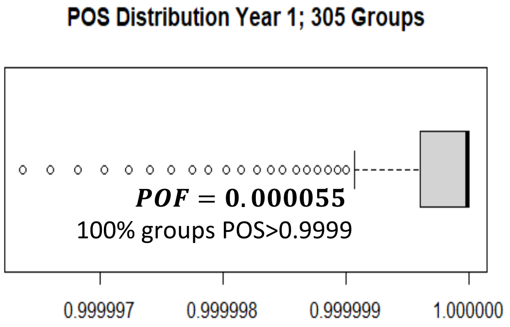
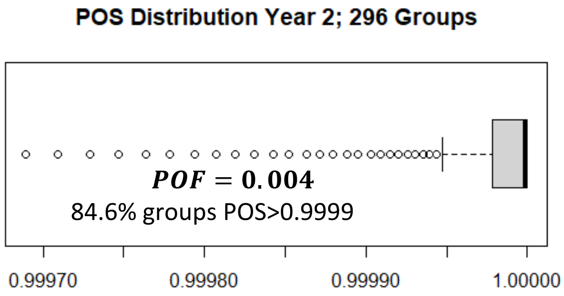


FIGURE 4: TEMPERATURE (K) AND FLUENCE (N/M^2) DISTRIBUTIONS AFTER 8 YEARS

The original block FE mesh had 625,650 volume elements and after removing elements based on the threshold parameter (stresses below which have no POF), the truncated mesh had 47,116 elements. Only 7.5% of the elements remained for analysis after year 8. The initial estimated threshold parameter was 7.26 MPa. The component stress distribution is closer to the original material strength distribution each subsequent year. Consequently, each year produces a smaller reduction of the threshold parameter. Because of the threshold reduction step: components with larger peak stresses are compared to stronger strength distributions, for the same material and components with lower peak stresses are compared to weaker strength distributions, for the same material. The threshold reduction based on the component's peak equivalent stress makes it harder to qualify components with low peak stresses (almost a penalty). This penalty less extensive and makes mathematical sense when updating the shape parameter when the threshold is reduced. This change is currently the balloting process to fix the full assessment in the ASME BPV. The penalty is necessary because there is no lower bound used for the threshold parameter and for the lack in ability to capture the disparate flaw mode. Figure 5 shows the distribution of groups and their corresponding POS's, along with the block POF, again for years 1, 2, 4, and 8.





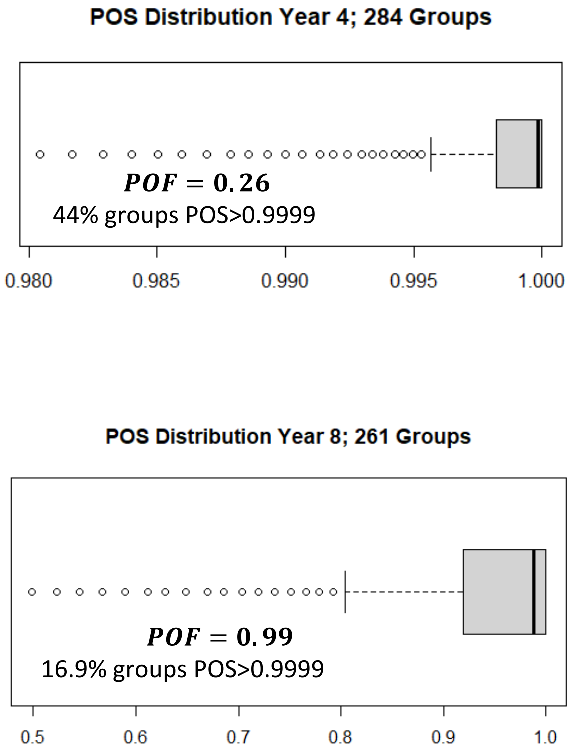


FIGURE 5: GROUP POS AND COMPONENT POF AFTER 1, 2, 4, AND 8 YEARS

Year	1	2	4	8
POF	0.000055	0.004	0.26	0.99

TABLE 3: POF BY YEAR

The POS distribution is displayed for year 8 in Figure 6, with red indicating POS≤0.9999 and blue POS>0.9999.

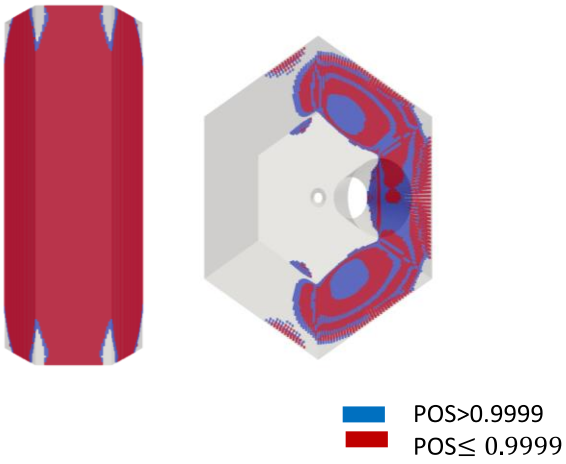


FIGURE 6*:* FINAL GROUP POS DISTRIBUTION

5. CONCLUSION

ASME BPVC nuclear graphite component design Codes provide semi-probabilistic reliability-based design approaches to determining whether a component is suitable to be used in a nuclear reactor. The assessments use common reliability-based design principles for weakest link systems, and modify Weibull's weakest link theory to apply to graphite material. Weibull distributions are chosen as the material reliability curves to estimate the ultimate tensile strength distribution. The failure mode considered is tensile fracture, or cracking. Choi et al. [2] categorize fracture as an ultimate limit state, which should be allowed to have only a very low probability of failure. We see this implemented for components classified as critical to safety (structural reliability class 1), as only allowed to have a $POF < 10^{-4}$. Choi et al. [2] also define ultimate limit-states as "*related to a structural collapse of part or all of the structure". It is conservative to treat all component tensile fractures as ultimate limit-states. It should be noted that some components, even components critical to safety, may not result in structural collapse from tensile fracture. The reflector brick used in the application of this paper is one example of this. The reflector brick would have failed the assessment after just one year if considered to be of structural reliability class 1.

The assessments do not account for changes in mechanical strength as a function of oxidation/irradiation (yet). The assessments account for uncertainty in the material tensile strength (aleatoric uncertainty), but not uncertainty in the loading conditions (epistemic uncertainty), hence making them semi-probabilistic methods. Two design by analysis options: the full and simplified assessments are options for determining design targets. Both require experimental data and FE stresses as inputs. Margin is built in by using ultimate tensile strengths, accounting for sampling uncertainty in the graphite's tensile strength distribution, and only allowing lower POFs on the material reliability curve based on the component's function in supporting the core.

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