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W. K. Terry
A. M. Ougouag

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Idaho National Engineering and Environmental Laboratory
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Hans D. Gougar
William K. Terry
Abderrafi M. Ougouag

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Idaho National Engineering and Environmental Laboratory
Idaho Falls, Idaho 83415

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ABSTRACT

The PEBBED technique provides a foundation for equilibrium fuel-cycle analysis and optimization in pebble-bed cores in which the fuel elements are continuously flowing and, if desired, recirculating. In addition to the modern analysis techniques used in, or being developed for, the code, PEBBED incorporates a novel nuclide-mixing algorithm that allows for sophisticated recirculation patterns using a matrix generated from basic core parameters. Derived from a simple partitioning of the pebble flow, the elements of the recirculation matrix are used to compute the spatially averaged density of each nuclide at the entry plane from the nuclide densities of pebbles emerging from the discharge conus. The order of the recirculation matrix is a function of the flexibility and sophistication of the fuel handling mechanism. This formulation for coupling pebble flow and neutronics enables core design and fuel cycle optimization to be performed by manipulating a few key core parameters. The formulation is amenable to modern optimization techniques.

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Matrix Formulation of Pebble Circulation in the PEBBED Code

1. INTRODUCTION

The PEBBED[1] code is a new tool for analyzing the asymptotic fuel cycle in recirculating pebble- bed reactors. Equations for neutron flux and nuclide distribution in a pebble-bed core are solved self-consistently by an iterative scheme, and the algorithm is shown to converge quickly to a solution unique to the pebble flow pattern. The neutronics solver may use a standard finite difference technique, but more advanced solution methods are suitable. The burnup solver uses a semianalytical method that guarantees convergence with accuracy. A key step in the algorithm is the computation of the entry-plane density of each nuclide of interest in each axial flow channel. These values depend upon the pebble loading and recirculation policy and the burnup accrued by pebbles on successive passes through the core. The current iterate of the flux is used to compute the exit-plane nuclide density in a pebble after one pass through the core in each channel, based on the density of that nuclide in a fresh pebble. Pebbles are then distributed according to the recirculation scheme to generate the entry-plane density in each channel on the next pass. This is repeated until the pebbles exceed the discharge burnup. The exit-plane values are then averaged according to the recirculation scheme in order to produce the actual entry-plane nuclide densities. The entry-plane nuclide flow rate is derived in the next section.

The homogenized entry plane nuclide density of a given nuclide for each flow channel, expressed here as the vector $\vec{N}$, is computed as a weighted average of the contributions from pebbles of various types and trajectories. The symbol ${}^{m}\vec{N}^{p}$ refers to the number density vector (the elements of which correspond to the flow channels) of that nuclide in a pebble of type *p* that has passed through the core *m* times. A *recirculation matrix* **R** stores the weight of each contribution so that

$$\vec{N} = \mathbf{R}\,{}^{m}\vec{N}^{p} \quad . \tag{1}$$

This report shows that the values of the elements of R depend on basic core parameters and, thus, can be computed manually, or generated using a suitable optimization algorithm.

2. THEORY

The flow rate of a given nuclide in pebble flow channel *i* is composed of contributions from pebbles of different types (*p*) and different prior histories. For a core with *P* pebble types each undergoing an average of M_p passes before discharge, the flow rate (atoms per second) of that nuclide at the entry plane of channel *i* is the sum of the flow rates of the nuclide in pebbles of all types and past histories at this location, expressed as

$$\dot{n}_i = \sum_{p=1}^{P}\sum_{m=1}^{M_p} {}^{m}\hat{N}_i^{p} f_i \cdot \alpha_i^{p} \cdot {}^{m}\alpha_i^{p} \tag{2}$$

where

α_i^p = the fraction of channel *i* flow that consists of type *p* pebbles

${}^{m}\alpha_i^p$ = the fraction of type *p* pebbles in channel *i* flow that are on their m^{th} PASS

${}^{m}\hat{N}_i^p$ = the number density of the nuclide of interest within pebbles of type *p*, entering channel *i*, starting their m^{th} pass

f_i = the volumetric flow rate of pebbles through channel *i*.

One can show that the channel-averaged nuclide density at the entry plane of channel i is given by

$$N_i = \sum_{p=1}^{P} \left\{ {}^1\widehat{N}_i^p \cdot {}^1\alpha_i^p \cdot \alpha_i^p + \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^m\widehat{N}_j^p \cdot \left[\frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_j^p \cdot {}^m\alpha_{ij}^p}{\alpha_i} \right] \right\} \tag{3}$$

where

α_i = the fraction of pebble flow that passes through channel i

${}^m\alpha_{ij}^p$ = the fraction of type p pebbles in flow channel j, and on pass m, transferred to channel i following this m^{th} pass

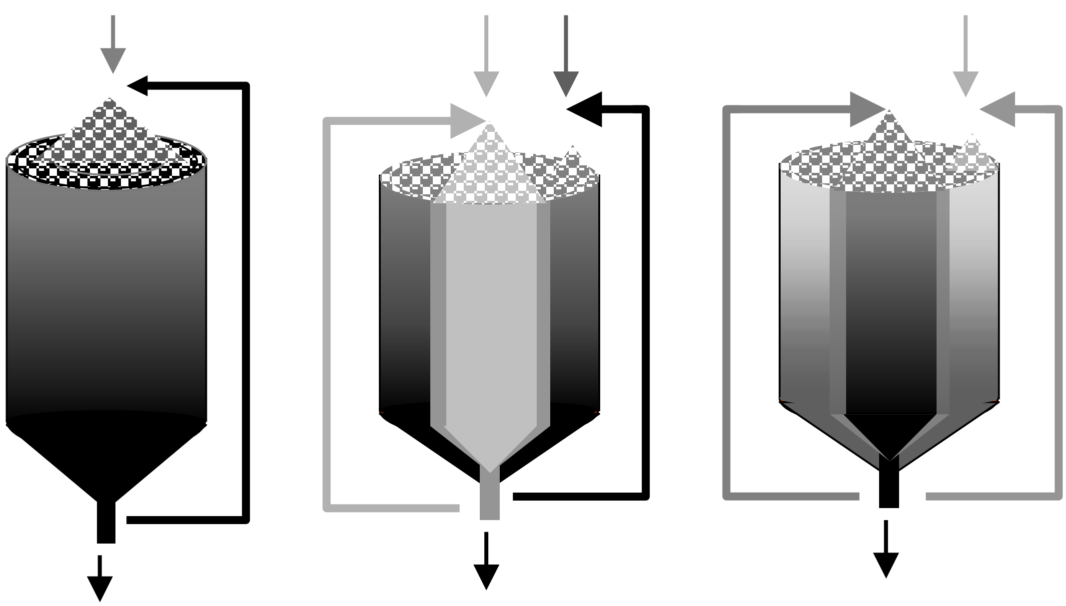
${}^m\widehat{N}_j^p$ = the number density of the nuclide of interest within pebbles of type p, exiting channel j after completing their m^{th} pass

f_j = the flow rate of pebbles through channel j.

Appendix A shows the complete derivation.

The values are shown here to be functions of the flow properties of the core and the fuel loading mechanism. Three models are discussed: the HTR Modul 200,[2] the PBMR,[3] and an alternative PBMR cycle.

The HTR Modul 200 (Figure 1a) possesses a single loading tube and a single discharge tube. The channel coefficients, $\alpha_{I,,}$ are determined by the channel boundaries and the radial flow distribution. There is only one pebble type ($P = 1$), thus $\alpha_j^p = 1$. The pebbles emerging from the bottom are randomly dropped back onto the bed, so the so-called *transfer* coefficient ${}^m\alpha_{ij}^p = \alpha_i$ for all i, j, p, and m. The random recirculation also implies that the burnup classes are equally represented in each zone, i.e.. ${}^m\alpha_j^p = (M_{\max})^{-1}$. The recirculation matrix for a core with J flow channels is then given by



(a) HTR Modul 200 (b) PBMR with Graphite Inner Reflector (c) PBMR with OUT-IN Fuel Cycle

Figure 1. Modular pebble-bed cores with different fuel cycles.

$$\mathbf{R} = \frac{1}{M_{\max}} \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_J \\ \alpha_1 & \alpha_2 & \cdots & \alpha_J \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \cdots & \alpha_J \end{bmatrix} \tag{4}$$

The value of M_{max} is fixed by the core power, geometry, heavy metal content of pebbles, and discharge burnup. Hence, the recirculation matrix elements are also entirely determined by these quantities and the partition of the core flow. The fuel handling mechanism in this design cannot be used to vary the asymptotic core nuclide distribution.

The pebble-bed modular reactor (PBMR) design under consideration by the South African utility Eskom uses two pebble types (graphite and fuel) flowing in separate regions of the core (Figure 1b). The graphite pebbles are dropped onto the bed via a central loading tube. The fuel pebbles are loaded by a number of loading tubes evenly spaced near the core periphery. There is no barrier between the two regions, so there is a zone between them in which the pebble types are mixed. The asymptotic core consists of a central reflector region composed entirely of graphite and a surrounding annulus of fuel pebbles. The size of the graphite region is regulated by the relative flow rates in the central and peripheral loading tubes. Many models of the PBMR core feature five concentric flow zones with roughly equivalent flow rates.[4] The volume of the graphite reflector is about 25% of the pebble bed volume. Thus, the innermost channel is composed of only graphite; the second channel contains roughly equal portions of fuel and graphite pebbles; and the outer three channels consist only of the fueled type. The radial placement and discharge of pebbles is not burnup (pass) dependent, so the burnup classes are equally represented in all channels. Though slightly more complicated than the HTR Modul 200, the partition coefficients are, again, simple functions of core flow properties, power, and discharge burnup. The recirculation matrix, R, can be expressed as two submatrices: one each for fuel, f, and graphite, g:

$$\mathbf{R}^f = \frac{1}{M_{\max}\alpha^f} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & .25\alpha_2 & .5\alpha_3 & .5\alpha_4 & .5\alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{bmatrix} , \quad \mathbf{R}^g = \frac{1}{M_{\max}\left(1-\alpha^f\right)} \begin{bmatrix} 0 & .5\alpha_2 & 0 & 0 & 0 \\ .5\alpha_1 & .25\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & .0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \tag{5}$$

The coefficient α^f refers to the fraction of total core flow consisting of fuel pebbles. This value is a simple function of the relative flow rates through the central and peripheral loading tubes and, thus, offers a degree of freedom in core design not available in the HTR Modul 200. By *tuning* the tube flow rates, one alters the sizes of the central reflector and active core annulus.

The fuel loading mechanism of the PBMR also allows for another type of two-region core. An "OUT-IN" cycle[5] is possible in which fresh fuel pebbles are loaded via the peripheral tubes, but no graphite pebbles are used (Figure 1c). The fuel circulates in the outer region until an intermediate burnup threshold is exceeded. This occurs after a specified number of passes, M_T, after which the pebbles are then loaded via the central tube. At equilibrium, the central region then consists of highly depleted elements, while the annulus is composed of relatively fresh elements. Like the HTR Modul, there is only one pebble type $(\alpha_j^p = 1)$ but the transfer coefficients vary with the pass number, *m*. To conserve pebble flow, only a fraction, α_T, of the pebbles completing pass M_T are diverted to the inner region; the remainder are circulated once more and diverted on the following pass. Defining α_j^o as the fraction of flow in channel j that is in the outer region, one can derive the following expressions for the transfer coefficients (see Appendix A-9):

$$ {}^{m}\alpha_{ij} = \frac{\alpha_i^o \alpha_i}{\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m < M_T \tag{6a} $$

$$ {}^{m}\alpha_{ij} = \alpha_T \alpha_j^o \frac{(1-\alpha_i^o)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} + (1-\alpha_T)\alpha_j^o \frac{\alpha_i^o \alpha_i}{\sum_{\text{all } i} \alpha_i^o \alpha_i} + (1-\alpha_j^o)\frac{(1-\alpha_i^o)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m = M_T \tag{6b} $$

$$ {}^{m}\alpha_{ij} = \frac{(1-\alpha_j^o)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m > M_T \tag{6c} $$

Furthermore, conservation of flow also fixes the values of the fraction of outer flow transferred, α_T, and the transfer pass number, M_T, according to

$$ \alpha_T = 1 + M_T - \frac{F\sum_{all\ j} \alpha_j^o \alpha_j}{{}^{1}F}, \ M_T = INT\left(\frac{F\sum_{all\ j} \alpha_j^o \alpha_j}{{}^{1}F} \right) \tag{7} $$

in which F is the total core pebble flow, and ${}^{1}F$ is the total fresh fuel injection rate.

Obtaining *pass* coefficients, ${}^{m}\alpha_j^p$, is less straightforward. The burnup-dependence of this recirculation scheme means that the burnup classes are not equally represented in each channel. Here, flow conservation is exploited to obtain a system of algebraic equations that represent the flow balance of all the channels. The flow of pebbles commencing their m^{th} pass in channel *i* consists of contributions from pebbles having completed *m-1* passes in all channels. This fact yields *m-1* equations for each channel, *I,* and pebble type *p* of the form

$$ \alpha_i \cdot \alpha_i^p \cdot {}^{m}\alpha_i^p = \sum_{j=1}^{J} \alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_j^p \, {}^{m-1}\alpha_{ij}^p \ . \tag{8} $$

The final equation required to determine the system completely is a direct consequence of flow conservation; i.e., the pass partition coefficients must sum to unity. Since all channels are coupled, the solution to this system involves inverting a matrix of order *Mp*J*, the product of the number of channels and the total number of passes traversed by each pebble type *p*.

The values of α_j are computed in the same manner as the type coefficients in the previous example, They are a function of the size of the inner region and thus can be *tuned* by adjusting the relative flow rates in the loading tubes. Tuning the size of the inner region in this way also changes the value of the intermediate threshold burnup above which the fuel is transferred from the outer to the inner channel. Although computing the coefficients in a burnup-dependent recirculation scheme such as this is rather more complicated than in the other core types, all of the coefficients can be easily computed from the basic core parameters of power, fuel content in the pebbles, discharge burnup, flow velocity, core height and radius, and loading tube flow rates.

3. RESULTS

Figure 2 illustrates the results of PEBBED calculations for the nominal PBMR (with graphite pebbles) and the same core with the OUT-IN fuel cycle described above. No attempt was made to optimize either core for a particular characteristic. Rather, the flow rate of the OUT-IN core was adjusted so that the high-burnup inner region was the same size as the graphite reflector region in the PBMR. This sets the transfer burnup threshold at about 62 MWD/kghm, which is attained during the tenth pass through the core. Each pebble then circulates four times in the inner region.

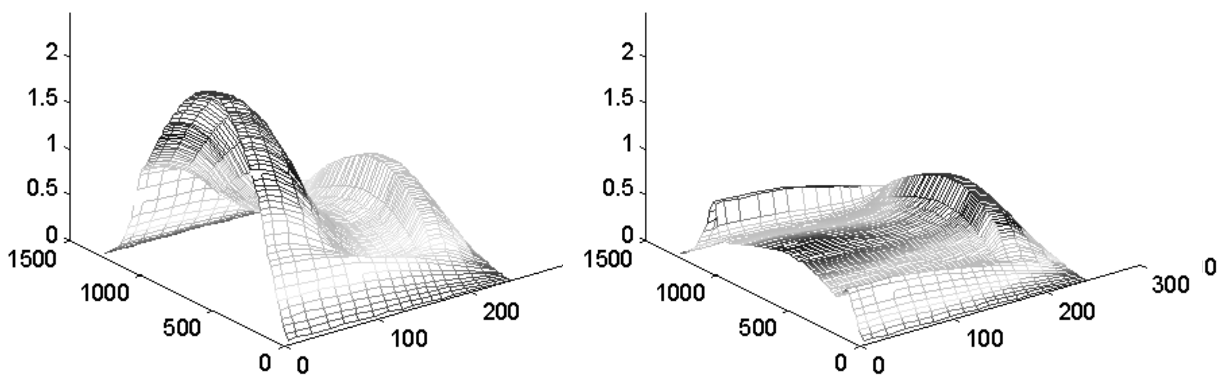


Figure 2. Thermal Flux in Nominal PBMR (left) and PBMR with OUT-IN Cycle (the origin corresponds to the top and center of the pebble bed)

The thermal flux peak in the graphite inner reflector clearly distinguishes the two cases. For the nominal PBMR, the peak fission power density is computed to be 5.57 W/cm^3. For the OUT-IN core, the peak fission power density is computed to be 4.78 W/cm^3. The lower peak power density was achieved at the expense of neutron economy. Compared to the nominal PBMR fuel enrichment of 8%, the OUT-IN core required a fresh pebble enrichment of 10% to maintain criticality.

Although an advanced optimization algorithm has yet to be added to the code, PEBBED has been used for some simple applications. In a study described elsewhere in these proceedings,[6] the code was used to assess some of the proliferation characteristics of a pebble bed reactor. The ability to model and track pebbles of different types and trajectories was also exploited to develop a fuel testing and qualification plan for PBMR fuel at the INEEL.[7] Small quantities of fuel pebbles were restricted to specific flow channels to determine the extreme conditions and operating envelope of the fuel. Figure 3 shows the accumulated burnup and fluence for average pebble and for pebbles restricted to channels 3 and 5 (all fuel).

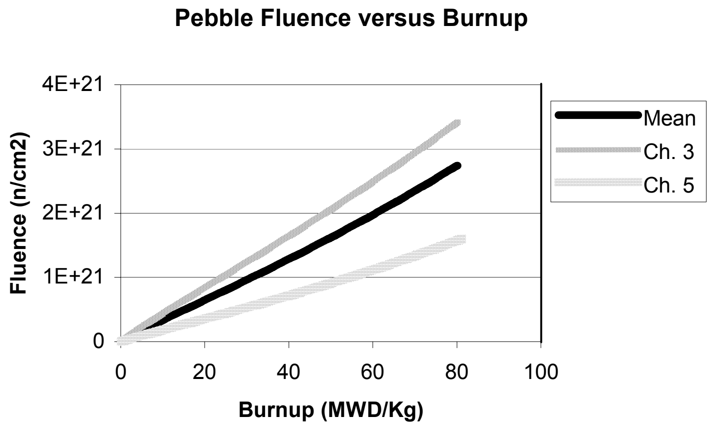


Figure 3. Fast fluence versus burnup for PBMR fuel.

4. CONCLUSIONS AND FURTHER WORK

The matrix formulation for coupling nuclide flow to neutronics in the PEBBED code provides an efficient method for accurately modeling all types of pebble bed cores and for performing advanced core design and fuel management. This work reveals how generalized flow coefficients are derived from basic core parameters so that different pebble types and trajectories can be modeled with ease. With this tool, advanced techniques such as genetic algorithms can be applied to perform rapid, accurate, and comprehensive scoping studies and core optimization.

Currently, the code assumes that pebble flow is strictly axial. For an accurate model of the discharge conus region, the pebble flow grid must be decoupled from the diffusion grid and generalized to two or three dimensions. This is not anticipated to invalidate the matrix approach. Future work will also include development of a three-dimensional nodal diffusion solver, and the matrix formulation will be expanded to allow for azimuthal variation in pebble placement. Advanced cross-section generation and thermal feedback parameterization must also be incorporated to capture spectral and thermal effects. All of these improvements are part of the work scope of PEBBED development.

5. ACKNOWLEDGEMENT

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Appendix A

Derivation of the PEBBEED Recirculation Matrix

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Derivation of the PEBBEED Recirculation Matrix

A-1. OBJECTIVE

The objective is to compute the channel mean entry plane nuclide density from the nuclide densities in pebbles at the exit plane and fresh pebbles. This entails computing a weighted average of the number density values for the nuclide for each pebble type, pass, and channel. The weights assigned to each nuclide contribution are the elements of the 4-dimensional recirculation matrix, $\Re$.

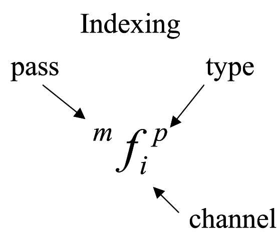
A-2. DEFINITIONS AND SYMBOLS

General

- A curved "hat" ($\frown$) over an *N* refers to a pebble nuclide density
- No hat refers to a channel average nuclide density
- A strike through *N* refers to an exit plane value (leaving the core)
- No strike refers to an entry plane value (entering the core).

Specific

$\dot{n}_i$	=	nuclide flow rate (atoms/sec) into channel *I*
N_i	=	mean entry plane density in *i*th channel
${}^m\hat{\not{N}}_j^p$	=	exit plane nuclide density in pebble of type type *p*, on pass *m*, in channel *j*
${}^mN_j^p$	=	channel—averaged entry plane nuclide density in pebble of type type *p*, on pass *m*, in channel *j*
N_j^p	=	channel—averaged entry plane nuclide density in pebble of type type *p*, in channel *j*, from all passes
$M_{\max}$	=	maximum number of passes per pebble (core mean)
P	=	total number of pebble types
J	=	total number of channels
F	=	total core pebble flow rate (cc/sec)
f_i	=	flow rate in channel *i* (cc/sec)
f_i^p	=	flow rate of type *p* in channel *i*
${}^mf_i$	=	flow rate of pebbles on pass *m* on channel *i*
${}^mf_i^p$	=	flow rate of pebbles of type *p,* on pass *m* in channel *i.*



If a quantity is missing one or more indices, then it is assumed that the quantity indicates the sum over all components specified by the missing index; e.g., ${}^{m}f_i$ refers to the flow rate into channel i of mth pass pebbles of *all* pebble types.

A-3. FLOW AND NUMBER DENSITY

Subdivide the flow of atoms into a channel as follows:

${}^{m}\dot{n}_i^p$ = atoms/sec of a nuclide flowing into channel *I* that are constituents of pebbles of type *p* and are starting their *m*th pass.

This is a *channel* flow rate. It is related to the nuclide density in the *pebbles* (smeared over the effective pebble volume). To relate the channel flow rate to the atom density of the nuclide within a pebble, one must account for the fact that only a fraction of the flow in *i* may be of type *p*, and only a fraction of the flow is undergoing its *m*th pass. Define the partition coefficents:

$\alpha_i = \dfrac{f_i}{F}$ (fraction of core flow in that is in channel *I*)

$\alpha_i^p = \dfrac{f_i^p}{f_i}$ (fraction of flow in *i* that us in type *p* pebbles)

${}^{m}\alpha_i^p = \dfrac{{}^{m}f_i^p}{f_i^p}$ (fraction of type *p* flow in *i* that is on pass *m*)

${}^{m}\alpha_{ij}^p = \dfrac{{}^{m}f_{i\leftarrow j}^p}{{}^{m}f_j^P}$ (fraction of flow of pebbles of type *p*, on pass *m*, in *j* that is diverted to *i*)

The flow rate of a nuclide into channel *i* due to pebbles of type *p* starting their *m*th pass is related to the nuclide density in the pebble by:

$$
\begin{aligned}
{}^{m}\dot{n}_i^p &= {}^{m}\widehat{N}_i^p \cdot f_i \cdot \frac{f_i^p}{f_i} \cdot \frac{{}^{m}f_i^p}{f_i^p} \qquad \text{atoms}/_{s} = \left(\text{atoms}/_{cc}\right)\left(cc/_{s}\right) \\
&= {}^{m}\widehat{N}_i^p \cdot f_i \cdot \alpha_i^p \cdot {}^{m}\alpha_i^p
\end{aligned}
\tag{1}
$$

To obtain the total flow of the nuclide into channel *i*, sum over M passes and types:

channel average value

$$
\sum_{p=1}^{P}\sum_{m=1}^{M} {}^{m}\dot{n}_i^p = \dot{n}_i = N_i f_i \tag{2}
$$

Substitute Equation (1) into (2) to obtain an expression for the total flow rate of the nuclide in terms of the contributions from all pebbles and passes.

$$\dot{n}_i = \sum_{p=1}^{P}\sum_{m=1}^{M} {}^{m}\dot{n}_i^{p} = \sum_{p=1}^{P}\sum_{m=1}^{M} {}^{m}\widehat{N}_i^{p} f_i \cdot \alpha_i^{p} \cdot {}^{m}\alpha_i^{p} \tag{3}$$

A-4. RECIRCULATION

The flow of a nuclide into channel i is composed of contributions from the channel exit plane flows and the fresh pebble injection flow rate, ${}^{1}\dot{n}_i^{p} = {}^{1}\widehat{N}_i^{p}\,{}^{1}f_i^{p}$:

$${}^{m+1}\dot{n}_i^{p} = \sum_{j=1}^{J} {}^{m}\dot{n}_j^{p} \cdot {}^{m}\alpha_{ij}^{p} \qquad (m = 1 \ldots M_{\max} - 1) \tag{4}$$

$$= \sum_{j=1}^{J} \left({}^{m}\widehat{N}_j^{p} \cdot f_j \cdot \alpha_j^{p} \cdot {}^{m}\alpha_j^{p} \right) {}^{m}\alpha_{ij}^{p} \tag{5}$$

Now sum this over m, add the fresh flow contribution, and sum over p to get the total flow of the nuclide into the channel:

$$\sum_{p=1}^{P}\sum_{m=1}^{M_{\max}} {}^{m}\dot{n}_i^{p} = \sum_{p=1}^{P} \left\{ {}^{1}\dot{n}_i^{p} + \sum_{j=1}^{J}\sum_{m=1}^{M_{\max}-1} {}^{m}\widehat{N}_j^{p} \cdot f_j \cdot \alpha_j^{p} \cdot {}^{m}\alpha_j^{p} \cdot {}^{m}\alpha_{ij}^{p} \right\} \tag{6}$$

From Equation (2), we know that:

$$\sum_{p=1}^{P}\sum_{m=1}^{M_{\max}} {}^{m}\dot{n}_i^{p} = N_i f_i$$

so, equate to the RHS of Equation (6) to get:

$$N_i \cdot f_i = \sum_{p=1}^{P} \left\{ {}^{1}\widehat{N}_i^{p}\,{}^{1}f_i^{p} + \sum_{j=1}^{J}\sum_{m=1}^{M_{\max}-1} {}^{m}\widehat{N}_j^{p} \cdot f_j \cdot \alpha_j^{p} \cdot {}^{m}\alpha_j^{p} \cdot {}^{m}\alpha_{ij}^{p} \right\} \tag{7}$$

and thus

$$N_i = \frac{1}{f_i} \sum_{p=1}^{P} \left\{ {}^{1}\widehat{N}_i^{p}\,{}^{1}f_i^{p} + \sum_{j=1}^{J}\sum_{m=1}^{M_{\max}-1} {}^{m}\widehat{N}_j^{p} \cdot f_j \cdot \alpha_j^{p} \cdot {}^{m}\alpha_j^{p} \cdot {}^{m}\alpha_{ij}^{p} \right\}. \tag{8}$$

The denominator can be brought inside the sums, and, noting that:

$${}^{1}f_i^{p} = {}^{1}\alpha_i^{p} \cdot \alpha_i^{p} \cdot f_i \tag{9}$$

then

$$N_i = \sum_{p=1}^{P} \left\{ \frac{{}^{1}\widehat{N}_i^{p} f_i\,{}^{1}\alpha_i^{p} \alpha_i^{p}}{f_i} + \sum_{j=1}^{J}\sum_{m=1}^{M_{\max}-1} {}^{m}\widehat{N}_j^{p} \cdot \frac{f_j}{f_i} \cdot \alpha_j^{p} \cdot {}^{m}\alpha_j^{p} \cdot {}^{m}\alpha_{ij}^{p} \right\} \tag{10}$$

Finally, given:

$$\frac{f_j}{f_i} = \frac{\frac{f_j}{F}}{\frac{f_i}{F}} = \frac{\alpha_j}{\alpha_i} \tag{11}$$

then

$$N_i = \sum_{p=1}^{P} \left\{ {}^1\widehat{N}_i^{p} \cdot {}^1\alpha_i^p \cdot \alpha_i^p + \sum_{j=1}^{J} \sum_{m=1}^{M_{\max}-1} {}^m\widehat{N}_j^p \cdot \frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_j^p \cdot {}^m\alpha_{ij}^p}{\alpha_i} \right\}. \tag{12}$$

This is the expression relating the nuclide density in *pebbles* from channels *j*, passes *m*, and types *p*, to the overall *channel* nuclide density in channel *i*. The inner bracketed term is thus the decomposed recirculation matrix element, ${}^m r_{ij}^p$:

$${}^m r_{ij}^p = \frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_j^p \cdot {}^m\alpha_{ij}^p}{\alpha_i} \tag{13}$$

This matrix element weights the contributions from the exit plane nuclide densities on channel, pebble type, and pass (burnup) basis to yield an overall entry plane channel nuclide density.

The partition coefficients, α_j, α_j^p , ${}^m\alpha_j^p$ and ${}^m\alpha_{ij}^p$, are functions of the core geometry and/or the pebble loading and recirculation policy. The fraction of total core flow in channel *j* (α_j) is a function of the total core dimensions and the fuel loading tube location. It can be considered fixed for a given core design. The pebble type fraction per channel (α_j^p) and the redistribution coefficient (${}^m\alpha_{ij}^p$) are both functions of the pebble loading mechanism. These may be considered to have 'user-specified' values in that they can be altered either in the core design process or, if the design allows, in the loading process itself. The fraction of pebbles of type *p* on pass *m* (${}^m\alpha_j^p$) is now shown to be a function of the other coefficients.

Except for $m = 1$ (fresh pebbles), the flow rate of pebbles of type *p* starting their m^{th} pass in channel *i* is given by (13):

$${}^m f_i^p = F \cdot \frac{f_i}{F} \cdot \frac{f_i^p}{f_i} \cdot \frac{{}^m f_i^p}{f_i^p} \tag{13a}$$

$$= F \cdot \alpha_i \cdot \alpha_i^p \cdot {}^m\alpha_i^p \tag{13b}$$

Here, *f* denotes the pebble flow rate (pebbles/sec) rather than the volumetric flow rate. Likewise, *F* denotes the total core pebble flow rate. The two rates are directly proportional if one assumes that the effective pebble volume (pebble plus surrounding coolant space) is constant.

Equation (14) reflects how this flow rate is evaluated from that of pebbles completing the *m-1*th pass:

$${}^m f_i^p = \sum_{j=1}^{J} F \cdot \alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_j^p \, {}^{m-1}\alpha_{ij}^p \tag{14}$$

Equating (13) and (14) and eliminating the total core flow rate *F* yields:

$$\alpha_i \cdot \alpha_i^p \cdot {}^m\alpha_i^p = \sum_{j=1}^{J} \alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_j^p \, {}^{m-1}\alpha_{ij}^p \tag{15}$$

Solve for ${}^m\alpha_j^p$ *:*

$${}^m\alpha_i^p = \frac{\sum_{j=1}^{J} \alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_j^p \, {}^{m-1}\alpha_{ij}^p}{\alpha_i \cdot \alpha_i^p} \tag{16}$$

This indicates that each pass-type partition coefficient ${}^m\alpha_j^p$ is a function of ${}^{m-1}\alpha_j^p$ *(j*=1..*J)*. Equation (16) is valid for $m = 2 \ldots M_{max}$. To obtain a fully determined set of linear equations, one more expression involving these coefficients is needed. This is obtained from the fact that, by definition, the sum of ${}^m\alpha_j^p$ over all passes *m* is unity.

$$\sum_{m=1}^{M_{max}} {}^m\alpha_j^p = 1 \quad \text{for all } j,p \tag{17}$$

The system of equations is more obvious if one substitutes the following into (16). Let:

$${}^{m-1}K_{ij}^p = \frac{\alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_{ij}^p}{\alpha_i \cdot \alpha_i^p} \tag{18}$$

so that

$${}^m\alpha_i^p = \sum_{j=1}^{J} {}^{m-1}K_{ij}^p \cdot {}^{m-1}\alpha_j^p \tag{19}$$

and Equation (16) is formulated as a linear equation with constant coefficients:

$${}^m\alpha_i^p - {}^{m-1}K_{i1}^p \cdot {}^{m-1}\alpha_1^p - {}^{m-1}K_{i2}^p \cdot {}^{m-1}\alpha_2^p \ldots - {}^{m-1}K_{iJ}^p \cdot {}^{m-1}\alpha_J^p = 0 \quad . \tag{20}$$

Combining Equation (17) with (20) yields the following system of linear equations of order J•M$_{max}$.

$$
\left[\begin{array}{ccccc|cccccc|c|cccccc}
1&1&1&\cdots&1 & 0&\cdots&&&&0 & \cdots & 0&\cdots&&&&0\\
-{}^1K^p_{1,1}&1&0&\cdots&0 & -{}^1K^p_{1,2}&&&&& & & -{}^1K^p_{1,J}&&&&&0\\
0&-{}^2K^p_{1,1}&1&0\cdots&0 & 0&\ddots&&&& & & \vdots&\ddots&&&&0\\
\vdots&&\cdots&\ddots&\vdots & \vdots&&&&& & & &&&&&0\\
0&&\cdots&-{}^{M_{max}-1}K^p_{1,1}&1 & 0&\cdots&0&-{}^{M_{max}-1}K^p_{1,2}&&0 & \cdots & 0&\cdots&0&-{}^{M_{max}-1}K^p_{1,J}&&0\\
\hline
0&0&\cdots&&0 & 1&1&1&\dots&&1 & \cdots & 0&\cdots&&&&0\\
-{}^1K^p_{2,1}&0&\cdots&&0 & -{}^1K^p_{2,2}&1&0&\dots&&0 & & -{}^1K^p_{2,J}&&&&&0\\
0&-{}^1K^p_{2,1}&\cdots&&0 & 0&&1&&&\vdots & & 0&\ddots&&&&0\\
\vdots&&\ddots&& & \vdots&&&\cdots&&0 & & \vdots&&&&&0\\
0&&\cdots&-{}^{M_{max}-1}K^p_{2,1}&0 & 0&\cdots&0&-{}^{M_{max}-1}K^p_{2,2}&&1 & \cdots & 0&\cdots&0&-{}^{M_{max}-1}K^p_{2,J}&&0\\
\hline
\vdots&&&& & \vdots&&&&& & \vdots & \vdots&&&&&\vdots\\
\hline
0&0&\cdots&& & 0&&\cdots&&&0 & \cdots & 1&1&&\cdots&&1\\
-{}^1K^p_{J,1}&&&& & -{}^1K^p_{J,2}&&&&&\vdots & & -{}^1K^p_{J,J}&1&&&&\\
0&\ddots&&& & 0&\ddots&&&& & & 0&&1&&&\\
&&&& & \vdots&&&&& & & \vdots&&&\ddots&&\vdots\\
0&&\cdots&-{}^{M_{max}-1}K^p_{J,1}&0 & 0&&\cdots&-{}^{M_{max}-1}K^p_{J,2}&&0 & \cdots & 0&\cdots&0&-{}^{M_{max}-1}K^p_{J,J}&&1
\end{array}\right]
\begin{bmatrix}
{}^1\alpha^p_1\\ {}^2\alpha^p_1\\ \vdots\\ \\ {}^{M_{max}}\alpha^p_1\\ {}^1\alpha^p_2\\ {}^2\alpha^p_2\\ \vdots\\ \\ {}^{M_{max}}\alpha^p_2\\ \vdots\\ {}^1\alpha^p_J\\ {}^2\alpha^p_J\\ \vdots\\ \\ {}^{M_{max}}\alpha^p_J
\end{bmatrix}
=
\begin{bmatrix}
1\\0\\0\\ \vdots\\0\\1\\0\\0\\ \vdots\\0\\ \vdots\\1\\0\\0\\ \vdots\\0
\end{bmatrix}
\tag{21}
$$

The coefficients K are unique and known for a specified recirculation scheme. The pass-type partition coefficients ${}^m\alpha^p_i$ can thus be computed *a priori* using a standard matrix inversion algorithm. This system is solved for each pebble type p.

This fully decomposed expression for the recirculation matrix element indicates the sensitivity of entry plane density to various partitioning schemes (i.e., pebble recirculation rules). It also lends itself to sophisticated optimization algorithms. The set of valid partition coefficients,

$$\left(\alpha_i, \alpha_j, \alpha^p_j, {}^m\alpha^p_{ij}\right)$$

would form the sample space over which a genetic algorithm would operate.

At first glance, in-core fuel management using the recirculation matrix appears to be complicated by the fact that the values of the partition coefficients are continuous over the interval [0,1]. This suggests that, unlike cores with stationary fuel elements, an infinite number of fuel loading and recirculation schemes is possible. However, one may argue that the change in the value of and ${}^m r^p_{ij}$ has a practical lower limit related to the size of a single pebble. This issue needs to be explored further, but it may reduce the range of core loadings to a finite set.

Although there an unlimited number of values for the partition coefficients, only sets that satisfy certain flow conservation rules are candidates for valid recirculation matrices. These rules are summarized next, followed by examples of matrices computed for specific reactor types and recirculation schemes.

A-5. RULES FOR RECIRCULATION MATRICES

Regardless of the values chosen for the elements of the recirculation matrix, total recirculated nuclide flow into or out of a channel must sum to the fraction of the core flow in the channel less the fresh injection flow rate. The following relations must hold.

$$\sum_{m=1}^{M_{\max}-1}\sum_{j=1}^{J} {}^m r_{ij}^p = \alpha_i^p\left(1-{}^1\alpha_i^p\right)$$

Flow of type *p* diverted from all channels to *i* must sum to the recirculated flow fraction (22a)

$$\sum_{p=1}^{P}\sum_{m=1}^{M_{\max}-1}\sum_{j=1}^{J} {}^m r_{ij}^p = 1-\sum_{p=1}^{P} {}^1\alpha_i^p$$

Total flow diverted from all channels to *i* must sum to recirculated flow fraction of *i* (22b)

$$\sum_{m=1}^{M_{\max}-1}\sum_{i=1}^{J} {}^m r_{ij}^p \alpha_i = \alpha_j^p \alpha_j \left(1-{}^1\alpha_j^p\right)$$

Flow of *p* diverted from channel *j* to other channels must sum to the recirculated flow fraction of *p* in *j* (22c)

$$\sum_{p=1}^{P}\sum_{m=1}^{M_{\max}-1}\sum_{i=1}^{J} {}^m r_{ij}^p \alpha_i = \alpha_j\left(1-\sum_{p=1}^{P} {}^1\alpha_j^p\right)$$

Total flow diverted from channel *j* to other channels must sum to the recirculated flow fraction of *j* (22d)

A-6. SPECIAL CASES

A fully reconfigurable reactor may divert pebbles to different channels simultaneously on the basis of exit channel, pebble type, and/or pass number (burnup). The fully decomposed recirc matrix element formulation is necessary to accurately compute entry plane nuclide densities from the fresh pebble nuclide densities.

However, currently planned PBR designs do not possess sophisticated recirculation capability and thus can use more simplified forms, such as are described here.

A-6.1. Special Case 1

Burnup (pass) Independent Recirculation $\Rightarrow {}^m\alpha_{ij}^p = \alpha_{ij}^p \Rightarrow {}^m\alpha_j^p = M_{\max}{}^{-1}$

(constant for all m)

Thus, when summing over passes, this coefficient can be taken out of the summation to yield the PEBBED 1.0 algorithm (add up exit plane number densities and divide by M_{max}).

$$N_i = \sum_{p=1}^{P}\left\{ {}^1\widehat{N}_i^p\, {}^1\alpha_i^p \cdot \alpha_i^p + \frac{1}{M_{\max}}\sum_{m=1}^{M_{\max}-1}\sum_{j=1}^{J} {}^m\widehat{N}_j^p \cdot \frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_{ij}^p}{\alpha_i}\right\} \tag{24a}$$

$${}^m r_{ij}^p = \frac{1}{M_{\max}}\left(\frac{\alpha_j \alpha_j^p \alpha_{ij}^p}{\alpha_i}\right) \tag{24b}$$

A-6.2. Special Case 2

Type Independent Recirculation $\Rightarrow \alpha_j^p = \left(\alpha^p\right)^{-1}$, ${}^m\alpha_{ij}^p = {}^m\alpha_{ij}$

(constant for all p)

(α_j is the fraction of the core pebble population consisting of type p pebbles, ${}^m\alpha_{ij}$ is the fraction of all m^{th} pass pebbles exiting j that are diverted to i.)

All pebbles are distributed uniformly, so simply multiply the summed exit plane number densities from type p by the fraction of type p in the core:

$$N_i = \sum_{p=1}^{P} \left\{ {}^1\widehat{N}_i^{p}\, {}^1\alpha_i^p \cdot \alpha_i^p + \frac{1}{\alpha^p} \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^m\widehat{N}_j^p \cdot \frac{\alpha_j \cdot {}^m\alpha_j \cdot {}^m\alpha_{ij}^p}{\alpha_i} \right\} \tag{25a}$$

$$ {}^m r_{ij}^p = \frac{1}{\alpha^p} \left(\frac{\alpha_j\, {}^m\alpha_j\, {}^m\alpha_{ij}}{\alpha_i} \right) \tag{25b}$$

Here, ${}^m\alpha_j$ is the fraction of all pebbles exiting j that are finishing their *m*th pass)

This is especially appropriate for single-pebble-type cores in which pebbles are redistributed on the basis of burnup ($\alpha^P = 1$).

A-6.3. Special Case 3

Pass *and* Type Independent Recirculation $\Rightarrow {}^m\alpha_{ij}^p = \alpha_{ij}$

(constant for all *p,m*)

Especially for single pebble type cores such as the MIT or HTR-Modul. Pebble recirculation is a function only of channel position:

$$N_i = \sum_{p=1}^{P} \left\{ {}^1\widehat{N}_i^{p}\, {}^1\alpha_i^p \cdot \alpha_i^p + \frac{1}{\alpha^p M_{\max}} \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^m\widehat{N}_j^p \cdot \frac{\alpha_j \alpha_{ij}}{\alpha_i} \right\} \tag{26a}$$

$$N_i = \sum_{p=1}^{P} {}^1\widehat{N}_i^{p}\, {}^1\alpha_i^p \cdot \alpha_i^p + \sum_{p=1}^{P} \frac{1}{\alpha^p M_{\max}} \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^m\widehat{N}_j^p \cdot \frac{\alpha_j \alpha_{ij}}{\alpha_i} \tag{26b}$$

$$ {}^m r_{ij}^p = \frac{1}{\alpha^p M_{\max}} \left(\frac{\alpha_j \alpha_{ij}}{\alpha_i} \right) \tag{26c}$$

There is only channel dependence in the recirculation matrix for this case; therefore, one may compute the channel-averaged entry plane nuclide density from the *channel*-averaged exit plane nuclide densities, rather than the exit plane *pebble* nuclide densities. To see this, reorder the sums in (16b) in the following way:

$$N_i = \sum_{p=1}^{P} {}^1\widehat{N}_i^{p}\, {}^1\alpha_i^p \alpha_i^p + \sum_{j=1}^{J} \frac{\alpha_j \alpha_{ij}}{\alpha_i} \left\{ \sum_{p=1}^{P} \frac{1}{\alpha^p M_{\max}} \sum_{m=1}^{M_{\max}-1} {}^m\widehat{N}_j^p \right\} \tag{27}$$

The term in brackets is simply the channel-averaged exit plane nuclide density of nuclides that will be recirculated:

$$N_i = \sum_{p=1}^{P} {}^1\widehat{N}_i^{p}\, {}^1\alpha_i^p \cdot \alpha_i^p + \sum_{j=1}^{J} \frac{\alpha_j \alpha_{ij}}{\alpha_i} N_j \tag{28}$$

<u>This is valid only if one is sorting neither by type nor pass number</u>.

A-6.4. Special Case 4

Fully Random (Drop in the Top) Recirculation $\Rightarrow {}^{m}\alpha_{ij}^{p} = \alpha_i$

If one simply drops pebbles at random into the core, then, in addition to the simplifications allowed in the previous cases, the probability of a pebble being dropped into channel *i* is just the fractional flow in *i*. α_{ij} cancels α_i in (26a) so that:

$$N_i = \sum_{p=1}^{P} \left\{ {}^{1}\hat{N}_i^{p}\, {}^{1}\alpha_i^{p} \cdot \alpha_i^{p} + \frac{1}{\alpha^{p} M_{\max}} \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^{m}\hat{N}_j^{p} \cdot \alpha_j \right\} \tag{29}$$

$${}^{m}r_{ij}^{p} = \frac{\alpha_j}{\alpha^{p} M_{\max}} \tag{30}$$

A-7. EXAMPLES OF RECIRCULATION MATRICES

A-7.1. Single Type, Fully Random Recirculation

("Drop in the Top")

Use Equation (20) with $P = 1$ ($\alpha^{P} = 1$)

$${}^{m}r_{ij}^{p} = \frac{\alpha_j}{M_{\max}} \qquad {}^{m}\Re^{1} = \frac{1}{M_{\max}} \begin{bmatrix} \alpha_1 & \alpha_2 & . & . & \alpha_J \\ \alpha_1 & \alpha_2 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \alpha_1 & . & . & . & \alpha_J \end{bmatrix}^{1}$$

A-7-2. Single Type, Pebble Channeling

Use Equation (26b), but note that: $P \Rightarrow 1 = \alpha^{P} = 1$

$$pass - independent \Rightarrow {}^{m}\alpha_i = \frac{1}{M_{\max}}$$

$$Channeled \Rightarrow {}^{m}\alpha_{ij}^{p} = \delta_{ij}$$

$${}^{m}r_{ij}^{1} = \frac{\alpha_j \delta_{ij}}{\alpha_i M_{\max}} \qquad {}^{m}\Re^{1} = \frac{1}{(M_{\max} - 1)} \begin{bmatrix} 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & 1 \end{bmatrix}^{1}$$

where

δ_{ij} = Kronecker delta

A-8. ESKOM PBMR

(P = 2, inner graphite channel, 1 mixed (50/50) channel, outer 3 all-fuel channels, pass-independent)

$$\alpha_j^f = [0 \quad .5 \quad 1 \quad 1 \quad 1] \qquad \alpha_j^g = [1 \quad .5 \quad 0 \quad 0 \quad 0]$$

The pebbles are distinguished by type (fuel or graphite) but not by pass or channel (Special Case 1). Fuel pebbles are distributed randomly over channels 2–5 while a graphite pebbles are distributed randomly over channels 1–2.

The channel diversion coefficients, α_{ij}^p , can be computed as follows:

$$\alpha_{ij}^p = \frac{\alpha_i^p \alpha_i}{\sum_{n=1}^{J} \alpha_n^p \alpha_n} \qquad \begin{array}{l} \text{<-- flow fraction of type } p \text{ in channel } i \\ \text{<-- total flow fraction of type } p \text{ in core } (\alpha^p) \end{array} \tag{31}$$

The expression for the recirculation matrix elements can thus be simplified to:

$$\begin{aligned} {}^m r_{ij}^p &= \frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_j \cdot {}^m\alpha_{ij}^p}{\alpha_i} = \frac{\alpha_j \cdot \alpha_j^p \cdot {}^m\alpha_{ij}^p}{M_{\max}\alpha_i} \\ &= \frac{\alpha_j \cdot \alpha_j^p \cdot \alpha_i^p}{M_{\max} \sum_{n=1}^{J} \alpha_n^p \cdot \alpha_n} \end{aligned} \tag{32}$$

The denominator of Equation (32) is constant for a given pebble type. For the PBMR fuel, it is:

$$M_{\max} \sum_{n=1}^{J} \alpha_n^f \alpha_n = 0.5\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = M_{\max}\alpha^f \tag{33}$$

thus

$$\Re^f = \frac{1}{M_{\max}\alpha^f} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & .25\alpha_2 & .5\alpha_3 & .5\alpha_4 & .5\alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{bmatrix}^f \tag{34}$$

The denominator of Equation (30) for the PBMR graphite matrix is:

$$M_{\max} \sum_{n=1}^{J} \alpha_n^g \alpha_n = \alpha_1 + 0.5\alpha_2 = M_{\max}\alpha^g \tag{35}$$

and

$$\Re^g = \frac{1}{M_{\max}\alpha^g} \begin{bmatrix} \alpha_1 & .5\alpha_2 & 0 & 0 & 0 \\ .5\alpha_1 & .25\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^g \tag{36}$$

A.9 TWO-ZONE SIMPLE TRANSFER

In this cycle, the core is divided into two zones, each of which contains one or more flow channels. One of the flow channels may be split between the two zones. A single type of fuel pebble is loaded into only one of the zones (the entry zone) and recirculated until an intermediate (transfer) threshold is attained. The pebbles are then transferred into the second (exit) zone and recirculated until the discharge burnup level is exceeded.

The radius of the boundary between the zones is determined by the relative rates of flow into these zones. This also fixes the transfer burnup threshold value, B_T,, and the number of passes each pebble undergoes before being transferred, M_T.

The rate of flow between the zones must equate to the fresh fuel injection rate in order to maintain flow conservation. Because the entry zone flow rate is not likely to be an integer multiple of the fresh fuel injection rate, only a fraction of the pebbles on pass M_T can be transferred. The remainder will be transferred on pass $M_T + 1$.

These parameters and the transfer coefficients are derived as follows. Define α_j^o as the fraction of the flow in flow channel *j* that is in the entry zone.

Also, let α_T be the fraction of the pebbles on pass M_T that are transferred from the entry zone to the exit zone. This number is the same for all all flow channels in the entry zone if there is only one discharge tube.

The flow rate of pebbles being transferred from the entry to the exit is:

$$F^T = F\sum_{j=1}^{J} {}^{M_T}\alpha_j \cdot \alpha_j \cdot \alpha_T \cdot \alpha_j^o + {}^{M_T+1}\alpha_j \cdot \alpha_j \cdot \alpha_j^o \tag{37a}$$

$$= F\sum_{j=1}^{J} \alpha_j \cdot \alpha_j^o \left({}^{M_T}\alpha_j \cdot \alpha_T + {}^{M_T+1}\alpha_j \right) \tag{37b}$$

$$= F\sum_{j=1}^{J} \alpha_j \cdot \alpha_j^o \left({}^{M_T}\alpha_j \cdot \alpha_T + (1-\alpha_T) \cdot {}^{M_T}\alpha_j \right) \tag{37c}$$

$$= F\sum_{j=1}^{J} \alpha_j \cdot \alpha_j^o \cdot {}^{M_T}\alpha_j \tag{37d}$$

In fact, this is just equal to the flow rate of pebbles on any pass *m*, including *m* = 1 (the fresh fuel injection rate), because pebbles are introduced to the core only when they are fresh and discharged only after pass M_{max}.

The total flow rate in the entry zone can be computed from

$$F^o = M_T \sum_{j=1}^{J} {}^{M_T}\alpha_j \cdot \alpha_j \cdot \alpha_j^o + \sum_{j=1}^{J} {}^{M_T+1}\alpha_j \cdot \alpha_j \cdot \alpha_j^o \tag{38}$$

This simplifies to

$$F^o = \left(M_T + 1 - \alpha^T \right) \sum_{j=1}^{J} {}^{M_T}\alpha_j \cdot \alpha_j \cdot \alpha_j^o \tag{39a}$$

$$= \left(M_T + 1 - \alpha^T\right) \sum_{j=1}^{J} {}^1\alpha_j \cdot \alpha_j \cdot \alpha_j^o \qquad (39b)$$

$$= \left(M_T + 1 - \alpha^T\right) {}^1F \qquad (39c)$$

${}^1F$ is the fresh fuel injection rate.

Equation (39c) can be solved for α^T to yield:

$$\alpha^T = M_T + 1 - \frac{F^o}{{}^1F} \qquad (40)$$

By the definition of a partition coefficient, 0 £ α_T £ 1, so that the transfer pass number M_T is fixed as the integer part of the quotient of the entry zone flow and fresh fuel flow,

$$M_T = \mathrm{int}\left(\frac{F^o}{{}^1F}\right) \qquad (41)$$

Thus, the transfer pass number and fraction of flow transferred in a two-zone simple transfer scheme are easily computed from the core flow distribution and fresh fuel injection rate.

These parameters are now used to compute the actual transfer coefficients for this fuel cycle.

For pretransfer flow, the pebbles in channel j are distributed according to the partition of the entry zone among the flow channels I:

$${}^m\alpha_{ij} = \frac{\alpha_i^o \alpha_i}{\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m < M_T \qquad (42)$$

For each channel j completing transfer pass M_T, there are three flow paths: an exit zone pebble will be transferred to another exit zone channel, an entry zone pebble will stay in an entry zone channel, or an entry zone pebble will be transferred to an exit zone channel. The transfer coefficient is the sum of the probabilities of these outcomes:

$$\begin{aligned} {}^m\alpha_{ij} = \left(1-\alpha_j^o\right)\frac{\left(1-\alpha_i^o\right)\alpha_i}{1-\sum_{\text{all } i}\alpha_i^o\alpha_i} + \left(1-\alpha_T\right)\alpha_j^o\frac{\alpha_i^o\alpha_i}{\sum_{\text{all } i}\alpha_i^o\alpha_i} + \ldots \\ \ldots + \alpha_T\alpha_j^o\frac{\left(1-\alpha_i^o\right)\alpha_i}{1-\sum_{\text{all } i}\alpha_i^o\alpha_i} \qquad m = M_T \end{aligned} \qquad (43)$$

Finally, for posttransfer flow, all of the pebbles are equivalently distributed among the exit zone flow channels,

$${}^m\alpha_{ij} = \frac{\left(1-\alpha_j^o\right)\alpha_i}{1-\sum_{\text{all } i}\alpha_i^o\alpha_i} \qquad m > M_T \qquad (44)$$

This expression can be obtained from Equation (43) by setting $\alpha_T = 1$.

The coefficients α_j^o are computed from the relative rate of flow of pebbles in the two zones, the location of the zone boundaries, and the flow profile in the Core, as observed by Bedenig et al. The results shown above indicate that a complete description of the recirculation scheme can be determined from a few simple core parameters.

A-10. FRESH NUCLIDE INJECTION RATE

The fresh nuclide injection rate can be expressed in terms of core power (P in MW) and discharge burnup (B_d in MWD/kg). First, express the mass flow rate (g/sec) of a heavy metal nuclide into the core per pebble per channel as ${}^1\dot{m}_i^p$ such that the total mass flow of the nuclide into the core is given by:

$$ {}_{hm}\dot{m} = \sum_{i=1}^{J}\sum_{p=1}^{P} \dot{m}_i^p = \sum_{i=1}^{J} \dot{m}_i \qquad (45)$$

where

${}^1\dot{m}_i$ = is the mass flow rate of the nuclide into channel i due to all pebble types.

${}^1m_i^p$ = may be converted to nuclide flow using the nuclide's molecular weight:

$$ \begin{aligned} {}^1\dot{m}_i^p &= {}^1\dot{n}_i^p * mol.wt. * 1.6604 * 10^{-24}\, g/amu \\ &= {}^1N_i^p\, {}^1f_i^p \cdot mol.wt. * 1.6604 * 10^{-24}\, g/amu \end{aligned} \qquad (46)$$

where

${}^1N_i^p$ = Fresh pebble nuclide density

${}^1f_i^p$ = Fresh nuclide volumetric injection rate due to type p pebbles

Analogous to nuclide flow, ${}^1\dot{m}_i^p$ may be expressed in terms of the core-wide mass injection rate and partition coefficients:

$$ {}^1\dot{m}_i^p = {}_{hm}\dot{m} \cdot \frac{{}^1\dot{m}_i}{{}_{hm}\dot{m}} \cdot \frac{{}^1\dot{m}_i^p}{{}^1\dot{m}_i} \qquad (47)$$

The nuclide density and conversion factors cancel so that:

$$ \begin{aligned} {}^1\dot{m}_i^p &= {}_{hm}\dot{m} \cdot \frac{{}^1f_i}{{}^1f} \cdot \frac{{}^1f_i^p}{{}^1f_i} \\ &= {}_{hm}\dot{m} \cdot {}^1\alpha_i \cdot {}^1\alpha_i^p \end{aligned} \qquad (48)$$

with the partition coefficients as previously defined. In terms of nuclide flow rate, the same partitioning applies:

$$ {}^1\dot{n}_i^p = \frac{{}_{hm}\dot{m}}{\left(mol.wt. \cdot 1.6604 \cdot 10^{-24}\right)} \cdot {}^1\alpha_i \cdot {}^1\alpha_i^p \qquad (49)$$

Izenson[51] relates core-wide heavy metal injection rate to the power and discharge burnup.

$$ {}_{hm}\dot{m} = \frac{P \text{ (MWD/day)}}{B_d \text{ (MWD/g) (86400sec/day)}} \qquad (50)$$

Let ${}^1_h n$ be the atom percent of nuclide h in fresh fuel (core-wide average) that contains H heavy metal isotopes, then

$$ {}_{H}^{1}\dot{n} = \sum_{h=1}^{H} {}_{h}^{1}\dot{n} $$

← new index!

and the hth fresh nuclide flow rate within type p pebbles into channel i thus is:

$$ {}_{h}^{1}\dot{n}_{i}^{p} = {}_{h}^{1}\widehat{N}_{i}^{p} \cdot {}_{h}^{1}f_{i}^{p} = \frac{P_{h}\alpha}{{}_{h}MB_{d}C} \cdot {}^{1}\alpha_{i} \cdot {}^{1}\alpha_{i}^{p} \tag{51} $$

where

${}_{h}M$ = molecular wt. of h

$$ C = \frac{86400(\sec/ day)}{1.6604 \cdot 10^{-24} (g / amu)} $$