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Prandtl Number Dependent Natural Convection With Internal Heat Sources

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Abstract – *Natural convection plays an important role in determining the thermal load from debris accumulated in the reactor vessel lower head during a severe accident. Recently, attention is being paid to the feasibility of external vessel flooding as a severe accident management strategy and to the phenomena affecting the success path for retaining the molten core material inside the vessel. The heat transfer inside the molten core material can be characterized by the strong buoyancy-induced flows resulting from internal heating due to decay of fission products. The thermo-fluid dynamic characteristics of such flow depend strongly on the thermal boundary conditions. The spatial and temporal variation of heat flux on the pool wall boundaries and the pool superheat are mainly characterized by the natural convection flow inside the molten pool. In general, the natural convection heat transfer phenomena involving the internal heat generation are represented by the modified Rayleigh number (Ra'), which quantifies the internal heat source and hence the strength of the buoyancy force. In this study, tests were conducted in a rectangular section 250 mm high, 500 mm long and 160 mm wide. Twenty-four T-type thermocouples were installed in the test section to measure temperatures. Four T-type thermocouples were used to measure the boundary temperatures. The thermocouples were placed in designated locations after calibration. A direct heating method was adopted in this test to simulate the uniform heat generation. The experiments covered a range of Ra' between* 1.5×10^{6} *and* 7.42×10^{15} *and the Prandtl number (Pr) between 0.7 and 6.5. Tests were conducted with water and air as simulant. The upper and lower boundary conditions were maintained uniform. The results demonstrated feasibility of the direct heating method to simulate uniform volumetric heat generation. Particular attentions were paid to the effect of Pr on natural convection heat transfer within the rectangular pool.*

I. INTRODUCTION

During a severe accident in a nuclear reactor, the core may melt and relocate to the lower plenum of reactor vessel to form a hemispherical pool. Should there be no effective cooling mechanism, the core debris may heat up, and a molten pool may form with natural convection due to internal heat sources. The high temperature of the molten core material may threaten the thermal and structural integrity of the reactor vessel. The extent and urgency of this threat depend primarily upon the intensity of the internal heat sources and the associated distribution of the heat fluxes on the vessel walls in contact with the molten core material. The feasibility of external vessel flooding as a severe accident management strategy, as well as the phenomena affecting the success in retaining the molten core material inside the vessel, has received wide attention.

One of the earliest study of natural convection in volumetrically heated fluid layers is that of Kulacki and Goldstein[1)]. The fluid layer was bounded by two isothermal upper and lower plates held at a constant temperature and four insulated side walls. The modified Rayleigh number (Ra') varied from 200 to 10^7. Data were obtained using interferometers and covered the laminar, transition, and turbulent regimes of natural convection. For Ra'>10^4, turbulent mixing effects begin to play a key role in the overall energy transport process, and any periodicity or near periodicity in the mean temperature fields evident at lower Ra' begins to disappear.

A numerical study of natural convection in internally heated pools contained in rectangular cavity was performed by Jahn and Reineke[2)]. They found that the temperature field suggested the presence of nonuniform eddies in the upper region of the pool with a stable and calm liquid layer in the lower region. They concluded that heat was transferred more effectively in the upper region as opposed to the lower region for this geometry.

Kulacki and Emara[3)] correlated upward and downward heat transfer in rectangular geometries under the constraint of relatively small temperature differences. In their study, a planar fluid layer was investigated, where only the top surface was cooled and the bottom surface was insulated. For these boundary conditions, heat transfer coefficients were obtained for Ra' up to 2×10^{12}. They also attempted to identify an independent effect of the Prandtl number (Pr) ranging from 2.75 to 6.86.

Kulacki and Nagle[4)] investigated natural convection with volumetric heating in a horizontal fluid layer with a rigid, insulated lower boundary and a rigid, isothermal upper boundary for Ra' from 114 to 1.8×10^6 times the critical value of the linear stability theory. A correlation for the mean Nusselt number (Nu) was obtained for steady heat transfer. Data were presented on fluctuating temperatures at high Ra' and on developing temperature distributions when the fluid layer was subjected to step change in power.

Hollands et al.[5)] reported on an experimental study on the natural convective heat transport through a horizontal layer of air, covering Ra' from subcritical to 4×10^6. Packets of a fluid with a high thermal diffusivity leaving the outer edge of the boundary layer will lose heat and take up a temperature equal to the surroundings within a much shorter distance from its starting point than a fluid with a low thermal diffusivity.

Correlation of the experimental data on heat transfer from non-boiling, horizontal fluid layers with internal heat generation was cast into a form suitable for analysis of post-accident heat removal in fast reactors by Baker et al.[6)] Available data on layers with equal boundary temperatures indicated that the downward heat transfer rate could be treated by conduction alone, while the upward heat transfer rate was largely controlled by convection.

A phenomenological model of eddy heat transfer in natural convection with volumetric energy sources at high Ra' was developed by Cheung[7)]. The model was applied to the problem of thermal convection in a horizontal heated fluid layer with an adiabatic lower boundary and an isothermal upper wall. At high Ra', the mean temperature was found to be essentially constant throughout the fluid layer except in a sublayer region near the upper wall. The thickness of such a region was observed to be inversely proportional to the mean Nu. Outside the sublayer region, the distribution of eddy heat flux was linear for all Ra'. Production of thermal variance was negligible in the lower 75-95% of the fluid layer and was greatest near the upper boundary. Comparison of the heat transfer predictions with measurements indicated an excellent agreement in the turbulent thermal convection regime.

Steinberner and Reineke[8)] investigated experimentally and numerically the buoyant convection with internal heat sources in a closed rectangular cavity for 10^7<Ra´<10^{14}. Their investigation of the structure and the dynamic behavior of the turbulent boundary layer at free convection with internal heat sources in rectangular cavities, revealed a number of equal properties in comparison to the turbulent boundary layer at the heated vertical plate.

Nourgaliev et al.[9)] investigated turbulence characteristics of the flow and thermal fields in an internally heated horizontal fluid layer for Ra' up to 5×10^8 using a finite-difference code for direct numerical simulations. Their results indicated nonequilibrium of turbulent kinetic energy and thermal variance. They found that important turbulence constants are remarkably nonuniformly distributed across the fluid layer and strongly dependent upon Ra' and fluid Pr.

Asfia et al.[10)] conducted experiments related to natural convection heat transfer in volumetrically heated spherical pools. Their experiments were performed in pyrex bell-jars, using Freon-113 as the simulant liquid. Microwave heating was used in the experiment, while a surrounding water pool cooled the experimental system. They concluded that a large variation in heat transfer coefficient existed along the vessel wall.

The BALI[11)] experiment was designed to study thermal-hydraulics of core material pool for in-vessel or ex-vessel situations. The core melt was represented by salt water. The pool, which was cooled from the bottom and the top, was heated electrically by the Joule effect with current supplies located on the sides.

Gabor et al.[12] conducted natural convection experiment with hemispherical pool containers. The pool container served both as a heat transfer surface and as an electrode. $ZnSO_4$-H_2O was used as the heat generating liquid.

In the study by Kulacki and Goldstein[1], energy transport was measured in a fluid layer of dilute electrolyte bounded horizontally by two rigid planes of constant and equal temperature. Joule heating by an alternating current passing horizontally through the fluid layer provided a volumetric energy source. Kulacki and Emara[3] and Kulacki and Nagle[4] used the same heating method. Kolb et al.[13] adopted a semicircular slice with a vertical section to study heat transfer at the boundaries of an internally heated molten salt pool with a boundary crust. Thin cable-type heaters, with a sheath diameter of 3 mm and 4 m in length, provided internal heating in the pool. Kymäläinen et al.[14] measured the heat flux distribution from a large volumetrically heated pool for the Loviisa nuclear power plant in Finland. Their experimental approach was based on a two-dimensional slice of the Loviisa lower head, including a portion of the cylindrical vessel wall. This allowed well-controlled uniform heating using the flats as the electrodes. The pool was filled with a conduction $ZnSO_4$-H_2O solution, and the current through each electrode was individually adjusted.

Previous investigations suffer from serious weakness in representing a molten core material behavior in the reactor vessel lower plenum involving specific conditions such as high Ra', turbulent boundary layers, a low height-to-diameter ratio, and a hemispherical geometry. Therefore, any extrapolations to different geometries and convection conditions have to be considered with reservation. Further, whereas in many previous studies only the average heat transfer from liquid to surrounding walls has been correlated, the heat flux profiles and their peak values are also needed for safety assessment concerning the external reactor vessel cooling.

II. EXPERIMENTAL APPARATUS

The test section is a rectangular cavity 500 mm long, 80 mm wide and 250 mm high, as shown in Fig. 1. Twenty thin cable-type heaters, 5 mm in diameter and 500 mm long, are used to simulate internal heating in the pool. They are uniformly distributed in the rectangular pool and supply a maximum of 2 kW power to the pool. A total of twenty-eight T-type thermocouples were installed in the test loop. Four T-type thermocouples were used to measure the boundary temperatures. Eight thermocouples were installed at mid-plane 0, 3, 40, 90, 140, 190, 247, 250 mm, respectively from the bottom plate. The remaining sixteen thermocouples were installed at the symmetry line, which is 150 mm off the centerline. The working fluid in the test section was heated up to reach a steady state. The average temperature increase rate was 0.6℃/min. The rate of temperature rise at any location in the pool did not differ by more than ±10% from the mean value. The results demonstrated the feasibility of the direct heating method to simulate uniform volumetric heat generation within the pool.

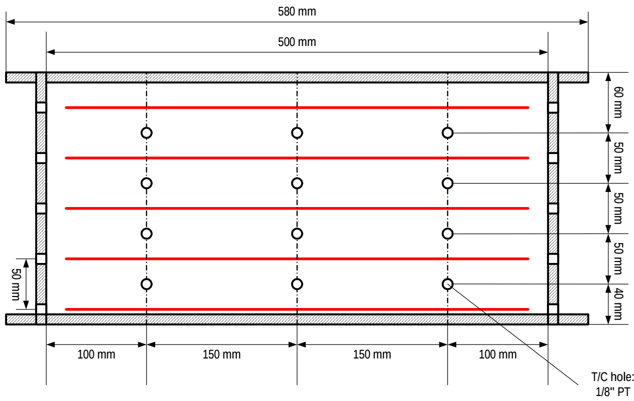

Fig. 1 Schematic diagram of the test section

Figure 2 shows the loop consisting of a demineralized water system, test section, heat exchanger, and a data acquisition system (DAS). The thermocouples were submerged sequentially in a constant temperature pool. The DAS bias error was calibrated so as to minimize the measurement error.

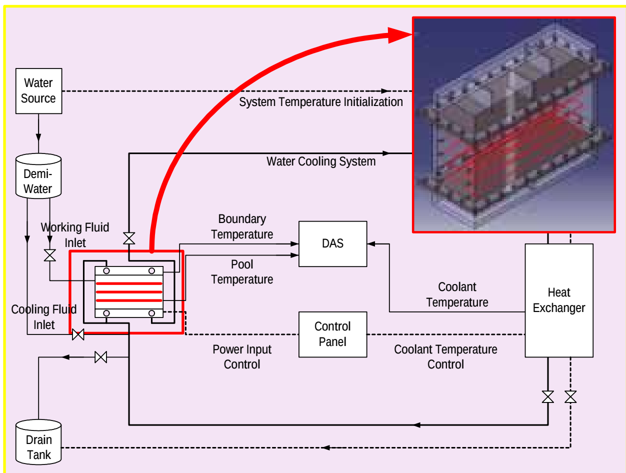

Fig. 2 Schematic diagram of the test loop

Once properly calibrated, thermocouples were placed at their designated locations. The water-cooling system, or heat exchanger, supplies the well-defined upper and lower boundary conditions. Performance test of this water cooling system showed the temperature difference of ±

0.5℃ ranging from 5℃ to 80℃. After following these procedures to check on proper functioning, the pool was allowed to heat up. Ra' can be calculated for the present experiment for varying input power (Q) and characteristic pool depth (L). The experiments covered a range of Ra' between 4.87×10^{7} and 2.32×10^{14} and of Pr between 0.7 and 3.98. Tests were performed with water and air as working fluids.

III. EXPERIMENTAL PROCEDURE AND HEATING METHOD

All the thermocouples were properly calibrated to be placed in their designated locations. The uniform heat generation is one of the most important conditions in the experiment. Thus, water temperatures were measured at different locations. After uniformity of the heat generation rate was checked, the demineralized working fluid was pumped into the test section through the heat exchanger. In this manner, the temperature of the working fluid in the test section and external cooling system reached an initial steady state. After assuring that everything was properly running, the power switch was turned on, and the pool was allowed to heat up. The DAS was adjusted to record the temperatures. Tests were conducted with water and air as listed in Table 1. The upper and lower boundary layers were maintained at a uniform temperature.

Table 1 Test matrix

Boundary Condition	Working Fluid	Test	Power [W]
Isothermal	Water	Case W1	38
		Case W2	295
		Case W3	832
		Case W4	1474
	Air	Case A1	15
		Case A2	40
		Case A3	81
		Case A4	149

IV. RESULTS AND DISCUSSION

The natural convection heat transfer involving internal heat generation is represented by Ra', which quantifies the internal heat source and hence the strength of the buoyancy force. Natural or free convection phenomena can be scaled in terms of the Grashof number, Gr, and the Dammkohler number, Da, as well as Pr, in the presence of volumetric heat sources. The dimensionless numbers are defined as

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2};\ \Pr = \frac{\nu}{\alpha};\ Da = \frac{QL^2}{k\Delta T} \tag{1}$$

Ra' can be used to characterize the heat transfer in natural or free convection problems, including those involving external heat sources or external heating such as heating from below. This dimensionless number is defined as

$$Ra = Gr\Pr = \frac{g\beta\Delta T L^3}{\alpha\nu};\ \alpha = \frac{k}{\rho c_p};\ \nu = \frac{\mu}{\rho} \tag{2}$$

The preceding equation relates the buoyancy and viscous forces, which are linearly related via the factor, Gr/Re, but other dependencies are also present. Ra' is germane to free or natural convection problems with internal heat sources, which is defined as

$$Ra' = RaDa = Gr\Pr Da = \frac{g\beta Q L^5}{\alpha\nu k} \tag{3}$$

Figure 3 illustrates the volumetrically heated fluid layer. There are three distinct regions: the upper boundary layer, turbulent mixing core, and lower boundary layer. In this test, both boundaries are kept at the same temperature. Thus, the fluid layer is stratified only above its mid-plane under pure conduction and the onset of convection is characterized in the upper half layer. The recirculating core region moves both upward and downward as Ra' increases. Also, a thin thermal boundary layer rapidly develops at the upper surface as Ra' increases. Thus, Nu has a strong dependency on Ra'. On the other hand, the lower region of the fluid layer is practically dominated by conduction at all Ra'.

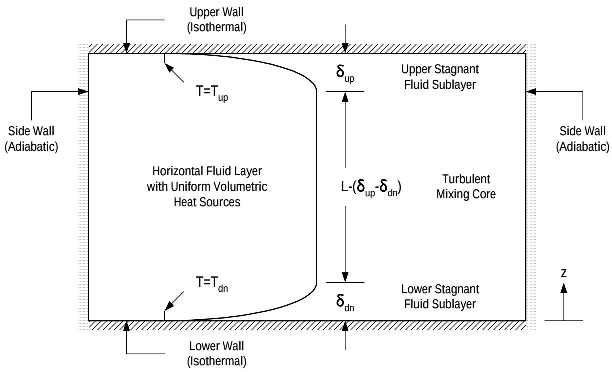

Fig. 3 Schematic of the volumetrically heated horizontal fluid layer

Downward plumes occur randomly and give the flow a locally unsteady character. At the lower surface, conduction is dominated by the downward plumes only occasionally, and convective transport is but a small fraction of the downward heat flux. The weak dependence of the lower surface Nu on Ra' is the result. For low values of Ra', the turbulence intensity is small and the turbulent or eddy viscosity is negligible in comparison with the molecular viscosity. Such flows can be characterized as laminar; and, furthermore, they would be steady if the internal heat source and boundary conditions vary slowly over the time scale of interest.

However, for much higher Ra', the frequency of plume occurrence increases, and the plumes become unstable as well. Cold plumes generated from the upper surface remove heat at the upper region. The flow is characterized as turbulent and unsteady for large values of Ra' at least in domains of vigorous mixing and high turbulent intensity. The molecular viscosity is small in relation to the eddy viscosity. The laminar, transition, and turbulent flows may coexist in the cavity due to the thermal stratification of the molten pool.

Figure 4 demonstrates the dimensionless temperature profiles. As Ra' is increased due to power, the temperature profiles are characterized by a well mixed nearly isothermal core and boundary layers on the upper and lower surfaces.

Heat transfer data in terms of Nu versus Ra' are represented in the familiar form as

$$Nu = C\left(Ra'\right)^m \quad (4)$$

When the Pr effects are taken into account, the modified form is

$$Nu = C\left(Ra'\right)^m (\Pr)^n \quad (5)$$

In this work, Equation (5) was not obtained because of insufficient experimental data. Figure 5 indicates that Nu is low at low Pr relative to some correlations with high Pr. Theofanous et al.[15] concluded that Pr has negligible effect on high Ra natural convection heat transfer in the range of 2.5 to 11. On the other hand, Nourgaliev et al. [9] reported that the Pr effect on Nu at the bottom surface of the enclosures is significant and amplifying with increasing Ra. Thus, the low Pr of the core melt could be an additional reason for excessive heat transfer in the downward region.

Figure 6 presents the Nu and Ra' relation. Experimental results with moderate Pr (~6.5) fluid are in fair agreement with other correlations. However, experimental results with low Pr (~0.7) fluid are considerably lower than predicted upward and downward Nu.

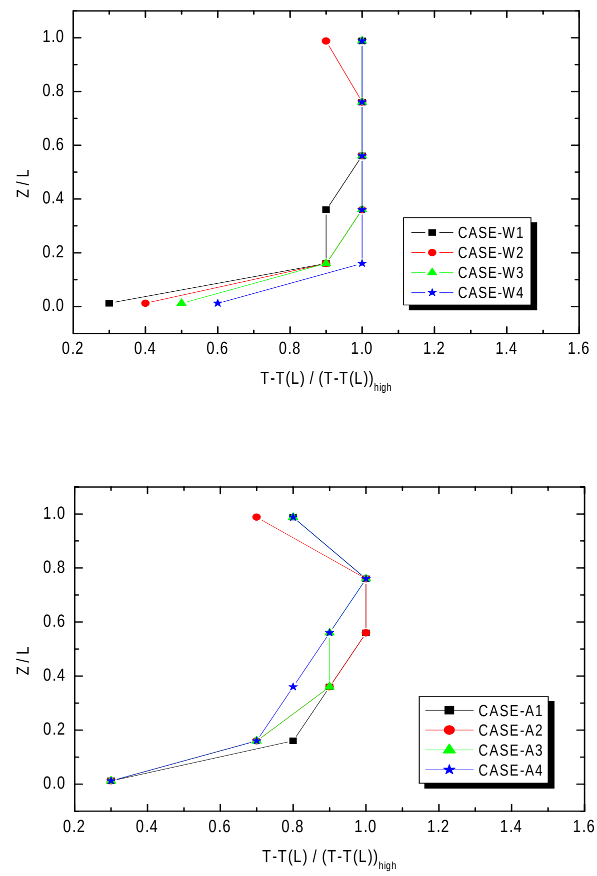

Fig. 4 Horizontal temperature profiles

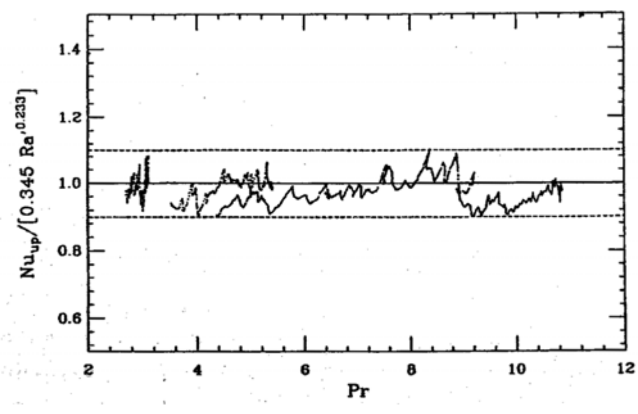

Fig. 5 Mini-ACOPO results showing the Pr effect (Taken from Ref. 15)

V. CONCLUSIONS

Comparison of the present results against the other correlations reported in the literature revealed that Nu is obviously underestimated. It was shown that dependence of the heat transfer characteristics on Pr is not minor in the rectangular pool. Although the low and high Pr data do not overlap in the current experimental results, the same range of Pr data will be obtained by changing the test conditions. Relatively limited literature exists for the Pr effect. More experimental and analytical efforts are needed to quantify the Pr effect on Nu.

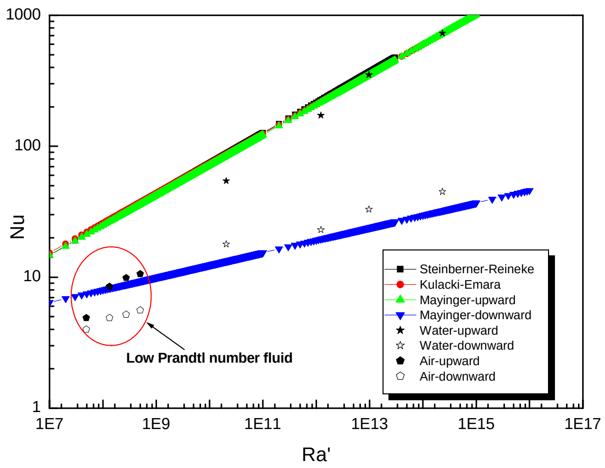

Fig. 6 Natural convection relation in pool

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NOMENCLATURE

C	coefficient in Equations (4) and (5)	
C_p	specific heat	[J/kg·K]
Da	Dammkohler number	
g	gravitational acceleration	[m/s^2]
Gr	Grashof number	
k	thermal conductivity	[W/m·K]
L	pool depth	[m]
Pr	Prandtl number	
Q	heat source	[W/m^3]
Ra	Rayleigh number	
Ra'	modified Rayleigh number	
ΔT	temperature difference	[K]

Greek Letters

α	thermal diffusivity	[m^2/s]
β	thermal expansion coefficient	[K^{-1}]
δ	boundary layer thickness	[m]
μ	dynamic viscosity	[N·s/m^2]
ν	kinematic viscosity	[m^2/s]
ρ	density	[kg/m^3]

Subscript

dn	downward
up	upward

Superscript

m	exponent in Equations (4) and (5)
n	exponent in Equation (5)

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