

# **Geochemical Signatures as a Tool for Vermiculite Provenance Determination**

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Carl D. Palmer

September 2008



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**<http://www.inl.gov>**

**Prepared for the  
U.S. Environmental Protection Agency  
Region 10  
and for the  
U.S. Department of Energy  
Under DOE Idaho Operations Office  
Contract DE-AC07-05ID14517**

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## **ACKNOWLEDGEMENTS**

We would like to thank Mr. John Barich, EPA Region 10, for providing the funding for this work. We would like to thank Mr. Jed Januch of EPA Region 10 for providing us with product samples, and for coordinating the efforts of EPA Region 8 and 4 to provide us with samples. We would also like to express our gratitude to Mr. Gary Taylor of the South Carolina Geological Survey for providing us with samples from the Enoree area. Without his assistance this work would not be possible. Finally we would like to thank Dr. Scott Hughes and Myles Miller from Idaho State University for providing expertise with regard to neutron activation analysis, and Ms Diane Johnson Cornelius, of Washington State University for her expertise with regard to X-ray fluorescence spectroscopy.



## ABSTRACT

Thirty-eight samples of known origin (China, Libby MT, South Africa, South Carolina) and 6 vermiculite product samples of unknown origin were analyzed for major and trace elements, including rare earth elements to determine the feasibility of distinguishing the provenance of the samples based upon a geochemical signature. Probability plots suggest that two of the four groups (Libby, South Carolina) were comprised of two subgroups. Results of hierarchical cluster analysis are highly sensitive to the linkage method chosen. Ward's method is the most useful for this data and suggests that there are five groups within the data set (South African samples, two subsets of the Libby samples, a subset of the South Carolina samples, and a second subset of the South Carolina samples combined with the China samples). Similar results were obtained using k-cluster analysis. Neither clustering method was able to distinguish samples from China from the South Carolina samples. Discriminant analysis was used on a four-category model comprised of the original four groups and on a six-category model comprised of the five categories identified from the cluster analysis but with the China samples grouped into a sixth category. The discriminant/classification model was able to distinguish all of the groups including the China samples from one another for both the four- and six-category models with 100% of the samples properly classified. The 6 unknown product samples were classified with a probability of consistency of 99%. Both discriminant models were also run with a subset of our analyte set to be consistent with the smaller Gunter et al., (2005) analyte set. Twenty vermiculite samples (nine of known origin and eleven of unknown origin) from their study were classified based on our discriminant models with the reduced set of analytes. Of the twenty samples, Gunter et al. (2005) was able to classify 16 with cluster analysis while our 4-category discriminant analysis model allowed us to classify all twenty. Of the 16 samples Gunter et al. (2005) classified using cluster analysis, all but one sample was assigned the same classification by our 4-category model. Of the nine samples with known origin, all were correctly classified. Similar results were obtained for the six-category model. Comparison of the plots of the canonical roots, the Wilks' L, and the square Mahalanobis distances suggest the full analyte set provides better discrimination of the groups than the reduced analyte set. The six-category model is more consistent with the results of the probability plots and the cluster analysis. Discriminant analysis of geochemical data from vermiculite ore is a powerful technique for determining the ore's provenance.

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## ACRONYMS

INL	Idaho National Laboratory
NAA	Neutron Activation Analysis
NIST	National Institute for Standards and Testing
ppm	Parts per million (mg/kg)
REE	Rare Earth Element
SCGS	South Carolina Geological Survey
USEPA	United States Environmental Protection Agency
USGS	United States Geological Survey
XRF	X-ray Fluorescence Spectroscopy



## 1. Introduction

Vermiculite is chemically similar to tri-octahedral smectites and micas, and structurally similar to phlogopite. It can be found as an alteration product of phlogopite or biotite, in a contact region between siliceous intrusives and mafic rocks, or in association with carbonatites (Deer et. al, 1993). When heated rapidly above 900°C, generation of steam results in “exfoliation”-- a process whereby the mineral expands perpendicular to its cleavage planes, resulting in a product with excellent insulating and absorption properties (Potter, 2000).

Vermiculite was used extensively as attic insulation, building materials, packing materials and gardening adjuncts. For example, the United States Environmental Protection Agency (EPA) states that 940,000 homes in the USA may have vermiculite as attic insulation (EPA, 1985). From 1924-1990, more than half of the world’s vermiculite supply came from Libby, Montana (USGS, 2006). Although vermiculite is no longer mined in Libby, it is still mined in the U.S. W.R. Grace & Co. currently mines vermiculite near Enoree, S.C., and Virginia Vermiculite Ltd. mines vermiculite in Louisa County, VA and operates a vermiculite subsidiary near Woodruff, S.C. In addition, substantial amounts of vermiculite are imported from the northern Transvaal region in South Africa, and China’s world supply continues to grow (Potter, 2000).

In 1999, Libby came to national attention when the Seattle Post-Intelligencer (Schneider, 1999) revealed the extent of lung-related illnesses both among workers and within the community, as a result of asbestos contamination of the vermiculite. A 2002 Agency for Toxic Substances and Disease Registry study (ATSDR, 2002) found that asbestosis mortality in the Libby area was 40 times higher than the rest of Montana and 60 times higher than the rest of the United States. Because Libby vermiculite can still be found (primarily as attic insulation), it would be useful to be able to distinguish the source (provenance) of vermiculite in an unknown sample. In addition, if data surfaces demonstrating that a particular mine is contaminated with asbestos, the ability to distinguish the source of an unknown sample would aid in determining relative risk, without the cost and complexity of asbestos sampling.

The objective of the current study is to determine if the provenance of vermiculite samples can be identified from their geochemical composition and to identify the combination of analytes that can best discriminate the provenances from one another.

## 2. Background

A 2001 feasibility study performed by EPA (Frank and Edmund, 2001) suggested several different approaches to the use of geochemical tracers to determine ore provenance. These include the use of accessory minerals, major, minor, and trace elements, and isotopic ratios.

Gunter and Singleton (2003) used whole rock chemical analysis for vermiculite provenance determination and found that Libby samples tend to be high in Cr, V, Ba, and FeO. Williams and others (2003) suggested Libby vermiculite samples might be distinguished from vermiculites from other locations using x-ray diffraction due to differences in relative intensities of 10, 12, and 14 angstrom peaks. In a study where samples from Libby, South Africa, South Carolina, and Virginia were compared to vermiculite products using electron microprobe data, Lowers and Meeker (2004) found that although samples from South Africa were distinct, Libby and South Carolina samples were more difficult to distinguish. This result is important because South Carolina is a major vermiculite supplier and its ore is reported to contain fibrous tremolite (Libby, 1975). Gunter and others (2005) used cluster analysis (average linkage)

on a data set that included 8 major element and 10 trace elements to distinguish ores from Libby, South Carolina, China, and South Africa.

Because rare-earth elements (REEs) commonly reflect the mineralogy and petrogenesis of the ore body, a statistical approach that includes these elements may be diagnostic of the ore's provenance. Boettcher (1967) described the mineralogy of the Rainy Creek Complex of Libby, from which Libby vermiculite was extracted, as containing vermiculite, biotite, apatite, pyroxene and tremolite, with trace amounts of sphene, zircon, carbonate, feldspar and microcline. Vermiculite ore bodies from South Carolina were described by Libby (1975) as containing vermiculite/hydrobiotite, biotite, augite, tremolite, microcline, apatite, tremolite, and talc with minor amounts of sphene/rutile, microperthite, zircon, and monazite. The Palabora complex in South Africa is mined not only for vermiculite, but also contains economic quantities of apatite, magnetite, baddeleyite ( $\text{ZrO}_2$ ), uranothorianite, linnaeite (for cobalt), and trace precious metals (Guilbert and Park, 1986). Mineralogy of the areas where the China samples were collected was not available to us.

In the current study, thirty eight vermiculite rock samples of known origin and six product samples are analyzed for 10 major and 28 trace elements including REEs. Cluster analysis is used to determine groupings of the samples, while discriminant/classification analysis is used to identify linear combinations of elements that can best discriminate the various sources of the samples. The results of these calculations allow us to classify product samples supplied to us by EPA and samples reported by Gunter et al. (2005).

### **3. Methods**

#### **3.1 Sample Collection**

Vermiculite samples were obtained from several sources. The U.S. Geological Survey (USGS) provided 13 samples from the A.L Bush collection including samples from Libby (3), South Carolina (2), South Africa (5), and China (3). Region 4 EPA provided three samples from South Africa and one South Carolina sample processed at Palmetto Vermiculite. Region 8 EPA sent 11 samples obtained from Libby. The South Carolina Geological Survey (SCGS) sent 10 samples from the Enoree, S.C. region. Finally, EPA Region 10 provided 6 vermiculite product samples (Table 1).

All the samples were "grab" samples. Some came from active or closed mining sites (e.g. Libby); others came from exposed outcrop (e.g., some samples from Enoree). No attempts were made to remove accessory minerals or otherwise alter the samples, except to reduce particle size.

#### **3.2 Sample Preparation and Analysis**

Because samples came from different sources and had different analytical requirements, several different preparation steps were employed. Samples provided by SCGS were easily pulverized to <200 mesh following about 2 minutes of grinding in a shatterbox using a zirconia grinding dish. The remaining vermiculite samples consisted of large flakes (and in the case of product, large exfoliated flakes). Methods used to reduce the particle size of these samples depended on the analytical method used. Splits (3g) intended for neutron activation analysis (NAA) were cut by steel bladed scissors to reduce the flakes to < 2 mm across. Splits (10-15g) intended for X-ray fluorescence (XRF) were pulverized by the GeoAnalytical Lab at Washington State University using a tungsten carbide swingmill. Loss on ignition (LOI) was measured on sample splits heated at 750° C. This temperature may not release all structural

water in vermiculite, but was chosen to minimize potential reaction with silica crucibles (Diane Johnson, personal communication).

All samples were analyzed for trace elements using NAA. Samples were irradiated at Oregon State University and counted by the Laboratory for Environmental Geochemistry at Idaho State University using methods described by Wright (1998). With the exception of samples from the USGS A.L. Bush collection, all samples were prepared and analyzed using XRF by the GeoAnalytical Lab at Washington State University using methods described by Johnson et. al (1999). Briefly, this involved mixing powdered samples with lithium tetraborate in a 1:2 ratio and heating in a graphite crucible at 1000° C for 5 minutes. Upon cooling, the samples were crushed and reground in a swingmill and re-fused for 5 minutes. Following cooling, resultant glass beads were polished with 600 grit silicon carbide and cleaned in an ultrasonic cleaner.

Table 1. Sample names, sources and brief description

Sample Name	Source/Description
USGS Samples from the A.L. Bush Collection	
GS2	Libby AB3, crude vermiculite
GS4	Libby AB4, no.1 sorted vermiculite
GS10	Libby AB111, no. 0 sorted vermiculite
GS15	South Carolina traveler's rest ore with 10% gangue donated by G. Breit, 1958
GS11	South Carolina no. 3c crude ore from 1967 or 1971
GS1	South Africa AB 35, crude no. 4 ore from Palabora
GS18	South Africa AB 117, classified vermiculite ore, no. 3 sort, Palabora
GS17	South Africa AB 118, classified vermiculite ore, no. 2 sort, Palabora
GS3	South Africa AB119, classified vermiculite ore, no. 1 sort, Palabora
GS13	South Africa AB 120, classified vermiculite ore, no. 0 sort, Palabora
GS12	China AB 36, Xinchang, processed vermiculite ore, 2.0-4.0mm
GS16	China AB 37, Xinchang, processed vermiculite ore, 0.5-2.0mm
GS7	China AB 38, Xinchang, crude vermiculite ore
Samples provided by EPA Region 4	
PVC1	South Africa, processed by Palmetto Vermiculite
PVC2	South Africa, processed by Palmetto Vermiculite
PVC3	South Africa, processed by Palmetto Vermiculite
PVC5	South Carolina, processed by Palmetto Vermiculite
Samples provided by EPA Region 8	
1A	Libby grab samples
2A	Libby grab samples
3A	Libby grab samples
4A	Libby grab samples
5A	Libby grab samples
6A	Libby grab samples
7A	Libby grab samples
8A	Libby grab samples
9A	Libby grab samples
10A	Libby grab samples
11A	Libby grab samples
Samples provided by South Carolina Geological Survey	
SC2	Enoree area grab samples
SC3	Enoree area grab samples
SC4	Enoree area grab samples
SC5	Enoree area grab samples
SC6	Enoree area grab samples
SC7	Enoree area grab samples
SC8	Enoree area grab samples
SC9	Enoree area grab samples
SC10	Enoree area grab samples
SC11	Enoree area grab samples
Product samples provided by EPA Region 10	
P1	Zonolite Chemical Packaging Vermiculite Product
P2	Thermorock Vermiculite Product
P3	Zonolite Industrial LAB Product
P4	Zonolite Industrial SPU Product
P5	Zonolite Plaster Aggregate Product
P6	Stronglite Fine Vermiculite Product

### 3.3 Statistics and Data Treatment

Major element data was normalized on an anhydrous basis (sum of major elements normalized to the sum of major elements plus the LOI) prior to statistical analysis or data plotting. Data gathered from the different sources and analytical procedures (NAA and XRF) were compiled into a single database for statistical analysis. X-ray fluorescence data was used for major element data. Samples provided by SCGS and EPA were analyzed by XRF by Washington State University, while the USGS provided XRF data for the A.L. Bush samples. Trace element data for Ce, Co, Cr, Eu, Hf, La, Lu, Nd, Sc, Sm, Ta, Tb, Th, and Yb were obtained using NAA. Ba, Cu, Ga, Nb, Ni, Pb, Rb, Sr, U, V, Y, Zn and Zr (Table 2) were obtained using X-ray fluorescence. In some cases (e.g., Pb), data from both the USGS for their samples, and from Washington State University for the remaining samples were used. Thus, two different detection limits are present for the same element, depending on where the analysis was performed. Some analytes, such as Ba, were duplicated with NAA and XRF analyses. When such duplication occurred, the decision to choose data from one technique versus the other was based on the accuracy of analyses of USGS schist standard SDC-1 and NIST coal-fly ash standard 1633-b using each technique (Appendix B). When the accuracy of the two techniques was similar, the technique with the lower detection limit was chosen.

Prior to statistical analysis, data was ln-transformed to render the data normally distributed and to reduce the influence of elements measured with different units (e.g. major elements expressed as weight percent and trace elements expressed as parts-per-million). This prevents elements such as Ba, which can be as high as 4000 ppm, from dominating the differences between populations. Statistical analyses were performed on the transformed data set using Statistica v. 7.1 (Statsoft, 2006). Not all analyzed elements were included in the statistical analysis. Zirconium and hafnium were excluded because samples pulverized with the zirconia grinder could be contaminated with these elements. Elements for which any of the samples were below the detection limit of the analytical method were excluded. This includes  $P_2O_5$ , Eu, Lu, Nd, Nb, Pb, Ta, Tb, Th, U, Y and Yb. Some elements were eliminated from the statistical analysis because they had properties that would skew the statistical analysis. Elements that are highly correlated can dominate the way likeness is measured (McPherson, 2001). Thus, a Pearson correlation matrix of the ln-transformed data was calculated. Another consideration to the validity of the results of the statistical analysis is the correlation of the variances (standard deviations) with the means across the categories (Statsoft, 2006). Such correlations can result in spurious statistical significance. To test for this possibility, Pearson product moment correlations between the group means and the group variance were calculated for each of the variables used in the analysis. All of the variables had insignificant correlations ( $p > 0.05$ ) except for Rb ( $p = 0.00313$ ), Sr ( $p = 0.0266$ ), and MgO ( $p = 0.0428$ ). Therefore Mg, Sr, and Rb were not used in further statistical analyses.

Cluster analysis was conducted using both hierarchical and non-hierarchical methods. The number of elements in the analyses was reduced by choosing a single element from groups of correlated elements with correlation coefficients  $> 0.7$ . For the tree clustering analysis (hierarchical), four different agglomeration techniques were applied: single linkage, complete linkage, unweighted pair-group average, and Ward's method. In all cases, a Euclidean distance measure was applied. For the k-cluster analysis (non-hierarchical), four sets of seeds were tested corresponding to 4, 5, 6, and 7 clusters. The choice of the number of clusters is based on the results of the hierarchical analysis. Additional tests are conducted based on the same seeds used in the five-cluster model plus an additional cluster using each of the China samples as seeds.

To determine which variables best differentiate vermiculite groups, discriminant analysis was performed using the module in the Statistica software package (Statsoft, 2006). The discriminant analysis was performed on the data set using two different sets of categories. The first set is a four-category model



based on the original grouping of the samples: Libby Montana (L), South Carolina (SC), South Africa (SA), and China (CH). The second set is a six-category mode which is based on the five categories identified in the cluster analysis (two categories from Libby, L1 and L2; one from South Africa, SA; two from South Carolina, SC1 and SC2 ) but with the China samples grouped into a sixth category (CH).

Both the four-category and the six-category model were analyzed using two subsets of the 38 measured analytes. For both variable sets, we eliminated 12 analytes described previously. Thus the first set of variables consists of the 21 analytes SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO\*, MnO, CaO, Na<sub>2</sub>O, K<sub>2</sub>O, Ba, Ce, Co, Cr, Cs, Cu, Ga, La, Ni, Sc, Sm, V, and Zn. The second set of variables is comprised of the 15 analytes that are common between variable set one and the element set used by Gunter (2003). These include SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO\*, MnO, CaO, K<sub>2</sub>O, Ba, Cr, Cu, Ga, Ni, Sc, V, and Zn.

As described in a previous paragraph, there are correlations between several of the analytes. Such correlations can result in ill-conditioning of the variance/covariance matrix which must be inverted during the solution process. We therefore proceeded using a forward stepping scheme to add the most discriminating analyte not currently in the model. The criterion to enter the model was an F statistic > 2 while the criterion to remove a variable from the model was an F statistic < 1. Once a variable is added to the model, analytes with correlation coefficients > 0.7 with the added variable were removed from further consideration. The process was then continued by adding the next most discriminating analyte from the remaining set. This procedure was repeated until an additional analyte resulted in a statistically insignificant ( $p > 0.05$ ) discriminant coefficient in the model.

## **4. Results and Discussion**

### **4.1 Data Description**

Analytical results are shown in Table 2. Statistical summaries of the data including means and standard deviations for ln(concentration) of each analyte by category, box and whisker plots for combined and individual groups, the Pearson correlation coefficient matrix, and categorized probability plots for each element are provided in Appendix A.

shows a spider diagram of all vermiculite samples in this study normalized to primitive mantle (Sun and McDonough, 1989). For illustrative purposes, samples that were below detection limit for a particular element were assigned a value of 0.001ppm for that element so that it could be plotted on the logarithmic diagram. From Figure 1, it appears that vermiculite from South Carolina is enriched in REE's relative to ores from South Africa and Libby; but its signature substantially overlaps that of ores from China. In contrast, vermiculite from Libby is notably depleted in Th, La, P, Sm, and Eu compared to ores from the other three localities. South African ores appear to have more geochemical variability than do ores from the other three localities; however they exhibit prominent depletions of Pb and Yb.

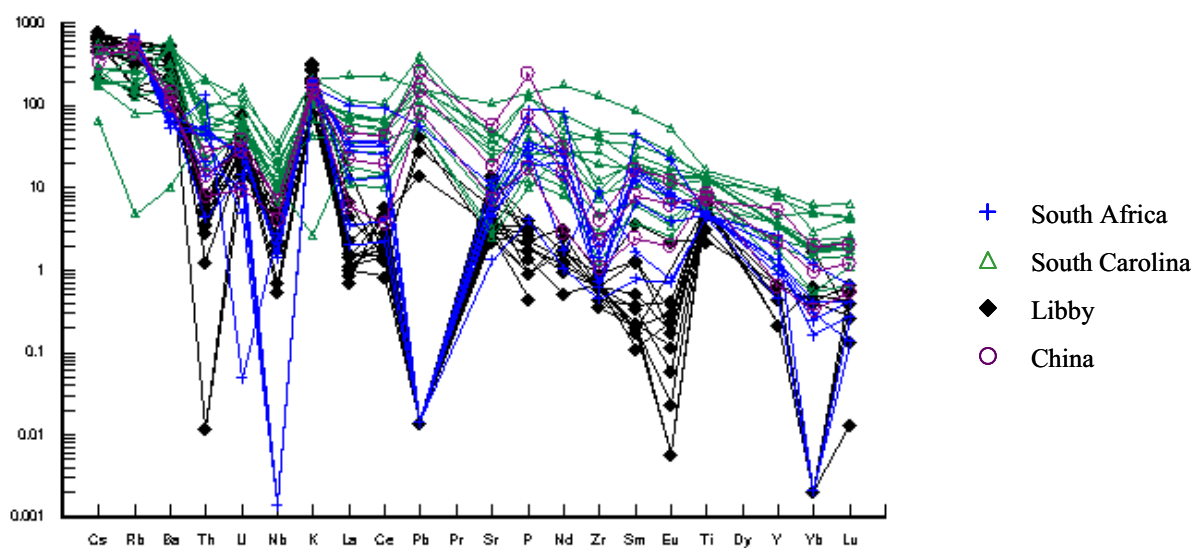


Figure 1. Spider diagram showing all vermiculite samples normalized to primitive mantle (Sun and McDonough, 1989).

Table 2. Chemical analysis of vermiculite samples. See Table 1 for sample descriptions and locations.

	GS-1	GS-3	GS-13	GS-17	GS-18	PVC1	PVC2	PVC3	GS-7	GS-12	GS-16	GS-11	GS-15	SC2	SC3	SC4
SiO <sub>2</sub>	43.0	38.3	37.8	39.1	38.9	38.7	39.4	39.2	41	38.4	40.2	38.9	42.6	44.2	47.7	44.3
TiO <sub>2</sub>	0.85	0.88	0.86	0.91	0.9	0.845	0.906	0.952	1.41	1.53	1.48	1.62	1.8	1.04	1.66	2.69
Al <sub>2</sub> O <sub>3</sub>	8.9	8.7	8.5	8.9	9.1	8.24	8.67	9.14	10.1	11.4	11.2	12.5	11.9	6.64	10.5	12.7
FeO*	6.11	6.74	6.74	6.65	6.38	5.92	6.42	6.94	7.1	9.35	8.72	8.27	6.92	9.26	7.60	9.26
MnO	0.04	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.08	0.08	0.09	0.12	0.92	0.15	0.12	0.12
MgO	22.5	22.1	21.8	22.8	23	23.1	23.0	23.8	12.4	16.3	16.2	17.8	17.2	22.9	14.0	11.1
CaO	0.6	4.0	4.5	3.1	2.5	5.16	3.79	2.30	7.8	2.5	3.8	1.2	1.8	2.65	3.93	5.16
Na <sub>2</sub> O	<0.15	<0.15	<0.15	<0.15	<0.15	0.06	0.07	0.05	0.56	<0.15	0.17	<0.15	0.27	0.18	0.91	1.30
K <sub>2</sub> O	5.27	4.34	4.31	4.59	4.86	4.78	4.83	5.16	4.23	3.69	4.72	3.84	4.95	0.07	3.15	3.13
P <sub>2</sub> O <sub>5</sub>	0.08	1.31	1.66	0.67	0.46	0.52	0.60	0.37	4.7	0.32	1.43	0.29	0.43	0.78	1.02	0.511
LOI (%)	10.1	11.1	11.1	10.8	10.8	8.98	8.04	7.63	8.1	13.8	10.5	13.0	8.80	8.57	6.64	6.64
Sum	98.1	98.2	98.1	98.3	97.6	96.4	95.7	95.6	98.4	98.4	98.4	98.5	97.5	96.4	97.2	96.9
Ba	643	366	359	370	432	644	520	447	1020	869	725	999	1640	71	1360	3970
Ce <sup>1</sup>	3.9	65.1	166	55.5	23.6	7.2	60.0	46.6	76.4	6.5	34.2	26.0	25.5	45.5	113	119
Co <sup>1</sup>	75.6	62.3	71.2	67.3	67.7	80.1	58.7	62.0	51.5	66.4	60.6	72.9	59.7	73.7	48.5	35.7
Cr <sup>1</sup>	210	251	282	235	215	263	198	238	601	776	1090	2150	2570	229	1100	442
Cs <sup>1</sup>	15.2	10.6	11.1	11.5	11.8	12.4	8.62	11.0	2.70	3.67	3.67	4.44	3.52	0.52	2.23	1.67
Cu	7	59	87	41	15	8	17	19	38	31	45	131	73	31	55	20
Eu <sup>1</sup>	0.12	1.46	3.73	1.28	0.66	0.13	1.33	1.05	2.07	0.33	1.07	0.58	0.70	1.24	2.16	2.94
Ga	16	12	14	14	14	12	12	13	12	8	13	22	18	8	15	16
Hf <sup>1</sup>	0.30	0.17	0.22	0.07	0.06	0.25	0.23	0.06	0.93	0.10	0.66	0.65	1.22	5.81	11.6	2.49
La <sup>1</sup>	1.4	25.1	66.9	22.1	8.8	2.4	25.0	19.1	31.6	4.1	15.3	12.4	8.7	19.6	50.3	54.9
Lu <sup>1</sup>	0.01	0.02	0.05	0.03	0.03	0.01	0.02	0.01	0.15	0.04	0.09	0.08	0.11	0.13	0.16	0.34
Nd <sup>1</sup>	1.4	37.7	114	37.0	18.8	2.3	31.8	27.1	41.9	4.0	20.0	14.2	17.2	23.9	52.1	91.0
Nb	<2	<2	2	<2	<2	1.0	1.5	1.1	5	3	5	7	8	7.6	15	7.8
Ni	152	201	223	172	157	148	157	176	458	499	575	331	412	606	535	73
Pb	<3	<3	4	<3	<3	<1	<1	<1	18	6	11	4	5	6	14	11
Rb	456	398	386	399	433	411	428	472	364	270	387	271	263	3	168	103
Sc <sup>1</sup>	5.0	5.2	8.7	9.4	8.8	5.9	10.6	7.5	9.3	6.6	10.7	11.8	11.3	10.6	13.0	25.1
Sm <sup>1</sup>	0.35	7.53	19.6	6.59	2.95	0.74	7.09	5.58	7.61	1.07	3.55	2.77	3.03	4.94	9.10	11.1
Sr	28	178	225	114	96	257	149	93	1210	158	384	66	139	171	489	1050
Ta <sup>1</sup>	0.10	0.07	<0.02	<0.02	0.04	0.33	0.43	0.40	0.03	0.01	0.04	0.04	0.66	0.32	0.74	0.35
Tb <sup>1</sup>	0.03	0.37	1.11	0.27	<0.05	0.06	0.51	0.38	0.73	<0.05	0.32	<0.05	0.31	0.53	0.67	0.78
Th <sup>1</sup>	0.37	4.57	11.1	3.72	1.74	1.16	4.29	3.63	2.27	0.69	1.20	18.0	1.57	4.43	9.08	5.06
U <sup>1</sup>	<0.4	0.5	<0.4	<0.4	<0.4	0.7	0.5	0.6	0.8	<0.4	0.6	1.2	1.1	1.5	2.1	1.3
V	31	41	35	39	34	13	18	19	161	165	176	194	190	89	124	241
Y	2	9	12	6	5	6	6	4	25	3	10	8	9	11	16	21
Yb <sup>1</sup>	<0.02	<0.02	0.58	0.20	0.12	0.15	0.08	0.15	0.99	0.16	0.48	0.27	0.66	0.85	0.86	2.46
Zn	67	84	81	77	73	61	68	78	109	144	144	138	107	92	94	97
Zr	5	8	7	8	7	101	16	21	46	12	26	14	53	92	469	101

<sup>1</sup>Analyses performed by INAA

Table 2. (continued).

	SC5	SC6	SC7	SC8	SC9	SC10	SC11	PVC5	GS-2	GS-4	GS-10	1A	2A	3A	4A	5A
SiO <sub>2</sub>	41.8	44.3	42.9	49.7	43.8	44.1	39.2	40.0	37.8	37.4	36.9	49.9	37.5	39.3	39.8	38.4
TiO <sub>2</sub>	3.30	2.65	2.38	1.23	2.31	2.73	2.92	1.90	1.09	1.24	1.22	0.48	1.23	1.15	1.09	1.47
Al <sub>2</sub> O <sub>3</sub>	11.6	12.4	11.4	9.9	15.1	11.5	9.88	12.4	11.6	12.0	12.1	3.71	12.5	13.1	9.54	13.7
FeO*	10.1	9.16	11.0	8.24	9.88	8.90	16.8	8.39	8.45	8.9	8.63	4.59	9.56	8.00	7.67	8.87
MnO	0.12	0.13	0.15	0.15	0.24	0.12	0.19	0.09	0.09	0.10	0.08	0.08	0.10	0.08	0.08	0.08
MgO	8.66	11.5	16.5	13.1	6.35	13.2	8.59	17.2	19.5	19.7	20.3	17.8	21.3	22.2	19.3	22.2
CaO	7.48	5.28	1.76	6.13	3.60	6.49	8.91	1.94	2.10	1.70	1.30	18.7	1.02	0.38	7.48	0.29
Na <sub>2</sub> O	0.65	1.25	0.11	1.28	0.26	1.28	0.91	0.41	<0.15	0.19	0.16	0.38	0.20	0.22	0.14	0.25
K <sub>2</sub> O	5.79	3.00	1.13	1.86	5.80	2.88	3.71	3.80	5.03	6.00	5.07	2.29	7.60	9.56	2.78	9.56
P <sub>2</sub> O <sub>5</sub>	2.87	0.50	0.19	0.51	2.45	0.36	2.59	0.23	0.06	<0.05	<0.05	0.04	0.03	0.02	0.02	0.02
LOI (%)	3.83	6.86	9.40	5.31	6.28	5.29	2.74	9.22	11.1	9.00	11.0	0.90	3.89	0.79	8.85	1.01
Sum	96.2	97.0	97.1	97.4	96.1	96.8	96.5	95.7	97.7	97.2	97.7	98.8	94.9	94.8	96.8	95.8
Ba	3720	3910	599	1090	4280	3710	2290	3580	2550	2510	2120	603	3300	2860	1620	3430
Ce <sup>1</sup>	404	93.6	66.8	76.7	187	86.3	122	17.8	10.6	8.8	2.7	8.8	8.2	3.9	1.5	2.3
Co <sup>1</sup>	36.8	38.9	58.5	48.7	59.2	38.6	50.0	51.6	63.4	63.3	67.2	33.1	67.1	55.1	81.6	76.1
Cr <sup>1</sup>	285	480	865	1290	102	512	12.3	1220	1950	1550	1250	1150	2210	1410	1180	825
Cs <sup>1</sup>	1.73	1.62	1.37	2.11	3.30	1.71	1.44	2.38	5.69	5.57	4.71	1.80	5.68	4.17	5.18	5.90
Cu	18	18	120	28	112	20	41	66	30	26	13	3	4	1	3	2
Eu <sup>1</sup>	8.89	2.42	1.85	1.77	4.64	2.17	3.85	0.42	0.05	0.07	0.05	0.38	<0.01	0.03	<0.01	0.04
Ga	19	17	16	13	21	15	18	16	16	18	18	6	16	16	13	16
Hf <sup>1</sup>	34.7	3.46	4.96	5.93	13.1	1.89	10.4	0.55	0.09	0.22	0.05	0.08	0.30	0.27	0.18	0.37
La <sup>1</sup>	157	41.8	24.7	33.9	78.2	42.8	49.3	7.6	0.5	0.6	0.0	1.6	1.0	1.0	0.7	4.7
Lu <sup>1</sup>	0.31	0.14	0.17	0.19	0.47	0.13	0.32	0.05	0.02	0.03	0.02	0.05	0.02	0.04	0.03	0.02
Nd <sup>1</sup>	243	49.4	38.7	36.9	103	42.3	68.8	10.9	<0.5	<0.5	<0.5	4.4	0.7	2.4	3.5	<0.5
Nb	25	7.0	11	13	20	5.8	13	4.4	4.0	4.0	3.0	1.3	2.0	1.3	1.7	1.3
Ni	243	74	360	277	51	68	11	440	211	232	206	112	186	264	129	221
Pb	11	11	4	23	28	8	8	4	<3	<3	3	<1	<1	<1	<1	<1
Rb	256	103	51	81	276	99	124	152	238	236	200	90	297	378	106	387
Sc <sup>1</sup>	24.1	25.2	14.5	18.3	27.6	26.3	41.6	10.7	9.8	14.0	12.9	74.2	16.9	38.0	11.9	17.6
Sm <sup>1</sup>	38.8	8.83	7.50	6.67	19.7	8.04	14.6	1.69	0.09	0.17	0.09	1.63	0.08	0.57	0.08	0.05
Sr	2260	1010	53	601	852	794	608	233	75	71	48	303	74	51	133	60
Ta <sup>1</sup>	1.40	0.36	0.62	0.62	1.40	0.31	0.55	0.60	0.17	0.16	0.03	0.82	0.51	0.45	0.16	0.42
Tb <sup>1</sup>	1.96	0.80	0.62	0.28	1.47	0.69	1.23	0.24	0.05	0.03	<0.05	<0.05	0.1	<0.05	<0.05	<0.05
Th <sup>1</sup>	17.1	6.49	3.69	4.61	7.55	4.24	1.98	1.47	<0.02	<0.02	0.11	0.71	0.52	0.25	0.43	0.30
U <sup>1</sup>	2.6	1.1	0.8	2.1	3.4	1.0	1.0	1.1	0.6	<0.4	0.5	1.0	1.7	0.8	0.6	0.6
V	302	239	141	133	230	261	480	182	122	134	123	51	96	85	72	94
Y	41	17	16	17	39	15	35	5	<2	<2	<2	3	<1	1	2	<1
Yb <sup>1</sup>	1.44	0.90	0.98	1.11	3.04	0.82	2.44	0.27	0.20	0.25	<0.02	0.20	0.86	0.32	<0.02	<0.02
Zn	140	97	193	103	151	84	139	103	65	76	67	23	69	51	42	53
Zr	1456	139	217	310	539	84	411	32	5.0	5.0	4	10	8	7	7	7

Table 2. (continued).

	6A	7A	8A	9A	10A	11A	P1	P2	P3	P4	P5	P6
SiO <sub>2</sub>	37.5	38.3	36.3	42.2	48.3	37.7	39.1	38.8	39.6	38.4	38.8	38.1
TiO <sub>2</sub>	1.42	1.28	1.28	0.90	0.66	1.22	1.39	0.77	2.24	1.26	1.28	1.21
Al <sub>2</sub> O <sub>3</sub>	13.2	12.8	12.1	8.0	4.9	12.1	13.4	6.8	13.3	11.9	12.8	12.1
FeO <sup>r</sup>	10.9	8.7	10.6	6.96	6.07	9.08	9.15	5.77	9.72	9.24	8.80	5.06
MnO	0.09	0.08	0.11	0.09	0.09	0.09	0.10	0.05	0.12	0.09	0.10	0.04
MgO	21.3	21.4	19.6	18.4	17.6	19.8	21.9	19.7	19.1	19.4	21.7	23.2
CaO	0.11	1.15	1.75	10.6	16.9	2.11	1.51	10.1	1.40	2.93	1.14	1.98
Na <sub>2</sub> O	0.25	0.22	0.16	0.23	0.30	0.28	0.14	0.10	0.09	0.28	0.22	1.45
K <sub>2</sub> O	9.02	8.03	5.63	2.94	2.40	4.58	5.57	4.06	4.05	4.62	6.29	4.38
P <sub>2</sub> O <sub>5</sub>	0.01	0.04	0.08	0.04	0.09	0.03	0.04	2.49	0.47	0.04	0.01	0.07
LOI (%)	1.59	3.05	7.23	6.16	1.64	9.24	4.19	7.26	6.34	7.96	4.27	7.89
Sum	95.5	95.0	95.0	96.5	99.0	96.3	96.5	95.9	96.4	96.2	95.5	95.5
Ba	3760	2670	3870	1450	1020	2140	2520	464	1210	2300	2440	2560
Ce <sup>1</sup>	5.1	3.1	3.4	2.6	2.5	2.6	9.9	354	51.3	7.0	11.1	14.1
Co <sup>1</sup>	78.6	77.5	81.8	69.6	66.2	71.3	84.9	48.9	66.0	79.3	81.7	65.3
Cr <sup>1</sup>	1840	1190	1280	1210	276	1000	1540	315	1140	821	1650	2220
Cs <sup>1</sup>	5.73	6.28	4.61	3.54	4.66	3.89	6.62	8.98	4.32	5.14	6.76	3.86
Cu	2	4	27	4	2	13	17	23	102	17	25	6
Eu <sup>1</sup>	<0.01	0.01	<0.01	0.03	0.05	0.02	0.04	7.72	1.87	0.01	<0.01	0.20
Ga	17	15	16	10	5.8	16	17	10	23	16	18	14
Hf <sup>1</sup>	0.24	0.49	0.48	0.24	0.69	0.26	0.35	0.56	3.17	0.26	0.30	0.58
La <sup>1</sup>	0.7	1.0	0.8	1.1	0.8	3.1	0.3	145	19.5	0.7	0.3	3.2
Lu <sup>1</sup>	0.02	0.02	0.01	<0.01	0.05	0.02	0.02	0.09	0.22	0.02	0.02	0.01
Nd <sup>1</sup>	<0.5	1.6	2.3	1.8	2.3	1.3	1.6	187	28.6	0.2	<0.5	4.3
Nb	1.5	1.3	1.6	0.5	0.4	1.4	3.0	1.5	9.2	2.0	2.7	3.0
Ni	143	240	154	146	81	180	235	129	493	166	233	382
Pb	<1	<1	2	<1	<1	<1	1	2	2	8	3	<1
Rb	337	309	364	226	103	202	257	365	282	203	276	241
Sc <sup>1</sup>	16.2	15.4	11.7	16.1	34.6	14.5	13.4	17.7	15.5	14.2	12.7	10.0
Sm <sup>1</sup>	0.10	0.15	0.09	0.23	0.58	0.10	0.10	41.9	7.34	0.13	0.06	0.83
Sr	55	72	125	215	269	91	52	430	92	91	61	206
Ta <sup>1</sup>	0.22	0.23	0.34	0.21	0.41	0.17	0.54	0.67	0.84	0.41	0.52	0.50
Tb <sup>1</sup>	<0.05	<0.05	<0.05	<0.05	<0.05	<0.05	0.07	2.56	0.84	<0.05	0.17	0.15
Th <sup>1</sup>	0.46	0.52	0.55	0.49	0.31	0.43	0.73	20.8	2.55	0.40	0.64	1.10
U <sup>1</sup>	0.9	0.5	1.0	0.8	0.6	0.4	0.6	1.1	1.6	0.7	0.6	0.9
V	103	93	89	69	64	86	95	18	172	94	103	55
Y	<1	<1	<1	2.8	2.2	<1	1.1	23	21	<1	<1	1.7
Yb <sup>1</sup>	<0.02	<0.02	<0.02	<0.02	<0.02	<0.02	0.22	0.74	1.44	0.25	0.27	0.24
Zn	58	51	75	40	30	108	70	60	171	63	79	38
Zr	7	7	7	7	10	7	7	22	46	8	6	12

A Pearson correlation matrix is provided in Appendix A. Elements with correlation coefficients exceeding 0.7 are shown in Table 3. Lanthanum was chosen for use in cluster analysis because its presence is associated with monazite, a mineral known to be present in South Carolina but not at Libby. While Ce and Sm are also associated with monazite, La is easily measured with XRF, a common laboratory instrument. Vanadium was chosen for use in statistical analyses over Ti and Mn. Manganese is in low concentration in vermiculite and may be difficult to detect in some samples. Titanium can substitute into the lattices of both biotite and clinopyroxene whereas V is more specific to substitution in clinopyroxene (Deer et al., 1992). Therefore the abundance of clinopyroxene or its trace element composition may be diagnostic of provenance. Cesium was chosen over sodium because sodium is in low concentration in vermiculite samples and may be difficult to detect in some samples.

Table 3. Table of highly correlated elements. Elements in **bold** were used in cluster analysis while the other correlated elements were removed from the analysis.

Element	Correlated Element	Correlation Coefficient
<b>La</b>	Ce	0.887
	Sm	0.927
<b>V</b>	Ti	0.809
	Mn	0.711
<b>Cs</b>	Na	0.701

The statistical methods that we employ have an underlying assumption of the data being normally distributed; therefore, it is worth reviewing this assumption with the data set employed in this study. Normality can be determined using statistical tests such as the Shapiro-Wilk (preferred) or Kolmogorov-Smirnov test and viewed graphically using probability plots (Appendix A) which should be linear for normal distributions. Although normality is required for the mathematical development of many statistical tests, the severity of the impact of using those tests when there is a violation of normality is not necessarily great (StatSoft, 2007). For the South African samples, every variable fits a normal distribution ( $p > 0.05$ ) using the Shapiro-Wilk test except for  $\text{SiO}_2$  and  $\text{CaO}$ . Both of these exceptions can be explained by an outlier (the same sample) with the remaining data fitting a normal distribution. All of the China samples fit a normal distribution except for  $\text{SiO}_2$ , Cs, and  $\text{CaO}$ . The latter two results are explained by duplicate results with only three samples in the category. For  $\text{SiO}_2$  in the China category, the data approximates a straight line and it fits a normal curve according to the Kolmogorov-Smirnov test. For the South Carolina samples, MnO,  $\text{K}_2\text{O}$ , Ba, Ni, and Rb failed the Shapiro-Wilk test. For all of the elements that failed the test, the probability plots appear to be comprised of two straight-line segments implying that the South Carolina samples are comprised of at least two subcategories that are normally distributed. However, the samples comprising the straight line segments are not consistent between analytes.

For the Libby category,  $\text{TiO}_2$ ,  $\text{Al}_2\text{O}_3$ , FeO, MgO, Co, Cr, Cs, Ga, La, Rb, Sc, and Sm failed the normality test. The results for Co, Cr, Cs, and La can be explained by a single outlier and are assumed to be normally distributed. For Rb, the deviations from a straight line are small and it does pass the Kolmogorov-Smirnov test. For the remaining elements that failed the test ( $\text{TiO}_2$ ,  $\text{Al}_2\text{O}_3$ , FeO, MgO, Ga, Sc, and Sm) the probability plots appear to be comprised of two line segments. Samples 1A, 4A, 9A, and 10A appear together in one of the line segments in a majority of those analytes. This is consistent with the origin of Libby from fractional crystallization from a single parent magma (Boettcher, 1966). Such paragenesis can be indicated by the non-normal distribution of trace elements (Kawabe, 1977).

## 4.2 Hierarchical Cluster Analysis

Cluster analysis is used to determine if there are characteristics of samples that allow them to be separated into different groups. Clustering methods can be applied when there is limited prior knowledge about the samples being analyzed. The results of hierarchical methods can be displayed as graphical output in the form of a dendrogram or tree that illustrates the distance between each of the clusters. There are several different agglomerative hierarchical cluster analysis methods based on the choice of linkage (amalgamation) method and the choice of measure of similarity (distance) between the clusters (e.g., Rencher, 2002). These hierarchical clustering techniques are non-unique in the sense that different clusters can be obtained depending on the choice of method. We chose to use four different linkage methods: single, complete, unweighted average, and Ward's methods. All of these methods are known to be monotonic, i.e., not show inversions (reversals) in the dendrogram (e.g., Morgan and Ray, 1995; Rencher, 2002). Single linkage defines the distance between two clusters as the minimum distance between a point A in one cluster and a point B in the other cluster. In contrast, complete linkage defines the distance between two clusters as the maximum distance between point A and point B. The average linkage approach defines the distance between two clusters A and B as the average of  $n_a n_b$  distances between the  $n_a$  points in A and the  $n_b$  points in B. Ward's method uses within-cluster and between-cluster distances to join clusters in a way that minimizes within-cluster distances (Rencher, 2002; Systat, 2007). In general, Ward's method and average linkage method are often preferred because they are relatively insensitive to outliers, although some types of data sets simply work better with some linkage methods than with others (Rencher, 2002).

In addition to the selected linkage method, it was necessary to choose a method to determine clustering distances. Clustering distance algorithms are specific to the type of data being analyzed. For example, some distance methods are appropriate for counts of objects while other methods are specific for quantitative data. Still other methods are more suitable for data that has been standardized. Absolute distance measure is appropriate for standardized, quantitative data (Systat, 2007) and it was used in our cluster analysis. Following the rationale outlined previously, the 15 elements used in the cluster analysis are: SiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO\*, CaO, K<sub>2</sub>O, Ba, Co, Cr, Cs, Cu, La, Ni, Sc, V and Zn.

Figure 2 (A-D) shows the cluster trees containing all the ore and product samples used in this study using single, complete, and unweighted average linkages and Ward's method, respectively. All of the dendrograms were computed with Euclidian distances. To determine the number of clusters from the dendrogram, we chose the number of clusters at the stage where

$$d_j > \bar{d} + ks_d \quad j = 1, 2, \dots, n \quad (1)$$

for the first time, where  $d_1, d_2, \dots, d_n$  are the distances when there are  $n, n-1, n-2, \dots, 1$  clusters and  $\bar{d}$  and  $s_d$  are the mean and standard deviation of the  $d$  (e.g., Rencher, 2002; Mojena, 1977). We used the recommended (Milligan and Cooper, 1985) value of 1.25 for the  $k$  parameter.

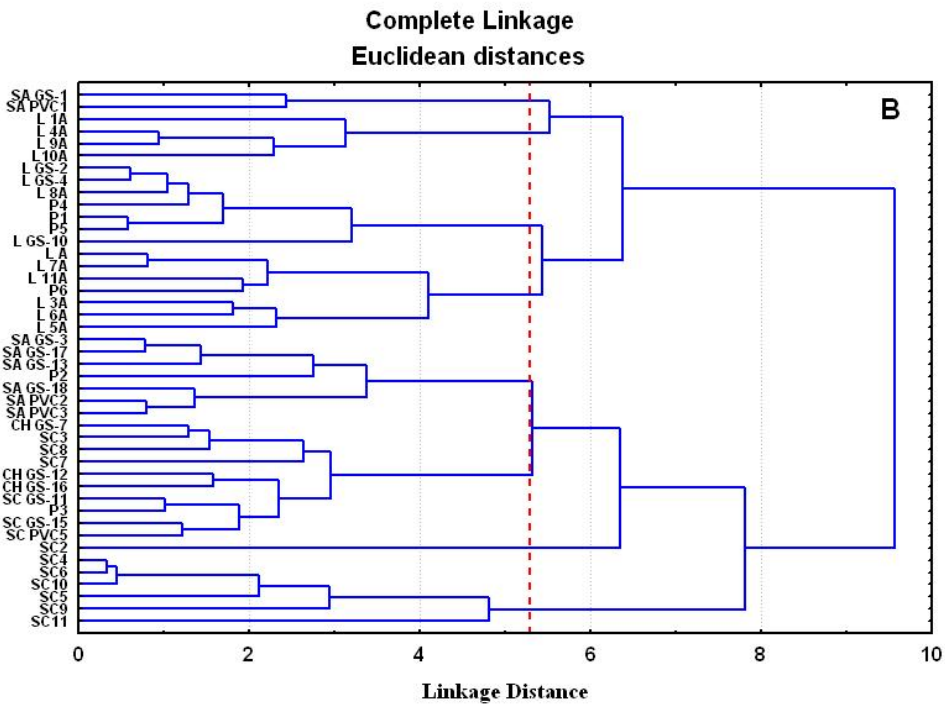
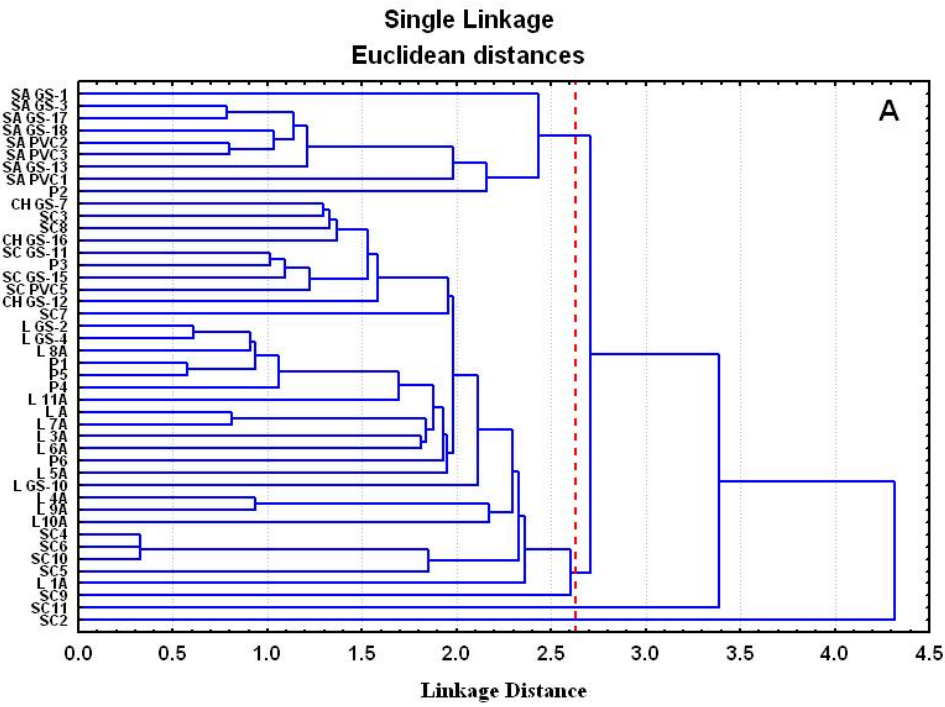
The samples in the single linkage method (Figure 2A) tend to remain separated (chaining), consistent with results that others have obtained using this linkage method (Rencher, 2002). Using the linkage distance cutoff from Eq. (1) there appear to be two orphans (SC2, SC11) and two groups (SA+P2, everything else). This method suggests only minimal clustering occurs and that it would be difficult to identify sources of vermiculite. In contrast, the complete linkage approach (Figure 2B) forms more clusters (7) with only 1 orphan (SC2). The clusters include 1) a subset of the SC samples; 2) a subset of SC samples,

CH samples, plus P3; 3) a subset of SA samples plus P2; 4) a group comprised of two SA samples (SA GS-1, SA PVC-1); 5) a subset of the L samples plus P1, P4, and P5; 6) a second subset of L samples plus P6; and 7) a third subset of L samples. The existence of subcategories for the L and SC categories is consistent with the observations of the probability plots discussed in previous paragraphs.

The unweighted average linkage (Figure 2C) and Ward's method (Figure 2D) generated 3 and 4 groups, respectively. The clusters in the unweighted average method were comprised of the L category plus P1, P2, P4, and P6; the SA category plus P5; and a subset of the SC category plus the CH category and P3. There were two orphans (SC2 and SC11). The four clusters generated in Ward's method were L with P1, P4, P5, and P6; SA with P2; a subset of SC; and another subset of SC with the CH samples and P3.

Except for the single linkage method, which only distinguishes SA samples, the cluster analyses used show that Libby and South Africa ores are distinct from each other and distinct from South Carolina and China ores, but that China ores and South Carolina ores cannot be distinguished from one another. Only Ward's method did not have any orphans. The particular samples that plot as orphans vary with the linkage method chosen. None of the methods distinguishes samples from China. While the general grouping of the six products is the same in all cluster trees, several of the known samples plot as orphans. Cluster analysis suggests that P1, P4, P5, and P6 are Libby products, whereas P2 is likely from South Africa and P3 is likely from South Carolina.





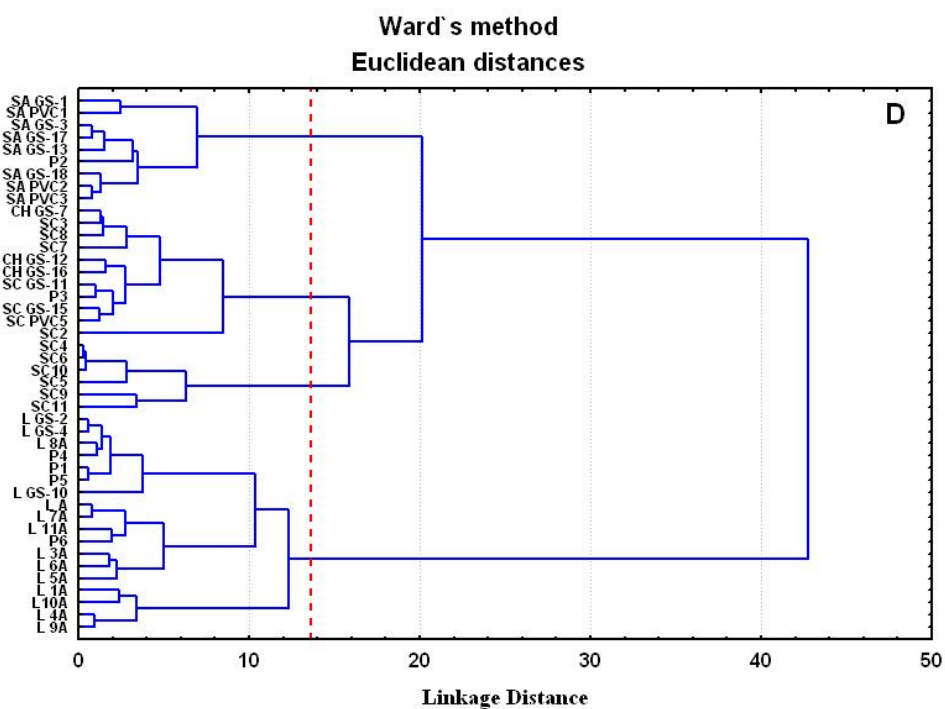
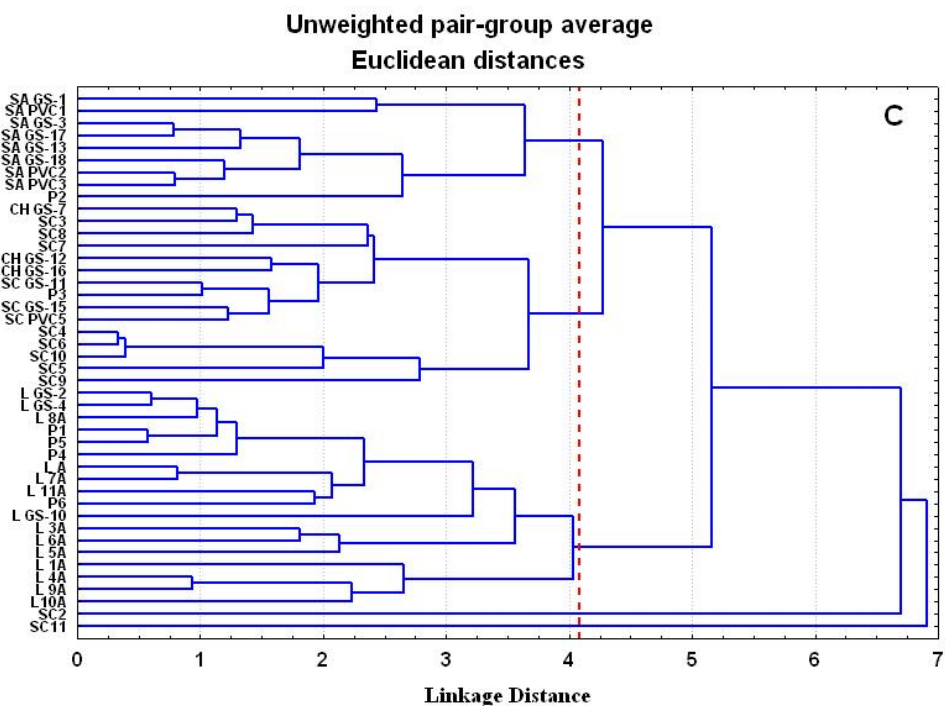


Figure 2. Cluster diagrams using single linkage (A), complete linkage (B), unweighted pair-group average (C), and Ward's method (D). All of the methods used Euclidean distances. All ores and products analyzed in this study are shown. Samples of known origin have their sample names preceded by a location designation (SC=South Carolina, SA=South Africa, CH=China, L=Libby, P=product), although *a priori* knowledge of the sample origin has no effect on the cluster analysis.

### 4.3 K-Cluster Analysis

We applied k-cluster analysis to test whether allowing the individual samples to be reclassified as the centroids of the clusters as identified would make a large difference in the perceived clusters. We considered 4, 5, 6, and 7 clusters with seeds chosen from the dendrogram for Ward's methods (Figure 2d, Table 4). The results yield clusters similar to those obtained from the hierachial analysis with the exception that sample SA GS-1 was assigned to one of the Libby subcategories rather than to the South African categories. To determine which of the four scenarios best represents the clustering, we calculated

$$c = \frac{tr(\mathbf{H})/(k-1)}{tr(\mathbf{E})/(n-k)} \quad (2)$$

where  $tr(\mathbf{H})$  and  $tr(\mathbf{E})$  are the traces (sum of the diagonal elements) of the  $p \times p$  between group and within group sum of product matrices,  $p$  is the number of variables (15),  $n$  is the total number of samples (44) and  $k$  is the number of clusters (Rencher, 2002). The ranks of  $\mathbf{E}$  and  $\mathbf{H}$  are  $p$  and  $k-1$ , respectively. The number of clusters that maximizes  $c$  is likely to be the best model. For the scenarios tested, the maximum  $c$  occurs with 5 clusters (Figure 3). This result, consistent with the probability distribution analysis discussed in the Data Description section (see also Appendix A) and similar to the hierarchial cluster analysis, suggests that the Libby samples and the South Carolina samples are both comprised of at least two subgroups, and the China samples remain indistinguishable from South Carolina samples. It should be emphasized that different results can be obtained if different seeds are used; nonetheless, the fact that we can reasonably replicate the hierarchial cluster analysis with the k-cluster analysis suggests that the clustering is reasonable.

As previously noted, with the hierarchial and k-clustering approach, the China samples are indistinguishable from a subset of the South Carolina samples. To see if we could separate out the China samples, we ran three scenarios of a 6 cluster model using the seeds from the previous 5 cluster model plus each of the three China samples as the sixth seed. All of these scenarios resulted in the creation of an orphan (SC 2) with the China samples clustered with the South Carolina in the same manner as seen in the 5 cluster scenario. Thus, the China samples cannot be differentiated from the South Carolina samples using the clustering techniques employed in this study.

Table 4. Seeds used in the k-cluster analysis.

No. of Clusters	Seeds
4	SA GS-17, SC GS-11, SC 10, L 7A
5	SA GS-17, SC GS-11, SC 10, L 7A, L 9A
6	SA GS-17, SC GS-11, SC 10, L 7A, L 9A, L GS-4
7	SA GS-17, SC GS-11, SC 10, L 7A, L 9A, L GS-4, SA GS-1

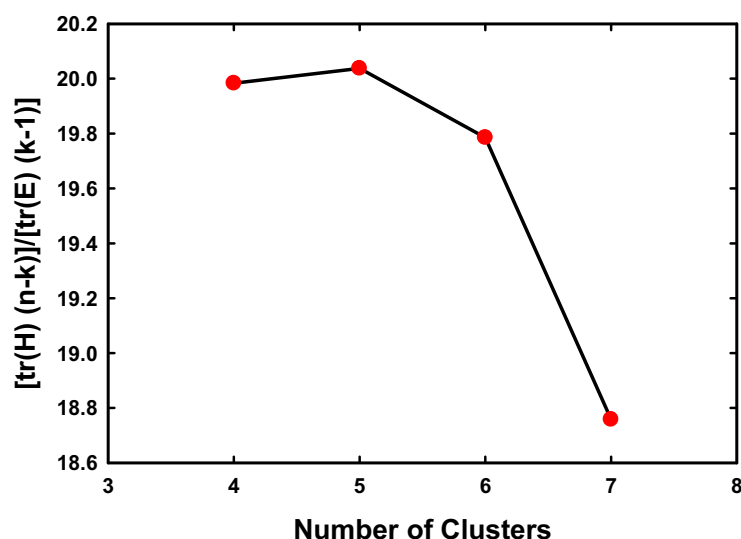


Figure 3. Plot of  $c$  versus number of clusters used in the  $k$ -cluster analysis. Maximum values of  $c$  reflect the "best" model.

#### 4.4 Discriminant Analysis

Discriminant analysis is a multivariate statistical tool that allows one to address the question of what factors differentiate one category from another. The cluster analysis tools we used in the previous sections consider the distance between a sample and clusters to assign the sample to the nearest cluster to build the groups. This distance is referred to as the Mahalanobis distance. Discriminant analysis determines what linear combinations of the variables (i.e., roots) best differentiate predefined groups from one another. Wilks'  $\Lambda$  is smaller when a variable's contribution to the discriminant function is larger. Once these discriminant functions are defined, they can be transformed to provide a method of classifying additional samples.

We investigate two sets of categories using discriminant analysis. The first set is a four-category model comprised of the original four groups of samples: L, SA, SC, and CH. This model allows us to compare the results with the previous work of Gunter et al. (2005). The second set of categories is a six-category model comprised of the five groups identified in the cluster analysis but with all of the China samples grouped into a sixth category. Both of these models were run with our set of analytes as well as a subset of these variables that are common with Gunter et al.'s (2005) analyte set, which allows for comparison of our models with than of Gunter et al. (2005).

##### 4.4.1 Four-Category Models

The first model that was run was the four-category model (L, SA, SC, CH) with our set of analytes. The sequence of variables added to the model and their corresponding statistics are summarized in Table 5. If the next variable (Zn) is added to model, its coefficient becomes insignificant ( $p = 0.110$ ). The smallest value of the Wilks' partial lambda ( $\Lambda$ ) is for V (Table 5) indicating that this variable contributes the most to discriminating between the groups. The other variables that contribute to discrimination are, in

increasing order of  $\Lambda$  and decreasing order of importance, Sc, Sm, Ni, K<sub>2</sub>O, Ba, Ga, SiO<sub>2</sub>, Co, and CaO. The tolerances were > 0.10 indicating a maximum of 90% redundancy in the discriminating power of the variables.

Table 5. Discriminant function analysis summary for the four category model.

Step	Element Added	Wilks' $\Lambda$					Correlated Variables Removed
		Wilks' $\Lambda$	Partial $\Lambda$	F-Remove	p-Level	Tolerance	
1	V	0.00228	0.1910	35.29	3.8443E-09	0.2960	TiO <sub>2</sub> , MnO, Na <sub>2</sub> O, Cs
2	Sm	0.00173	0.2522	24.71	1.1951E-07	0.2305	La, Ce
3	Sc	0.00221	0.1972	33.93	5.6882E-09	0.1084	none
4	Ni	0.00153	0.2843	20.98	5.2351E-07	0.1600	none
5	Ga	0.00074	0.5843	5.93	3.3631E-03	0.2003	Al <sub>2</sub> O <sub>3</sub>
6	K <sub>2</sub> O	0.00125	0.3478	15.62	6.2577E-06	0.1174	none
7	Ba	0.00087	0.5018	8.27	5.4278E-04	0.1009	none
8	SiO <sub>2</sub>	0.00064	0.6794	3.93	1.9897E-02	0.3953	none
9	Co	0.00063	0.6952	3.65	2.5998E-02	0.2398	none
10	CaO	0.00061	0.7110	3.39	3.3710E-02	0.2979	none

The discriminant functions were calculated using canonical analysis. There are three roots that are calculated (number of categories – 1). All three of the roots are highly significant ( $p < 0.05$ ) and therefore all three roots (Table 6) should be considered.

Table 6. Chi-square tests for successive roots removed for the four-category model.

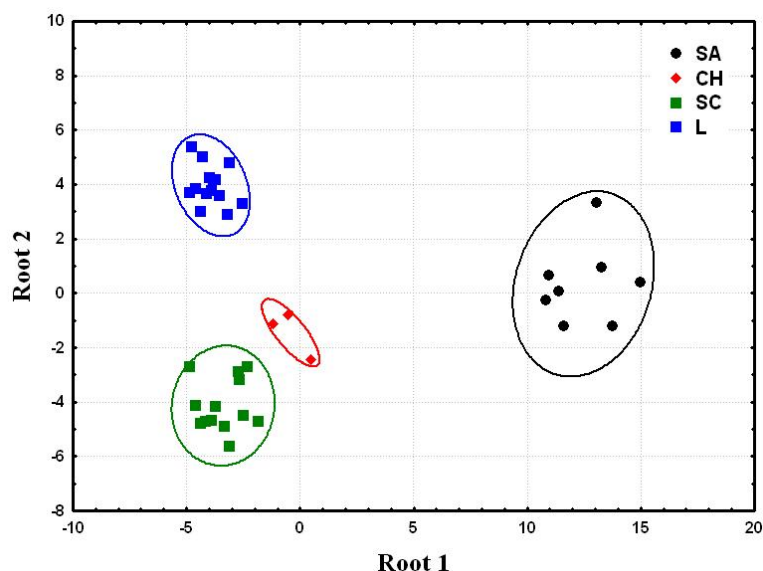
Roots Removed	Eigen-value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	47.36	0.9896	0.00044	232.19	30	0.000E+00
1	13.23	0.9642	0.02105	115.83	18	2.220E-16
2	2.34	0.8369	0.29958	36.16	8	1.641E-05

The standardized discriminant function coefficients (Table 7) suggest that the first root is dominated by Sc, Ni, and Ba. The second root is dominated primarily by Sm while the third root is dominated by Ga and K<sub>2</sub>O. The first discriminant function (root) accounts for 75% of the discriminatory power of the model. The addition of the second function accounts an additional 21% of the discriminatory power and the addition of the third function adds another ~4%. Thus the three roots account for nearly 100% of the variability.

Table 7. Raw and standardized discriminant function coefficients for the four category model.

Variable	Raw Coefficients for Canonical Variables			Standardized Coefficients for Canonical Variables		
	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3
V	-3.8913	-1.2663	2.52540	-1.41103	-0.45918	0.91575
Sm	1.0846	-1.4575	0.05126	1.10442	-1.48419	0.05220
Sc	-5.4607	2.1399	-1.63977	-2.49940	0.97945	-0.75053
Ni	-2.8046	0.3588	0.50912	-2.09650	0.26821	0.38058
Ga	0.0862	-0.7480	-5.86921	0.02502	-0.21703	-1.70306
K <sub>2</sub> O	2.5027	1.1602	1.81737	1.89425	0.87812	1.37552
Ba	-2.6596	-0.9911	-0.89945	-2.03836	-0.75957	-0.68935
SiO <sub>2</sub>	-4.2602	-13.5984	-5.34365	-0.26495	-0.84572	-0.33234
Co	-5.3605	-0.6265	-0.02755	-1.13199	-0.13230	-0.00582
CaO	-0.6967	0.6461	0.07810	-0.73666	0.68318	0.08258
Constant	100.7097	59.9839	29.89091	47.35652	13.23402	2.33797
Eigenval	47.3565	13.2340	2.33797	--	--	--
Cum.Prop	0.7525	0.9628	1.00000	0.7525	0.9628	1.00000

From the discriminant equations, the canonical scores for each of the samples can be calculated and plotted (Figure 4). The ellipses are based on the mean  $\pm 1.5$  times the range of the analytes as defined in the Statistica software package (StatSoft, 2006). From the scatter plots, one can see that root 1 is very good at discriminating South Africa samples from the others while root 2 is good at discriminating Libby samples from the South Carolina samples. The samples from China are best discriminated by root 3.



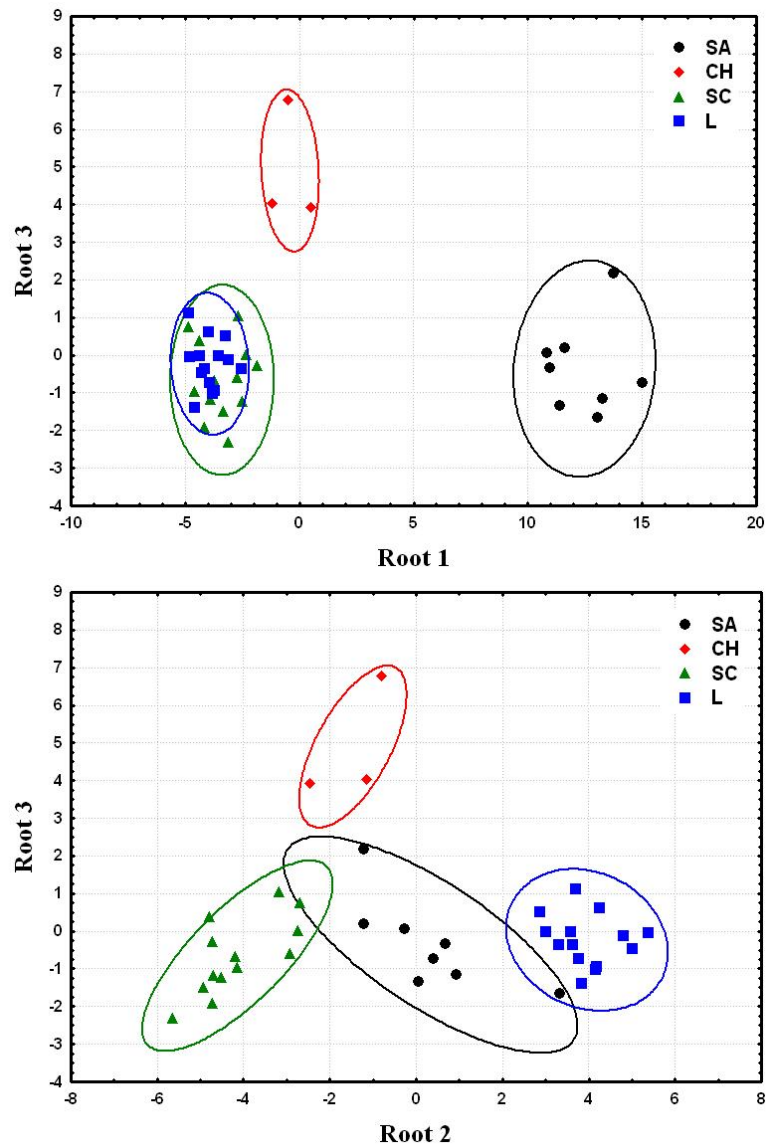


Figure 4. Scatter plots of canonical scores for data from the four-category method.

From the classification functions (Table 8), we can classify each of the samples used in discriminant analysis and calculate *post hoc* probabilities of the samples belonging to each of the categories. The result of these calculations is that 100% of the samples are correctly placed into each of their identified categories with probabilities close to unity. The 6 product samples discussed in the Cluster Analysis section were also classified. Samples P2 and P3 have near unit probabilities of belonging to categories SA and SC (South Africa and South Carolina), respectively. The remaining product samples (P1, P4, P5, P6) were identified as belonging to category L (Libby) (Table 9).

Table 8. Coefficients for the classification functions for the four-category model.

	<b>SA</b> <b>p=.21053</b>	<b>CH</b> <b>p=.07895</b>	<b>SC</b> <b>p=.34211</b>	<b>L</b> <b>p=.36842</b>
<b>V</b>	359.9	425.6	426.6	419.5
<b>Sm</b>	-5.5	-16.6	-16.2	-28.5
<b>Sc</b>	486.2	544.1	563.7	583.3
<b>Ni</b>	224.5	262.7	267.2	271.9
<b>Ga</b>	224.8	194.1	228.5	219.9
<b>K<sub>2</sub>O</b>	-237.9	-262.7	-283.4	-274.6
<b>Ba</b>	311.1	342.4	357.9	351.0
<b>SiO<sub>2</sub></b>	3064.2	3115.5	3194.3	3084.1
<b>Co</b>	991.0	1061.1	1078.9	1076.7
<b>CaO</b>	22.9	31.1	31.0	36.7
<b>Constant</b>	-10634.1	-11818.6	-12444.9	-12001.1

Table 9. Classification probabilities for the six products for the four-category model.

<b>Sample</b>	<b>SA</b> <b>p=.21053</b>	<b>CH</b> <b>p=.07895</b>	<b>SC</b> <b>p=.34211</b>	<b>L</b> <b>p=.36842</b>
<b>P1</b>	0.00000	0.00000	0.00000	1.00000
<b>P2</b>	1.00000	0.00000	0.00000	0.00000
<b>P3</b>	0.00000	0.00000	1.00000	0.00000
<b>P4</b>	0.00000	0.00000	0.00000	1.00000
<b>P5</b>	0.00000	0.00000	0.00000	1.00000
<b>P6</b>	0.00000	0.00089	0.00110	0.99800

There are twenty additional vermiculite samples from Gunter et al. (2005) that could be used to test the general approach used in this study. Because that work used a different set of elements than our present study, the discriminant model was reconstructed using data generated from our samples with a common set of elements. The twenty additional samples were then classified and compared to the results reported by Gunter et al. (2005). The common elements between our data set and the two data sets provided by Gunter et al. (2005) are SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, MgO, CaO, Na<sub>2</sub>O, K<sub>2</sub>O, P<sub>2</sub>O<sub>5</sub>, Ba, Cr, Cu, Ga, Nb, Ni, Pb, Rb, Sr, V, Y, Zn, and Zr. The elements P<sub>2</sub>O<sub>5</sub>, Nb, Pb, and Y were not included in the analyses because some of the samples had concentrations of these analytes below the detection limit. Zr and Hf were not included because the samples were potentially contaminated by the zirconia grinder. Rb, Sr, and MgO were not included in the analysis because of the significant correlations between the means and the variances for the elements. Therefore, the final set of elements used in the analysis were SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, CaO, Na<sub>2</sub>O, K<sub>2</sub>O, Ba, Cr, Cu, Ga, Ni, Sr, V, and Zn. The values used were the ln-transformed concentrations reported in Table 2.

The elements were sequentially added to determine the smallest subset of variables that would present information useful for classification of those samples. Details of the model can be found in Appendix D. The selected elements included by increasing values of Wilks' partial  $\Lambda$  (hence, in descending order of importance in the model) are Cr, V, Ga, Zn, Sc, FeO\*, K<sub>2</sub>O and SiO<sub>2</sub>. All of the coefficients are statistically significant ( $p < 0.05$ ). All three roots (number of categories – 1) are highly significant ( $p \ll 0.05$ ) indicating that all three roots need to be considered. The standardized discriminant function



coefficients suggest that the first root is dominated by V and Cr and accounts for 65% of the discriminatory power of the model. The second root is dominated by Zn and FeO and accounts for an additional 26% to the discriminatory power of the model while the inclusion of the third root, which is dominated by Ga provides an additional 9% to the discriminatory power. The first root discriminates the SA and SC samples from the CH and L samples while the second root discriminates the L samples from the other samples. The third root discriminated the CH samples from the others. *Post hoc* classification of the samples used to define the model showed that 100% of the samples were correctly classified.

The twenty additional samples from the Gunter data set included two vermiculite samples known to be from Libby, one known to be from South Africa, five known to be from South Carolina, four samples of attic insulation and seven samples of garden products. Results of the *posteriori* classification of these samples are shown in Table 10. Using discriminant analysis, samples of definite known origin from Libby, South Africa, and South Carolina are classified correctly with a probability of model agreement near unity. Garden samples are all ascribed a South African origin, while three of four attic samples are attributed to Libby. The fourth sample, at3UI was classified as originating from South Carolina.

Most of the results of the discriminant analysis results concur with those of Gunter's cluster analysis with a few exceptions. Gunter et al.'s (2005) cluster analysis had several outliers, indicated by a "?" under "Gunter Classification" in Table 10. Discriminant analysis showed that samples SC3UI and SC5UI came from South Carolina, consistent with their reported origin. Samples ga5UI and ga6UI come from South Africa, which is consistent with the remaining garden samples, although the origin of these products cannot be confirmed by the manufacturers. Discriminant analysis of samples at1UI, at2UI and at4UI agree with Gunter's assessment of a Libby origin; however sample at3UI differs. Discriminant analysis suggests this sample comes from South Carolina while Gunter's cluster analysis suggests a Libby origin. One of the key differences between the ore body in Libby versus the ore bodies in South Carolina is the presence of monazite in the latter. Because La, Ce, and Sm strongly partition into Monazite, the use of one of these elements in discriminant analysis should more accurately separate Libby samples from those originating from South Carolina. The use of Libby vermiculite as attic insulation is relatively common in the Pacific Northwest and Intermountain West. Therefore it is more likely that sample at3UI is from Libby, although this cannot be confirmed.

While both of these models appear to do very well in the *post-posteriori* classification of the samples, there are differences between the two. The first model, which uses more analytes, shows greater separation of the groups in the plots of the various roots than does the second model. In addition, the square of the Mahalanabis distances between the groups are greater for the first model than for the second model. Further, the eigen values, canonical R values, and  $\chi^2$  values for the first model are larger and the L values are smaller than the model based on the set of elements that are common with Gunter et al. (2005). A key finding is that the China samples can be differentiated from the other three groups using discriminant analysis, regardless of which of the two sets of analytes are used.

Table 10. *Posteriori* classification probabilities for the samples analyzed by Gunter et al. (2005) using discriminant analysis. Li= samples from Libby; at= attic samples; ga= garden products; SA= samples from South Africa, and SC= samples from South Carolina.

Sample	Reported Category	Gunter Class-ification	This Work Class-ification	SA p=.21053	CH p=.07895	SC p=.34211	L p=.36842
Li1UI	L	L	L	0.000000	0.000000	0.000001	0.999999
Li2UI	L	L	L	0.000000	0.000000	0.000002	0.999998
at1UI	?	L	L	0.000000	0.000000	0.000224	0.999776
at2UI	?	L	L	0.000000	0.000000	0.015272	0.984728
at3UI	?	L	SC	0.000000	0.000707	0.999140	0.000152
at4UI	?	L	L	0.000000	0.000000	0.000055	0.999945
ga1UI	?	SA	SA	1.000000	0.000000	0.000000	0.000000
ga2UI	?	SA	SA	1.000000	0.000000	0.000000	0.000000
ga3UI	?	SA	SA	1.000000	0.000000	0.000000	0.000000
ga4UI	?	SA	SA	1.000000	0.000000	0.000000	0.000000
ga5UI	?	?	SA	1.000000	0.000000	0.000000	0.000000
ga6UI	?	?	SA	1.000000	0.000000	0.000000	0.000000
ga7UI	?	SA	SA	1.000000	0.000000	0.000000	0.000000
SA1UI	SA	SA	SA	1.000000	0.000000	0.000000	0.000000
SC1UI	SC	SC	SC	0.000000	0.000000	0.999998	0.000002
SC2UI	SC	SC	SC	0.000000	0.000000	1.000000	0.000000
SC3UI	SC	?	SC	0.000000	0.000059	0.999927	0.000014
SC4UI	SC	SC	SC	0.000000	0.000000	0.999999	0.000001
SC5UI	SC	?	SC	0.000000	0.000000	1.000000	0.000000
SC6UI	SC	SC	SC	0.000000	0.000000	1.000000	0.000000

#### 4.4.2 Six-Category Models

Our four-category discriminant analysis approach to classify vermiculite samples as being from Libby, South Carolina, South Africa or China is useful; however, it does appear to contradict results from the probability plots and the cluster analysis, which suggest that the Libby and South Carolina groups are comprised of two subgroups. To develop a more consistent model, we performed discriminant analysis on the five groups defined by the k-cluster analysis. In addition, we considered the China samples to be a sixth group to determine if it was possible to discriminate these samples from the other groups. More detailed results of the discriminant analysis are provided in Appendix E. The sequence of variables added to the model (Table 11) is similar to that of the four-category system except that Zn is added and Co and SiO<sub>2</sub> are removed. When the next variable (SiO<sub>2</sub>) is added, both SiO<sub>2</sub> and Co become insignificant ( $p > 0.05$ ). The Wilks' partial  $\Lambda$  is smallest for K<sub>2</sub>O indicating that this variable has the greatest discriminating power. This is followed by Sm, Ba, Ni, Sc, and V in decreasing discriminatory power. In general, the tolerances are much larger in the six category model than in the four-category model suggesting less redundancy in the variables. All five roots calculated by the canonical analysis (Table 12) are highly significant ( $p \ll 0.05$ ) and therefore all five roots should be considered.

Table 11. Discriminant function analysis summary for six-category model.

Step	Element	Wilks'		F-Remove	p-Level	Tolerance	Correlated Variables
	Added	Wilks' $\Lambda$	Partial $\Lambda$				Removed
1	V	0.000071	0.3299	9.75	3.4945E-05	0.5914	TiO <sub>2</sub> , MnO, Na <sub>2</sub> O, Cs
2	Sm	0.000130	0.1804	21.81	3.2768E-08	0.2678	La, Ce
3	Sc	0.000079	0.2975	11.33	1.0769E-05	0.2285	none
4	Ni	0.000083	0.2846	12.07	6.4804E-06	0.5591	none
5	Ga	0.000049	0.4821	5.16	2.3657E-03	0.3182	Al <sub>2</sub> O <sub>3</sub>
6	K <sub>2</sub> O	0.000140	0.1672	23.90	1.3487E-08	0.0971	none
7	Ba	0.000088	0.2673	13.16	3.1512E-06	0.1182	none
8	Zn	0.000038	0.6215	2.92	3.3653E-02	0.5474	Cu
9	CaO	0.000037	0.6376	2.73	4.3387E-02	0.6756	none

Table 12. Chi-square tests for successive roots removed for the six-category model.

Roots Removed	Eigen-value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	58.62	0.9916	0.000023	314.44	45	0.000E+00
1	14.56	0.9673	0.001400	193.84	32	0.000E+00
2	6.10	0.9269	0.021797	112.87	21	1.432E-14
3	2.52	0.8459	0.154733	55.05	12	1.774E-07
4	0.84	0.6753	0.543986	17.96	5	2.996E-03

The standardized discriminant function coefficients (Table 13) suggest that the first root is dominated by K<sub>2</sub>O and Ba while the second root is dominated by Sm. The third root appears to be an even combination of Ga, Ni, K<sub>2</sub>O, and Zn. The fourth and fifth roots are dominated by Ga and K<sub>2</sub>O, respectively. The first discriminant function accounts for 71% of the discriminatory power of the model. The successive addition of the second through the fifth roots respectively adds about 18, 7, 3, and 1% to the discriminatory power of the model. One obvious feature of the six-category model is that both Libby and South Carolina are split into two subgroups while China and South Africa remain as intact groups. This discriminatory power is particularly well-shown in the scatter plots of root 2 versus root 1 and root 3 versus root 1 (Figure 5). The first root helps to discriminate SA and SC-1 from the other groups, and CH from SC-1, SC-2, and SA. The second root helps to discriminate L-1 and L-2 from the other categories and to a lesser extent SC-1 from the other categories. Root 3 helps to discriminate CH from the other categories, L-2 from L-1, and SC-1 from SC-2.

Table 13. Raw and standardized discriminant function coefficients for six-category model.

Variable	Raw Coefficients for Canonical Variables					Standardized Coefficients for Canonical Variables				
	Root 1	Root 2	Root 3	Root 4	Root 5	Root 1	Root 2	Root 3	Root 4	Root 5
V	3.3563	0.39857	-1.44983	-1.5702	-0.47662	0.92563	0.10992	-0.399840	-0.433042	-0.131446
Sm	-0.4869	-1.87879	0.28619	-0.0510	0.10142	-0.44814	-1.72909	0.263382	-0.046966	0.093335
Sc	3.8683	2.80671	0.34535	0.7660	0.57451	1.42188	1.03168	0.126943	0.281567	0.211174
Ni	1.1534	1.47644	-1.77744	0.2454	0.97284	0.51028	0.65321	-0.786381	0.108565	0.430406
Ga	1.0671	-0.59144	4.31377	4.5631	3.39753	0.22201	-0.12305	0.897485	0.949367	0.706860
K <sub>2</sub> O	-3.8471	1.30642	-1.13204	-0.9936	-1.48704	-2.58650	0.87832	-0.761091	-0.668008	-0.999762
Ba	3.9162	-1.55981	0.93788	0.4717	-0.04087	2.28060	-0.90835	0.546171	0.274676	-0.023798
Zn	1.7426	-0.56622	-3.24252	0.6700	-1.79642	0.38576	-0.12534	-0.717797	0.148309	-0.397674
CaO	0.0815	0.65764	0.37499	-0.8376	0.33780	0.05726	0.46218	0.263539	-0.588639	0.237399
Constant	-64.4706	-3.63909	12.18640	-12.1094	-3.46947	58.62018	14.56459	6.098748	2.515637	0.838283
Eigenval	58.6202	14.56459	6.09875	2.5156	0.83828	--	--	--	--	--
Cum.Prob.	0.7094	0.88561	0.95941	0.9899	1.00000	0.70937	0.88561	0.959414	0.989856	1.000000

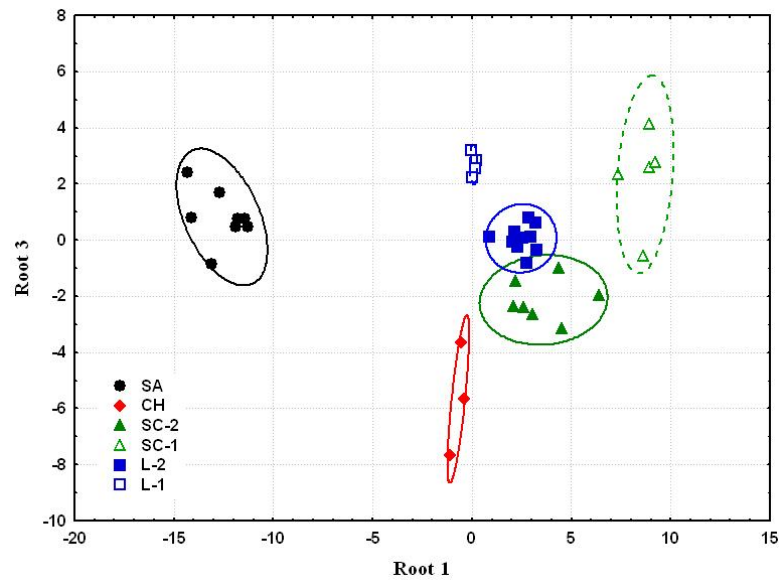
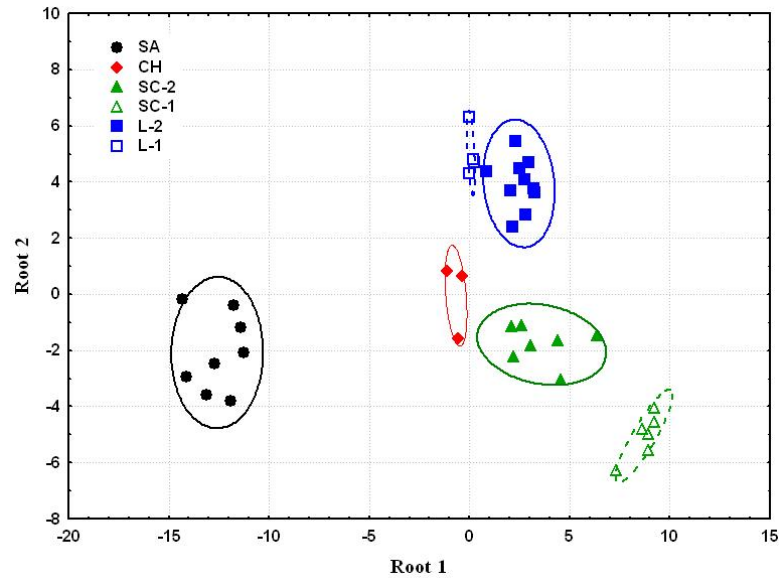


Figure 5. Scatter plots of canonical scores for data from six-category model. Ellipses define the compositional fields of each category based on their roots.

Using the six-category classification functions (Table 14), we classified *posteriori* probabilities for each of the samples used in the discriminant analysis and calculated probabilities of the 6 product samples belonging to each of the categories (Table 15). Included in the analysis was sample SA GS-1, which was classified as a Libby sample under some of the cluster analysis scenarios. As with the four-category model, 100% of the six-category samples were correctly identified with the probabilities in all cases being very close to unity. Thus, the model is able to distinguish each of the groups extremely well. The results for the products were identical to the results obtained by both cluster analysis and the four-category discriminant analysis.

Table 14. Coefficients for the classification functions for the six-category model.

	SA p=.21053	CH p=.07895	SC-2 p=.18421	SC-1 p=.15789	L-2 p=.26316	L-1 p=.10526
V	128.17	183.47	185.69	197.76	180.63	175.41
Sm	-40.14	-51.57	-49.28	-44.53	-59.11	-58.86
Sc	256.31	302.87	319.79	329.99	331.81	323.98
Ni	106.25	132.60	132.11	123.01	133.71	128.13
Ga	97.86	64.02	108.68	123.44	110.78	103.78
K <sub>2</sub> O	-199.61	-230.72	-260.67	-285.53	-249.28	-239.38
Ba	205.23	241.25	265.47	294.33	254.64	243.96
Zn	197.56	238.29	232.72	231.91	224.41	205.64
CaO	11.73	13.67	12.38	12.56	15.54	21.17
Constant	-1732.97	-2486.32	-2752.38	-3037.80	-2677.25	-2456.60

Table 15. *Posteriori* calculations of classification probabilities for the six products and SA GS-1 using the six-category model.

Posterior Probabilities (fprintrace1n1) Incorrect classifications are marked with *							
Samples	Observed Classif.	CH p=.07895	L1 p=.10526	L2 p=.26316	SA p=.21053	SC1 p=.15789	SC2 p=.18421
P1	---	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
P2	---	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
P3	---	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
P4	---	0.00000	0.00007	0.99993	0.00000	0.00000	0.00000
P5	---	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
P6	---	0.00000	0.15155	0.84141	0.00000	0.00000	0.00703

The six-category model was also run using the Gunter et al., (2005) analyte set. The analytes in the model in order of decreasing importance (increasing Wilks' partial  $\Lambda$ ) are Ni, Zn, V, SiO<sub>2</sub>, FeO, Ga, Sc, and Ba. All five roots are highly significant ( $p < 0.05$ ) and therefore all five should be considered. The standardized coefficients for the canonical variables suggest that the first, second, third, and fourth roots are dominated by FeO, Zn, Ni, and Ga, respectively while the fifth root is evenly influenced by SiO<sub>2</sub>, Ga, FeO, and Zn. For the samples used to define the model, 100% were properly classified. For the 20 Gunter et al. (2005) samples, all of the samples that were classified as SA in the four-category model

were also classified as SA in the six-category model. The samples that were classified as L samples in the four-category model were classified as L2 in the six-category model. The samples previously classified as SC were classified as SC2 in the six-category model except for SC5UI, which was classified as SC1. Sample at3UI which was classified as SC in the four-category model has a 61% chance of being classified as CH, 35% of being SC2, and 4% of being L2 in the six-category model, thus it likely remains incorrectly classified.

As with the four-category model, the six-category model using the analyte set in this study appears to have greater discriminating power than the model based on the Gunter et al. (2005) analyte set. Again, this conclusion is based on the better separation of the groups in the plots of the various roots, the larger squared Mahalanobis distances, and the smaller Wilks'  $\Lambda$ .

## 5. Conclusions

Several conclusions can be drawn from this work:

1. Probability plots suggest that there may be at least two subcategories in the Libby and the South Carolina samples.
2. While cluster analysis is useful for group classification when there is no prior knowledge about a sample's origin, it is highly sensitive to the linkage algorithm chosen. Ward's linkage provides the best classification for this data set based upon knowledge of the origin of the samples.
3. Because of the differences in accessory minerals between the deposits, REE's such as La, Ce, Sc, and Sm are highly diagnostic in provenance determination. Samarium is particularly useful for distinguishing samples from Libby as Sm is highly depleted in Libby samples.
4. Hierarchical cluster analysis and k-cluster analysis both support that notion that there are subcategories in the Libby and South Carolina samples.
5. Cluster analysis suggests that there are five clusters in the data corresponding to the South African samples, two subsets of the Libby samples, a subset of the South Carolina samples, and a second subset of the South Carolina samples combined with the China samples. Neither the hierarchical nor the non-hierarchical clustering methods tested in this work was capable of distinguishing the China samples from the South Carolina samples.
6. Discriminant analysis shows that the four ore provenances can be uniquely differentiated from one another using three roots (factors). The first root is dominated by Sc, Ni, and Ba, the second root by Sm, and the third root by Ga and  $K_2O$ . Unknown samples can be assigned a provenance with a probability computed from the model.
7. Discrimination can also be achieved using a subset of our analytes that is consistent with that used by Gunter et al. (2005). The first root is dominated by V and Cr, the second by Zn and FeO, and the third root is dominated by Ga. The discrimination is better with our full analyte set than with the smaller analyte set of Gunter et al. (2005).
8. Groups from a six-category model comprised of one group of South Africa samples, one group of China samples, two groups of Libby samples, and two groups of South Carolina samples can be differentiated using five roots for both the full analyte set and the reduced analyte set.
9. The six-category discriminant model developed from the full analyte set best represents the data since it is consistent with the results of the probability plots and the cluster analysis. For the five

roots describing this model, the first root is dominated by  $K_2O$  and Ba while the second root is dominated by Sm. The third root appears to be an even combination of Ga, Ni,  $K_2O$ , and Zn. The fourth and fifth roots are dominated by Ga and  $K_2O$ , respectively.

10. Because discriminant analysis allows for clear differentiation between the primary suppliers of vermiculite to the United States, this technique can be used to determine the relative risk of asbestos contamination based on the ore's origin.



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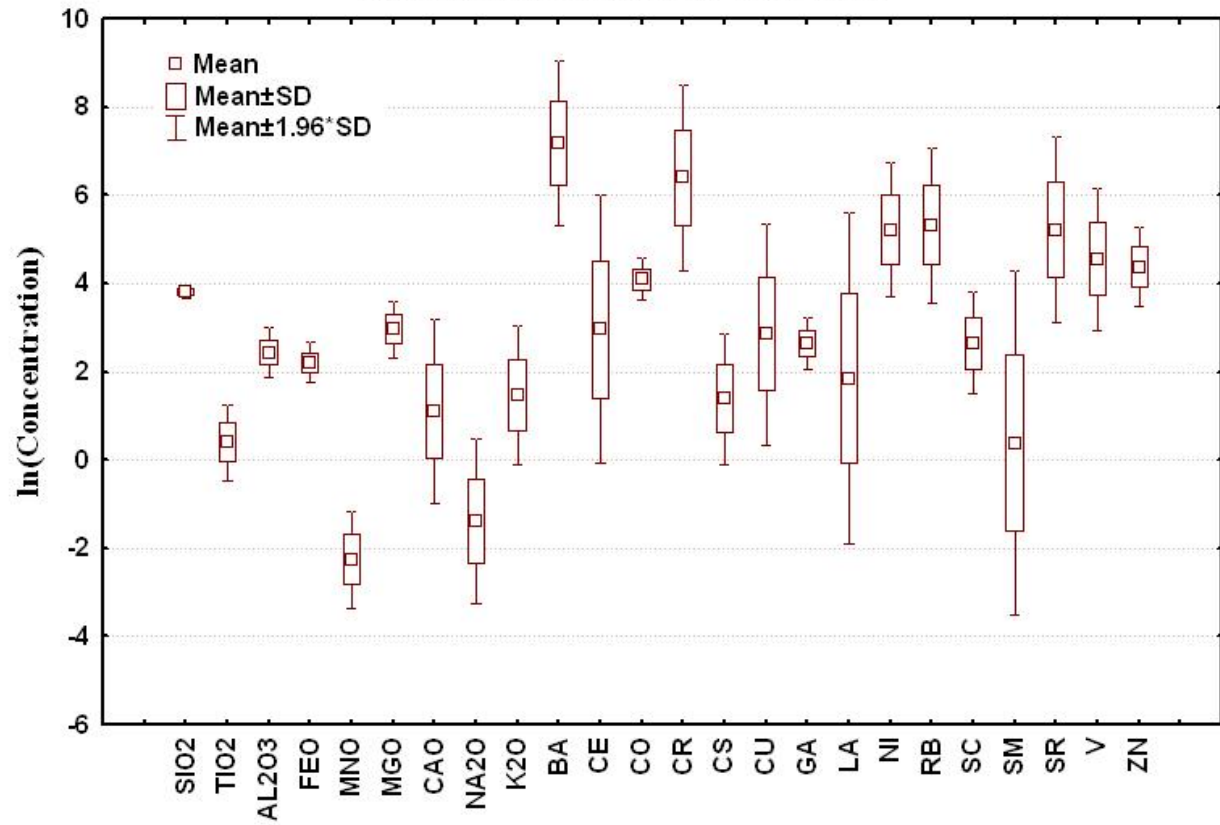
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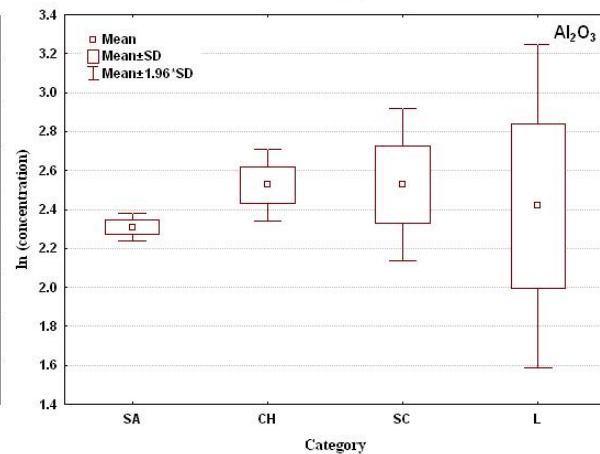
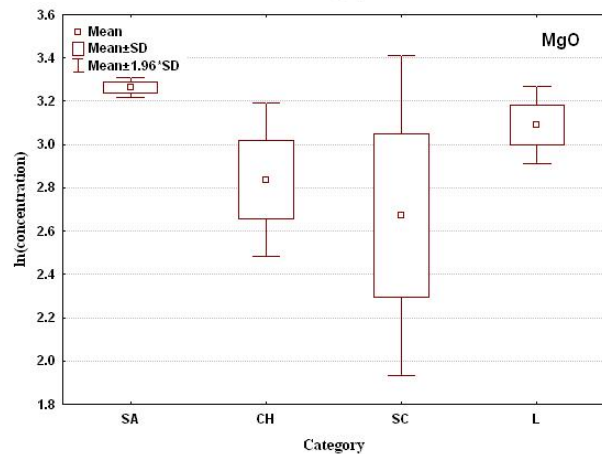
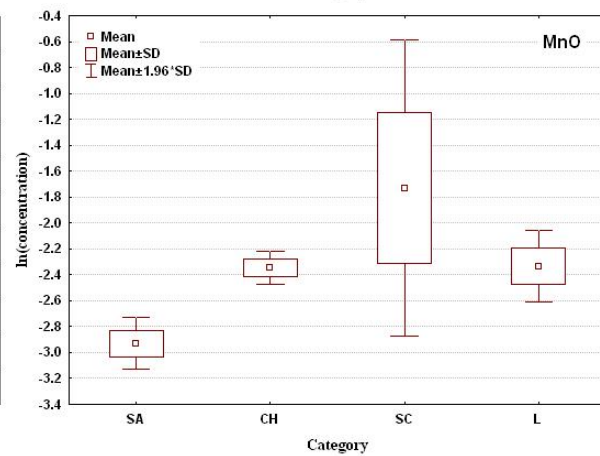
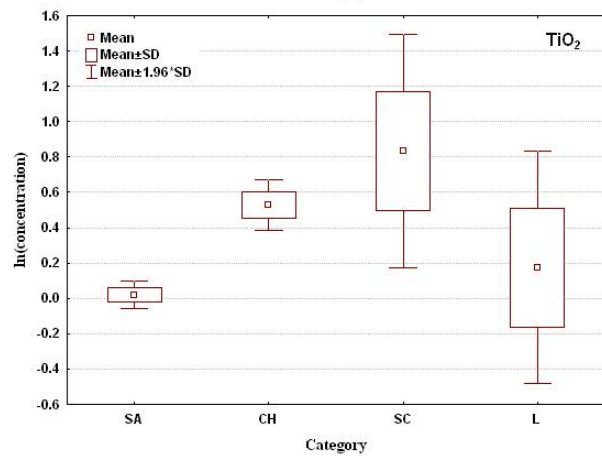
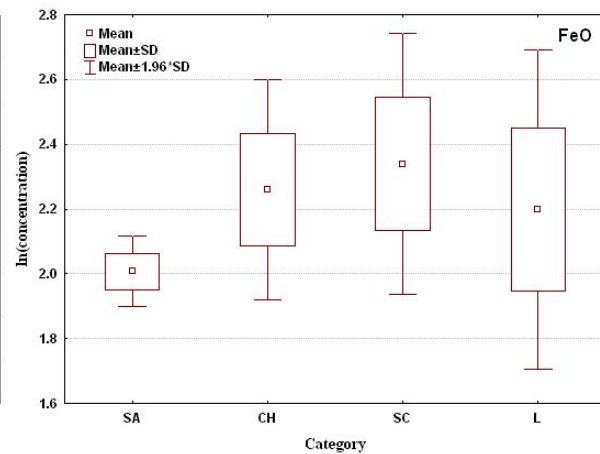
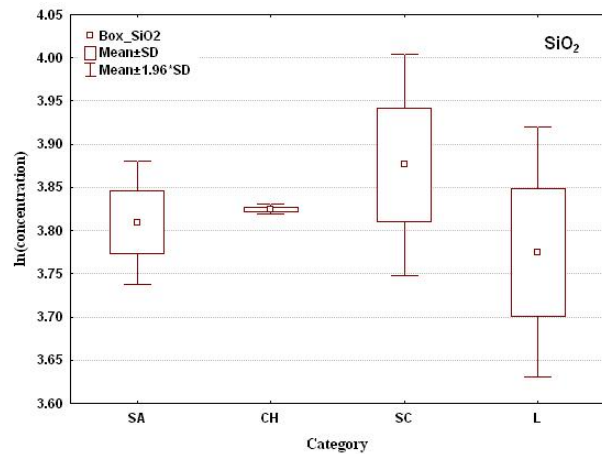
## **Appendix A**

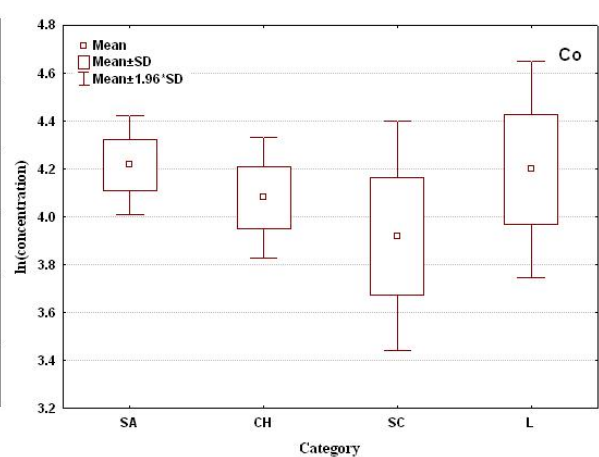
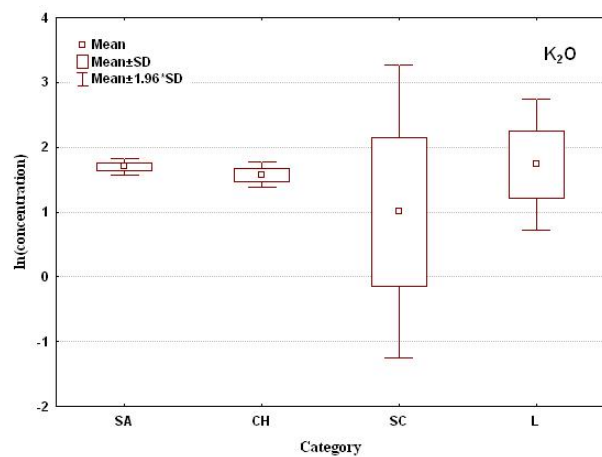
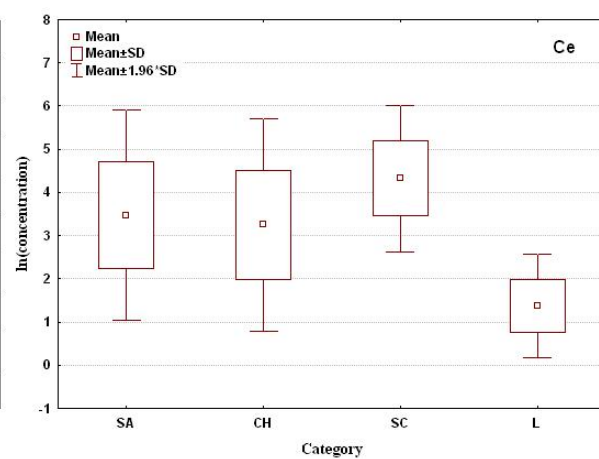
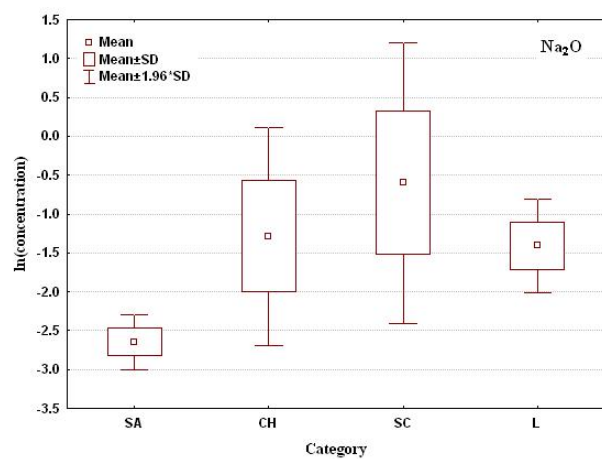
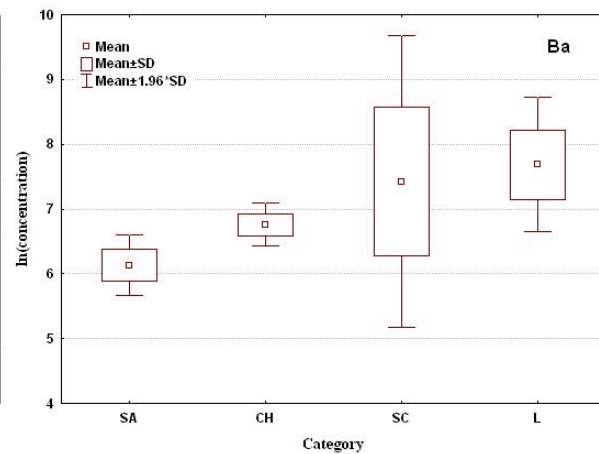
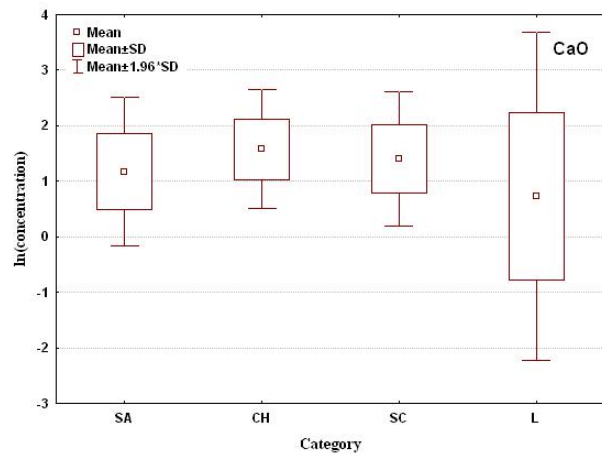
### **Summary Statistics**

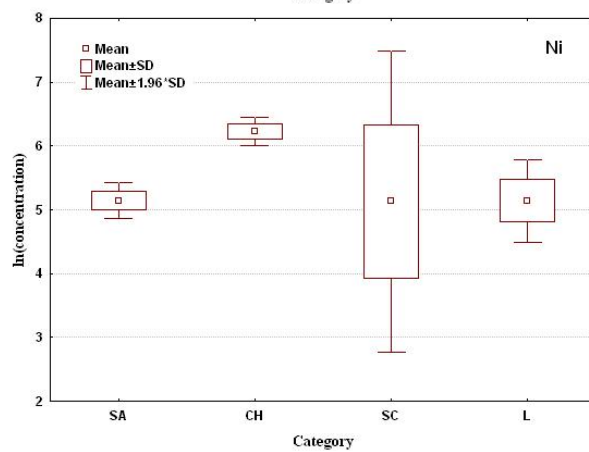
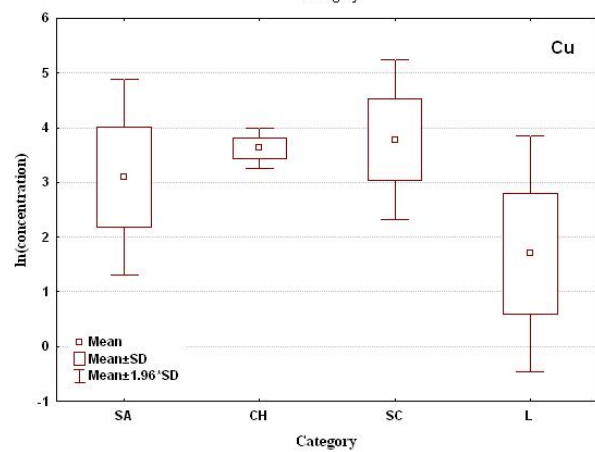
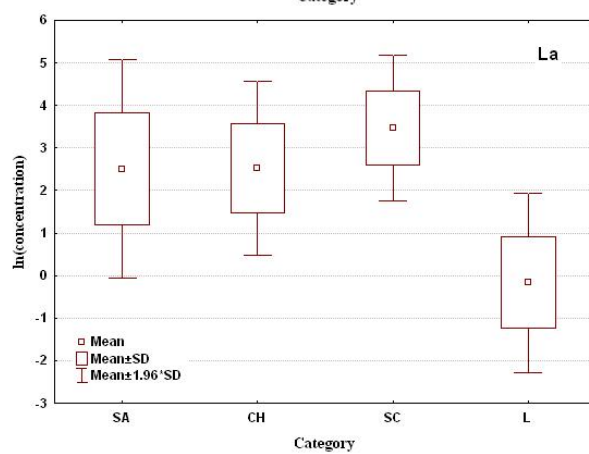
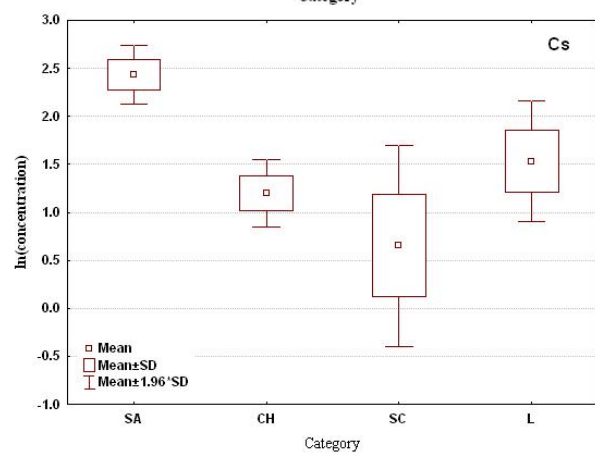
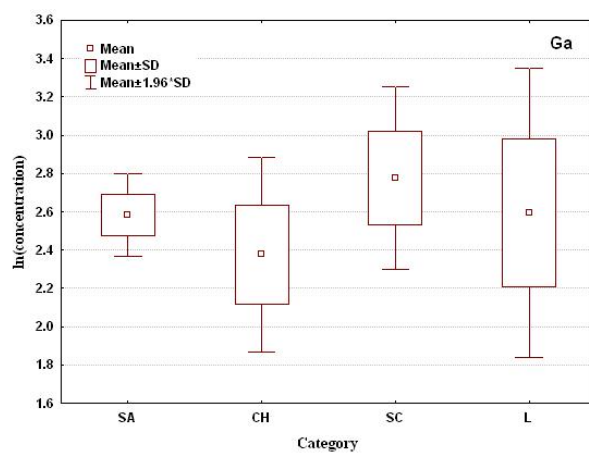
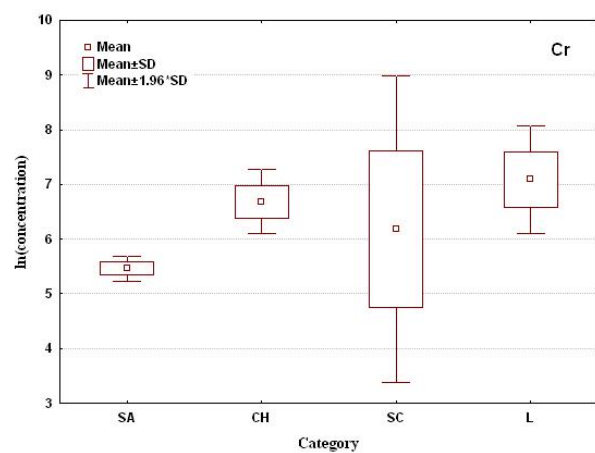
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	SA	CH	SC	L	All Grps	SA	CH	SC	L	All Grps
SiO <sub>2</sub>	3.809336	3.824437	3.876127	3.775172	3.820791	0.036393	0.002959	0.065377	0.073868	0.073805
TiO <sub>2</sub>	0.019161	0.526237	0.832966	0.174501	0.394831	0.038896	0.073812	0.336246	0.334960	0.440907
Al <sub>2</sub> O <sub>3</sub>	2.309454	2.526623	2.527845	2.418164	2.441362	0.035489	0.094341	0.198985	0.423765	0.289549
FeO	2.007325	2.259633	2.339286	2.197073	2.210717	0.055912	0.172765	0.205457	0.251465	0.230499
MnO	-2.92791	-2.347197	-1.731358	-2.333723	-2.253806	0.101930	0.066166	0.583250	0.139198	0.562687
MgO	3.263843	2.83723	2.6718	3.091087	2.963975	0.023249	0.181455	0.376797	0.090707	0.328133
CaO	1.168229	1.574407	1.402339	0.721355	1.115748	0.683581	0.544937	0.615059	1.509077	1.064260
Na <sub>2</sub> O	-2.645446	-1.28382	-0.6024	-1.405935	-1.382351	0.181115	0.715569	0.920588	0.306553	0.949446
K <sub>2</sub> O	1.698371	1.572483	1.003054	1.732347	1.463079	0.067733	0.098832	1.152936	0.516970	0.800711
Ba	6.132254	6.760357	7.419586	7.684082	7.19397	0.240951	0.170802	1.149478	0.529929	0.952631
Ce	3.468545	3.24667	4.328862	1.37216	2.972995	1.239657	1.256664	0.864760	0.609245	1.556853
Co	4.216251	4.080523	3.918926	4.198216	4.097175	0.105580	0.128716	0.244393	0.230095	0.243089
Cr	5.459674	6.682223	6.175684	7.087224	6.400765	0.119716	0.298661	1.431814	0.498026	1.074054
Cs	2.433655	1.197877	0.650945	1.532221	1.394112	0.158796	0.177211	0.533973	0.320387	0.757001
Cu	3.091361	3.62608	3.777596	1.696444	2.854424	0.915008	0.186601	0.747994	1.098601	1.277897
Ga	2.582509	2.376433	2.776348	2.593058	2.63644	0.108983	0.260299	0.243696	0.385449	0.301780
La	2.507134	2.527397	3.471813	-0.169074	1.852784	1.310820	1.040573	0.873723	1.077766	1.906974
Ni	5.144616	6.231283	5.132414	5.139552	5.224366	0.145111	0.114894	1.203971	0.331659	0.776365
Rb	6.044889	5.817997	4.669662	5.413933	5.324046	0.070449	0.192611	1.162617	0.495495	0.901794
Sc	1.997867	2.165887	2.901339	2.897751	2.651751	0.286453	0.245814	0.450425	0.554214	0.590661
Sm	1.287143	1.121356	2.027684	-1.822001	0.381924	1.329488	0.988971	0.842625	0.976860	1.990945
Sr	4.788851	6.037207	5.977637	4.569326	5.213218	0.697986	1.020647	1.142948	0.615129	1.077622
V	3.289914	5.119277	5.285472	4.481825	4.556153	0.422829	0.046011	0.435091	0.269377	0.825948
Zn	4.293389	4.87699	4.743639	3.985201	4.379952	0.110496	0.160769	0.246413	0.400474	0.454791
Valid N	8	3	13	14	38	8	3	13	14	38

**Box & Whisker Plot**  
**Summary for all groups combined**

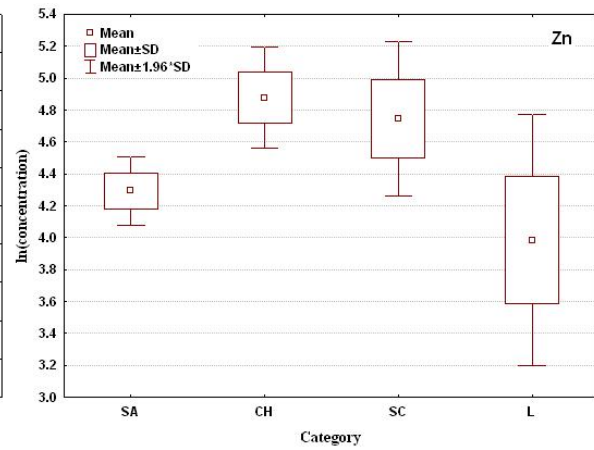
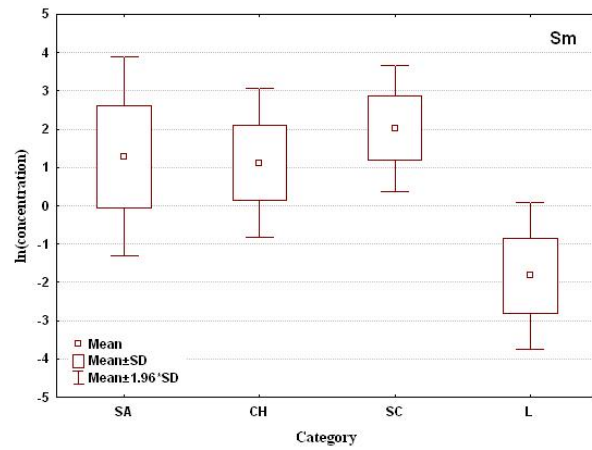
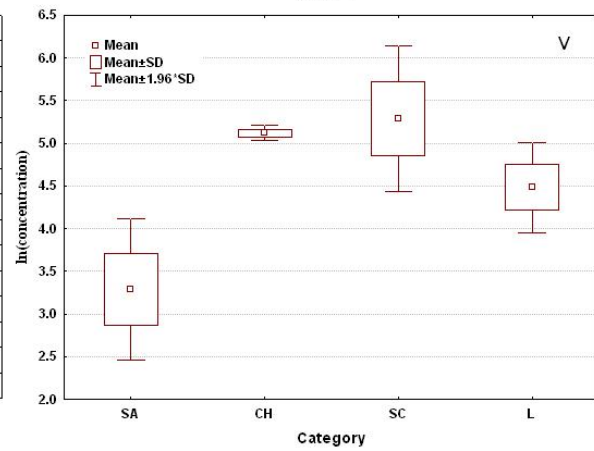
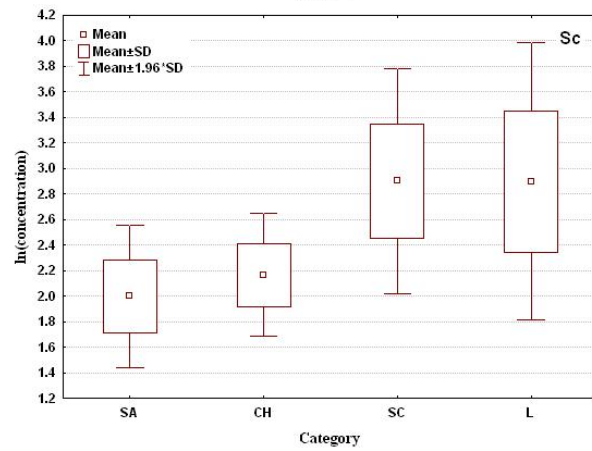
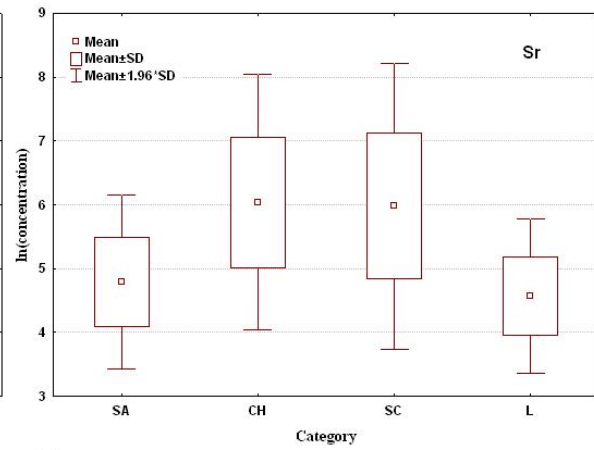
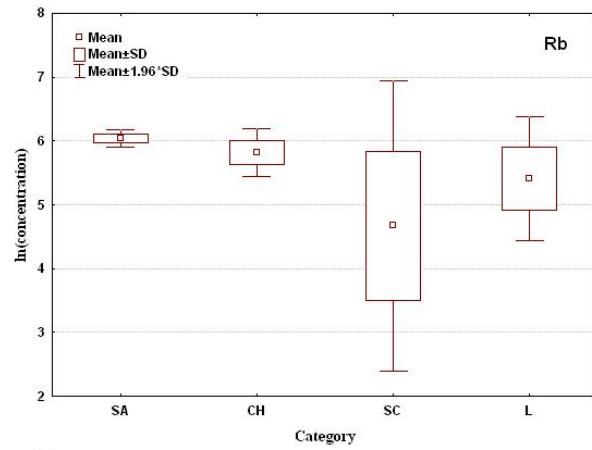








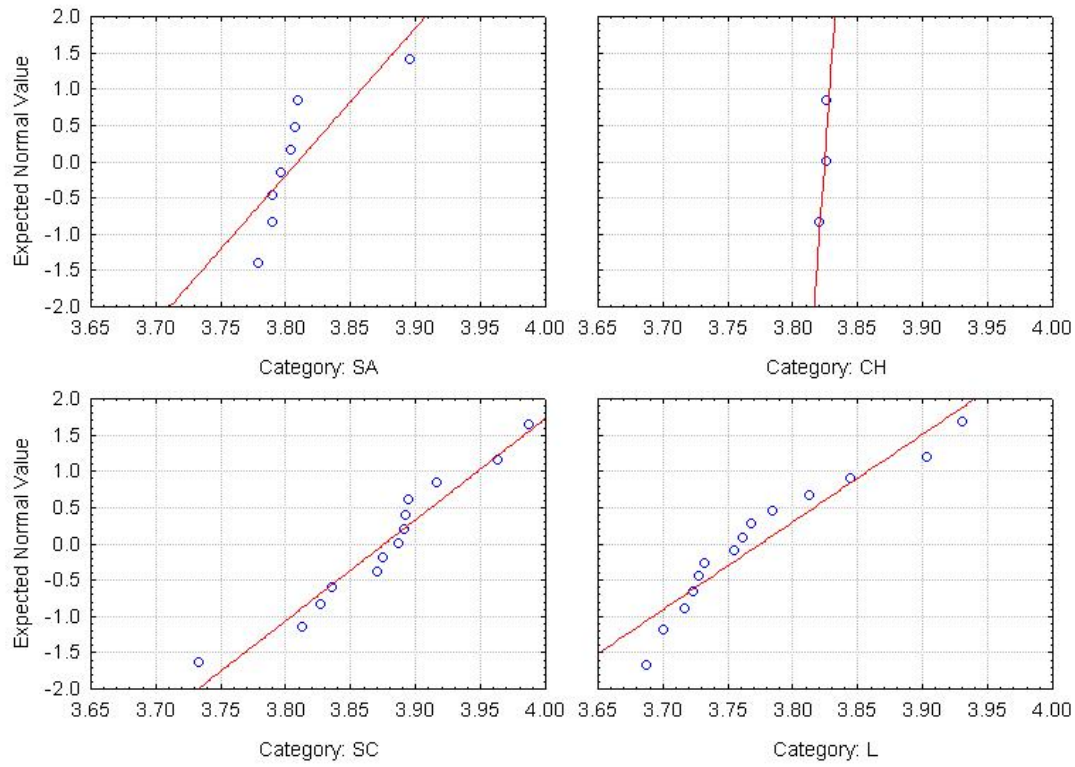




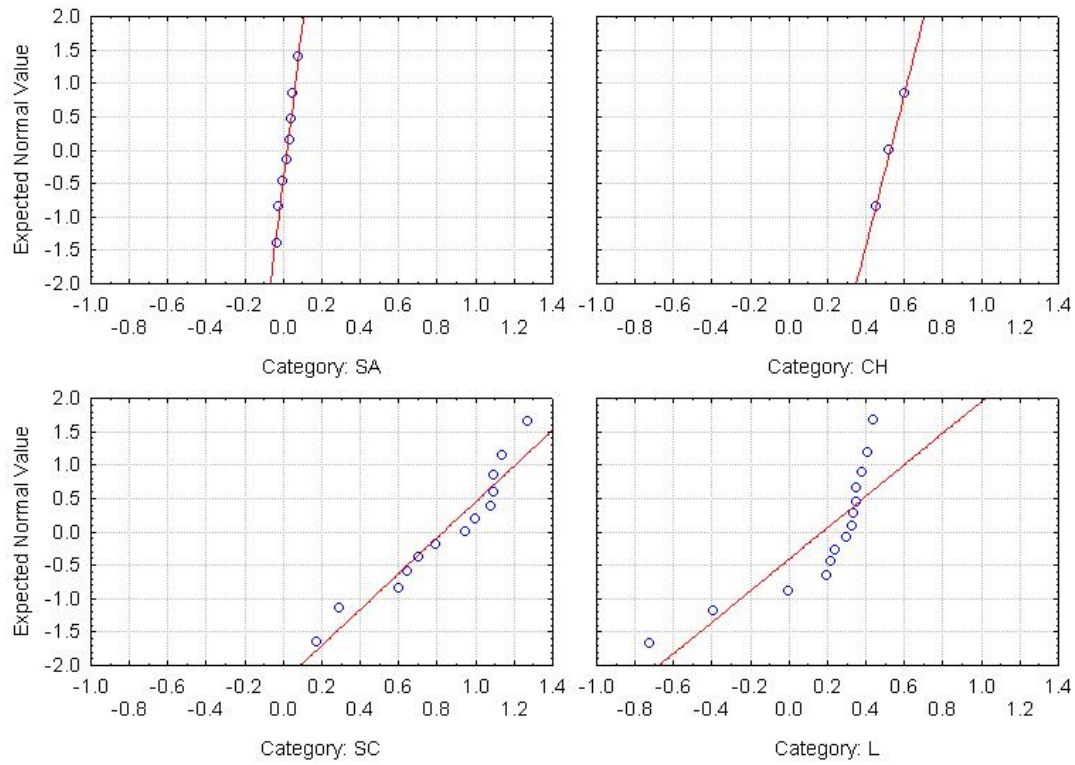
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SiO <sub>2</sub>	1.000																					
TiO <sub>2</sub>	0.029	1.000																				
Al <sub>2</sub> O <sub>3</sub>	-0.391	0.696	1.000																			
FeO	-0.259	0.746	0.620	1.000																		
MnO	0.280	0.576	0.282	0.486	1.000																	
MgO	-0.385	-0.636	-0.114	-0.399	-0.541	1.000																
CaO	0.514	-0.116	-0.582	-0.276	0.011	-0.402	1.000															
Na <sub>2</sub> O	0.298	0.489	0.135	0.197	0.399	-0.633	0.236	1.000														
K <sub>2</sub> O	-0.584	0.040	0.431	-0.032	-0.208	0.047	-0.353	-0.126	1.000													
P <sub>2</sub> O <sub>5</sub>	0.369	0.289	-0.108	0.043	0.126	-0.461	0.517	0.054	-0.226	1.000												
Ba	-0.293	0.534	0.592	0.390	0.316	-0.426	-0.257	0.560	0.550	-0.380	1.000											
Ce	0.355	0.417	-0.003	0.113	0.191	-0.511	0.395	0.213	-0.211	0.851	-0.217	1.000										
Co	-0.512	-0.322	0.226	0.079	-0.200	0.634	-0.493	-0.594	0.140	-0.450	-0.142	-0.597	1.000									
Cr	-0.089	-0.052	0.305	-0.125	0.134	0.350	-0.381	0.125	0.166	-0.587	0.321	-0.459	0.191	1.000								
Cs	-0.476	-0.522	0.023	-0.393	-0.584	0.596	-0.263	-0.701	0.635	-0.186	-0.156	-0.281	0.549	-0.011	1.000							
Cu	0.244	0.453	0.298	0.355	0.386	-0.257	0.173	-0.059	-0.177	0.651	-0.208	0.658	-0.134	-0.147	-0.168	1.000						
Eu	0.497	0.305	-0.154	-0.048	0.131	-0.455	0.464	0.145	-0.280	0.893	-0.350	0.867	-0.646	-0.498	-0.297	0.591	1.000					
Ga	-0.347	0.668	0.841	0.568	0.321	-0.185	-0.504	0.081	0.435	-0.015	0.532	0.158	0.119	0.129	0.064	0.371	-0.025	1.000				
Hf	0.425	0.675	0.161	0.422	0.578	-0.749	0.212	0.574	-0.351	0.483	0.203	0.586	-0.442	-0.248	-0.686	0.357	0.490	0.274	1.000			
La	0.403	0.388	-0.060	0.062	0.155	-0.495	0.393	0.200	-0.240	0.853	-0.266	0.887	-0.568	-0.508	-0.296	0.528	0.869	0.038	0.600	1.000		
Lu	0.441	0.592	0.072	0.341	0.525	-0.726	0.263	0.431	-0.289	0.616	0.042	0.706	-0.609	-0.302	-0.592	0.484	0.654	0.179	0.759	0.679	1.000	
Nd	0.498	0.204	-0.226	-0.109	0.094	-0.385	0.569	0.093	-0.314	0.758	-0.348	0.675	-0.467	-0.458	-0.262	0.409	0.742	-0.144	0.479	0.801	0.520	1.000
Nb	0.102	0.536	0.275	0.423	0.576	-0.496	0.095	0.554	-0.174	0.079	0.407	0.221	-0.291	0.240	-0.637	0.206	0.112	0.203	0.562	0.142	0.471	0.024
Ni	0.070	-0.071	0.195	-0.160	-0.018	0.435	-0.303	-0.177	-0.141	-0.033	-0.274	-0.085	0.239	0.624	0.001	0.215	-0.029	-0.008	-0.089	-0.078	-0.098	-0.071
Pb	0.328	0.641	0.293	0.485	0.567	-0.581	0.215	0.399	-0.267	0.510	0.127	0.571	-0.339	-0.058	-0.554	0.653	0.444	0.277	0.636	0.436	0.675	0.370
Rb	-0.522	-0.136	0.283	-0.202	-0.362	0.195	-0.256	-0.367	0.920	-0.027	0.228	-0.108	0.224	0.061	0.787	-0.059	-0.133	0.294	-0.434	-0.109	-0.365	-0.146
Sc	0.150	0.171	-0.237	0.107	0.338	-0.575	0.237	0.556	-0.068	-0.116	0.409	0.086	-0.564	-0.077	-0.535	-0.284	0.069	-0.092	0.381	0.075	0.420	0.069
Sm	0.483	0.297	-0.196	-0.041	0.139	-0.486	0.493	0.144	-0.286	0.899	-0.361	0.920	-0.641	-0.533	-0.293	0.554	0.937	-0.048	0.540	0.927	0.699	0.838
Sr	0.460	0.357	-0.182	0.002	0.219	-0.765	0.707	0.615	-0.197	0.682	0.109	0.654	-0.731	-0.374	-0.501	0.213	0.616	-0.161	0.608	0.679	0.602	0.608
Ta	0.148	0.244	0.013	0.095	0.368	-0.338	0.015	0.385	-0.098	-0.153	0.392	0.040	-0.276	0.069	-0.394	-0.171	-0.048	0.096	0.500	0.017	0.205	0.004
Tb	0.277	0.324	-0.060	0.060	0.216	-0.423	0.291	0.267	-0.206	0.640	-0.100	0.811	-0.498	-0.385	-0.282	0.456	0.652	0.101	0.591	0.687	0.539	0.497
Th	0.352	0.244	-0.109	-0.055	0.069	-0.309	0.295	0.063	-0.242	0.625	-0.294	0.660	-0.325	-0.373	-0.197	0.356	0.597	-0.017	0.474	0.753	0.472	0.796
U	0.246	0.347	0.104	0.175	0.418	-0.432	0.034	0.425	-0.161	-0.013	0.349	0.060	-0.313	0.119	-0.499	-0.064	0.025	0.117	0.504	0.054	0.289	0.041
V	0.131	0.809	0.477	0.695	0.711	-0.698	-0.033	0.651	-0.069	0.089	0.575	0.180	-0.350	0.131	-0.700	0.308	0.124	0.433	0.597	0.130	0.581	0.006
Y	0.541	0.205	-0.247	-0.106	0.127	-0.406	0.595	0.106	-0.345	0.737	-0.341	0.616	-0.431	-0.450	-0.288	0.395	0.708	-0.168	0.501	0.751	0.501	0.983
Yb	0.346	0.446	0.099	0.149	0.335	-0.462	0.292	0.323	-0.237	0.591	-0.071	0.762	-0.532	-0.188	-0.420	0.585	0.669	0.179	0.512	0.601	0.670	0.474
Zn	0.130	0.743	0.542	0.664	0.465	-0.425	-0.065	0.045	-0.093	0.571	0.001	0.566	-0.143	-0.186	-0.304	0.807	0.508	0.529	0.532	0.540	0.600	0.360
Zr	0.519	0.626	0.061	0.317	0.491	-0.771	0.370	0.486	-0.330	0.653	0.069	0.708	-0.583	-0.385	-0.622	0.419	0.658	0.169	0.885	0.713	0.762	0.587

	Nb	Ni	Pb	Rb	Sc	Sm	Sr	Ta	Tb	Th	U	V	Y	Yb	Zn	Zr
Nb	1.000															
Ni	0.083	1.000														
Pb	0.587	0.097	1.000													
Rb	-0.350	-0.014	-0.302	1.000												
Sc	0.422	-0.497	0.142	-0.305	1.000											
Sm	0.115	-0.088	0.464	-0.136	0.120	1.000										
Sr	0.362	-0.275	0.492	-0.200	0.363	0.676	1.000									
Ta	0.463	-0.165	0.089	-0.260	0.472	-0.017	0.176	1.000								
Tb	0.262	-0.097	0.453	-0.137	0.136	0.715	0.570	0.254	1.000							
Th	0.061	-0.027	0.469	-0.097	0.043	0.739	0.497	0.018	0.460	1.000						
U	0.397	-0.056	0.199	-0.285	0.372	0.037	0.268	0.810	0.141	0.077	1.000					
V	0.583	-0.058	0.622	-0.310	0.428	0.091	0.335	0.206	0.117	-0.019	0.377	1.000				
Y	0.038	-0.074	0.388	-0.176	0.065	0.795	0.607	0.042	0.460	0.769	0.096	0.033	1.000			
Yb	0.464	0.054	0.615	-0.220	0.211	0.678	0.500	0.145	0.649	0.365	0.133	0.332	0.463	1.000		
Zn	0.318	0.184	0.673	-0.058	-0.188	0.487	0.223	-0.058	0.363	0.362	0.106	0.533	0.355	0.518	1.000	
Zr	0.509	-0.137	0.607	-0.361	0.295	0.699	0.726	0.426	0.633	0.557	0.478	0.455	0.606	0.641	0.570	1.000

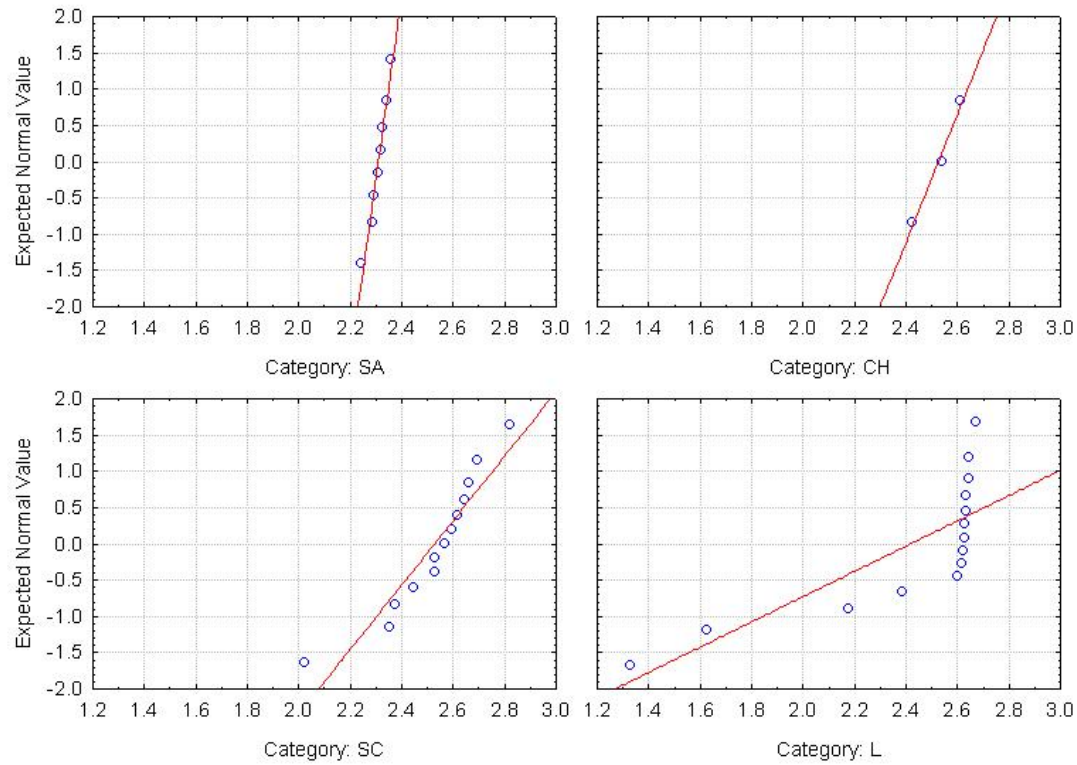
Categorized Probability Plot: SiO2



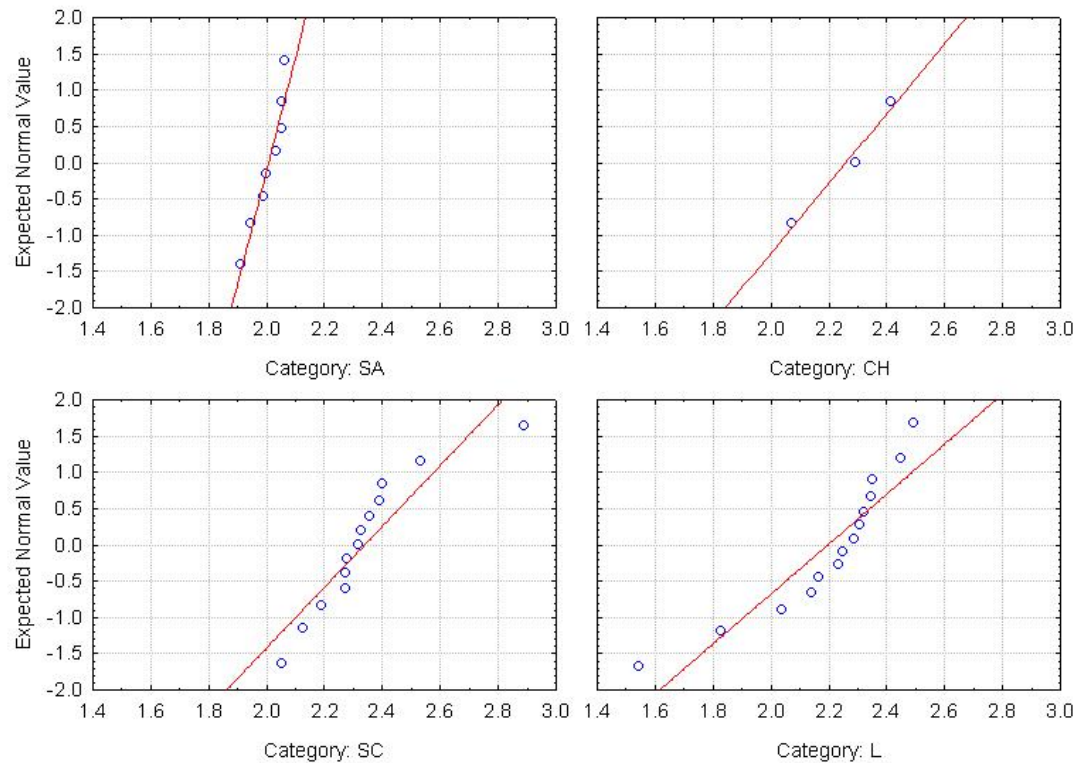
Categorized Probability Plot: TiO2



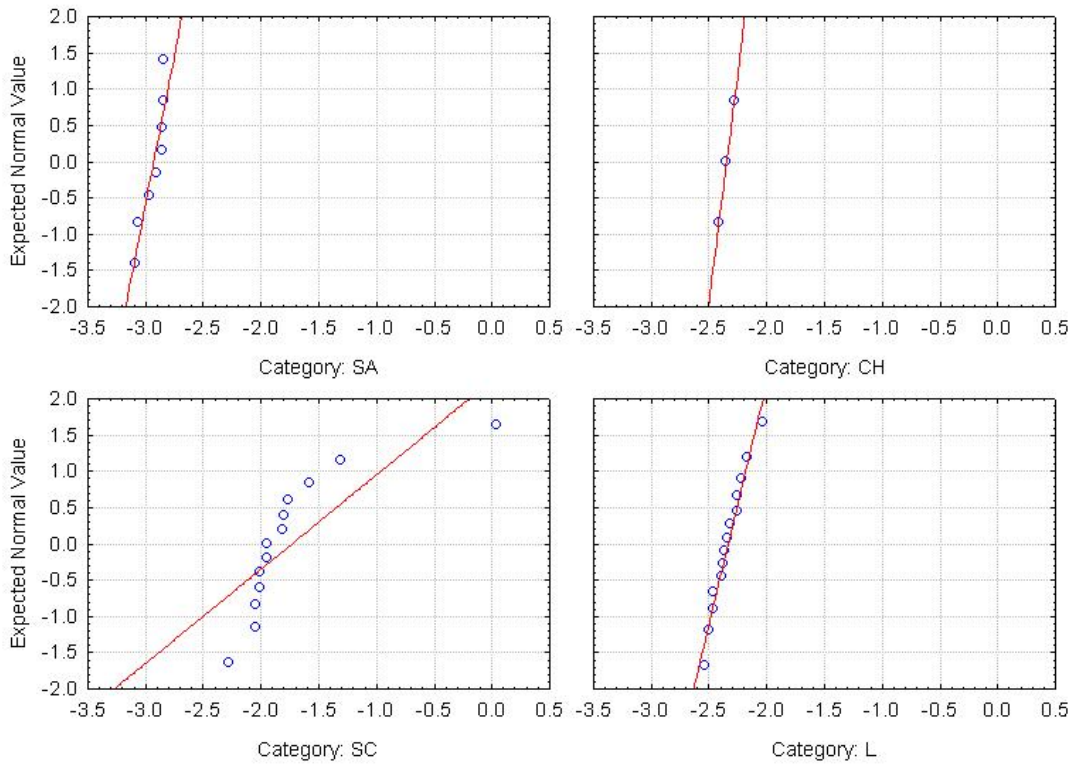
Categorized Probability Plot: AL2O3



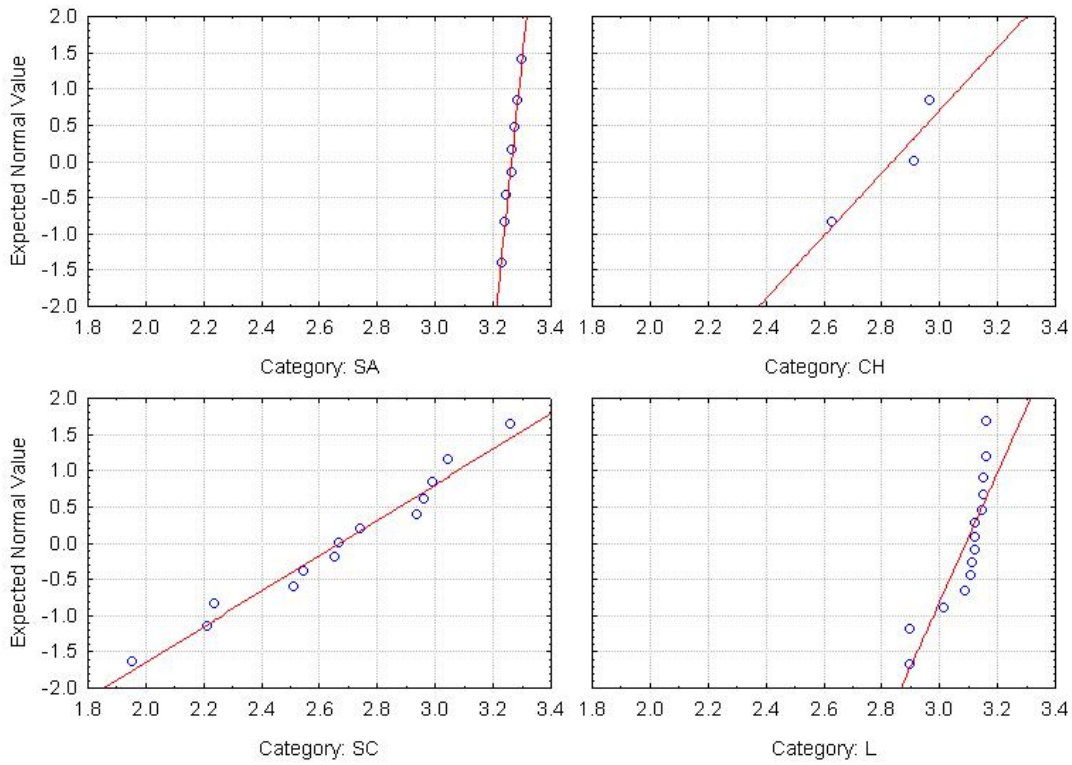
Categorized Probability Plot: FEO



Categorized Probability Plot: MNO

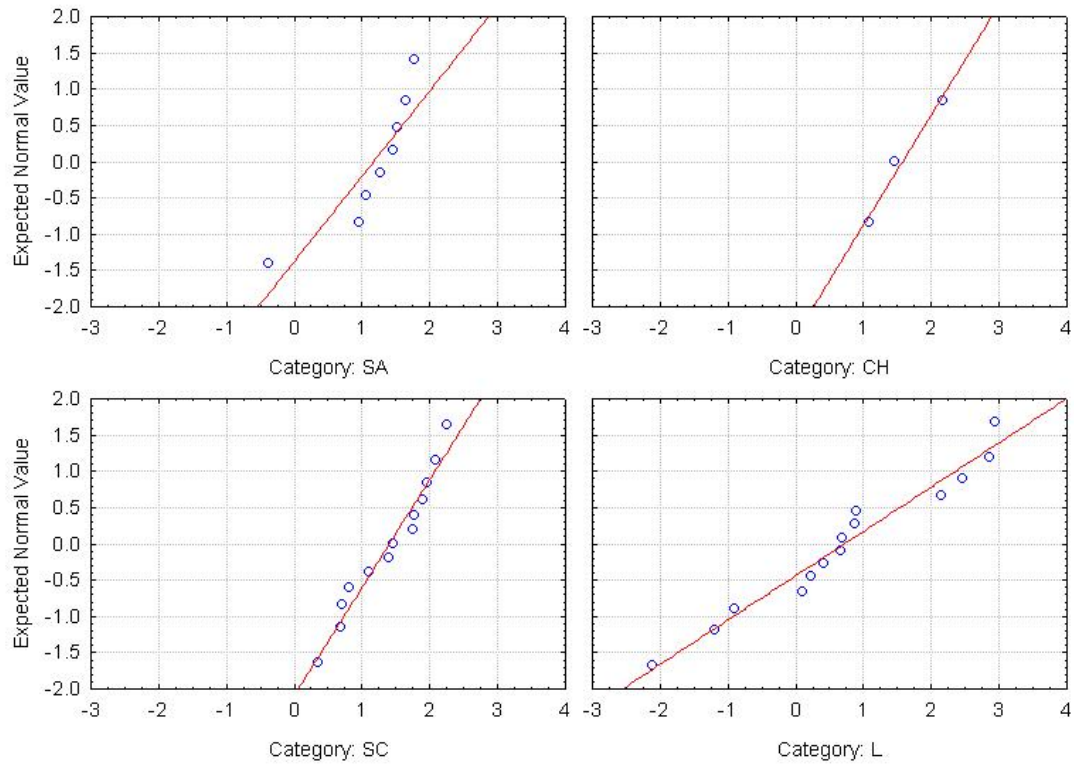


Categorized Probability Plot: MGO

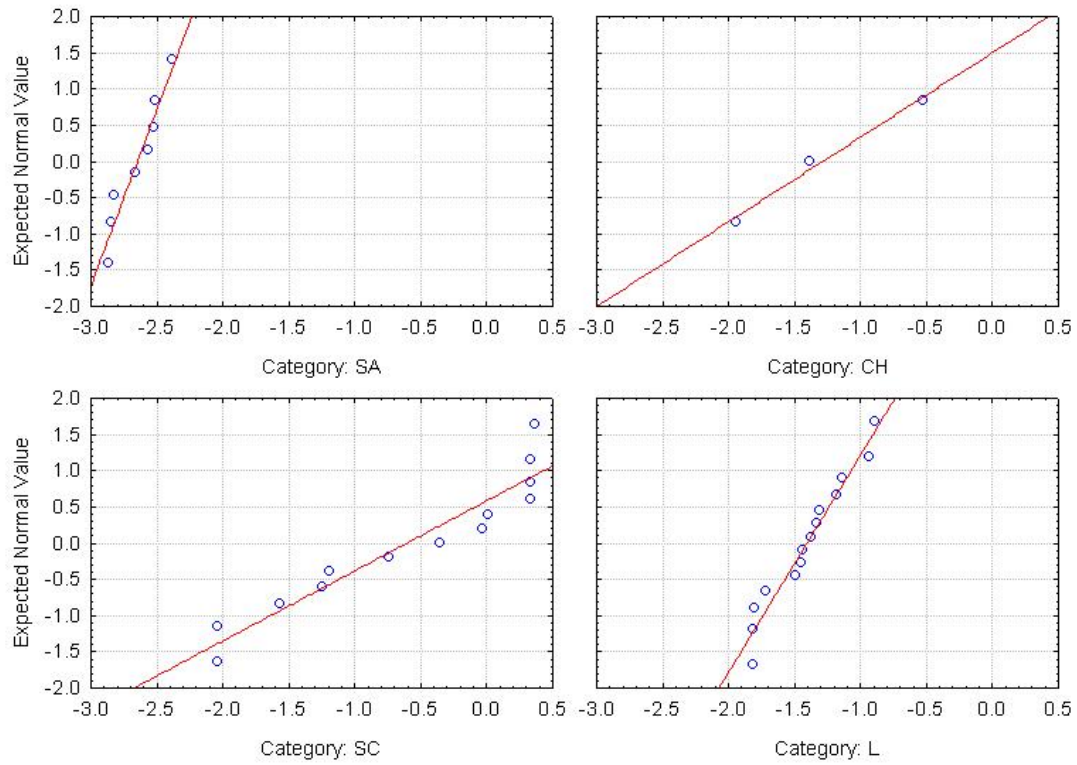




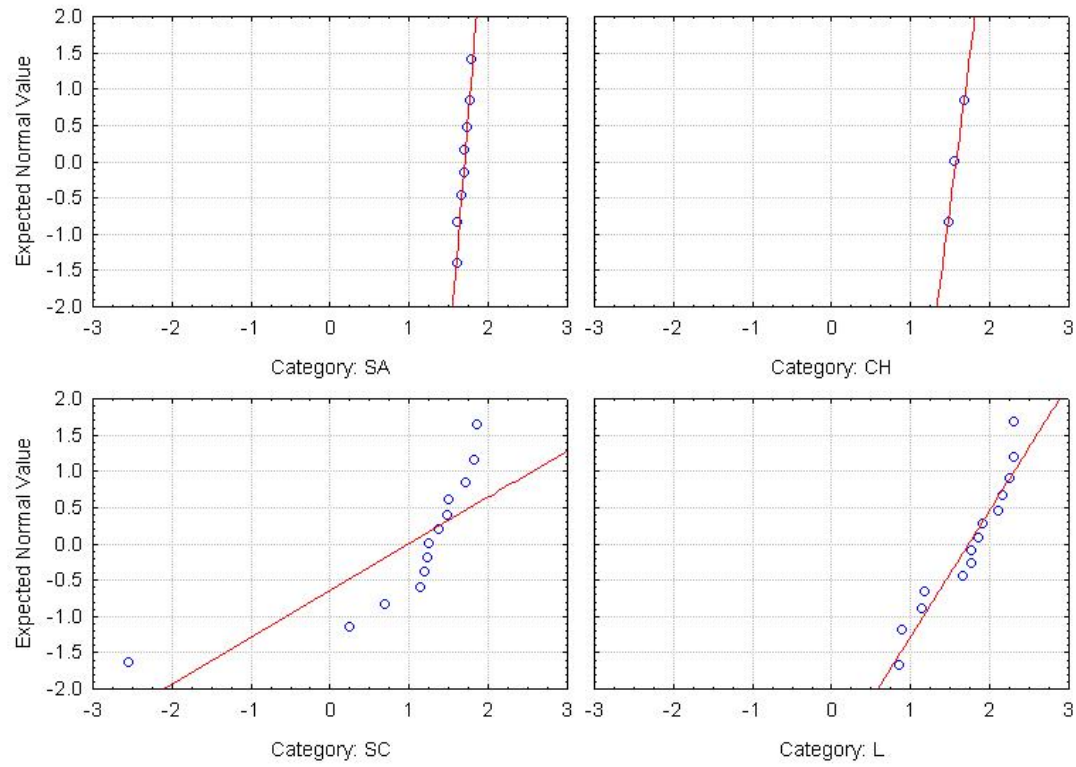
Categorized Probability Plot: CAO



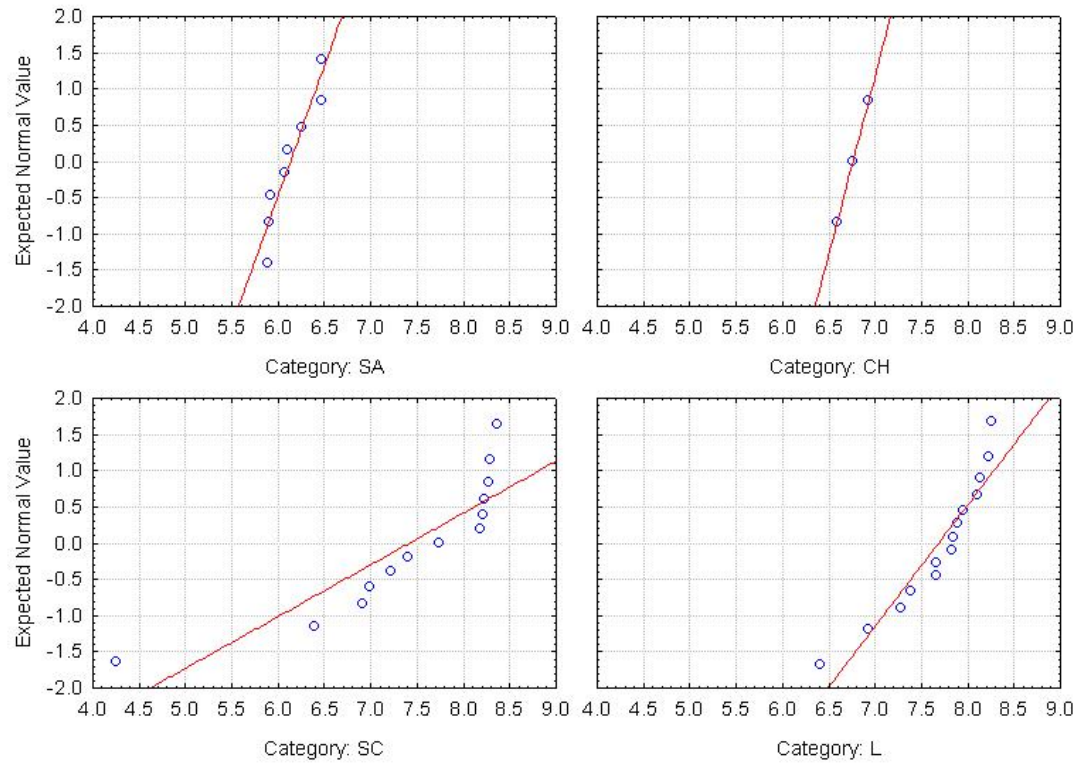
Categorized Probability Plot: NA2O



Categorized Probability Plot: K2O

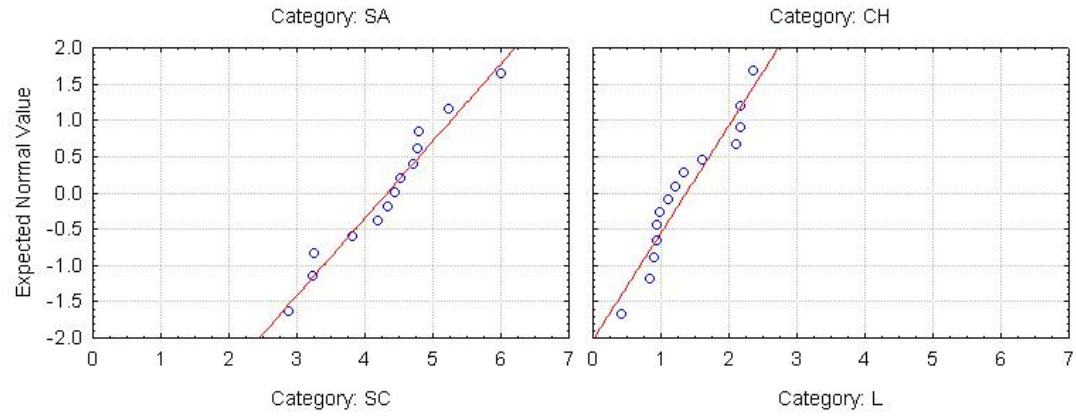
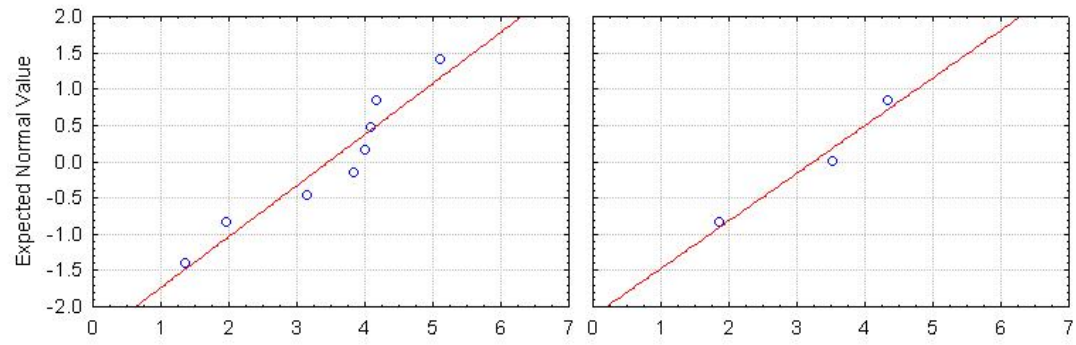


Categorized Probability Plot: BA

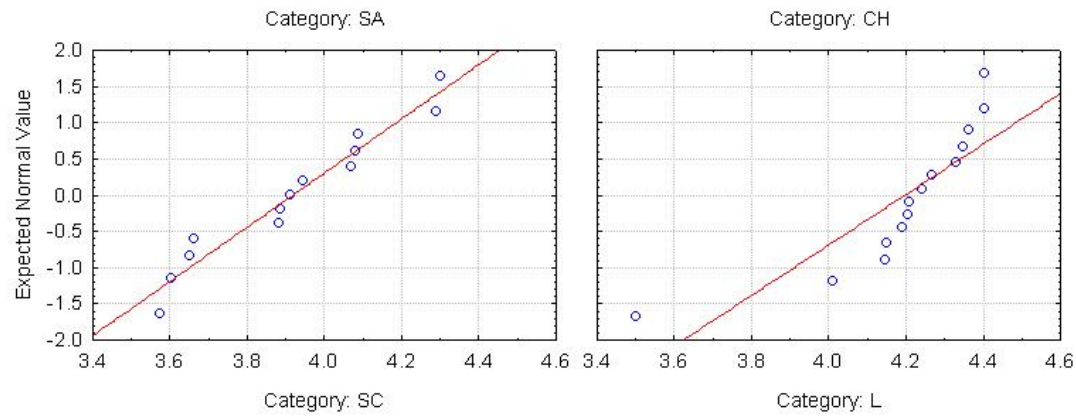
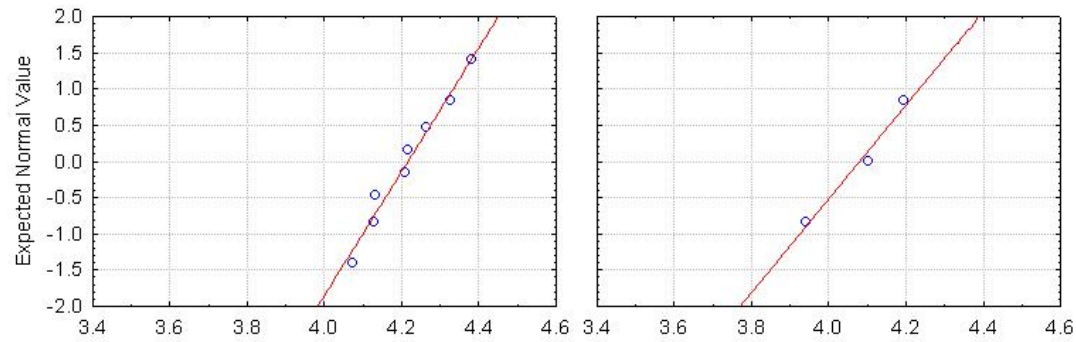




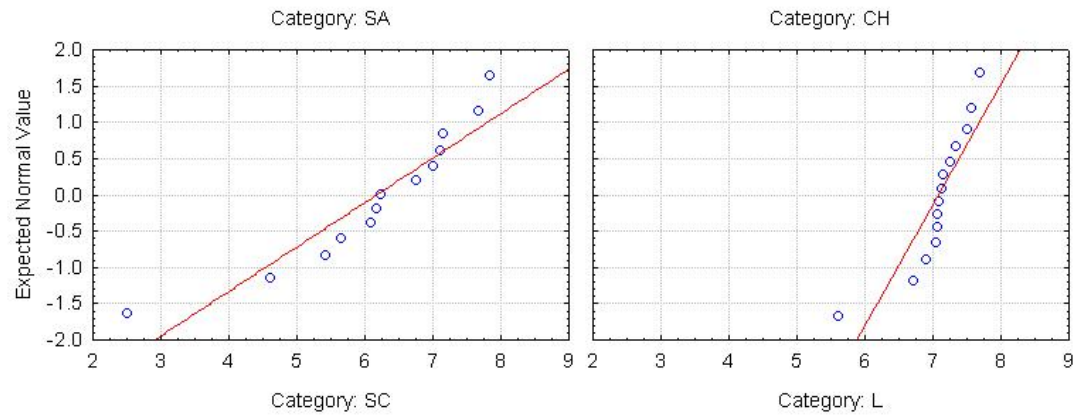
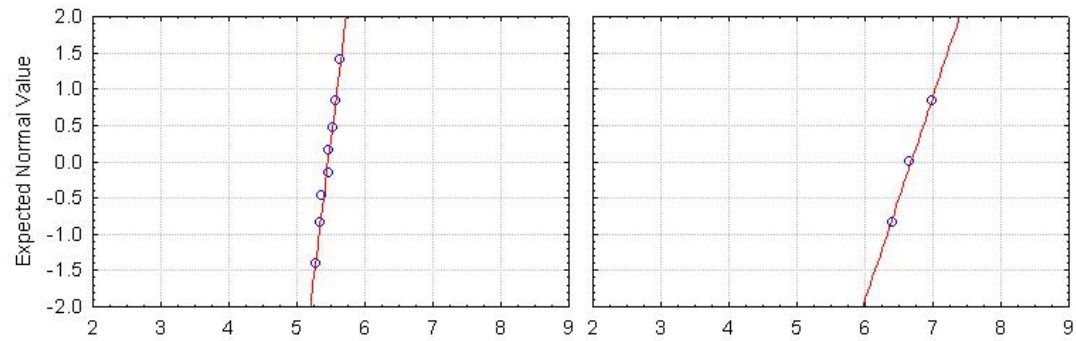
Categorized Probability Plot: CE



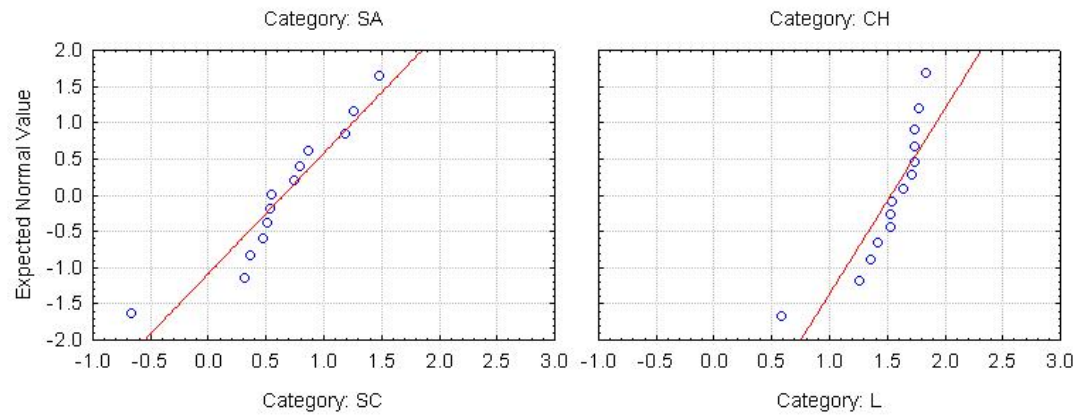
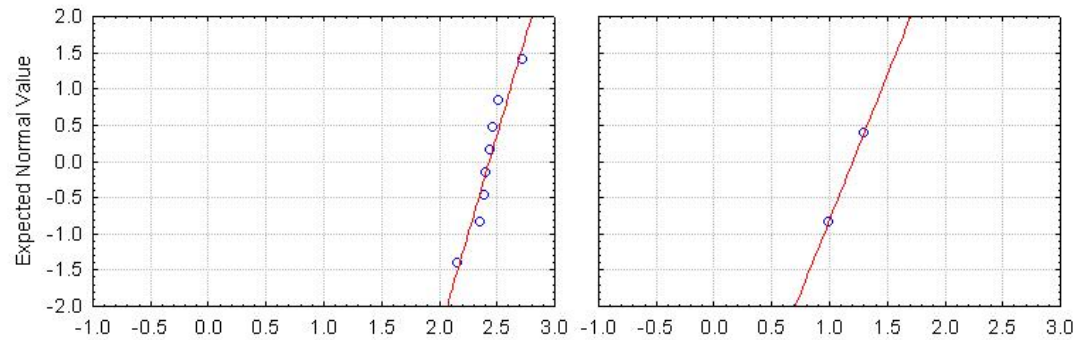
Categorized Probability Plot: CO



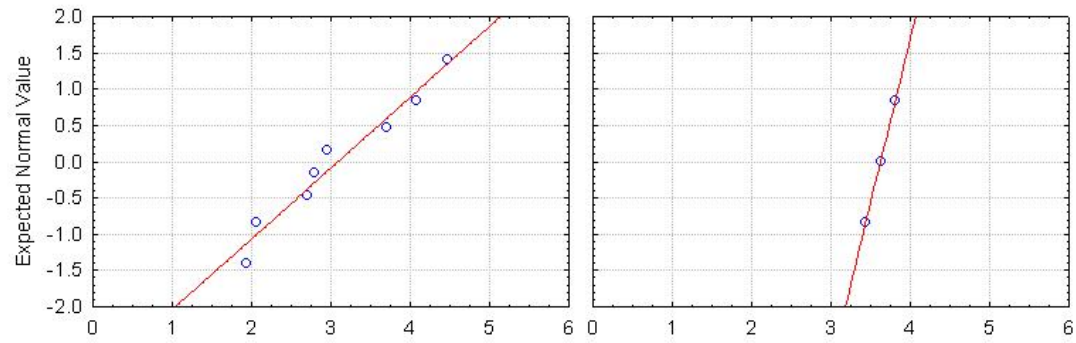
Categorized Probability Plot: CR



Categorized Probability Plot: CS

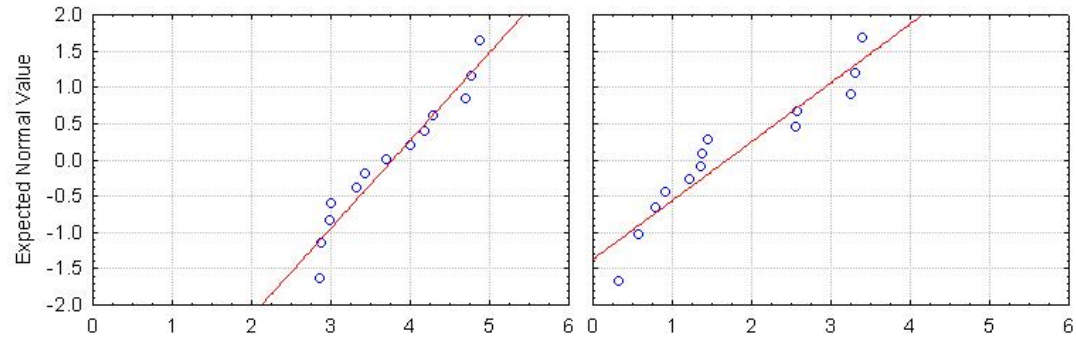


Categorized Probability Plot: CU



Category: SA

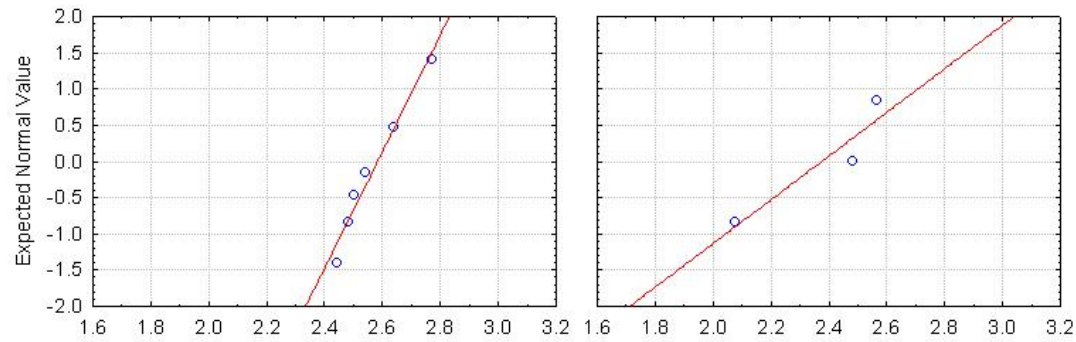
Category: CH



Category: SC

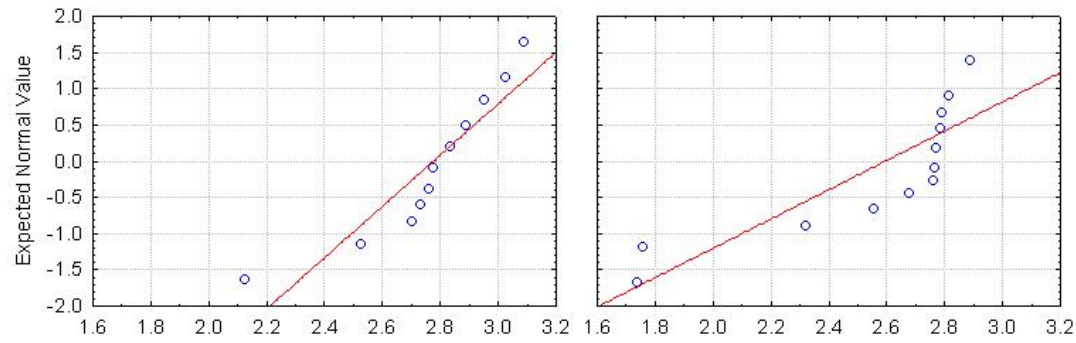
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Categorized Probability Plot: GA



Category: SA

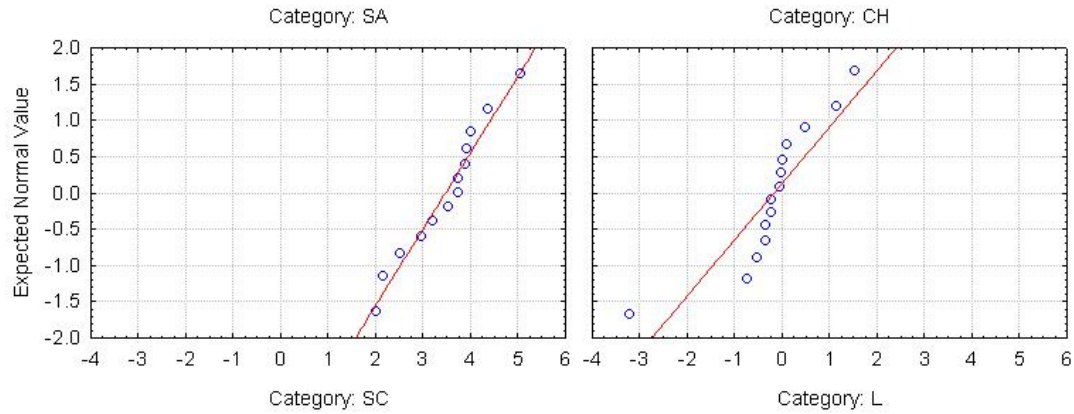
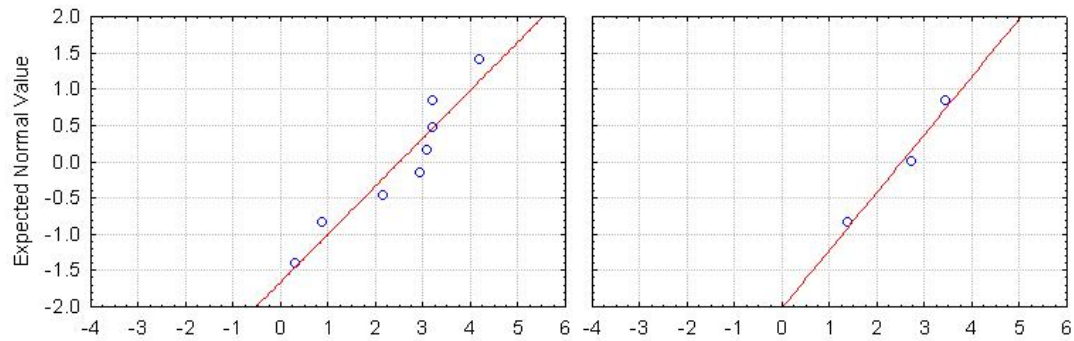
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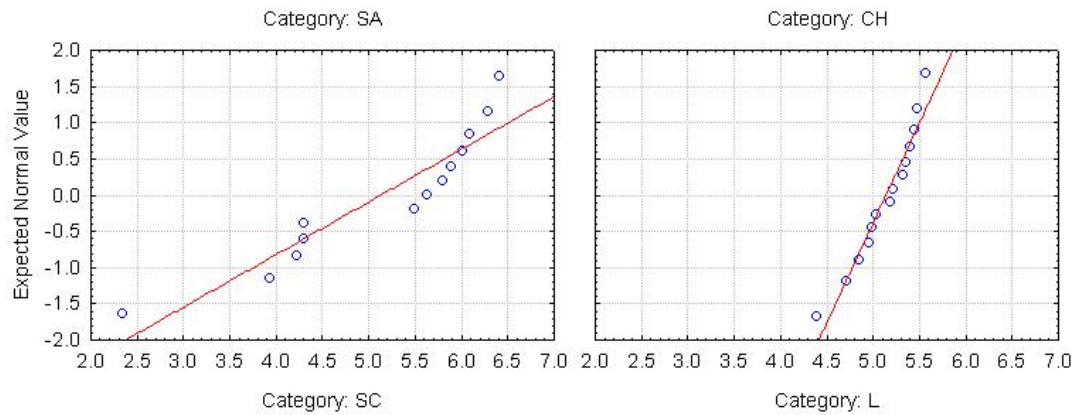
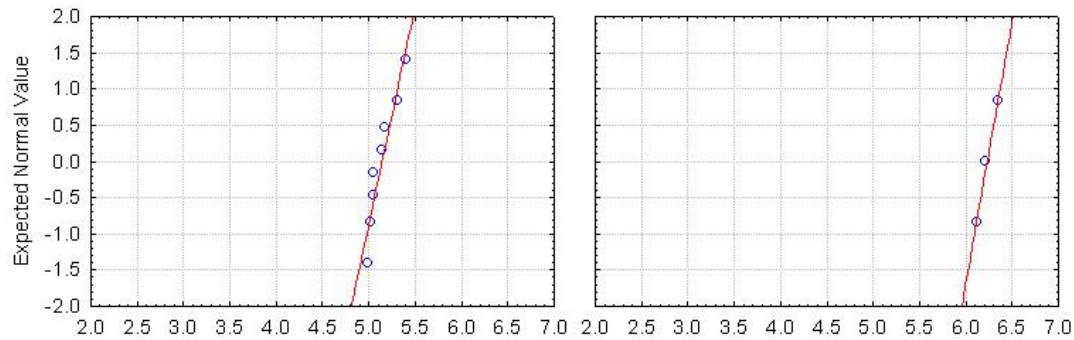
Category: SC

Category: L

Categorized Probability Plot: LA

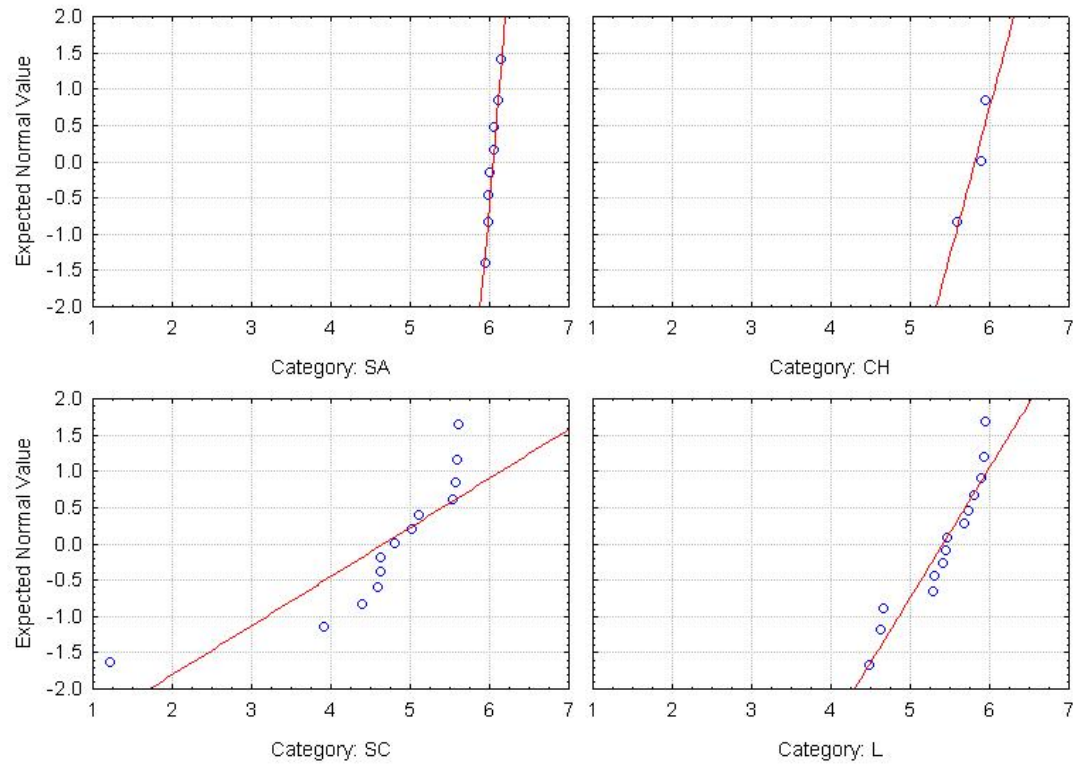


Categorized Probability Plot: NI

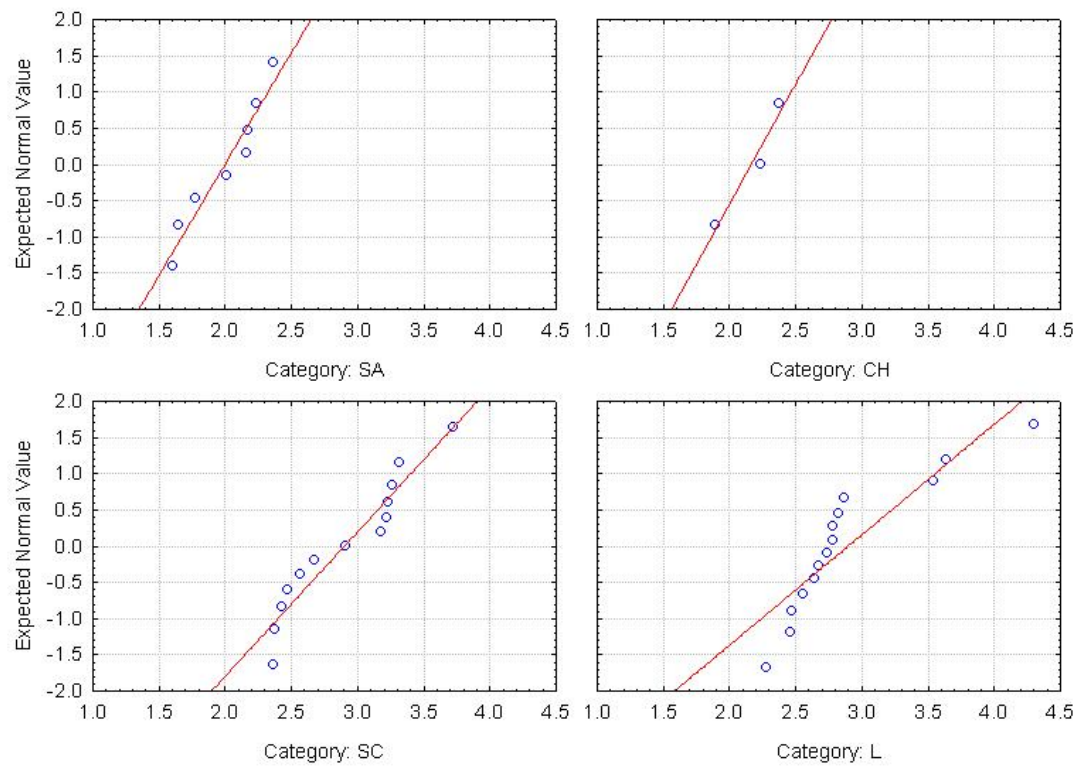




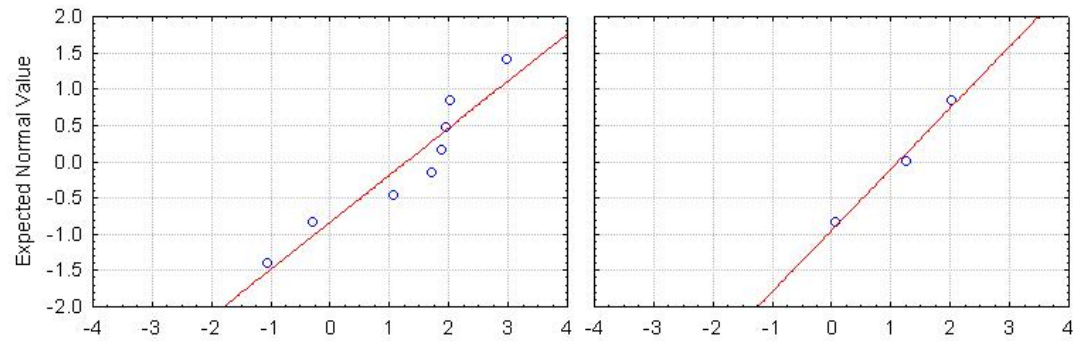
Categorized Probability Plot: RB



Categorized Probability Plot: SC

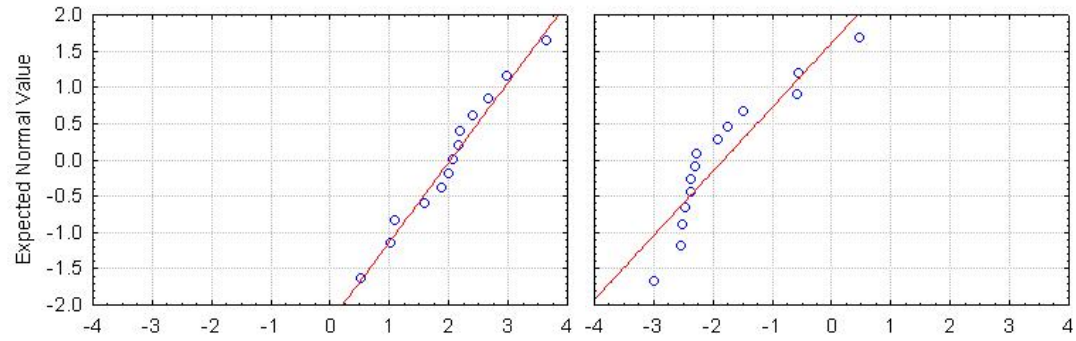


Categorized Probability Plot: SM



Category: SA

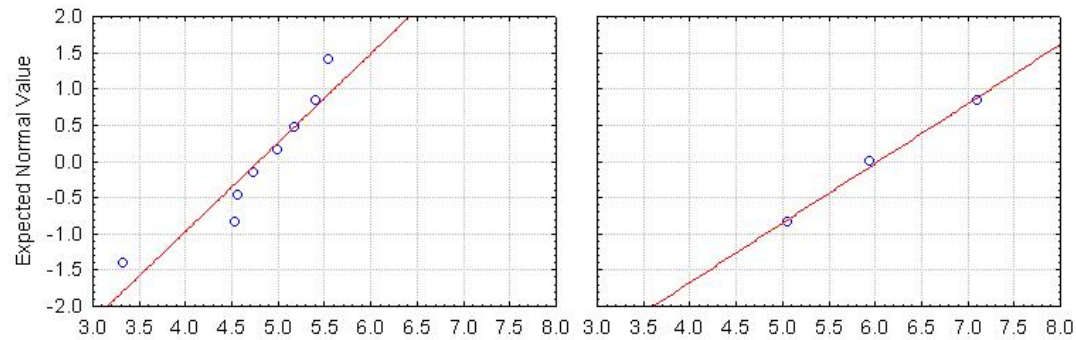
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Category: SC

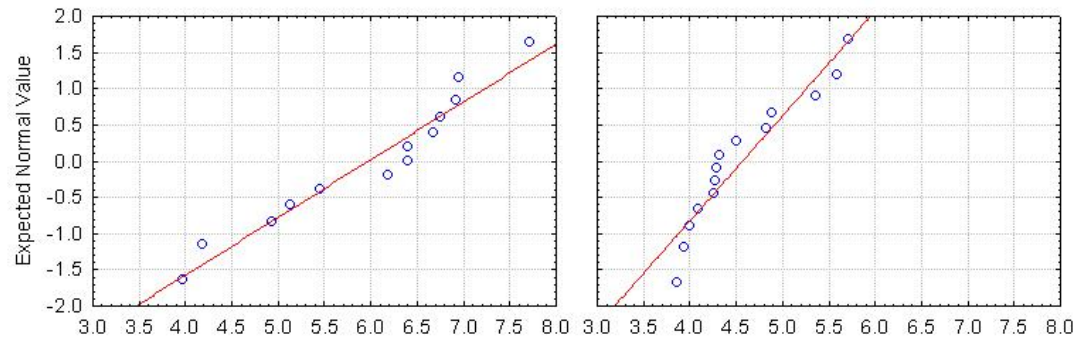
Category: L

Categorized Probability Plot: SR



Category: SA

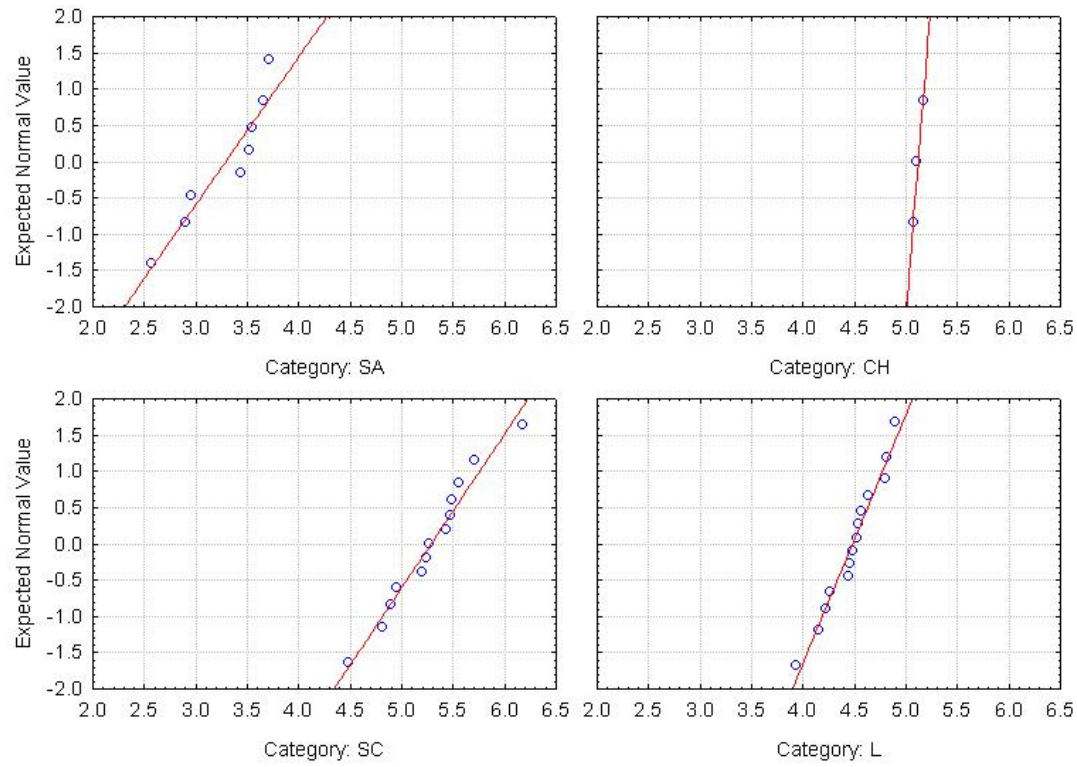
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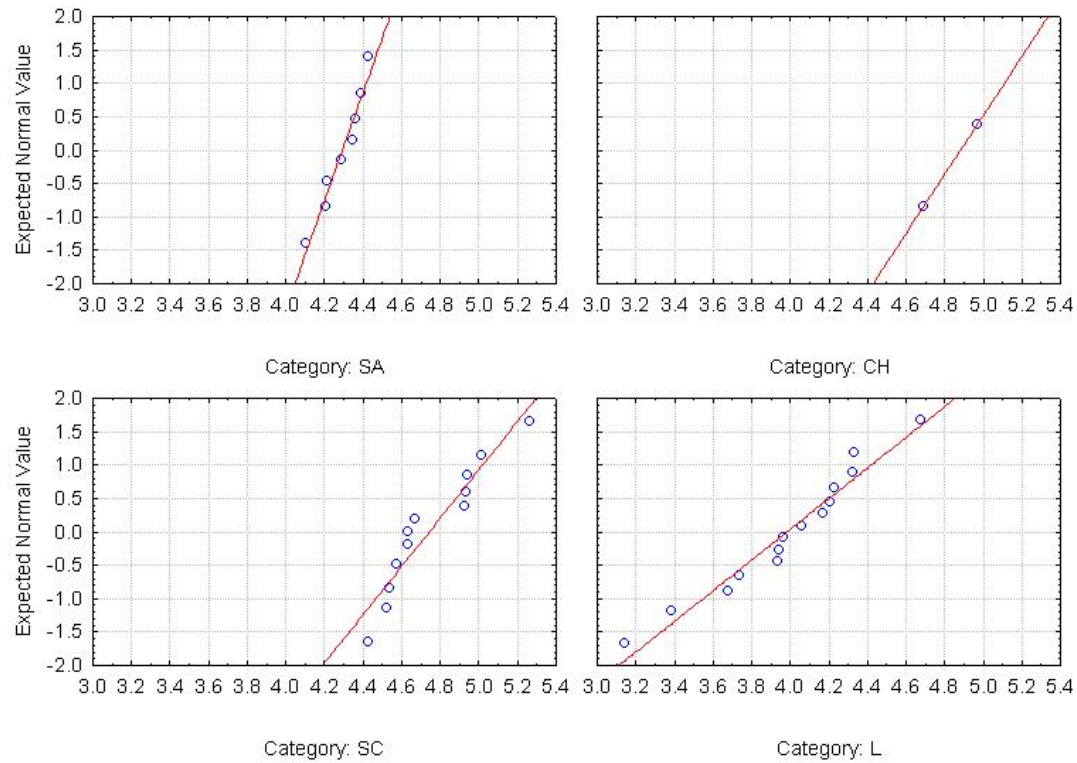
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Category: L

Categorized Probability Plot: V



Categorized Probability Plot: ZN









**Appendix B**  
**Analytical Data for Duplicates and Blind Standards**

Neutron Activation Analysis Data for Duplicates and Blind Standards

Sample	FeO %	Na2O %	Sc ppm	Cr ppm	Co ppm	Ni ppm	Rb ppm	Cs ppm	Sr ppm	Ba ppm	La ppm	Ce ppm
SC-7	11.5	0.12	14.5	865	58.5	295	47	1.37	210	600	24.7	66.8
SC-13 (SC7dup)	11.6	0.11	14.5	864	58.6	290	49	1.36	270	570	24.7	69.9
PVC5	8.32	0.37	10.7	1220	51.6	545	148	2.38	320	3820	7.6	17.8
12A (PVC5 dup)	8.57	0.35	11.9	1130	57.9	370	142	2.26	30	3390	7.0	16.6
NIST 1633-b values	10.01	0.37	41	198.2	50	120.6	140	11	1041	709	94	190
SC-12(1633b)	10.3	0.28	34.7	193	41.3	113	140	11.0	1070	660	84.3	198
13A(1633b)	10.6	0.29	42.5	209	49.5	8	145	10.8	1130	725	84.7	196
average	10.5	0.3	38.6	201	45.4	60.5	142.5	10.9	1100	693	84.5	197
dev from accept%	4.40	-24.2	-5.85	1.41	-9.20	-49.8	1.79	-0.91	5.67	-2.33	-10.11	3.68
USGS SDC-1 values	n/a	2.05	17	64	18	38	127	4	180	630	42	93
14A(mica schist)	6.53	2.03	15.8	66	18.5	5	119	3.21	180	625	41.7	95.1
SDC-1 (mica schist)	6.70	2.03	13.3	65	17.3	19	133	3.43	250	630	39.7	100
average	6.615	2.03	14.6	65.8	17.9	12.1	126	3.32	215	628	40.7	97.6
dev from accept%		-0.98	-14.4	2.73	-0.56	-68.3	-0.79	-17.0	19.4	-0.40	-3.10	4.89

Neutron Activation Analysis Data for Duplicates and Blind Standards

Sample	Nd ppm	Sm ppm	Eu ppm	Tb ppm	Yb ppm	Lu ppm	Zr ppm	Hf ppm	Ta ppm	Th ppm	U ppm
SC-7	38.7	7.50	1.85	0.62	0.98	0.17	130	4.96	0.62	3.69	0.8
SC-13 (SC7dup)	38.4	7.55	1.90	0.78	1.03	0.14	150	5.24	0.60	3.55	0.7
PVC5	10.9	1.69	0.42	0.24	0.27	0.05	65	0.55	0.60	1.47	1.1
12A (PVC5 dup)	7.8	1.60	0.40	0.05	0.06	0.06	0	0.31	0.14	0.63	0.7
NIST 1633-b values	85	20	4.1	2.6	7.6	1.2	n/a	6.8	1.8	25.7	8.79
SC-12(1633b)	101	18.9	4.10	2.44	7.68	1.00	275	7.03	1.79	27.0	7.9
13A(1633b)	90.9	18.8	4.09	2.99	8.35	1.06	530	6.31	1.86	27.7	7.9
average	96.0	18.9	4.1	2.72	8.02	1.03	403	6.67	1.83	27.4	7.87
dev from accept%	12.9	-5.75	-0.12	4.42	5.46	-14.2		-1.91	1.39	6.42	-10.5
USGS SDC-1 values	40	8.2	1.7	1.2	4	n/a	290	8.3	1.2	12	3.10
14A(mica schist)	45.7	8.35	1.65	1.18	4.32	0.58	365	9.02	1.27	13.2	2.64
SDC-1 (mica schist)	37.9	7.81	1.74	1.14	3.99	0.60	265	9.31	1.26	12.4	2.59
average	41.8	8.08	1.70	1.16	4.155	0.59	315	9.17	1.27	12.8	2.62
dev from accept%	4.50	-1.46	-0.29	-3.33	3.88		8.62	10.42	5.42	6.67	-15.6

X-ray Fluorescence Analytical Data for Duplicates and Blind Standards

	SiO <sub>2</sub> wt. %	TiO <sub>2</sub> wt %	Al <sub>2</sub> O <sub>3</sub> wt %	FeO* wt %	MnO wt %	MgO wt %	CaO wt %	Na <sub>2</sub> O wt %	K <sub>2</sub> O wt %	P <sub>2</sub> O <sub>5</sub> wt %	Ni ppm	Cr ppm
USGS SDC-1 values	65.8	1.01	15.8	n/a	0.088	1.69	1.4	2.05	3.28	0.16	38	64
SC-1	65.24	0.993	15.65	6.32	0.115	1.70	1.42	2.02	3.27	0.147	36	59
deviation from accepted %	-0.85	-1.70	-0.96		30.8	0.50	1.31	-1.54	-0.43	-8.09	-3.99	-8.13
SC-7	42.95	2.375	11.44	11.02	0.151	16.54	1.76	0.11	1.13	0.193	360	810
SC-13 (SC-7 dup)	43.00	2.404	11.63	11.08	0.148	16.55	1.73	0.11	1.14	0.197	361	821
	Sc ppm	V ppm	Ba ppm	Rb ppm	Sr ppm	Zr ppm	Y ppm	Nb ppm	Ga ppm	Cu ppm	Zn ppm	Pb ppm
USGS SDC-1 values	17	102	630	127	180	290	n/a	21	21	30	103	25
SC-1	15	99	641	124	183	313	39	18.1	20	28	101	22
deviation from accepted %	-14.1	-3.33	1.81	-2.05	1.39	7.86		-13.8	-4.29	-8.00	-1.55	-13.6
SC-7	16	141	599	51	53	217	16	11	16	120	193	4
SC-13 (SC-7 dup)	17	143	605	51	52	179	16	12	15	121	195	4
	La ppm	Ce ppm	Th ppm	Nd ppm	U ppm	Cs ppm						
USGS SDC-1 values	42	93	12	40	3.1	4						
SC-1	43	96	10	42	4	4						
deviation from accepted %	3.33	3.01	-15.8	5.00	35.5	0.00						
SC-7	26	64	2	37	1	3						
SC-13 (SC-7 dup)	26	66	4	37	1	3						

## **Appendix C**

### **Results of k-Cluster Analyses**

**Number of Clusters:** 4  
**Seeds:** SA GS-17, SC GS-11, SC 10, L 7A  
**Number of Cases:** 44  
**Number of Variables:** 15  
**Number of Iterations:** 2

Cluster #	Members	Distance From Mean
1	L GS-4	0.412480
	L 5A	0.755415
	L 3A	0.656409
	SA GS-1	0.783291
	L 9A	0.548220
	L GS-2	0.474476
	L 7A	0.239307
	L GS-10	0.813468
	L 1A	1.017736
	L A	0.321739
	L 4A	0.514271
	L 6A	0.836704
	L 8A	0.427209
	L10A	0.909868
	L 11A	0.438278
	P1	0.382675
	P4	0.307546
	P5	0.503818
	P6	0.498646
2	SC6	0.319303
	SC9	0.469735
	SC10	0.325862
	SC4	0.275086
	SC5	0.484033
	SC11	0.885084
3	SC GS-11	0.434205
	SC7	0.390553
	CH GS-16	0.262405
	CH GS-7	0.406576
	CH GS-12	0.465453
	SC GS-15	0.425174
	SC2	1.277526
	SC3	0.354497
	SC8	0.398839
	SC PVC5	0.452305
4	P3	0.347564
	SA GS-17	0.189418
	SA PVC1	0.689490
	SA GS-13	0.449926
	SA GS-3	0.296435
	SA GS-18	0.313724
	SA PVC2	0.181403
	SA PVC3	0.188881
	P2	0.599834

Variable	Cluster Means (Copy4 of fprinttraceln1)			
	Cluster No. 1	Cluster No. 2	Cluster No. 3	Cluster No. 4
<b>SIO2</b>	3.778331	3.849233	3.867960	3.794851
<b>AL2O3</b>	2.459212	2.583347	2.511979	2.274363
<b>FEO</b>	2.177033	2.433307	2.269980	1.998000
<b>CAO</b>	0.654885	1.867485	1.108066	1.518997
<b>K2O</b>	1.739451	1.444872	0.962538	1.663782
<b>BA</b>	7.645436	8.182878	6.794402	6.091486
<b>CO</b>	4.236341	3.747972	4.080889	4.161819
<b>CR</b>	7.038855	5.215080	6.916371	5.510035
<b>CS</b>	1.629600	0.606405	0.897954	2.367243
<b>CU</b>	1.909796	3.366312	4.038102	3.241873
<b>LA</b>	-0.197733	4.134732	2.807034	3.092121
<b>NI</b>	5.207242	4.101848	6.091285	5.124131
<b>SC</b>	2.750628	3.324568	2.455241	2.156190
<b>V</b>	4.416292	5.640415	5.034225	3.219415
<b>ZN</b>	4.021447	4.745518	4.815015	4.279053

Cluster Number	Euclidean Distances between Clusters (Copy4 of fprinttraceln1)			
	Distances below diagonal Squared distances above diagonal			
	No. 1	No. 2	No. 3	No. 4
No. 1	0.000000	2.067298	1.171056	1.371823
No. 2	1.437810	0.000000	0.877303	1.185436
No. 3	1.082154	0.936645	0.000000	0.716966
No. 4	1.171248	1.088778	0.846738	0.000000

Variable	Analysis of Variance (Copy4 of fprinttraceln1)					
	Between SS	df	Within SS	df	F	signif. p
<b>SIO2</b>	0.0666	3	0.15109	40	5.87464	0.002017
<b>AL2O3</b>	0.3959	3	3.07712	40	1.71547	0.179226
<b>FEO</b>	0.7166	3	1.63255	40	5.85262	0.002062
<b>CAO</b>	8.7000	3	36.65877	40	3.16430	0.034771
<b>K2O</b>	4.4964	3	19.58718	40	3.06079	0.039011
<b>BA</b>	21.1820	3	14.97753	40	18.85672	0.000000
<b>CO</b>	1.1181	3	1.39249	40	10.70608	0.000027
<b>CR</b>	25.1554	3	21.78276	40	15.39772	0.000001
<b>CS</b>	14.9624	3	7.35482	40	27.12492	0.000000
<b>CU</b>	35.1041	3	29.95315	40	15.62623	0.000001
<b>LA</b>	132.8996	3	36.66223	40	48.33298	0.000000
<b>NI</b>	15.8496	3	8.10448	40	26.07544	0.000000
<b>SC</b>	5.2903	3	7.81160	40	9.02978	0.000109
<b>V</b>	24.1772	3	4.51721	40	71.36324	0.000000
<b>ZN</b>	5.3980	3	3.51269	40	20.48969	0.000000



Variable	Descriptive Statistics for Cluster 1 (Copy4 of fprinttraceln1)		
	Cluster contains 19 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.778331	0.069376	0.004813
<b>AL2O3</b>	2.459212	0.373250	0.139316
<b>FEO</b>	2.177033	0.248929	0.061966
<b>CAO</b>	0.654885	1.317984	1.737082
<b>K2O</b>	1.739451	0.443557	0.196743
<b>BA</b>	7.645436	0.535989	0.287284
<b>CO</b>	4.236341	0.211589	0.044770
<b>CR</b>	7.038855	0.619416	0.383676
<b>CS</b>	1.629600	0.400257	0.160206
<b>CU</b>	1.909796	1.047873	1.098038
<b>LA</b>	-0.197733	1.037418	1.076235
<b>NI</b>	5.207242	0.349360	0.122053
<b>SC</b>	2.750628	0.572148	0.327353
<b>V</b>	4.416292	0.351433	0.123505
<b>ZN</b>	4.021447	0.369734	0.136703

Variable	Descriptive Statistics for Cluster 2 (Copy4 of fprinttraceln1)		
	Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.849233	0.064647	0.004179
<b>AL2O3</b>	2.583347	0.154465	0.023859
<b>FEO</b>	2.433307	0.227939	0.051956
<b>CAO</b>	1.867485	0.303384	0.092042
<b>K2O</b>	1.444872	0.323651	0.104750
<b>BA</b>	8.182878	0.224827	0.050547
<b>CO</b>	3.747972	0.202223	0.040894
<b>CR</b>	5.215080	1.455548	2.118621
<b>CS</b>	0.606405	0.295308	0.087207
<b>CU</b>	3.366312	0.730764	0.534016
<b>LA</b>	4.134732	0.505349	0.255378
<b>NI</b>	4.101848	1.011453	1.023037
<b>SC</b>	3.324568	0.203011	0.041213
<b>V</b>	5.640415	0.278753	0.077703
<b>ZN</b>	4.745518	0.246920	0.060970

Variable	Descriptive Statistics for Cluster 3 (Copy4 of fprinttraceln1)		
	Cluster contains 11 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.867960	0.065264	0.004259
<b>AL2O3</b>	2.511979	0.194600	0.037869
<b>FEO</b>	2.269980	0.148755	0.022128
<b>CAO</b>	1.108066	0.586298	0.343746
<b>K2O</b>	0.962538	1.244080	1.547736
<b>BA</b>	6.794402	0.962912	0.927199
<b>CO</b>	4.080889	0.152748	0.023332
<b>CR</b>	6.916371	0.642794	0.413184
<b>CS</b>	0.897954	0.625877	0.391722
<b>CU</b>	4.038102	0.561030	0.314755
<b>LA</b>	2.807034	0.740200	0.547896
<b>NI</b>	6.091285	0.241221	0.058188
<b>SC</b>	2.455241	0.270979	0.073429
<b>V</b>	5.034225	0.232013	0.053830
<b>ZN</b>	4.815015	0.250971	0.062987

Variable	Descriptive Statistics for Cluster 4 (Copy4 of fprinttraceln1)		
	Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.794851	0.011730	0.000138
<b>AL2O3</b>	2.274363	0.101026	0.010206
<b>FEO</b>	1.998000	0.071824	0.005159
<b>CAO</b>	1.518997	0.461925	0.213375
<b>K2O</b>	1.663782	0.079901	0.006384
<b>BA</b>	6.091486	0.200601	0.040241
<b>CO</b>	4.161819	0.145818	0.021263
<b>CR</b>	5.510035	0.147189	0.021665
<b>CS</b>	2.367243	0.129757	0.016837
<b>CU</b>	3.241873	0.790193	0.624405
<b>LA</b>	3.092121	1.226735	1.504879
<b>NI</b>	5.124131	0.173400	0.030067
<b>SC</b>	2.156190	0.373953	0.139841
<b>V</b>	3.219415	0.441959	0.195328
<b>ZN</b>	4.279053	0.129461	0.016760

**Number of Clusters:** 6  
**Seeds:** SA GS-17, SC GS-11, SC 10, L 7A, L 9A  
**Number of Cases:** 44  
**Number of Variables:** 15  
**Number of Iterations:** 2

Cluster #	Members	Distance From Mean
1	L 9A	0.244654
	L 1A	0.472058
	L 4A	0.362217
	L10A	0.353303
2	L GS-4	0.353766
	L 5A	0.702337
	L 3A	0.629236
	SA GS-1	0.775860
	L GS-2	0.436138
	L 7A	0.217604
	L GS-10	0.785601
	L A	0.281378
	L 6A	0.736967
	L 8A	0.375317
	L 11A	0.466517
	P1	0.318551
	P4	0.360606
	P5	0.411252
	P6	0.542717
3	SC6	0.319303
	SC9	0.469735
	SC10	0.325862
	SC4	0.275086
	SC5	0.484033
	SC11	0.885084
4	SC GS-11	0.434205
	SC7	0.390553
	CH GS-16	0.262405
	CH GS-7	0.406576
	CH GS-12	0.465453
	SC GS-15	0.425174
	SC2	1.277526
	SC3	0.354497
	SC8	0.398839
	SC PVC5	0.452305
5	P3	0.347564
	SA GS-17	0.189418
	SA PVC1	0.689490
	SA GS-13	0.449926
	SA GS-3	0.296435
	SA GS-18	0.313724
	SA PVC2	0.181403
	SA PVC3	0.188881
	P2	0.599834

Variable	Cluster Means (Copy2 of fprinttraceln1)				
	Cluster No. 1	Cluster No. 2	Cluster No. 3	Cluster No. 4	Cluster No. 5
<b>SIO2</b>	3.873115	3.753055	3.849233	3.867960	3.794851
<b>AL2O3</b>	1.880023	2.613662	2.583347	2.511979	2.274363
<b>FEO</b>	1.895712	2.252052	2.433307	2.269980	1.998000
<b>CAO</b>	2.601408	0.135813	1.867485	1.108066	1.518997
<b>K2O</b>	1.020488	1.931175	1.444872	0.962538	1.663782
<b>BA</b>	6.999165	7.817775	8.182878	6.794402	6.091486
<b>CO</b>	4.084200	4.276913	3.747972	4.080889	4.161819
<b>CR</b>	6.709892	7.126579	5.215080	6.916371	5.510035
<b>CS</b>	1.258937	1.728443	0.606405	0.897954	2.367243
<b>CU</b>	1.028537	2.144799	3.366312	4.038102	3.241873
<b>LA</b>	0.005218	-0.251854	4.134732	2.807034	3.092121
<b>NI</b>	4.738605	5.332212	4.101848	6.091285	5.124131
<b>SC</b>	3.276493	2.610398	3.324568	2.455241	2.156190
<b>V</b>	4.145607	4.488474	5.640415	5.034225	3.219415
<b>ZN</b>	3.488330	4.163612	4.745518	4.815015	4.279053

Cluster Number	Euclidean Distances between Clusters (Copy2 of fprinttraceln1)				
	Distances below diagonal Squared distances above diagonal				
	No. 1	No. 2	No. 3	No. 4	No. 5
No. 1	0.000000	0.758079	2.161219	1.663291	1.504669
No. 2	0.870677	0.000000	2.201848	1.199389	1.495993
No. 3	1.470109	1.483863	0.000000	0.877303	1.185436
No. 4	1.289686	1.095166	0.936645	0.000000	0.716966
No. 5	1.226650	1.223108	1.088778	0.846738	0.000000

Variable	Analysis of Variance (Copy2 of fprinttraceln1)					
	Between SS	df	Within SS	df	F	signif. p
<b>SIO2</b>	0.1121	4	0.10557	39	10.35219	0.000008
<b>AL2O3</b>	2.0956	4	1.37745	39	14.83304	0.000000
<b>FEO</b>	1.1176	4	1.23157	39	8.84761	0.000035
<b>CAO</b>	27.8973	4	17.46143	39	15.57711	0.000000
<b>K2O</b>	7.1154	4	16.96818	39	4.08855	0.007317
<b>BA</b>	23.2982	4	12.86135	39	17.66203	0.000000
<b>CO</b>	1.2354	4	1.27521	39	9.44551	0.000019
<b>CR</b>	25.7037	4	21.23446	39	11.80208	0.000002
<b>CS</b>	15.6585	4	6.65870	39	22.92799	0.000000
<b>CU</b>	39.0390	4	26.01829	39	14.62932	0.000000
<b>LA</b>	133.1083	4	36.45353	39	35.60165	0.000000
<b>NI</b>	16.9623	4	6.99173	39	23.65405	0.000000
<b>SC</b>	6.6914	4	6.41050	39	10.17720	0.000010
<b>V</b>	24.5484	4	4.14598	39	57.73003	0.000000
<b>ZN</b>	6.8381	4	2.07267	39	32.16675	0.000000

Variable	Descriptive Statistics for Cluster 1 (Copy2 of fprinttraceln1)		
	Cluster contains 4 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.873115	0.053706	0.002884
<b>AL2O3</b>	1.880023	0.486421	0.236606
<b>FEO</b>	1.895712	0.271284	0.073595
<b>CAO</b>	2.601408	0.372423	0.138699
<b>K2O</b>	1.020488	0.168720	0.028466
<b>BA</b>	6.999165	0.445148	0.198157
<b>CO</b>	4.084200	0.399849	0.159879
<b>CR</b>	6.709892	0.726625	0.527984
<b>CS</b>	1.258937	0.475329	0.225938
<b>CU</b>	1.028537	0.352594	0.124322
<b>LA</b>	0.005218	0.376154	0.141492
<b>NI</b>	4.738605	0.250995	0.062999
<b>SC</b>	3.276493	0.820677	0.673511
<b>V</b>	4.145607	0.150538	0.022662
<b>ZN</b>	3.488330	0.276401	0.076397

Variable	Descriptive Statistics for Cluster 2 (Copy2 of fprinttraceln1)		
	Cluster contains 15 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.753055	0.048153	0.002319
<b>AL2O3</b>	2.613662	0.083753	0.007015
<b>FEO</b>	2.252052	0.187771	0.035258
<b>CAO</b>	0.135813	0.912377	0.832431
<b>K2O</b>	1.931175	0.244508	0.059784
<b>BA</b>	7.817775	0.419223	0.175748
<b>CO</b>	4.276913	0.122166	0.014924
<b>CR</b>	7.126579	0.583947	0.340994
<b>CS</b>	1.728443	0.328391	0.107841
<b>CU</b>	2.144799	1.050743	1.104062
<b>LA</b>	-0.251854	1.156938	1.338505
<b>NI</b>	5.332212	0.252870	0.063943
<b>SC</b>	2.610398	0.420095	0.176480
<b>V</b>	4.488474	0.356959	0.127420
<b>ZN</b>	4.163612	0.237764	0.056532

Variable	Descriptive Statistics for Cluster 3 (Copy2 of fprinttraceln1)		
	Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.849233	0.064647	0.004179
<b>AL2O3</b>	2.583347	0.154465	0.023859
<b>FEO</b>	2.433307	0.227939	0.051956
<b>CAO</b>	1.867485	0.303384	0.092042
<b>K2O</b>	1.444872	0.323651	0.104750
<b>BA</b>	8.182878	0.224827	0.050547
<b>CO</b>	3.747972	0.202223	0.040894
<b>CR</b>	5.215080	1.455548	2.118621
<b>CS</b>	0.606405	0.295308	0.087207
<b>CU</b>	3.366312	0.730764	0.534016
<b>LA</b>	4.134732	0.505349	0.255378
<b>NI</b>	4.101848	1.011453	1.023037
<b>SC</b>	3.324568	0.203011	0.041213
<b>V</b>	5.640415	0.278753	0.077703
<b>ZN</b>	4.745518	0.246920	0.060970

Variable	Descriptive Statistics for Cluster 4 (Copy2 of fprinttraceln1)		
	Cluster contains 11 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.867960	0.065264	0.004259
<b>AL2O3</b>	2.511979	0.194600	0.037869
<b>FEO</b>	2.269980	0.148755	0.022128
<b>CAO</b>	1.108066	0.586298	0.343746
<b>K2O</b>	0.962538	1.244080	1.547736
<b>BA</b>	6.794402	0.962912	0.927199
<b>CO</b>	4.080889	0.152748	0.023332
<b>CR</b>	6.916371	0.642794	0.413184
<b>CS</b>	0.897954	0.625877	0.391722
<b>CU</b>	4.038102	0.561030	0.314755
<b>LA</b>	2.807034	0.740200	0.547896
<b>NI</b>	6.091285	0.241221	0.058188
<b>SC</b>	2.455241	0.270979	0.073429
<b>V</b>	5.034225	0.232013	0.053830
<b>ZN</b>	4.815015	0.250971	0.062987

Variable	Descriptive Statistics for Cluster 5 (Copy2 of fprinttraceIn1)		
	Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.794851	0.011730	0.000138
<b>AL2O3</b>	2.274363	0.101026	0.010206
<b>FEO</b>	1.998000	0.071824	0.005159
<b>CAO</b>	1.518997	0.461925	0.213375
<b>K2O</b>	1.663782	0.079901	0.006384
<b>BA</b>	6.091486	0.200601	0.040241
<b>CO</b>	4.161819	0.145818	0.021263
<b>CR</b>	5.510035	0.147189	0.021665
<b>CS</b>	2.367243	0.129757	0.016837
<b>CU</b>	3.241873	0.790193	0.624405
<b>LA</b>	3.092121	1.226735	1.504879
<b>NI</b>	5.124131	0.173400	0.030067
<b>SC</b>	2.156190	0.373953	0.139841
<b>V</b>	3.219415	0.441959	0.195328
<b>ZN</b>	4.279053	0.129461	0.016760

**Number of Clusters:** 6  
**Seeds:** SA GS-17, SC GS-11, SC 10, L 7A, L 9A, L GS-4  
**Number of Cases:** 44  
**Number of Variables:** 15  
**Number of Iterations:** 2

Cluster #	Members	Distance From Mean
1	L 9A	0.244654
	L 1A	0.472058
	L 4A	0.362217
	L10A	0.353303
2	L 7A	0.212526
	L 5A	0.421248
	L 3A	0.438004
	SA GS-1	0.739775
	L A	0.277824
	L 6A	0.593011
	L 11A	0.525206
	P6	0.493268
3	L GS-4	0.177267
	L GS-2	0.186228
	L GS-10	0.576599
	L 8A	0.279981
	P1	0.116036
	P4	0.289965
	P5	0.165021
4	SC6	0.319303
	SC9	0.469735
	SC10	0.325862
	SC4	0.275086
	SC5	0.484033
	SC11	0.885084
5	SC GS-11	0.434205
	SC7	0.390553
	CH GS-16	0.262405
	CH GS-7	0.406576
	CH GS-12	0.465453
	SC GS-15	0.425174
	SC2	1.277526
	SC3	0.354497
	SC8	0.398839
	SC PVC5	0.452305
	P3	0.347564
6	SA GS-17	0.189418
	SA PVC1	0.689490
	SA GS-13	0.449926
	SA GS-3	0.296435
	SA GS-18	0.313724
	SA PVC2	0.181403
	SA PVC3	0.188881
	P2	0.599834



Variable	Cluster Means (Copy2 of fprinttraceln1)					
	Cluster No. 1	Cluster No. 2	Cluster No. 3	Cluster No. 4	Cluster No. 5	Cluster No. 6
<b>SIO2</b>	3.873115	3.750070	3.75647	3.849233	3.867960	3.794851
<b>AL2O3</b>	1.880023	2.598485	2.63101	2.583347	2.511979	2.274363
<b>FEO</b>	1.895712	2.182992	2.33098	2.433307	2.269980	1.998000
<b>CAO</b>	2.601408	-0.317481	0.65386	1.867485	1.108066	1.518997
<b>K2O</b>	1.020488	2.030982	1.81711	1.444872	0.962538	1.663782
<b>BA</b>	6.999165	7.788103	7.85169	8.182878	6.794402	6.091486
<b>CO</b>	4.084200	4.254299	4.30276	3.747972	4.080889	4.161819
<b>CR</b>	6.709892	7.028946	7.23816	5.215080	6.916371	5.510035
<b>CS</b>	1.258937	1.744077	1.71058	0.606405	0.897954	2.367243
<b>CU</b>	1.028537	1.346354	3.05731	3.366312	4.038102	3.241873
<b>LA</b>	0.005218	0.471740	-1.07882	4.134732	2.807034	3.092121
<b>NI</b>	4.738605	5.349915	5.31198	4.101848	6.091285	5.124131
<b>SC</b>	3.276493	2.678719	2.53232	3.324568	2.455241	2.156190
<b>V</b>	4.145607	4.324445	4.67594	5.640415	5.034225	3.219415
<b>ZN</b>	3.488330	4.082861	4.25590	4.745518	4.815015	4.279053

Cluster Number	Euclidean Distances between Clusters (Copy2 of fprinttraceln1)					
	Distances below diagonal Squared distances above diagonal					
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
No. 1	0.000000	0.838567	0.899746	2.161219	1.663291	1.504669
No. 2	0.915733	0.000000	0.438098	2.122024	1.285821	1.418282
No. 3	0.948549	0.661890	0.000000	2.526728	1.334262	1.818456
No. 4	1.470109	1.456717	1.589568	0.000000	0.877303	1.185436
No. 5	1.289686	1.133941	1.155103	0.936645	0.000000	0.716966
No. 6	1.226650	1.190917	1.348501	1.088778	0.846738	0.000000

Variable	Analysis of Variance (Copy2 of fprinttraceln1)					
	Between SS	df	Within SS	df	F	signif. p
<b>SIO2</b>	0.1122	5	0.10542	38	8.09210	0.000028
<b>AL2O3</b>	2.0995	5	1.37350	38	11.61725	0.000001
<b>FEO</b>	1.1993	5	1.14981	38	7.92739	0.000034
<b>CAO</b>	31.4197	5	13.93900	38	17.13105	0.000000
<b>K2O</b>	7.2862	5	16.79741	38	3.29664	0.014341
<b>BA</b>	23.3133	5	12.84626	38	13.79243	0.000000
<b>CO</b>	1.2442	5	1.26644	38	7.46623	0.000058
<b>CR</b>	25.8671	5	21.07105	38	9.32985	0.000007
<b>CS</b>	15.6627	5	6.65451	38	17.88811	0.000000
<b>CU</b>	49.9678	5	15.08950	38	25.16683	0.000000
<b>LA</b>	142.0841	5	27.47775	38	39.29867	0.000000
<b>NI</b>	16.9677	5	6.98636	38	18.45805	0.000000
<b>SC</b>	6.7714	5	6.33048	38	8.12933	0.000027
<b>V</b>	25.0097	5	3.68474	38	51.58401	0.000000
<b>ZN</b>	6.9498	5	1.96089	38	26.93617	0.000000

Variable	Descriptive Statistics for Cluster 1 (Copy2 of fprinttraceln1)		
	Cluster contains 4 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.873115	0.053706	0.002884
<b>AL2O3</b>	1.880023	0.486421	0.236606
<b>FEO</b>	1.895712	0.271284	0.073595
<b>CAO</b>	2.601408	0.372423	0.138699
<b>K2O</b>	1.020488	0.168720	0.028466
<b>BA</b>	6.999165	0.445148	0.198157
<b>CO</b>	4.084200	0.399849	0.159879
<b>CR</b>	6.709892	0.726625	0.527984
<b>CS</b>	1.258937	0.475329	0.225938
<b>CU</b>	1.028537	0.352594	0.124322
<b>LA</b>	0.005218	0.376154	0.141492
<b>NI</b>	4.738605	0.250995	0.062999
<b>SC</b>	3.276493	0.820677	0.673511
<b>V</b>	4.145607	0.150538	0.022662
<b>ZN</b>	3.488330	0.276401	0.076397

Variable	Descriptive Statistics for Cluster 2 (Copy2 of fprinttraceln1)		
	Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.750070	0.065589	0.004302
<b>AL2O3</b>	2.598485	0.113176	0.012809
<b>FEO</b>	2.182992	0.232292	0.053960
<b>CAO</b>	-0.317481	1.035240	1.071723
<b>K2O</b>	2.030982	0.295395	0.087258
<b>BA</b>	7.788103	0.563756	0.317820
<b>CO</b>	4.254299	0.119839	0.014361
<b>CR</b>	7.028946	0.769757	0.592526
<b>CS</b>	1.744077	0.442181	0.195524
<b>CU</b>	1.346354	0.748535	0.560305
<b>LA</b>	0.471740	0.703330	0.494673
<b>NI</b>	5.349915	0.319077	0.101810
<b>SC</b>	2.678719	0.572141	0.327345
<b>V</b>	4.324445	0.408673	0.167014
<b>ZN</b>	4.082861	0.301427	0.090858

Variable	Descriptive Statistics for Cluster 3 (Copy2 of fprinttraceln1)		
	Cluster contains 7 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.75647	0.019132	0.000366
<b>AL2O3</b>	2.63101	0.027667	0.000765
<b>FEO</b>	2.33098	0.075427	0.005689
<b>CAO</b>	0.65386	0.323923	0.104926
<b>K2O</b>	1.81711	0.096092	0.009234
<b>BA</b>	7.85169	0.191760	0.036772
<b>CO</b>	4.30276	0.128871	0.016608
<b>CR</b>	7.23816	0.277735	0.077137
<b>CS</b>	1.71058	0.151061	0.022819
<b>CU</b>	3.05731	0.317789	0.100990
<b>LA</b>	-1.07882	1.024741	1.050094
<b>NI</b>	5.31198	0.171833	0.029526
<b>SC</b>	2.53232	0.128637	0.016548
<b>V</b>	4.67594	0.159971	0.025591
<b>ZN</b>	4.25590	0.085297	0.007276

Variable	Descriptive Statistics for Cluster 4 (Copy2 of fprinttraceln1)		
	Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.849233	0.064647	0.004179
<b>AL2O3</b>	2.583347	0.154465	0.023859
<b>FEO</b>	2.433307	0.227939	0.051956
<b>CAO</b>	1.867485	0.303384	0.092042
<b>K2O</b>	1.444872	0.323651	0.104750
<b>BA</b>	8.182878	0.224827	0.050547
<b>CO</b>	3.747972	0.202223	0.040894
<b>CR</b>	5.215080	1.455548	2.118621
<b>CS</b>	0.606405	0.295308	0.087207
<b>CU</b>	3.366312	0.730764	0.534016
<b>LA</b>	4.134732	0.505349	0.255378
<b>NI</b>	4.101848	1.011453	1.023037
<b>SC</b>	3.324568	0.203011	0.041213
<b>V</b>	5.640415	0.278753	0.077703
<b>ZN</b>	4.745518	0.246920	0.060970

Variable	Descriptive Statistics for Cluster 5 (Copy2 of fprinttraceln1)		
	Cluster contains 11 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.867960	0.065264	0.004259
<b>AL2O3</b>	2.511979	0.194600	0.037869
<b>FEO</b>	2.269980	0.148755	0.022128
<b>CAO</b>	1.108066	0.586298	0.343746
<b>K2O</b>	0.962538	1.244080	1.547736
<b>BA</b>	6.794402	0.962912	0.927199
<b>CO</b>	4.080889	0.152748	0.023332
<b>CR</b>	6.916371	0.642794	0.413184
<b>CS</b>	0.897954	0.625877	0.391722
<b>CU</b>	4.038102	0.561030	0.314755
<b>LA</b>	2.807034	0.740200	0.547896
<b>NI</b>	6.091285	0.241221	0.058188
<b>SC</b>	2.455241	0.270979	0.073429
<b>V</b>	5.034225	0.232013	0.053830
<b>ZN</b>	4.815015	0.250971	0.062987

Variable	Descriptive Statistics for Cluster 6 (Copy2 of fprinttraceln1)		
	Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.794851	0.011730	0.000138
<b>AL2O3</b>	2.274363	0.101026	0.010206
<b>FEO</b>	1.998000	0.071824	0.005159
<b>CAO</b>	1.518997	0.461925	0.213375
<b>K2O</b>	1.663782	0.079901	0.006384
<b>BA</b>	6.091486	0.200601	0.040241
<b>CO</b>	4.161819	0.145818	0.021263
<b>CR</b>	5.510035	0.147189	0.021665
<b>CS</b>	2.367243	0.129757	0.016837
<b>CU</b>	3.241873	0.790193	0.624405
<b>LA</b>	3.092121	1.226735	1.504879
<b>NI</b>	5.124131	0.173400	0.030067
<b>SC</b>	2.156190	0.373953	0.139841
<b>V</b>	3.219415	0.441959	0.195328
<b>ZN</b>	4.279053	0.129461	0.016760

**Number of Clusters:** 7  
**Seeds:** SA GS-17, SC GS-11, SC 10, L 7A, L 9A, L GS-4, SS GS-1  
**Number of Cases:** 44  
**Number of Variables:** 15  
**Number of Iterations:** 2

Cluster #	Members	Distance From Mean
1	L 3A	0.321940
	L A	0.255551
	L 5A	0.395374
	L 6A	0.499871
	L 7A	0.261557
	P6	0.541052
2	SC6	0.319303
	SC9	0.469735
	SC10	0.325862
	SC4	0.275086
	SC5	0.484033
	SC11	0.885084
3	SA GS-13	0.312145
	SA GS-3	0.261966
	SA GS-17	0.195952
	SA PVC2	0.258850
	SA PVC3	0.294314
	P2	0.498900
4	SC GS-11	0.434205
	SC2	1.277526
	CH GS-16	0.262405
	CH GS-7	0.406576
	CH GS-12	0.465453
	SC GS-15	0.425174
	SC3	0.354497
	SC7	0.390553
	SC8	0.398839
	SC PVC5	0.452305
	P3	0.347564
5	L GS-4	0.140977
	L GS-2	0.183521
	L GS-10	0.642763
	L 8A	0.229516
	L 11A	0.542762
	P1	0.148714
	P4	0.235555
	P5	0.216429
6	L 9A	0.244654
	L 1A	0.472058
	L 4A	0.362217
	L10A	0.353303
7	SA GS-1	0.398374
	SA GS-18	0.338640
	SA PVC1	0.309404

Variable	Cluster Means (fprinttraceln1)						
	Cluster No. 1	Cluster No. 2	Cluster No. 3	Cluster No. 4	Cluster No. 5	Cluster No. 6	Cluster No. 7
<b>SIO2</b>	3.722787	3.849233	3.793150	3.867960	3.757902	3.873115	3.831930
<b>AL2O3</b>	2.638675	2.583347	2.265747	2.511979	2.631519	1.880023	2.307050
<b>FEO</b>	2.195737	2.433307	2.011653	2.269980	2.332720	1.895712	1.952893
<b>CAO</b>	-0.508148	1.867485	1.552011	1.108066	0.682779	2.601408	0.821253
<b>K2O</b>	2.131755	1.444872	1.646838	0.962538	1.797547	1.020488	1.741980
<b>BA</b>	8.028680	8.182878	6.032575	6.794402	7.828551	6.999165	6.334193
<b>CO</b>	4.240338	3.747972	4.116030	4.080889	4.298275	4.084200	4.307943
<b>CR</b>	7.329450	5.215080	5.522915	6.916371	7.196859	6.709892	5.429966
<b>CS</b>	1.645485	0.606405	2.325357	0.897954	1.666555	1.258937	2.569033
<b>CU</b>	1.042053	3.366312	3.526678	4.038102	2.996719	1.028537	2.240273
<b>LA</b>	0.388797	4.134732	3.611703	2.807034	-0.801337	0.005218	1.122284
<b>NI</b>	5.430375	4.101848	5.157128	6.091285	5.297132	4.738605	5.024720
<b>SC</b>	2.858697	3.324568	2.215982	2.455241	2.550046	3.276493	1.852350
<b>V</b>	4.451203	5.640415	3.277335	5.034225	4.648238	4.145607	3.175100
<b>ZN</b>	3.963297	4.745518	4.306278	4.815015	4.308714	3.488330	4.199813

Cluster Number	Euclidean Distances between Clusters (fprinttraceln1)						
	Distances below diagonal						
	Squared distances above diagonal						
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7
No. 1	0.000000	2.363313	2.061789	1.519284	0.474652	0.955291	0.951489
No. 2	1.537307	0.000000	1.109993	0.877303	2.315574	2.161219	1.923806
No. 3	1.435893	1.053562	0.000000	0.702903	1.951840	1.807591	0.584609
No. 4	1.232592	0.936645	0.838393	0.000000	1.189627	1.663291	1.167020
No. 5	0.688950	1.521701	1.397083	1.090700	0.000000	0.832038	0.897492
No. 6	0.977390	1.470109	1.344467	1.289686	0.912161	0.000000	0.933194
No. 7	0.975443	1.387013	0.764597	1.080287	0.947360	0.966020	0.000000

Variable	Analysis of Variance (fprinttraceln1)					
	Between SS	df	Within SS	df	F	signif. p
<b>SIO2</b>	0.1321	6	0.08555	37	9.52275	0.000002
<b>AL2O3</b>	2.1892	6	1.28384	37	10.51533	0.000001
<b>FEO</b>	1.2876	6	1.06159	37	7.47939	0.000027
<b>CAO</b>	30.9164	6	14.44234	37	13.20084	0.000000
<b>K2O</b>	7.5201	6	16.56346	37	2.79979	0.023943
<b>BA</b>	25.4501	6	10.70944	37	14.65459	0.000000
<b>CO</b>	1.2993	6	1.21132	37	6.61446	0.000082
<b>CR</b>	29.1503	6	17.78781	37	10.10581	0.000001
<b>CS</b>	16.7236	6	5.59368	37	18.43664	0.000000
<b>CU</b>	53.9977	6	11.05953	37	30.10852	0.000000
<b>LA</b>	143.7562	6	25.80563	37	34.35284	0.000000
<b>NI</b>	17.1512	6	6.80287	37	15.54721	0.000000
<b>SC</b>	8.0973	6	5.00459	37	9.97749	0.000002
<b>V</b>	25.8529	6	2.84153	37	56.10566	0.000000
<b>ZN</b>	7.2667	6	1.64404	37	27.25686	0.000000

Variable	Descriptive Statistics for Cluster 1 (fprintraceln1) Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.722787	0.028518	0.000813
<b>AL2O3</b>	2.638675	0.017218	0.000296
<b>FEO</b>	2.195737	0.242360	0.058738
<b>CAO</b>	-0.508148	1.080191	1.166813
<b>K2O</b>	2.131755	0.267516	0.071565
<b>BA</b>	8.028680	0.151743	0.023026
<b>CO</b>	4.240338	0.137213	0.018828
<b>CR</b>	7.329450	0.390541	0.152522
<b>CS</b>	1.645485	0.203293	0.041328
<b>CU</b>	1.042053	0.548337	0.300673
<b>LA</b>	0.388797	0.767337	0.588806
<b>NI</b>	5.430375	0.329535	0.108594
<b>SC</b>	2.858697	0.433768	0.188155
<b>V</b>	4.451203	0.229305	0.052581
<b>ZN</b>	3.963297	0.190107	0.036140

Variable	Descriptive Statistics for Cluster 2 (fprintraceln1) Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.849233	0.064647	0.004179
<b>AL2O3</b>	2.583347	0.154465	0.023859
<b>FEO</b>	2.433307	0.227939	0.051956
<b>CAO</b>	1.867485	0.303384	0.092042
<b>K2O</b>	1.444872	0.323651	0.104750
<b>BA</b>	8.182878	0.224827	0.050547
<b>CO</b>	3.747972	0.202223	0.040894
<b>CR</b>	5.215080	1.455548	2.118621
<b>CS</b>	0.606405	0.295308	0.087207
<b>CU</b>	3.366312	0.730764	0.534016
<b>LA</b>	4.134732	0.505349	0.255378
<b>NI</b>	4.101848	1.011453	1.023037
<b>SC</b>	3.324568	0.203011	0.041213
<b>V</b>	5.640415	0.278753	0.077703
<b>ZN</b>	4.745518	0.246920	0.060970

Variable	Descriptive Statistics for Cluster 3 (fprintraceln1) Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.793150	0.011894	0.000141
<b>AL2O3</b>	2.265747	0.112460	0.012647
<b>FEO</b>	2.011653	0.074342	0.005527
<b>CAO</b>	1.552011	0.493022	0.243071
<b>K2O</b>	1.646838	0.086425	0.007469
<b>BA</b>	6.032575	0.153988	0.023712
<b>CO</b>	4.116030	0.129906	0.016876
<b>CR</b>	5.522915	0.159604	0.025473
<b>CS</b>	2.325357	0.122083	0.014904
<b>CU</b>	3.526678	0.666112	0.443705
<b>LA</b>	3.611703	0.804021	0.646450
<b>NI</b>	5.157128	0.191000	0.036481
<b>SC</b>	2.215982	0.403888	0.163125
<b>V</b>	3.277335	0.406110	0.164926
<b>ZN</b>	4.306278	0.128219	0.016440

Variable	Descriptive Statistics for Cluster 4 (fprintraceln1) Cluster contains 11 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.867960	0.065264	0.004259
<b>AL2O3</b>	2.511979	0.194600	0.037869
<b>FEO</b>	2.269980	0.148755	0.022128
<b>CAO</b>	1.108066	0.586298	0.343746
<b>K2O</b>	0.962538	1.244080	1.547736
<b>BA</b>	6.794402	0.962912	0.927199
<b>CO</b>	4.080889	0.152748	0.023332
<b>CR</b>	6.916371	0.642794	0.413184
<b>CS</b>	0.897954	0.625877	0.391722
<b>CU</b>	4.038102	0.561030	0.314755
<b>LA</b>	2.807034	0.740200	0.547896
<b>NI</b>	6.091285	0.241221	0.058188
<b>SC</b>	2.455241	0.270979	0.073429
<b>V</b>	5.034225	0.232013	0.053830
<b>ZN</b>	4.815015	0.250971	0.062987



Variable	Descriptive Statistics for Cluster 5 (fprintraceIn1) Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.757902	0.018173	0.000330
<b>AL2O3</b>	2.631519	0.025656	0.000658
<b>FEO</b>	2.332720	0.070006	0.004901
<b>CAO</b>	0.682779	0.310847	0.096626
<b>K2O</b>	1.797547	0.104765	0.010976
<b>BA</b>	7.828551	0.189212	0.035801
<b>CO</b>	4.298275	0.119983	0.014396
<b>CR</b>	7.196859	0.282423	0.079763
<b>CS</b>	1.666555	0.187249	0.035062
<b>CU</b>	2.996719	0.340483	0.115929
<b>LA</b>	-0.801337	1.231278	1.516046
<b>NI</b>	5.297132	0.164537	0.027072
<b>SC</b>	2.550046	0.129222	0.016698
<b>V</b>	4.648238	0.167548	0.028072
<b>ZN</b>	4.308714	0.168973	0.028552

Variable	Descriptive Statistics for Cluster 6 (fprintraceIn1) Cluster contains 4 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.873115	0.053706	0.002884
<b>AL2O3</b>	1.880023	0.486421	0.236606
<b>FEO</b>	1.895712	0.271284	0.073595
<b>CAO</b>	2.601408	0.372423	0.138699
<b>K2O</b>	1.020488	0.168720	0.028466
<b>BA</b>	6.999165	0.445148	0.198157
<b>CO</b>	4.084200	0.399849	0.159879
<b>CR</b>	6.709892	0.726625	0.527984
<b>CS</b>	1.258937	0.475329	0.225938
<b>CU</b>	1.028537	0.352594	0.124322
<b>LA</b>	0.005218	0.376154	0.141492
<b>NI</b>	4.738605	0.250995	0.062999
<b>SC</b>	3.276493	0.820677	0.673511
<b>V</b>	4.145607	0.150538	0.022662
<b>ZN</b>	3.488330	0.276401	0.076397

Variable	Descriptive Statistics for Cluster 7 (fprintraceIn1) Cluster contains 3 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.831930	0.056218	0.003160
<b>AL2O3</b>	2.307050	0.057905	0.003353
<b>FEO</b>	1.952893	0.045313	0.002053
<b>CAO</b>	0.821253	1.096079	1.201389
<b>K2O</b>	1.741980	0.049725	0.002473
<b>BA</b>	6.334193	0.230160	0.052973
<b>CO</b>	4.307943	0.085452	0.007302
<b>CR</b>	5.429966	0.123695	0.015300
<b>CS</b>	2.569033	0.134179	0.018004
<b>CU</b>	2.240273	0.409596	0.167768
<b>LA</b>	1.122284	0.958304	0.918347
<b>NI</b>	5.024720	0.031119	0.000968
<b>SC</b>	1.852350	0.291895	0.085203
<b>V</b>	3.175100	0.530420	0.281345
<b>ZN</b>	4.199813	0.093181	0.008683

**Number of Clusters:** 6  
**Seeds:** SA GS-17, SC GS-11, SC 10, L 7A, L 9A, and CH GS-7 or CH GS-12 or Ch GS-16  
**Number of Cases:** 44  
**Number of Variables:** 15  
**Number of Iterations:** 2

Cluster #	Members	Distance From Mean
1	L GS-4	0.353766
	L 5A	0.702337
	L 3A	0.629236
	SA GS-1	0.775860
	L GS-2	0.436138
	L 7A	0.217604
	L GS-10	0.785601
	L A	0.281378
	L 6A	0.736967
	L 8A	0.375317
	L 11A	0.466517
	P1	0.318551
	P4	0.360606
	P5	0.411252
	P6	0.542717
2	SC6	0.319303
	SC9	0.469735
	SC10	0.325862
	SC4	0.275086
	SC5	0.484033
	SC11	0.885084
3	SA GS-17	0.189418
	SA PVC1	0.689490
	SA GS-13	0.449926
	SA GS-3	0.296435
	SA GS-18	0.313724
	SA PVC2	0.181403
	SA PVC3	0.188881
	P2	0.599834
4	SC GS-11	0.369949
	CH GS-7	0.403518
	CH GS-12	0.456103
	CH GS-16	0.223736
	SC GS-15	0.331403
	SC3	0.346930
	SC7	0.472495
	SC8	0.431193
	SC PVC5	0.376759
	P3	0.283669
5	L 9A	0.244654
	L 1A	0.472058
	L 4A	0.362217
	L10A	0.353303
6	SC2	0.000000

Variable	Cluster Means (fprinttraceln1)					
	Cluster No. 1	Cluster No. 2	Cluster No. 3	Cluster No. 4	Cluster No. 5	Cluster No. 6
<b>SIO2</b>	3.753055	3.849233	3.794851	3.863054	3.873115	3.91702
<b>AL2O3</b>	2.613662	2.583347	2.274363	2.560960	1.880023	2.02217
<b>FEO</b>	2.252052	2.433307	1.998000	2.261441	1.895712	2.35537
<b>CAO</b>	0.135813	1.867485	1.518997	1.108516	2.601408	1.10356
<b>K2O</b>	1.931175	1.444872	1.663782	1.312626	1.020488	-2.53834
<b>BA</b>	7.817775	8.182878	6.091486	7.047997	6.999165	4.25845
<b>CO</b>	4.276913	3.747972	4.161819	4.058978	4.084200	4.30000
<b>CR</b>	7.126579	5.215080	5.510035	7.064636	6.709892	5.43372
<b>CS</b>	1.728443	0.606405	2.367243	1.053334	1.258937	-0.65585
<b>CU</b>	2.144799	3.366312	3.241873	4.098513	1.028537	3.43399
<b>LA</b>	-0.251854	4.134732	3.092121	2.790184	0.005218	2.97553
<b>NI</b>	5.332212	4.101848	5.124131	6.059717	4.738605	6.40697
<b>SC</b>	2.610398	3.324568	2.156190	2.464680	3.276493	2.36085
<b>V</b>	4.488474	5.640415	3.219415	5.088672	4.145607	4.48976
<b>ZN</b>	4.163612	4.745518	4.279053	4.844230	3.488330	4.52287

Cluster Number	Euclidean Distances between Clusters (fprinttraceln1)					
	Distances below diagonal			Squared distances above diagonal		
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
No. 1	0.000000	2.201848	1.495993	1.125890	0.758079	3.729663
No. 2	1.483863	0.000000	1.185436	0.857319	2.161219	2.872424
No. 3	1.223108	1.088778	0.000000	0.741711	1.504669	2.264799
No. 4	1.061080	0.925915	0.861226	0.000000	1.693049	1.974809
No. 5	0.870677	1.470109	1.226650	1.301172	0.000000	3.160991
No. 6	1.931234	1.694823	1.504925	1.405279	1.777918	0.000000

Variable	Analysis of Variance (fprinttraceln1)					
	Between SS	df	Within SS	df	F	signif. p
<b>SIO2</b>	0.1147	5	0.10292	38	8.47248	0.000019
<b>AL2O3</b>	2.3595	5	1.11355	38	16.10348	0.000000
<b>FEO</b>	1.1256	5	1.22355	38	6.99163	0.000102
<b>CAO</b>	27.8973	5	17.46141	38	12.14218	0.000000
<b>K2O</b>	20.5972	5	3.48641	38	44.89960	0.000000
<b>BA</b>	30.3724	5	5.78719	38	39.88636	0.000000
<b>CO</b>	1.2882	5	1.22240	38	8.00908	0.000031
<b>CR</b>	28.1217	5	18.81638	38	11.35847	0.000001
<b>CS</b>	18.3143	5	4.00296	38	34.77136	0.000000
<b>CU</b>	39.4404	5	25.61684	38	11.70118	0.000001
<b>LA</b>	133.1395	5	36.42231	38	27.78133	0.000000
<b>NI</b>	17.0720	5	6.88211	38	18.85277	0.000000
<b>SC</b>	6.7012	5	6.40070	38	7.95678	0.000033
<b>V</b>	24.8745	5	3.81989	38	49.49002	0.000000
<b>ZN</b>	6.9319	5	1.97879	38	26.62377	0.000000

Variable	Descriptive Statistics for Cluster 1 (fprintraceIn1) Cluster contains 15 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.753055	0.048153	0.002319
<b>AL2O3</b>	2.613662	0.083753	0.007015
<b>FEO</b>	2.252052	0.187771	0.035258
<b>CAO</b>	0.135813	0.912377	0.832431
<b>K2O</b>	1.931175	0.244508	0.059784
<b>BA</b>	7.817775	0.419223	0.175748
<b>CO</b>	4.276913	0.122166	0.014924
<b>CR</b>	7.126579	0.583947	0.340994
<b>CS</b>	1.728443	0.328391	0.107841
<b>CU</b>	2.144799	1.050743	1.104062
<b>LA</b>	-0.251854	1.156938	1.338505
<b>NI</b>	5.332212	0.252870	0.063943
<b>SC</b>	2.610398	0.420095	0.176480
<b>V</b>	4.488474	0.356959	0.127420
<b>ZN</b>	4.163612	0.237764	0.056532

Variable	Descriptive Statistics for Cluster 2 (fprintraceIn1) Cluster contains 6 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.849233	0.064647	0.004179
<b>AL2O3</b>	2.583347	0.154465	0.023859
<b>FEO</b>	2.433307	0.227939	0.051956
<b>CAO</b>	1.867485	0.303384	0.092042
<b>K2O</b>	1.444872	0.323651	0.104750
<b>BA</b>	8.182878	0.224827	0.050547
<b>CO</b>	3.747972	0.202223	0.040894
<b>CR</b>	5.215080	1.455548	2.118621
<b>CS</b>	0.606405	0.295308	0.087207
<b>CU</b>	3.366312	0.730764	0.534016
<b>LA</b>	4.134732	0.505349	0.255378
<b>NI</b>	4.101848	1.011453	1.023037
<b>SC</b>	3.324568	0.203011	0.041213
<b>V</b>	5.640415	0.278753	0.077703
<b>ZN</b>	4.745518	0.246920	0.060970

Variable	Descriptive Statistics for Cluster 3 (fprintraceln1) Cluster contains 8 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.794851	0.011730	0.000138
<b>AL2O3</b>	2.274363	0.101026	0.010206
<b>FEO</b>	1.998000	0.071824	0.005159
<b>CAO</b>	1.518997	0.461925	0.213375
<b>K2O</b>	1.663782	0.079901	0.006384
<b>BA</b>	6.091486	0.200601	0.040241
<b>CO</b>	4.161819	0.145818	0.021263
<b>CR</b>	5.510035	0.147189	0.021665
<b>CS</b>	2.367243	0.129757	0.016837
<b>CU</b>	3.241873	0.790193	0.624405
<b>LA</b>	3.092121	1.226735	1.504879
<b>NI</b>	5.124131	0.173400	0.030067
<b>SC</b>	2.156190	0.373953	0.139841
<b>V</b>	3.219415	0.441959	0.195328
<b>ZN</b>	4.279053	0.129461	0.016760

Variable	Descriptive Statistics for Cluster 4 (fprintraceln1) Cluster contains 10 cases		
	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.863054	0.066622	0.004438
<b>AL2O3</b>	2.560960	0.112934	0.012754
<b>FEO</b>	2.261441	0.153934	0.023696
<b>CAO</b>	1.108516	0.618011	0.381937
<b>K2O</b>	1.312626	0.470885	0.221733
<b>BA</b>	7.047997	0.494170	0.244204
<b>CO</b>	4.058978	0.141621	0.020057
<b>CR</b>	7.064636	0.436369	0.190418
<b>CS</b>	1.053334	0.374386	0.140165
<b>CU</b>	4.098513	0.552379	0.305123
<b>LA</b>	2.790184	0.778013	0.605303
<b>NI</b>	6.059717	0.229069	0.052473
<b>SC</b>	2.464680	0.283724	0.080499
<b>V</b>	5.088672	0.153556	0.023579
<b>ZN</b>	4.844230	0.244036	0.059554

Descriptive Statistics for Cluster 5 (fprintraceIn1) Cluster contains 4 cases			
Variable	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.873115	0.053706	0.002884
<b>AL2O3</b>	1.880023	0.486421	0.236606
<b>FEO</b>	1.895712	0.271284	0.073595
<b>CAO</b>	2.601408	0.372423	0.138699
<b>K2O</b>	1.020488	0.168720	0.028466
<b>BA</b>	6.999165	0.445148	0.198157
<b>CO</b>	4.084200	0.399849	0.159879
<b>CR</b>	6.709892	0.726625	0.527984
<b>CS</b>	1.258937	0.475329	0.225938
<b>CU</b>	1.028537	0.352594	0.124322
<b>LA</b>	0.005218	0.376154	0.141492
<b>NI</b>	4.738605	0.250995	0.062999
<b>SC</b>	3.276493	0.820677	0.673511
<b>V</b>	4.145607	0.150538	0.022662
<b>ZN</b>	3.488330	0.276401	0.076397

Descriptive Statistics for Cluster 6 (fprintraceIn1) Cluster contains 1 cases			
Variable	Mean	Standard Deviation	Variance
<b>SIO2</b>	3.91702	0.00	0.00
<b>AL2O3</b>	2.02217	0.00	0.00
<b>FEO</b>	2.35537	0.00	0.00
<b>CAO</b>	1.10356	0.00	0.00
<b>K2O</b>	-2.53834	0.00	0.00
<b>BA</b>	4.25845	0.00	0.00
<b>CO</b>	4.30000	0.00	0.00
<b>CR</b>	5.43372	0.00	0.00
<b>CS</b>	-0.65585	0.00	0.00
<b>CU</b>	3.43399	0.00	0.00
<b>LA</b>	2.97553	0.00	0.00
<b>NI</b>	6.40697	0.00	0.00
<b>SC</b>	2.36085	0.00	0.00
<b>V</b>	4.48976	0.00	0.00
<b>ZN</b>	4.52287	0.00	0.00

## **Appendix D**

### **Results of Four-Category Discriminant Analyses**



## Four-Category Model, Original Element Set

### Element Set:

SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, CaO, Na<sub>2</sub>O, K<sub>2</sub>O, Ba, Ce, Co, Cr, Cs, Cu, Ga, La, Ni, Sc, Sm V, Zn

### Categories:

Category	Abbreviation	Sample IDs	No. Samples
South Africa	SA	GS-1, GS-3, GS-13, GS-17, GS-18,PVC1, PVC2, PVC3	8
China	CH	GS-7, GS-12, GS-16	3
South Carolina	SC	GS-11, SG-15, SC2, SC3, SC4, SC5, SC6, SC7, SC8, SC9, SC10, SC11, PVC5	13
Libby	L	GS-2, GS-4, GS-10, 1A, 2A, 3A, 4A, 5A, 6A, 7A, 8A, 9A, 10A, 11A	14
Total Number of Samples =			38

### Discriminant Function Analysis.

N=38	Discriminant Function Analysis Summary (fprintrace1n1) Step 10, N of vars in model: 10; Grouping: Category (4 grps) Wilks' Lambda: .00044 approx. F (30,74)=32.014 p<0.0000						
	Element Added	Wilks' $\Lambda$	Partial $\Lambda$	F-Remove	p-Level	Tolerance	Correlated Variables Removed
1	V	0.00228	0.1910	35.29	3.8443E-09	0.2960	TiO <sub>2</sub> , MnO, Na <sub>2</sub> O, Cs
2	Sm	0.00173	0.2522	24.71	1.1951E-07	0.2305	La, Ce
3	Sc	0.00221	0.1972	33.93	5.6882E-09	0.1084	none
4	Ni	0.00153	0.2843	20.98	5.2351E-07	0.1600	none
5	Ga	0.00074	0.5843	5.93	3.3631E-03	0.2003	Al <sub>2</sub> O <sub>3</sub>
6	K <sub>2</sub> O	0.00125	0.3478	15.62	6.2577E-06	0.1174	none
7	Ba	0.00087	0.5018	8.27	5.4278E-04	0.1009	none
8	SiO <sub>2</sub>	0.00064	0.6794	3.93	1.9897E-02	0.3953	none
9	Co	0.00063	0.6952	3.65	2.5998E-02	0.2398	none
10	CaO	0.00061	0.7110	3.39	3.3710E-02	0.2979	none

n = 38	Variables currently not in the model (fprintrace1n1) Df for all F-tests: 3,24					
	Element	Wilks' $\Lambda$	Wilks' Partial $\Lambda$	F to Enter	p-level	Toler. 1-Toler.
	FeO	0.000397	0.9125	0.7675	0.5235	0.1931 0.8069
	Cr	0.000380	0.8734	1.1592	0.3458	0.2338 0.7662
	Cu	0.000399	0.9156	0.7372	0.5402	0.4438 0.5562
	Zn	0.000340	0.7820	2.2300	0.1107	0.3498 0.6502

	Squared Mahalanobis Distances (fprintraceIn1)			
Category	SA	CH	SC	L
SA	0.0000	196.9609	271.7414	282.0565
CH	196.9609	0.0000	46.9349	68.1226
SC	271.7414	46.9349	0.0000	66.1183
L	282.0565	68.1226	66.1183	0.0000

	F-values; df = 10,26 (fprintraceIn1)			
Category	SA	CH	SC	L
SA		31.598	98.953	105.583
CH	31.598		8.412	12.375
SC	98.953	8.412		32.771
L	105.583	12.375	32.771	

	p-levels (fprintraceIn1)			
Category	SA	CH	SC	L
SA		5.036E-12	4.389E-18	1.941E-18
CH	5.036E-12		6.282E-06	1.536E-07
SC	4.389E-18	6.282E-06		3.281E-12
L	1.941E-18	1.536E-07	3.281E-12	

Summary of Stepwise Analysis (fprintraceIn1)											
Variable Enter/Remove	Step	F to enter/ remove	df 1	df 2	p-level	No. of Vars. In Model	Lambda	F-value	df 1	df 2	p-level
V -(E)	1	52.653	3	34	7.209E-13	1	0.177120	52.653	3	34	7.209E-13
Sm -(E)	2	33.598	3	33	3.825E-10	2	0.043686	41.629	6	66	1.391E-20
Sc -(E)	3	22.999	3	32	3.990E-08	3	0.013841	41.656	9	78	3.281E-26
Ni -(E)	4	10.419	3	31	6.715E-05	4	0.006892	38.149	12	82	1.712E-28
Ga -(E)	5	8.367	3	30	3.428E-04	5	0.003752	36.411	15	83	3.898E-30
K <sub>2</sub> O -(E)	6	9.704	3	29	1.350E-04	6	0.001873	37.641	18	83	0.000E+00
Ba -(E)	7	6.926	3	28	1.245E-03	7	0.001075	37.817	21	81	0.000E+00
SiO <sub>2</sub> -(E)	8	2.751	3	27	6.211E-02	8	0.000823	34.767	24	79	0.000E+00
Co -(E)	9	2.989	3	26	4.930E-02	9	0.000612	32.885	27	77	0.000E+00
CaO -(E)	10	3.387	3	25	3.371E-02	10	0.000435	32.014	30	74	0.000E+00

Chi-Square Tests with Successive Roots Removed (fprintraceIn1)						
Roots Removed	Eigen- value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	47.35652	0.989606	0.000435	232.1880	30	0.000E+00
1	13.23402	0.964233	0.021047	115.8299	18	2.220E-16
2	2.33797	0.836909	0.299583	36.1609	8	1.641E-05

Variable	Raw Coefficients for Canonical Variables			Standardized Coefficients for Canonical Variables		
	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3
V	-3.8913	-1.2663	2.52540	-1.41103	-0.45918	0.91575
Sm	1.0846	-1.4575	0.05126	1.10442	-1.48419	0.05220
Sc	-5.4607	2.1399	-1.63977	-2.49940	0.97945	-0.75053
Ni	-2.8046	0.3588	0.50912	-2.09650	0.26821	0.38058
Ga	0.0862	-0.7480	-5.86921	0.02502	-0.21703	-1.70306
K <sub>2</sub> O	2.5027	1.1602	1.81737	1.89425	0.87812	1.37552
Ba	-2.6596	-0.9911	-0.89945	-2.03836	-0.75957	-0.68935
SiO <sub>2</sub>	-4.2602	-13.5984	-5.34365	-0.26495	-0.84572	-0.33234
Co	-5.3605	-0.6265	-0.02755	-1.13199	-0.13230	-0.00582
CaO	-0.6967	0.6461	0.07810	-0.73666	0.68318	0.08258
Constant	100.7097	59.9839	29.89091	--	--	--
Eigenval	47.3565	13.2340	2.33797	47.35652	13.23402	2.33797
Cum.Prob	0.7525	0.9628	1.00000	0.75254	0.96285	1.00000

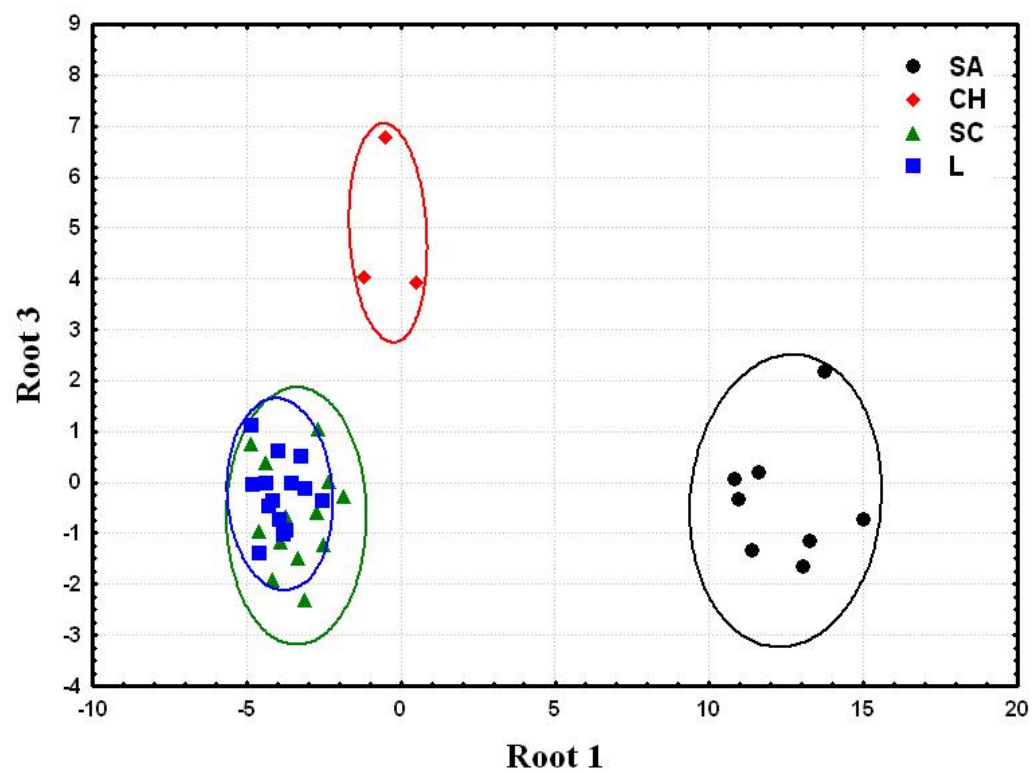
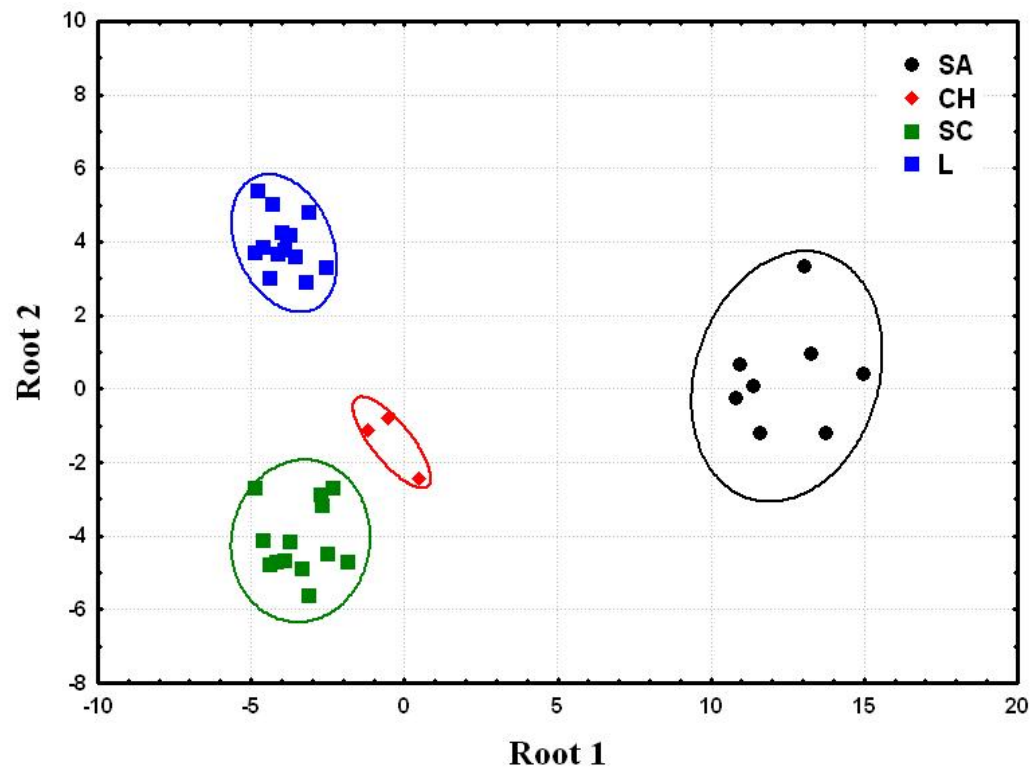
Factor Structure Matrix (fprinttraceln1)  
Correlations Variables - Canonical Roots  
(Pooled-within-groups correlations)

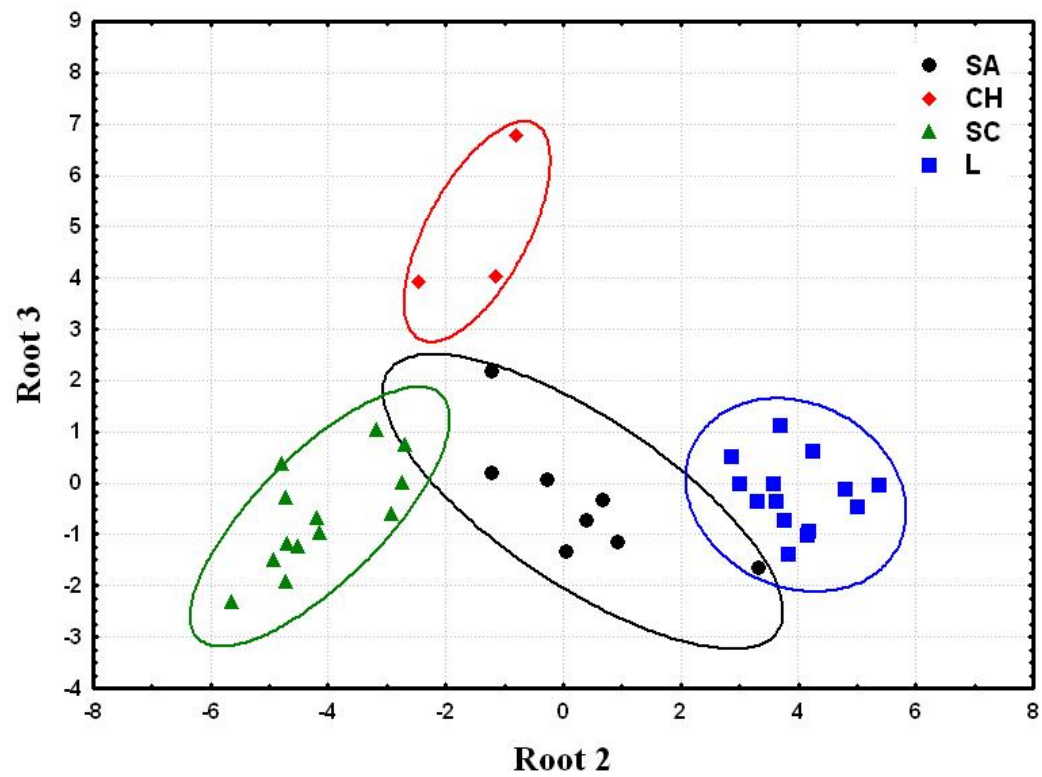
Variable	Root 1	Root 2	Root 3
V	-0.265650	-0.301819	0.205141
Sm	0.084162	-0.461940	0.017361
Sc	-0.120982	0.002864	-0.225644
Ni	-0.000411	-0.013043	0.270923
Ga	-0.018921	-0.068479	-0.204791
K <sub>2</sub> O	0.023647	0.117008	0.066308
Ba	-0.115295	0.040881	-0.112695
SiO <sub>2</sub>	-0.010777	-0.198531	-0.048099
Co	0.041778	0.164090	0.036244
CaO	0.008053	-0.082343	0.064625

Means of Canonical Variables  
(fprinttraceln1)

Category	Root 1	Root 2	Root 3
SA	12.48530	0.33682	-0.358611
CH	-0.40133	-1.46090	4.901036
SC	-3.37724	-4.13881	-0.658425
L	-3.91245	3.96376	-0.233907

		Unstandardized Canonical Scores (fprintraceIn1)		
Samples	Group	Root 1	Root 2	Root 3
SA GS-1	SA	11.38517	0.06275	-1.35223
SA GS-3	SA	13.77015	-1.21673	2.16048
SA GS-13	SA	11.59856	-1.22209	0.18014
SA GS-17	SA	10.84693	-0.26074	0.04967
SA GS-18	SA	10.96063	0.66855	-0.33234
SA PVC1	SA	13.06462	3.32121	-1.66963
SA PVC2	SA	13.25851	0.93341	-1.16515
SA PVC3	SA	14.99782	0.40820	-0.73982
CH GS-7	CH	0.49530	-2.44887	3.90816
CH GS-12	CH	-0.50687	-0.79155	6.77699
CH GS-16	CH	-1.19242	-1.14229	4.01795
SC GS-11	SC	-2.29402	-2.72258	0.00284
SC GS-15	SC	-2.67490	-3.17300	1.04691
SC2	SC	-3.89127	-4.68218	-1.19594
SC3	SC	-1.82741	-4.72547	-0.27658
SC4	SC	-3.31516	-4.92198	-1.50356
SC5	SC	-4.36647	-4.78101	0.36413
SC6	SC	-4.13514	-4.71356	-1.92307
SC7	SC	-2.48276	-4.51315	-1.24989
SC8	SC	-3.69649	-4.19009	-0.67063
SC9	SC	-3.08595	-5.63920	-2.32574
SC10	SC	-4.59665	-4.13545	-0.98107
SC11	SC	-2.68803	-2.91241	-0.59866
SC PVC5	SC	-4.84989	-2.69445	0.75174
L GS-2	L	-3.21066	2.87755	0.50267
L GS-4	L	-4.38172	3.00500	-0.02110
L GS-10	L	-4.12813	3.64101	-0.37085
L 1A	L	-3.11477	4.79748	-0.11592
L A	L	-4.29443	5.02010	-0.48149
L 3A	L	-4.57087	3.84173	-1.39136
L 4A	L	-3.69845	4.18178	-0.94517
L 5A	L	-4.78087	5.37838	-0.04997
L 6A	L	-2.55196	3.29835	-0.37827
L 7A	L	-3.97981	4.25121	0.62305
L 8A	L	-3.90601	3.76343	-0.72574
L 9A	L	-3.52560	3.57936	-0.02611
L10A	L	-4.83719	3.69514	1.12010
L 11A	L	-3.79381	4.16212	-1.01455
P1	----	-5.22578	3.87869	-0.84295
P2	----	13.31965	0.46538	-1.33576
P3	----	-3.25215	-2.73771	-0.70710
P4	----	-4.51610	3.65506	-0.72381
P5	----	-4.94768	4.44159	-0.39023
P6	----	0.07617	0.99367	-0.53219





## Classification

	Classification Functions; grouping: Category (fprinttraceln1)			
Variable	SA p=.21053	CH p=.07895	SC p=.34211	L p=.36842
V	359.9	425.6	426.6	419.5
Sm	-5.5	-16.6	-16.2	-28.5
Sc	486.2	544.1	563.7	583.3
Ni	224.5	262.7	267.2	271.9
Ga	224.8	194.1	228.5	219.9
K <sub>2</sub> O	-237.9	-262.7	-283.4	-274.6
Ba	311.1	342.4	357.9	351.0
SiO <sub>2</sub>	3064.2	3115.5	3194.3	3084.1
Co	991.0	1061.1	1078.9	1076.7
CaO	22.9	31.1	31.0	36.7
Constant	-10634.1	-11818.6	-12444.9	-12001.1

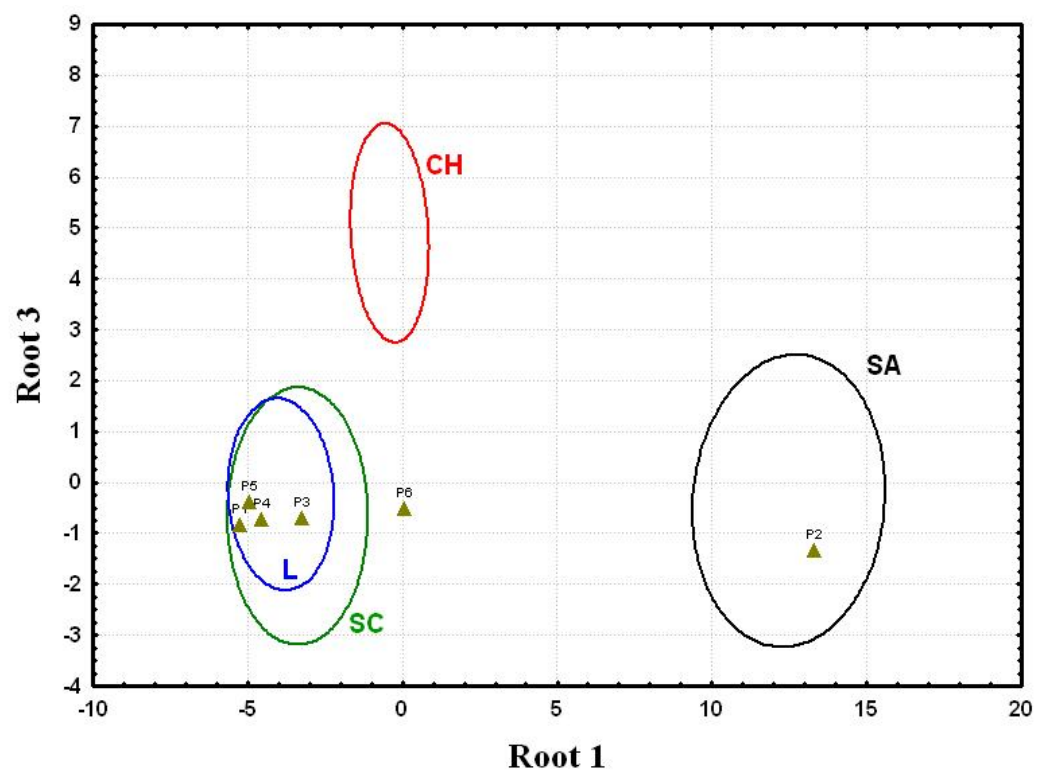
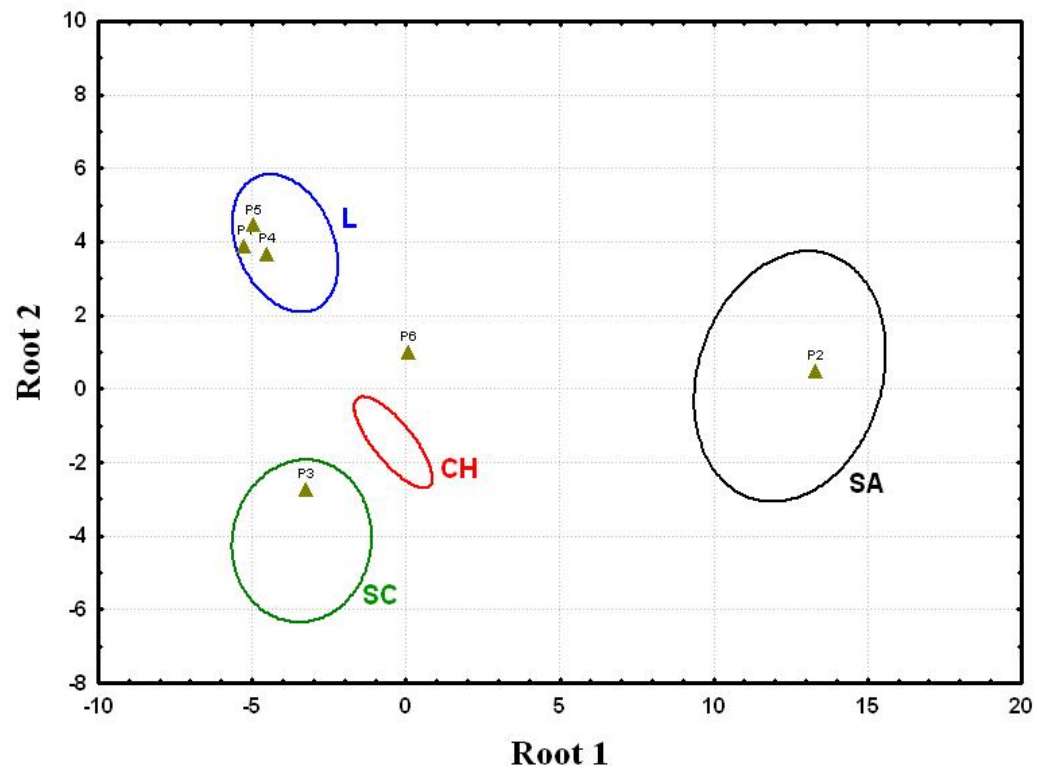
Classification Matrix (fprinttraceln1.sta) Rows: Observed classifications Columns: Predicted classifications					
	Percent Correct	SA p=.21053	CH p=.07895	SC p=.34211	L p=.36842
SA	100.00	8	0	0	0
CH	100.00	0	3	0	0
SC	100.00	0	0	13	0
L	100.00	0	0	0	14
Total	100.00	8	3	13	14

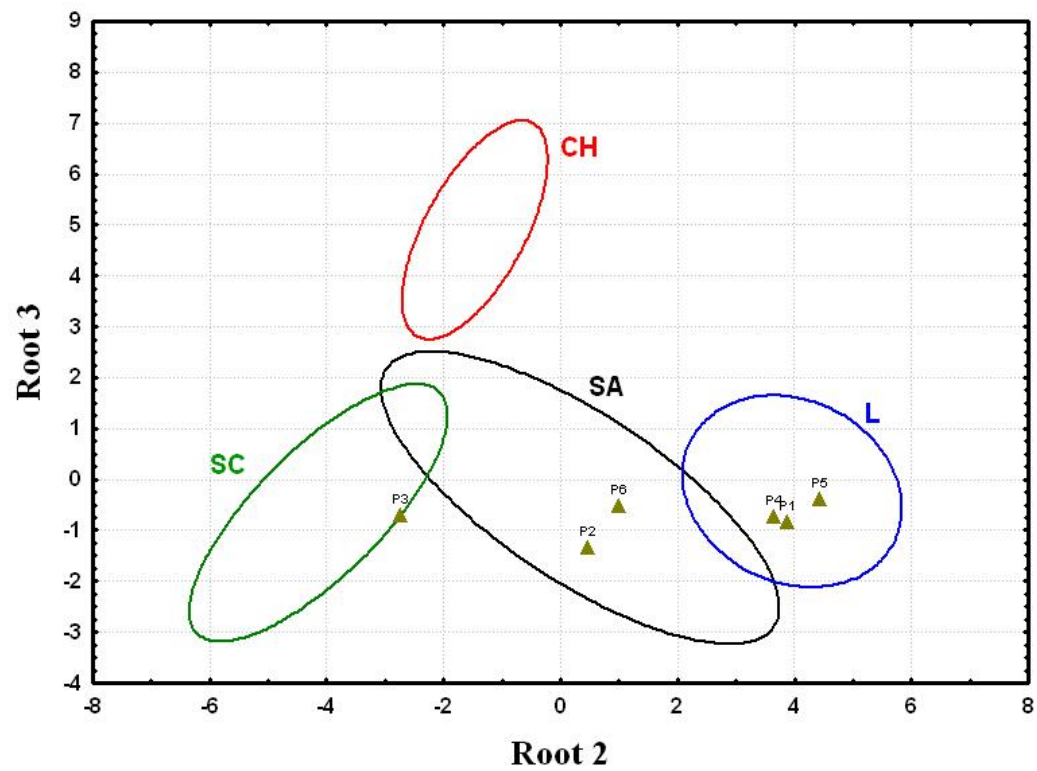
		Classification of Cases (fprinttraceln1) Incorrect classifications are marked with *			
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	SA	CH	SC	L
SA GS-3	SA	SA	CH	SC	L
SA GS-13	SA	SA	CH	SC	L
SA GS-17	SA	SA	CH	SC	L
SA GS-18	SA	SA	CH	SC	L
SA PVC1	SA	SA	CH	SC	L
SA PVC2	SA	SA	CH	SC	L
SA PVC3	SA	SA	CH	SC	L
CH GS-7	CH	CH	SC	L	SA
CH GS-12	CH	CH	SC	L	SA
CH GS-16	CH	CH	SC	L	SA
SC GS-11	SC	SC	CH	L	SA
SC GS-15	SC	SC	CH	L	SA
SC2	SC	SC	CH	L	SA
SC3	SC	SC	CH	L	SA
SC4	SC	SC	CH	L	SA
SC5	SC	SC	CH	L	SA
SC6	SC	SC	CH	L	SA
SC7	SC	SC	CH	L	SA
SC8	SC	SC	CH	L	SA
SC9	SC	SC	CH	L	SA
SC10	SC	SC	CH	L	SA
SC11	SC	SC	CH	L	SA
SC PVC5	SC	SC	CH	L	SA
L GS-2	L	L	CH	SC	SA
L GS-4	L	L	SC	CH	SA
L GS-10	L	L	SC	CH	SA
L 1A	L	L	CH	SC	SA
L A	L	L	SC	CH	SA
L 3A	L	L	SC	CH	SA
L 4A	L	L	SC	CH	SA
L 5A	L	L	SC	CH	SA
L 6A	L	L	SC	CH	SA
L 7A	L	L	CH	SC	SA
L 8A	L	L	SC	CH	SA
L 9A	L	L	SC	CH	SA
L10A	L	L	CH	SC	SA
L 11A	L	L	SC	CH	SA
P1	---	L	SC	CH	SA
P2	---	SA	CH	SC	L
P3	---	SC	CH	L	SA
P4	---	L	SC	CH	SA
P5	---	L	SC	CH	SA
P6	---	L	SC	CH	SA



Squared Mahalanobis Distances from Group Centroids (fprinttraceln1)					
Incorrect classifications are marked with *					
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	16.9700	195.0438	250.7606	265.1830
SA GS-3	SA	11.8906	209.8816	311.9984	346.7255
SA GS-13	SA	8.1718	171.0063	238.1502	272.3211
SA GS-17	SA	4.9162	153.2077	219.5761	237.4744
SA GS-18	SA	3.5377	162.1192	229.8939	233.1789
SA PVC1	SA	15.1546	251.5678	331.2030	294.8888
SA PVC2	SA	4.1219	231.6402	305.2500	307.4096
SA PVC3	SA	7.5382	273.5217	359.4000	371.5714
CH GS-7	CH	171.1337	4.1741	40.1145	79.1151
CH GS-12	CH	225.2874	8.2792	79.0294	87.6644
CH GS-16	CH	211.1628	4.2479	38.3618	54.2894
SC GS-11	SC	235.7968	37.0443	11.4942	55.2603
SC GS-15	SC	246.9409	25.7696	7.1492	56.9203
SC2	SC	320.2600	85.9058	27.0247	101.8546
SC3	SC	235.8648	44.8765	8.2697	85.2296
SC4	SC	283.4905	66.3584	6.2015	85.7952
SC5	SC	318.8942	55.5265	10.6342	85.2324
SC6	SC	306.9500	73.8467	5.2612	80.9559
SC7	SC	251.1997	54.3228	4.1304	77.7746
SC8	SC	286.4762	53.3855	4.1402	70.7581
SC9	SC	292.7213	87.5667	15.7910	107.9508
SC10	SC	316.0582	63.2296	5.4675	70.5000
SC11	SC	262.8518	59.5893	23.9895	70.9208
SC PVC5	SC	316.3979	43.9955	11.7110	51.6496
L GS-2	L	258.1226	50.6223	55.1674	6.7772
L GS-4	L	294.4542	62.7401	55.1740	3.9096
L GS-10	L	290.9650	71.7523	65.2131	4.2103
L 1A	L	283.8763	92.2576	100.7781	21.9029
L A	L	304.5414	87.1649	85.7919	2.3568
L 3A	L	314.2932	95.1264	75.6802	11.8173
L 4A	L	283.3036	83.1510	75.6796	6.8613
L 5A	L	327.1675	94.0028	96.4515	6.3234
L 6A	L	247.2584	67.5150	68.4391	14.6828
L 7A	L	289.0733	65.4224	74.0852	2.5090
L 8A	L	285.1882	75.8738	67.3663	4.9188
L 9A	L	271.1152	63.5837	64.1337	4.4822
L10A	L	327.7320	74.7554	80.8637	16.9590
L 11A	L	281.4329	79.4817	70.5659	2.0230
P1	---	329.3152	87.6333	70.5848	4.9563
P2	---	14.1618	243.3677	312.9378	322.8923
P3	---	267.6032	51.5702	12.3427	55.9312
P4	---	302.6873	77.2389	64.5413	3.1954
P5	---	322.7456	85.4927	78.1479	3.3110
P6	---	165.9113	47.2360	49.7475	36.2826

		Posterior Probabilities (fprintraceln1) Incorrect classifications are marked with *			
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	1.00000	0.00000	0.00000	0.00000
SA GS-3	SA	1.00000	0.00000	0.00000	0.00000
SA GS-13	SA	1.00000	0.00000	0.00000	0.00000
SA GS-17	SA	1.00000	0.00000	0.00000	0.00000
SA GS-18	SA	1.00000	0.00000	0.00000	0.00000
SA PVC1	SA	1.00000	0.00000	0.00000	0.00000
SA PVC2	SA	1.00000	0.00000	0.00000	0.00000
SA PVC3	SA	1.00000	0.00000	0.00000	0.00000
CH GS-7	CH	0.00000	1.00000	0.00000	0.00000
CH GS-12	CH	0.00000	1.00000	0.00000	0.00000
CH GS-16	CH	0.00000	1.00000	0.00000	0.00000
SC GS-11	SC	0.00000	0.00000	1.00000	0.00000
SC GS-15	SC	0.00000	0.00002	0.99998	0.00000
SC2	SC	0.00000	0.00000	1.00000	0.00000
SC3	SC	0.00000	0.00000	1.00000	0.00000
SC4	SC	0.00000	0.00000	1.00000	0.00000
SC5	SC	0.00000	0.00000	1.00000	0.00000
SC6	SC	0.00000	0.00000	1.00000	0.00000
SC7	SC	0.00000	0.00000	1.00000	0.00000
SC8	SC	0.00000	0.00000	1.00000	0.00000
SC9	SC	0.00000	0.00000	1.00000	0.00000
SC10	SC	0.00000	0.00000	1.00000	0.00000
SC11	SC	0.00000	0.00000	1.00000	0.00000
SC PVC5	SC	0.00000	0.00000	1.00000	0.00000
L GS-2	L	0.00000	0.00000	0.00000	1.00000
L GS-4	L	0.00000	0.00000	0.00000	1.00000
L GS-10	L	0.00000	0.00000	0.00000	1.00000
L 1A	L	0.00000	0.00000	0.00000	1.00000
L A	L	0.00000	0.00000	0.00000	1.00000
L 3A	L	0.00000	0.00000	0.00000	1.00000
L 4A	L	0.00000	0.00000	0.00000	1.00000
L 5A	L	0.00000	0.00000	0.00000	1.00000
L 6A	L	0.00000	0.00000	0.00000	1.00000
L 7A	L	0.00000	0.00000	0.00000	1.00000
L 8A	L	0.00000	0.00000	0.00000	1.00000
L 9A	L	0.00000	0.00000	0.00000	1.00000
L10A	L	0.00000	0.00000	0.00000	1.00000
L 11A	L	0.00000	0.00000	0.00000	1.00000
P1	---	0.00000	0.00000	0.00000	1.00000
P2	---	1.00000	0.00000	0.00000	0.00000
P3	---	0.00000	0.00000	1.00000	0.00000
P4	---	0.00000	0.00000	0.00000	1.00000
P5	---	0.00000	0.00000	0.00000	1.00000
P6	---	0.00000	0.00089	0.00110	0.99800





## Four-Category Model, Gunter Element Set

### Element Set:

SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, CaO, K<sub>2</sub>O, Ba, Cr, Cu, Ga, Ni, Sc, V, Zn

### Categories:

Category	Abbreviation	Sample IDs	No. Samples
South Africa	SA	GS-1, GS-3, GS-13, GS-17, GS-18, PVC1, PVC2, PVC3	8
China	CH	GS-7, GS-12, GS-16	3
South Carolina	SC	GS-11, SG-15, SC2, SC3, SC4, SC5, SC6, SC7, SC8, SC9, SC10, SC11, PVC5	13
Libby	L	GS-2, GS-4, GS-10, 1A, 2A, 3A, 4A, 5A, 6A, 7A, 8A, 9A, 10A, 11A	14
Total Number of Samples =			38

### Discriminant Function Analysis.

N = 38	Discriminant Function Analysis Summary (fprinttraceln1) Step 8, N of vars in model: 8; Grouping: Category (4 grps) Wilks' Lambda: .00198 approx. F (24,78)=24.821 p<0.00						
	Step	Element Added	Wilks' $\Lambda$	Wilks' Partial $\Lambda$	F-Remove	p-Level	Tolerance
	1	V	0.00542	0.3658	15.60	4.3979E-06	0.4099
	2	Zn	0.00500	0.3966	13.69	1.2793E-05	0.2403
	3	SiO <sub>2</sub>	0.00265	0.7493	3.01	4.7439E-02	0.2878
	4	Ga	0.00520	0.3811	14.62	7.5508E-06	0.1734
	5	Cr	0.00576	0.3441	17.16	1.9500E-06	0.3630
	6	FeO	0.00300	0.6610	4.61	9.8700E-03	0.1390
	7	Sc	0.00329	0.6016	5.96	2.9592E-03	0.4186
	8	K <sub>2</sub> O	0.00280	0.7085	3.70	2.3671E-02	0.3029

n = 38	Variables currently not in the model (fprinttraceln1) Df for all F-tests: 3,26					
	Element	Wilks' $\Lambda$	Wilks' Partial $\Lambda$	F to Enter	p-level	Toler. 1-Toler.
	CaO	0.001938	0.9776	0.1988	0.8962	0.4941 0.5059
	Ba	0.001464	0.7386	3.0665	0.0456	0.1442 0.8558
	Ni	0.001816	0.9164	0.7908	0.5101	0.2033 0.7967

	Squared Mahalanobis Distances (fprinttraceln1)			
Category	SA	CH	SC	L
SA	0.0000	102.5790	117.4226	77.9210
CH	102.5790	0.0000	39.9000	63.8787
SC	117.4226	39.9000	0.0000	38.7489
L	77.9210	63.8787	38.7489	0.0000

	F-values; df = 8,28 (fprinttraceln1)			
Category	SA	CH	SC	L
SA		22.21631	57.72453	39.37717
CH	22.21631		9.65412	15.66576
SC	57.72453	9.65412		25.92757
L	39.37717	15.66576	25.92757	

	p-levels (fprinttraceln1)			
Category	SA	CH	SC	L
SA		3.3858E-10	2.2966E-15	3.1267E-13
CH	3.3858E-10		2.5782E-06	1.7819E-08
SC	2.2966E-15	2.5782E-06		5.4161E-11
L	3.1267E-13	1.7819E-08	5.4161E-11	

Summary of Stepwise Analysis (fprinttraceln1)											
Variable Enter/Remove	Step	F to enter/ remove	df 1	df 2	p-level	No. of Vars. In Model	Lambda	F-value	df 1	df 2	p-level
V-(E)	1	52.653	3	34	7.209E-13	1	0.177120	52.653	3	34	7.209E-13
Zn-(E)	2	15.013	3	33	2.470E-06	2	0.074898	29.194	6	66	8.613E-17
SiO <sub>2</sub> -(E)	3	19.567	3	32	2.174E-07	3	0.026424	29.914	9	78	8.320E-22
Ga-(E)	4	8.146	3	31	3.828E-04	4	0.014776	26.879	12	82	1.735E-23
Cr-(E)	5	10.542	3	30	6.744E-05	5	0.007193	27.600	15	83	5.342E-26
FeO-(E)	6	7.228	3	29	9.154E-04	6	0.004116	27.380	18	83	1.797E-27
Sc-(E)	7	4.397	3	28	1.177E-02	7	0.002798	26.011	21	81	1.427E-27
K <sub>2</sub> O-(E)	8	3.703	3	27	2.367E-02	8	0.001982	24.821	24	79	1.790E-27

Chi-Square Tests with Successive Roots Removed (fprinttraceln1)						
Roots Removed	Eigen- value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	17.368	0.972397	0.001982	192.9299	24	0.000E+00
1	7.088	0.936143	0.036408	102.7017	14	1.443E-15
2	2.396	0.839954	0.294478	37.8991	6	1.176E-06

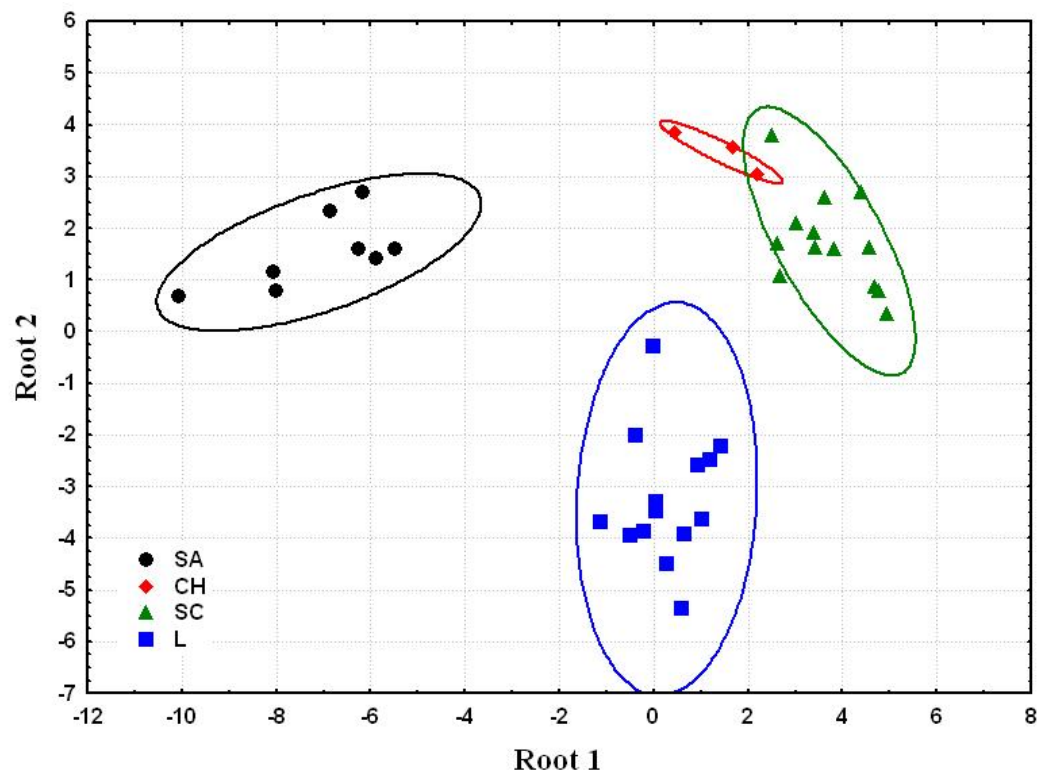
	Raw Coefficients for Canonical Variables			Standardized Coefficients for Canonical Variables		
Variable	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3
V	3.1420	0.8886	-1.5701	1.13933	0.32224	-0.569353
Zn	-0.2183	5.4974	-1.8441	-0.06432	1.61951	-0.543257
SiO <sub>2</sub>	9.9705	4.2941	12.7728	0.62010	0.26706	0.794381
Ga	-0.3122	-1.1279	7.6398	-0.09059	-0.32729	2.216844
Cr	1.1384	-0.9852	-0.3952	1.03500	-0.89570	-0.359268
FeO	3.7137	-7.1198	1.3745	0.75631	-1.44998	0.279928
Sc	1.5325	-1.1312	1.3047	0.70142	-0.51774	0.597164
K <sub>2</sub> O	-0.8796	0.0145	-1.1590	-0.66576	0.01095	-0.877241
Constant	-68.9045	-16.5362	-55.9869	--	--	--
Eigenval	17.3676	7.0882	2.3958	17.36758	7.08820	2.395840
Cum.Prob	0.6468	0.9108	1.0000	0.64680	0.91077	1.000000

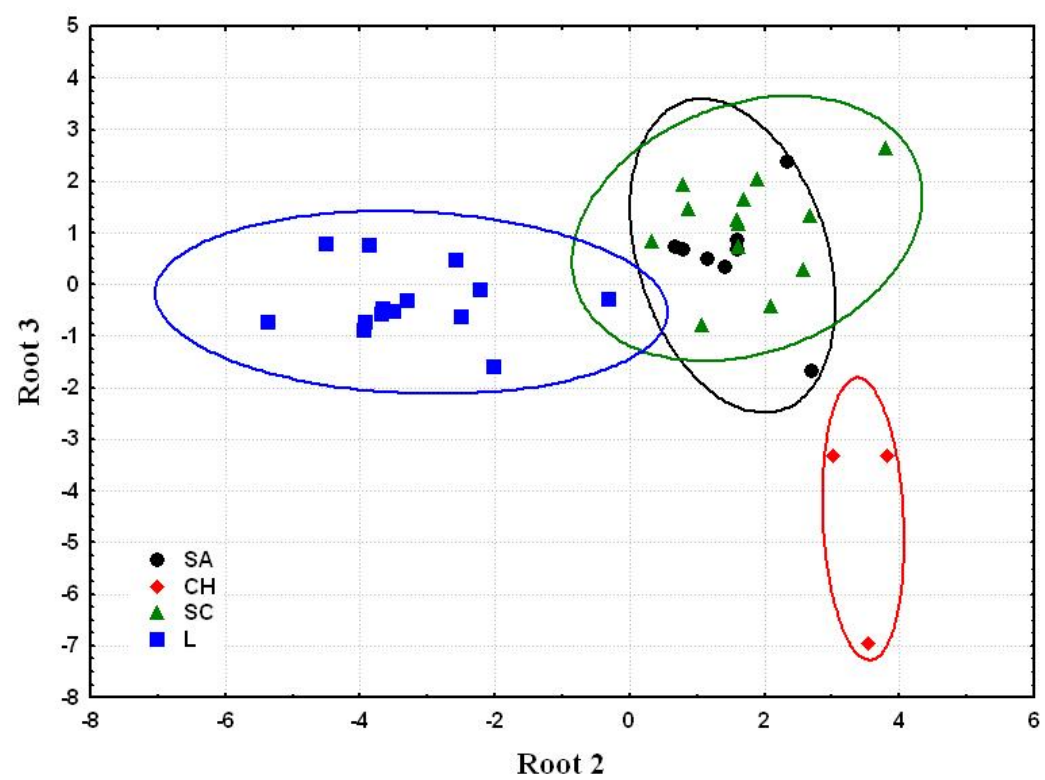
Factor Structure Matrix (fprintraceIn1) Correlations Variables - Canonical Roots (Pooled-within-groups correlations)			
Variable	Root 1	Root 2	Root 3
V	0.510621	0.117634	-0.090200
Zn	0.132942	0.426040	-0.004085
SiO <sub>2</sub>	0.086252	0.214834	0.178307
Ga	0.049883	0.025378	0.232298
Cr	0.086460	-0.204432	-0.196677
FeO	0.149094	0.031691	0.018553
Sc	0.177869	-0.168057	0.161675
K <sub>2</sub> O	-0.076324	-0.102075	-0.133844

Means of Canonical Variables (fprintraceIn1)			
Category	Root 1	Root 2	Root 3
SA	-7.09263	1.52883	0.55594
CH	1.43957	3.46738	-4.54528
SC	3.72849	1.74464	1.08437
L	0.28228	-3.23665	-0.35061

		Unstandardized Canonical Scores		
Samples	Group	Root 1	Root 2	Root 3
SA GS-1	SA	-6.8691	2.33157	2.38055
SA GS-3	SA	-6.1658	2.69496	-1.66991
SA GS-13	SA	-5.8720	1.41615	0.32302
SA GS-17	SA	-5.4680	1.59278	0.67047
SA GS-18	SA	-6.2442	1.58909	0.84623
SA PVC1	SA	-10.0534	0.66692	0.71975
SA PVC2	SA	-8.0082	0.78816	0.68410
SA PVC3	SA	-8.0604	1.15101	0.49331
CH GS-7	CH	0.4479	3.83101	-3.32757
CH GS-12	CH	1.6861	3.54792	-6.97361
CH GS-16	CH	2.1847	3.02322	-3.33467
SC GS-11	SC	3.4216	1.62454	0.72880
SC GS-15	SC	3.0204	2.10385	-0.42831
SC2	SC	3.3885	1.90711	2.02638
SC3	SC	2.6183	1.70263	1.64760
SC4	SC	4.6889	0.87046	1.47019
SC5	SC	3.6269	2.58079	0.28463
SC6	SC	4.7837	0.78886	1.93148
SC7	SC	4.4059	2.68683	1.33839
SC8	SC	4.5627	1.62100	1.18345
SC9	SC	2.5128	3.79450	2.63122
SC10	SC	4.9455	0.33025	0.83793
SC11	SC	3.8224	1.59912	1.23626
SC PVC5	SC	2.6728	1.07044	-0.79116
L GS-2	L	1.2039	-2.49136	-0.62790
L GS-4	L	1.4127	-2.22190	-0.11227
L GS-10	L	0.9458	-2.58119	0.46281
L 1A	L	1.0292	-3.64762	-0.47766
L A	L	0.6493	-3.91384	-0.72750
L 3A	L	0.2832	-4.50200	0.76912
L 4A	L	-0.2016	-3.86424	0.73905
L 5A	L	-1.1407	-3.68284	-0.58859
L 6A	L	0.5812	-5.36939	-0.73092
L 7A	L	-0.5014	-3.94066	-0.90610
L 8A	L	0.0362	-3.48799	-0.51717
L 9A	L	0.0617	-3.30043	-0.31072
L10A	L	-0.3914	-2.00923	-1.59197
L 11A	L	-0.0162	-0.30047	-0.28870
P1	----	0.2267	-2.74608	0.27492
P2	----	-7.2611	-0.07846	0.17481
P3	----	2.5951	2.02858	1.04453
P4	----	0.1824	-2.90728	0.49128
P5	----	0.2457	-1.82842	-0.29813
P6	----	-2.9812	-2.32754	-0.25845
Li1UI	----	0.7445	-2.82027	1.11412
Li2UI	----	-0.3713	-1.86869	0.80540
at1UI	----	0.0384	-1.11334	0.56251
at2UI	----	0.5234	-0.42104	-0.05394
at3UI	----	1.0941	1.86280	-0.32419
at4UI	----	0.1955	-2.14112	2.76890
ga1UI	----	-7.0706	2.07781	1.37983
ga2UI	----	-7.3793	2.42123	1.51806
ga3UI	----	-6.4877	1.85108	-0.37271
ga4UI	----	-7.5112	1.71044	0.43326
ga5UI	----	-8.9533	2.52884	-4.40406
ga6UI	----	-7.1461	2.42040	0.29110
ga7UI	----	-7.9248	1.50284	2.10292
SA1UI	----	-15.6962	-0.95550	6.50161
SC1UI	----	3.0122	0.94970	1.18751
SC2UI	----	2.5889	1.86007	1.10042
SC3UI	----	1.6612	1.88958	-0.10643
SC4UI	----	3.3381	0.95770	1.08347
SC5UI	----	2.9061	3.38219	8.42554
SC6UI	----	7.7752	0.06021	2.64416







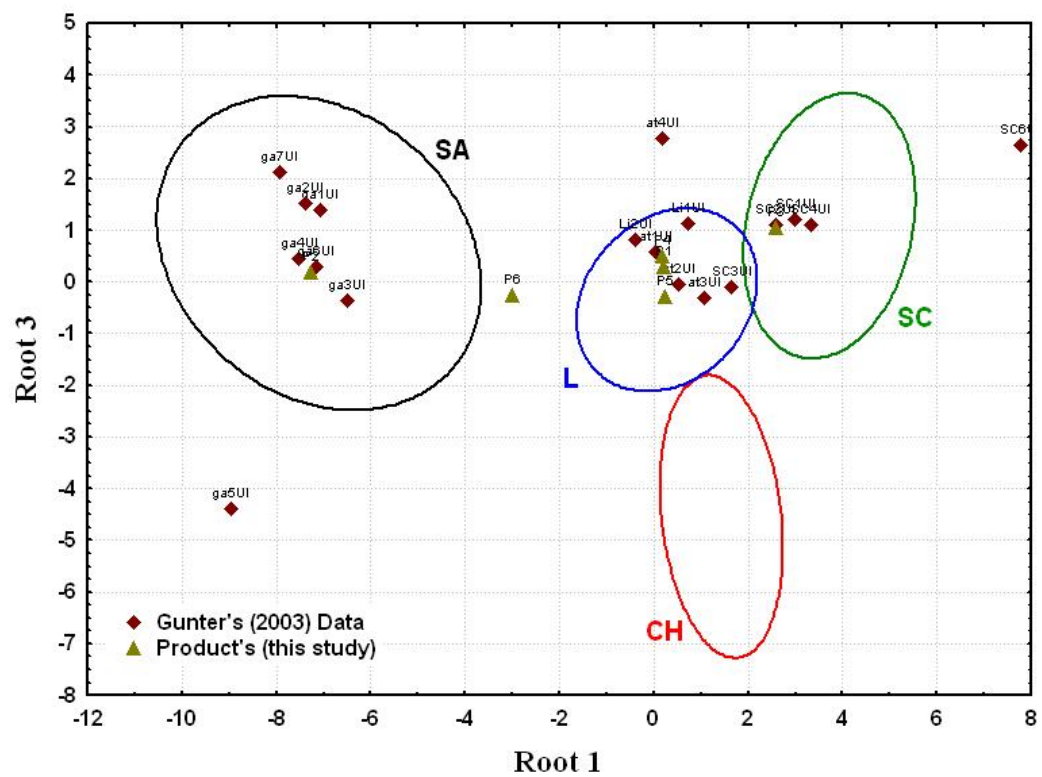
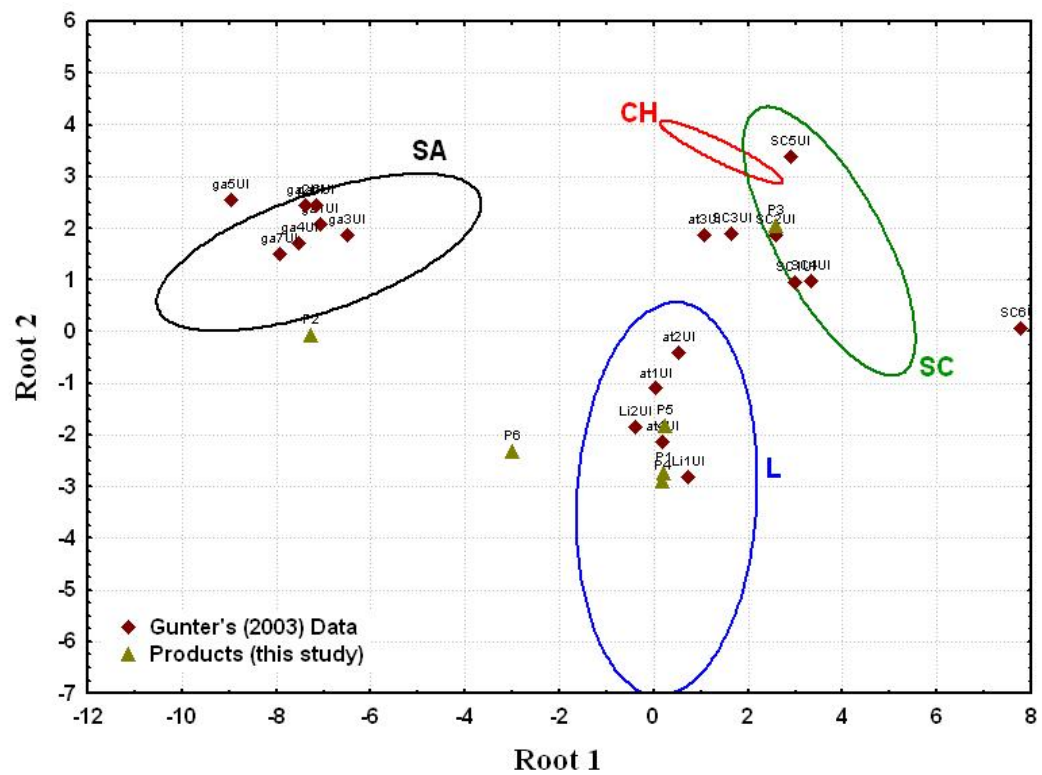
	Classification Functions; grouping: Category (fprinttraceln1)			
Variable	SA p=.21053	CH p=.07895	SC p=.34211	L p=.36842
V	2.35	38.89	35.71	22.71
Zn	-154.45	-136.25	-156.60	-180.59
SiO <sub>2</sub>	3865.52	3893.76	3981.09	3907.01
Ga	227.17	183.35	227.59	223.32
Cr	56.55	66.37	68.45	70.00
FeO	1049.38	1060.25	1088.76	1109.45
Sc	116.90	121.13	133.93	132.41
K <sub>2</sub> O	41.94	40.38	31.82	36.44
Constant	-8689.72	-9015.97	-9450.60	-9046.63

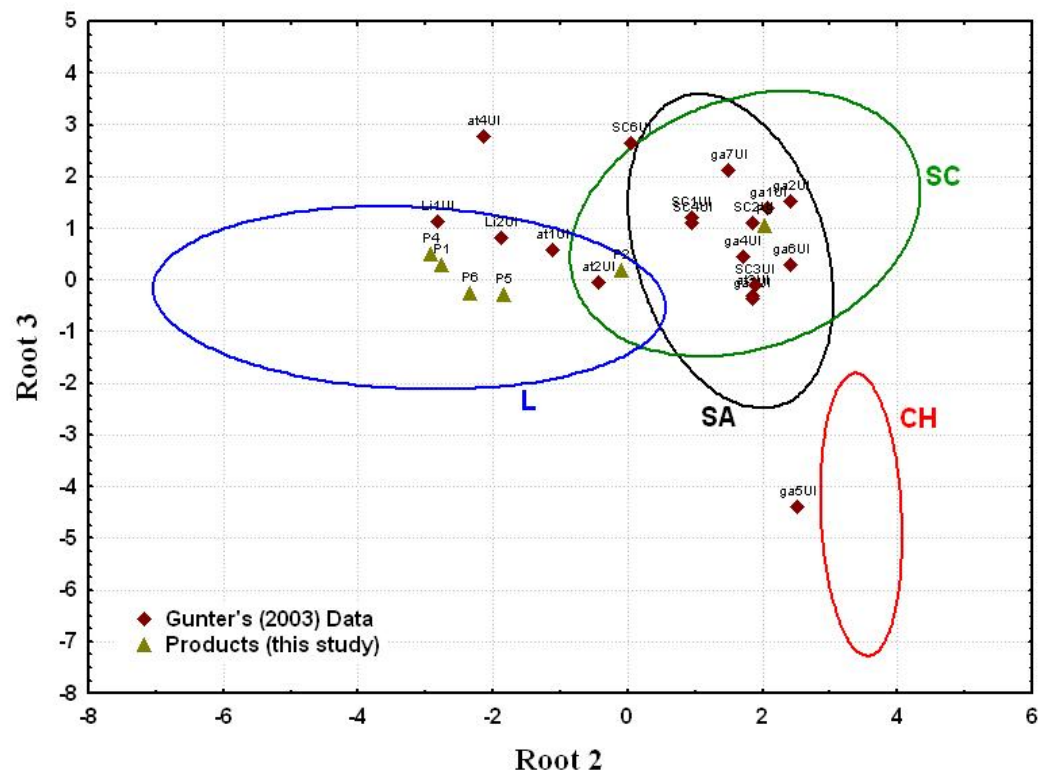
	Classification Matrix (fprinttraceln1.sta) Rows: Observed classifications Columns: Predicted classifications				
Category	Percent Correct	SA p=.21053	CH p=.07895	SC p=.34211	L p=.36842
SA	100.00	8	0	0	0
CH	100.00	0	3	0	0
SC	100.00	0	0	13	0
L	100.00	0	0	0	14
Total	100.00	8	3	13	14

		Classification of Cases (fprinttraceln1) Incorrect classifications are marked with *			
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	SA	L	SC	CH
SA GS-3	SA	SA	CH	L	SC
SA GS-13	SA	SA	L	CH	SC
SA GS-17	SA	SA	L	CH	SC
SA GS-18	SA	SA	L	CH	SC
SA PVC1	SA	SA	L	CH	SC
SA PVC2	SA	SA	L	CH	SC
SA PVC3	SA	SA	L	CH	SC
CH GS-7	CH	CH	SC	L	SA
CH GS-12	CH	CH	SC	L	SA
CH GS-16	CH	CH	SC	L	SA
SC GS-11	SC	SC	L	CH	SA
SC GS-15	SC	SC	CH	L	SA
SC2	SC	SC	L	CH	SA
SC3	SC	SC	L	CH	SA
SC4	SC	SC	L	CH	SA
SC5	SC	SC	CH	L	SA
SC6	SC	SC	L	CH	SA
SC7	SC	SC	CH	L	SA
SC8	SC	SC	L	CH	SA
SC9	SC	SC	CH	L	SA
SC10	SC	SC	L	CH	SA
SC11	SC	SC	L	CH	SA
SC PVC5	SC	SC	CH	L	SA
L GS-2	L	L	SC	CH	SA
L GS-4	L	L	SC	CH	SA
L GS-10	L	L	SC	CH	SA
L 1A	L	L	SC	CH	SA
L A	L	L	SC	CH	SA
L 3A	L	L	SC	SA	CH
L 4A	L	L	SC	SA	CH
L 5A	L	L	SC	SA	CH
L 6A	L	L	SC	CH	SA
L 7A	L	L	SC	CH	SA
L 8A	L	L	SC	CH	SA
L 9A	L	L	SC	CH	SA
L10A	L	L	SC	CH	SA
L 11A	L	L	SC	CH	SA
P1	----	L	SC	CH	SA
P2	----	SA	L	CH	SC
P3	----	SC	L	CH	SA
P4	----	L	SC	CH	SA
P5	----	L	SC	CH	SA
P6	----	L	SA	SC	CH
Li1UI	----	L	SC	CH	SA
Li2UI	----	L	SC	SA	CH
at1UI	----	L	SC	CH	SA
at2UI	----	L	SC	CH	SA
at3UI	----	SC	CH	L	SA
at4UI	----	L	SC	SA	CH
ga1UI	----	SA	L	CH	SC
ga2UI	----	SA	L	CH	SC
ga3UI	----	SA	L	CH	SC
ga4UI	----	SA	L	CH	SC
ga5UI	----	SA	CH	L	SC
ga6UI	----	SA	L	CH	SC
ga7UI	----	SA	L	SC	CH
SA1UI	----	SA	CH	SC	L
SC1UI	----	SC	L	CH	SA
SC2UI	----	SC	L	CH	SA
SC3UI	----	SC	CH	L	SA
SC4UI	----	SC	L	CH	SA
SC5UI	----	SC	L	SA	CH
SC6UI	----	SC	L	CH	SA

		Classification of Cases (fprinttraceln1) Incorrect classifications are marked with *			
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	11.1117	125.3794	121.4217	96.6947
SA GS-3	SA	8.3850	67.9171	107.5970	79.7133
SA GS-13	SA	2.7725	82.5833	94.0735	61.1936
SA GS-17	SA	3.0235	78.7991	85.1361	57.7981
SA GS-18	SA	1.1484	91.9767	99.8753	67.6551
SA PVC1	SA	9.8663	167.9820	191.5654	123.5401
SA PVC2	SA	3.7948	126.1767	141.2167	88.3934
SA PVC3	SA	2.8505	122.7704	141.4478	91.3319
CH GS-7	CH	81.4099	6.7667	38.7485	63.0099
CH GS-12	CH	142.3667	10.4934	76.8835	96.3946
CH GS-16	CH	104.1872	2.9667	24.2947	52.4585
SC GS-11	SC	115.7849	40.3367	5.4312	39.8480
SC GS-15	SC	110.0448	27.7790	9.3897	42.4956
SC2	SC	137.7911	75.0506	26.6603	67.3881
SC3	SC	100.6208	47.9525	6.6486	38.9433
SC4	SC	140.8570	54.2716	2.6192	40.3856
SC5	SC	119.2895	32.0986	4.5490	48.6329
SC6	SC	144.0491	60.8686	3.3072	42.2379
SC7	SC	143.0818	52.9384	10.3236	63.8570
SC8	SC	143.0960	52.8292	7.5690	51.1198
SC9	SC	105.9211	56.9778	12.2894	67.5205
SC10	SC	147.4364	52.1159	4.5461	36.8848
SC11	SC	139.6550	62.6446	20.1034	58.4854
SC PVC5	SC	98.6788	22.6492	6.3760	25.7494
L GS-2	L	89.2079	53.7196	30.0607	4.2934
L GS-4	L	87.9762	53.1415	23.6497	3.4857
L GS-10	L	83.1192	63.5124	28.4451	3.1338
L 1A	L	110.6936	84.2032	55.6687	17.6088
L A	L	93.1076	71.5831	46.6834	2.6359
L 3A	L	95.8384	98.1095	56.0082	7.8734
L 4A	L	80.1615	87.9270	50.5813	5.3720
L 5A	L	65.6302	75.1713	57.6976	4.0131
L 6A	L	110.0684	95.3134	65.7487	6.7213
L 7A	L	76.0780	72.4708	54.7573	1.9990
L 8A	L	80.6614	70.0934	47.0994	3.6725
L 9A	L	77.4773	67.8541	43.0653	2.2749
L10A	L	72.1252	52.1556	48.3158	13.5891
L 11A	L	57.7561	38.0551	23.7113	12.3347
P1	----	72.9457	64.3315	34.1033	1.6540
P2	----	12.0333	119.8296	134.1975	76.4284
P3	----	103.5883	43.8990	10.6143	44.2659
P4	----	73.7223	68.6969	35.6801	1.9403
P5	----	67.3673	49.0251	28.3240	3.5031
P6	----	48.2126	87.2758	79.1804	27.2596
Li1UI	----	92.0576	83.4579	41.1551	13.9438
Li2UI	----	67.1427	70.7435	40.3028	13.9955
at1UI	----	64.4927	55.6954	28.7167	12.0614
at2UI	----	76.7734	50.7261	30.8531	22.6687
at3UI	----	83.9088	36.5117	24.9381	42.6642
at4UI	----	75.9851	91.0034	34.9221	15.4421
ga1UI	----	13.3154	121.7965	129.1545	97.6383
ga2UI	----	8.3196	122.1456	130.5433	100.7181
ga3UI	----	8.6169	90.1484	113.7889	79.0022
ga4UI	----	21.6859	129.4524	148.2193	107.2897
ga5UI	----	122.6431	202.4916	285.1446	228.5462
ga6UI	----	4.6190	101.9510	123.0930	91.3454
ga7UI	----	11.0538	143.7163	144.8618	103.8056
SA1UI	----	153.2063	472.8933	451.6190	345.1311
SC1UI	----	119.9616	58.7965	18.2754	44.4636
SC2UI	----	107.5166	49.1569	14.6909	46.7811
SC3UI	----	78.6327	23.6763	7.1471	29.6737
SC4UI	----	111.6998	43.8807	3.0666	31.2822
SC5UI	----	171.7661	176.8258	63.6761	134.1399
SC6UI	----	266.5673	142.4352	60.6444	114.9799

		Classification of Cases (fprintraceln1) Incorrect classifications are marked with *			
Case	Observed Classif.	1 p=0.21053	2 p=0.07895	3 p=0.34211	4 p=0.36842
SA GS-1	SA	1.00000	0.00000	0.00000	0.00000
SA GS-3	SA	1.00000	0.00000	0.00000	0.00000
SA GS-13	SA	1.00000	0.00000	0.00000	0.00000
SA GS-17	SA	1.00000	0.00000	0.00000	0.00000
SA GS-18	SA	1.00000	0.00000	0.00000	0.00000
SA PVC1	SA	1.00000	0.00000	0.00000	0.00000
SA PVC2	SA	1.00000	0.00000	0.00000	0.00000
SA PVC3	SA	1.00000	0.00000	0.00000	0.00000
CH GS-7	CH	0.00000	1.00000	0.00000	0.00000
CH GS-12	CH	0.00000	1.00000	0.00000	0.00000
CH GS-16	CH	0.00000	0.99990	0.00010	0.00000
SC GS-11	SC	0.00000	0.00000	1.00000	0.00000
SC GS-15	SC	0.00000	0.00002	0.99998	0.00000
SC2	SC	0.00000	0.00000	1.00000	0.00000
SC3	SC	0.00000	0.00000	1.00000	0.00000
SC4	SC	0.00000	0.00000	1.00000	0.00000
SC5	SC	0.00000	0.00000	1.00000	0.00000
SC6	SC	0.00000	0.00000	1.00000	0.00000
SC7	SC	0.00000	0.00000	1.00000	0.00000
SC8	SC	0.00000	0.00000	1.00000	0.00000
SC9	SC	0.00000	0.00000	1.00000	0.00000
SC10	SC	0.00000	0.00000	1.00000	0.00000
SC11	SC	0.00000	0.00000	1.00000	0.00000
SC PVC5	SC	0.00000	0.00007	0.99987	0.00007
L GS-2	L	0.00000	0.00000	0.00000	1.00000
L GS-4	L	0.00000	0.00000	0.00004	0.99996
L GS-10	L	0.00000	0.00000	0.00000	1.00000
L 1A	L	0.00000	0.00000	0.00000	1.00000
L A	L	0.00000	0.00000	0.00000	1.00000
L 3A	L	0.00000	0.00000	0.00000	1.00000
L 4A	L	0.00000	0.00000	0.00000	1.00000
L 5A	L	0.00000	0.00000	0.00000	1.00000
L 6A	L	0.00000	0.00000	0.00000	1.00000
L 7A	L	0.00000	0.00000	0.00000	1.00000
L 8A	L	0.00000	0.00000	0.00000	1.00000
L 9A	L	0.00000	0.00000	0.00000	1.00000
L10A	L	0.00000	0.00000	0.00000	1.00000
L 11A	L	0.00000	0.00000	0.00313	0.99687
P1	----	0.00000	0.00000	0.00000	1.00000
P2	----	1.00000	0.00000	0.00000	0.00000
P3	----	0.00000	0.00000	1.00000	0.00000
P4	----	0.00000	0.00000	0.00000	1.00000
P5	----	0.00000	0.00000	0.00000	1.00000
P6	----	0.00002	0.00000	0.00000	0.99998
Li1UI	----	0.00000	0.00000	0.00000	1.00000
Li2UI	----	0.00000	0.00000	0.00000	1.00000
at1UI	----	0.00000	0.00000	0.00022	0.99978
at2UI	----	0.00000	0.00000	0.01527	0.98473
at3UI	----	0.00000	0.00071	0.99914	0.00015
at4UI	----	0.00000	0.00000	0.00005	0.99995
ga1UI	----	1.00000	0.00000	0.00000	0.00000
ga2UI	----	1.00000	0.00000	0.00000	0.00000
ga3UI	----	1.00000	0.00000	0.00000	0.00000
ga4UI	----	1.00000	0.00000	0.00000	0.00000
ga5UI	----	1.00000	0.00000	0.00000	0.00000
ga6UI	----	1.00000	0.00000	0.00000	0.00000
ga7UI	----	1.00000	0.00000	0.00000	0.00000
SA1UI	----	1.00000	0.00000	0.00000	0.00000
SC1UI	----	0.00000	0.00000	1.00000	0.00000
SC2UI	----	0.00000	0.00000	1.00000	0.00000
SC3UI	----	0.00000	0.00006	0.99993	0.00001
SC4UI	----	0.00000	0.00000	1.00000	0.00000
SC5UI	----	0.00000	0.00000	1.00000	0.00000
SC6UI	----	0.00000	0.00000	1.00000	0.00000







## **Appendix E**

### **Results of Six-Category Discriminant Analyses**

## Six-Category Model, Original Element Set

### Element Set:

SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, CaO, Na<sub>2</sub>O, K<sub>2</sub>O, Ba, Ce, Co, Cr, Cs, Cu, Ga, La, Ni, Sc, Sm V, Zn

### Categories:

Category	Abbreviation	Sample IDs	No. Samples
South Africa	SA	GS-1, GS-3, GS-13, GS-17, GS-18,PVC1, PVC2, PVC3	8
China	CH	GS-7, GS-12, GS-16	3
South Carolina Group 1	SC-1	SC4, SC5, SC6, SC9, SC10, SC11	6
South Carolina Group 2	SC-2	GS-11, SG-15, SC2, SC3, SC7, SC8, PVC5	7
Libby Group 1	L-1	1A, 4A, 9A, 10A	4
Libby Group 2	L-2	GS-2, GS-4, GS-10, 2A, 3A, 5A, 6A, 7A, 8A, 11A	10
Total Number of Samples =			38

### Discriminant Function Analysis.

N=38	Discriminant Function Analysis Summary (fprintrace1n1) Step 9, N of vars in model: 9; Grouping: Category (6 grps) Wilks' Lambda: .00002 approx. F (45,110)=24.143 p<0.0000						
	Element Added	Wilks' $\Lambda$	Partial $\Lambda$	F-Remove	p-Level	Tolerance	Correlated Variables Removed
1	V	0.000071	0.3299	9.75	3.4945E-05	0.5914	TiO <sub>2</sub> , MnO, Na <sub>2</sub> O, Cs
2	Sm	0.000130	0.1804	21.81	3.2768E-08	0.2678	La, Ce
3	Sc	0.000079	0.2975	11.33	1.0769E-05	0.2285	none
4	Ni	0.000083	0.2846	12.07	6.4804E-06	0.5591	none
5	Ga	0.000049	0.4821	5.16	2.3657E-03	0.3182	Al <sub>2</sub> O <sub>3</sub>
6	K <sub>2</sub> O	0.000140	0.1672	23.90	1.3487E-08	0.0971	none
7	Ba	0.000088	0.2673	13.16	3.1512E-06	0.1182	none
8	Zn	0.000038	0.6215	2.92	3.3653E-02	0.5474	Cu
9	CaO	0.000037	0.6376	2.73	4.3387E-02	0.6756	none

n = 38	Variables currently not in the model (fprinttraceln1) Df for all F-tests: 5,23					
Element	Wilks' $\Lambda$	Wilks' Partial $\Lambda$	F to Enter	p-level	Toler.	1-Toler.
SiO <sub>2</sub>	0.000015	0.647741	2.501605	0.059903	0.737388	0.262612
TiO <sub>2</sub>	0.000018	0.750680	1.527776	0.220233	0.325465	0.674536
Al <sub>2</sub> O <sub>3</sub>	0.000020	0.830799	0.936839	0.475802	0.178166	0.821834
FeO	0.000015	0.658278	2.387927	0.069568	0.255005	0.744995
MnO	0.000015	0.645487	2.526403	0.057988	0.788902	0.211099
Na <sub>2</sub> O	0.000016	0.687560	2.090322	0.103344	0.413220	0.586780
Ce	0.000015	0.647546	2.503743	0.059735	0.198209	0.801791
Co	0.000016	0.701345	1.958823	0.123280	0.333443	0.666557
Cr	0.000021	0.895060	0.539323	0.744482	0.443738	0.556262
Cs	0.000020	0.860765	0.744087	0.598548	0.179826	0.820174
Cu	0.000015	0.643308	2.550539	0.056185	0.331555	0.668445
La	0.000023	0.986088	0.064898	0.996762	0.548343	0.451657

Squared Mahalanobis Distances (fprinttraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
CH	0.000	100.830	75.667	197.590	182.152	51.436
L1	100.830	0.000	34.884	225.900	184.275	95.104
L2	75.667	34.884	0.000	265.031	128.534	43.349
SA	197.590	225.900	265.031	0.000	465.624	273.045
SC1	182.152	184.275	128.534	465.624	0.000	62.073
SC2	51.436	95.104	43.349	273.045	62.073	0.000

F-values; df = 9,25 (fprinttraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
CH		14.404	14.551	35.926	30.359	9.001
L1	14.404		8.306	50.200	36.855	20.174
L2	14.551	8.306		98.160	40.167	14.875
SA	35.926	50.200	98.160		133.035	84.947
SC1	30.359	36.855	40.167	133.035		16.712
SC2	9.001	20.174	14.875	84.947	16.712	

p-levels (fprinttraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
CH		7.6467E-08	6.9097E-08	4.1375E-12	2.7638E-11	6.3904E-06
L1	7.6467E-08		1.2859E-05	8.7353E-14	3.0934E-12	2.3847E-09
L2	6.9097E-08	1.2859E-05		3.0074E-17	1.1558E-12	5.5456E-08
SA	4.1375E-12	8.7353E-14	3.0074E-17		7.5376E-19	1.7131E-16
SC1	2.7638E-11	3.0934E-12	1.1558E-12	7.5376E-19		1.7007E-08
SC2	6.3904E-06	2.3847E-09	5.5456E-08	1.7131E-16	1.7007E-08	

Summary of Stepwise Analysis (fprintrace1n1)											
Variable Enter/ Remove	Step	F to enter/ remove	df 1	df 2	p-level	No. of vars. In Model	Lambda	F-value	df 1	df 2	p-level
V-(E)	1	59.974	5	32	2.609E-15	1	0.096424	59.974	5	32	2.609E-15
Sm-(E)	2	26.239	5	31	2.779E-10	2	0.018429	39.471	10	62	3.886E-23
Sc-(E)	3	19.221	5	30	1.493E-08	3	0.004384	34.111	15	83	3.828E-29
Ni-(E)	4	15.907	5	29	1.492E-07	4	0.001171	32.309	20	97	0.000E+00
Ga-(E)	5	8.383	5	28	6.060E-05	5	0.000469	29.002	25	106	0.000E+00
K <sub>2</sub> O-(E)	6	5.910	5	27	8.174E-04	6	0.000224	26.305	30	110	0.000E+00
Ba-(E)	7	14.198	5	26	9.724E-07	7	0.000060	29.011	35	112	0.000E+00
Zn-(E)	8	3.149	5	25	2.434E-02	8	0.000037	26.273	40	112	0.000E+00
CaO-(E)	9	2.728	5	24	4.339E-02	9	0.000023	24.143	45	110	0.000E+00

Chi-Square Tests with Successive Roots Removed (fprintrace1n1)						
Roots Removed	Eigen-value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	58.62	0.9916	0.000023	314.44	45	0.000E+00
1	14.56	0.9673	0.001400	193.84	32	0.000E+00
2	6.10	0.9269	0.021797	112.87	21	1.432E-14
3	2.52	0.8459	0.154733	55.05	12	1.774E-07
4	0.84	0.6753	0.543986	17.96	5	2.996E-03

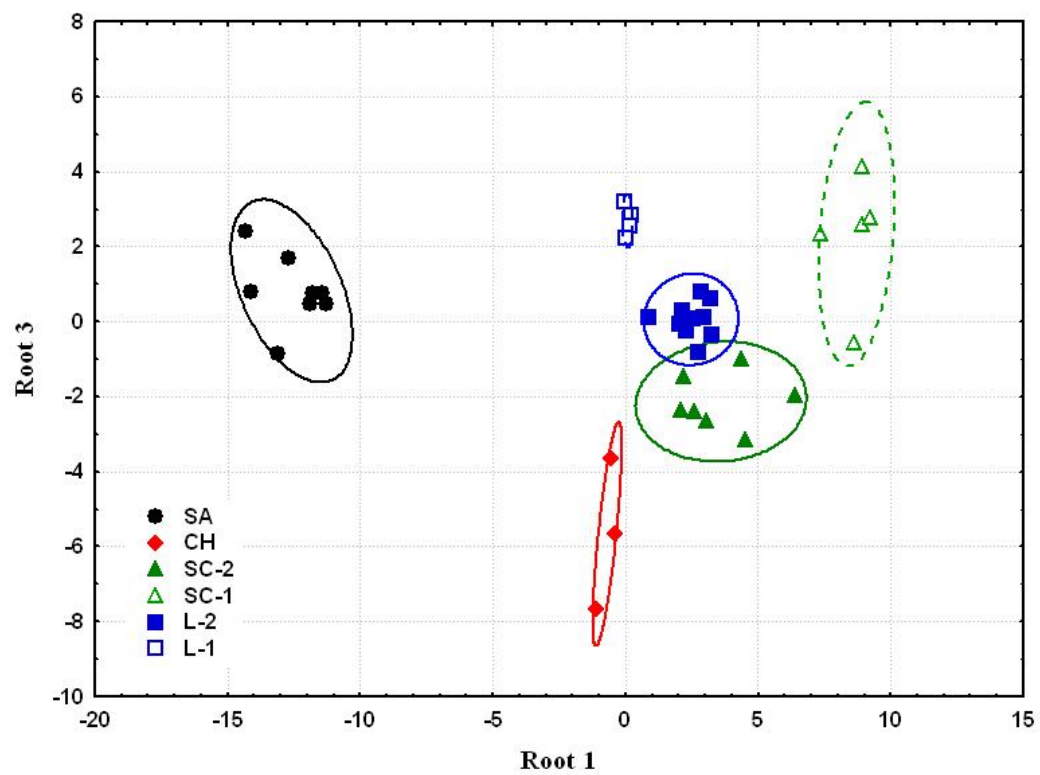
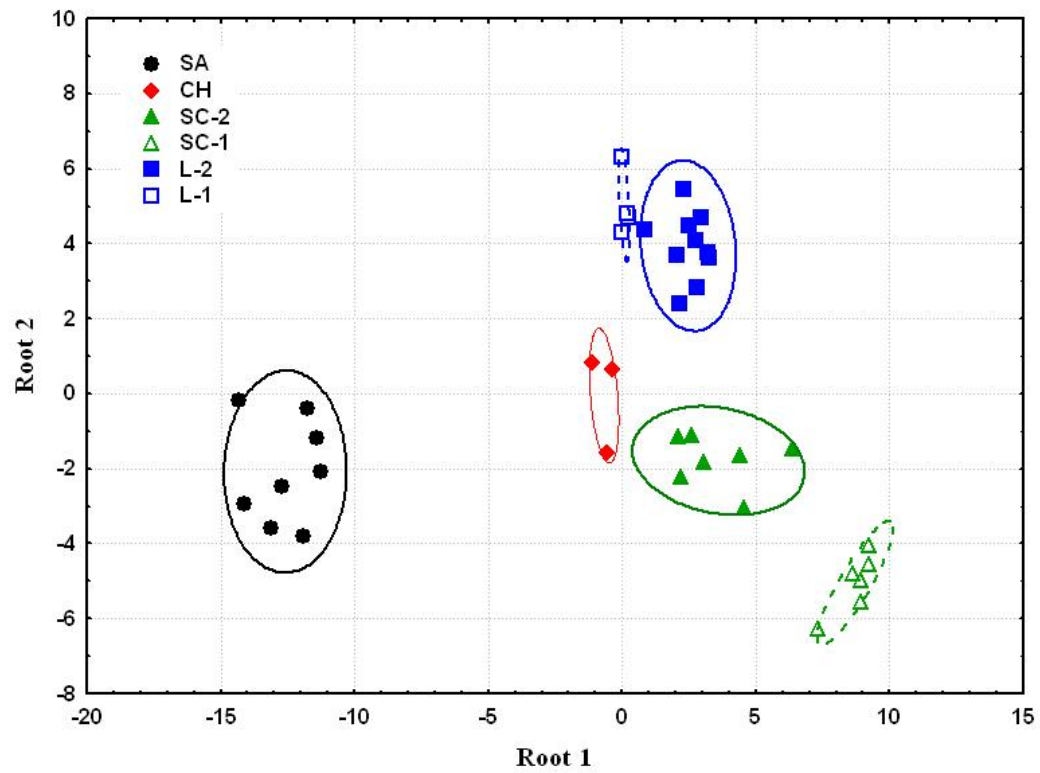
Raw Coefficients (fprintrace1n1) for Canonical Variables						Standardized Coefficients (fprintrace1n1) for Canonical Variables					
Variable	Root 1	Root 2	Root 3	Root 4	Root 5	Variable	Root 1	Root 2	Root 3	Root 4	Root 5
V	3.3563	0.39857	-1.44983	-1.5702	-0.47662	V	0.92563	0.10992	-0.399840	-0.433042	-0.131446
Sm	-0.4869	-1.87879	0.28619	-0.0510	0.10142	SM	-0.44814	-1.72909	0.263382	-0.046966	0.093335
Sc	3.8683	2.80671	0.34535	0.7660	0.57451	SC	1.42188	1.03168	0.126943	0.281567	0.211174
Ni	1.1534	1.47644	-1.77744	0.2454	0.97284	NI	0.51028	0.65321	-0.786381	0.108565	0.430406
Ga	1.0671	-0.59144	4.31377	4.5631	3.39753	GA	0.22201	-0.12305	0.897485	0.949367	0.706860
K <sub>2</sub> O	-3.8471	1.30642	-1.13204	-0.9936	-1.48704	K2O	-2.58650	0.87832	-0.761091	-0.668008	-0.999762
Ba	3.9162	-1.55981	0.93788	0.4717	-0.04087	BA	2.28060	-0.90835	0.546171	0.274676	-0.023798
Zn	1.7426	-0.56622	-3.24252	0.6700	-1.79642	ZN	0.38576	-0.12534	-0.717797	0.148309	-0.397674
CaO	0.0815	0.65764	0.37499	-0.8376	0.33780	CAO	0.05726	0.46218	0.263539	-0.588639	0.237399
Constant	-64.4706	-3.63909	12.18640	-12.1094	-3.46947	Constant	---	---	---	---	---
Eigenval	58.6202	14.56459	6.09875	2.5156	0.83828	Eigenval	58.62018	14.56459	6.098748	2.515637	0.838283
Cum.Prob.	0.7094	0.88561	0.95941	0.9899	1.00000	Cum.Prob.	0.70937	0.88561	0.959414	0.989856	1.000000

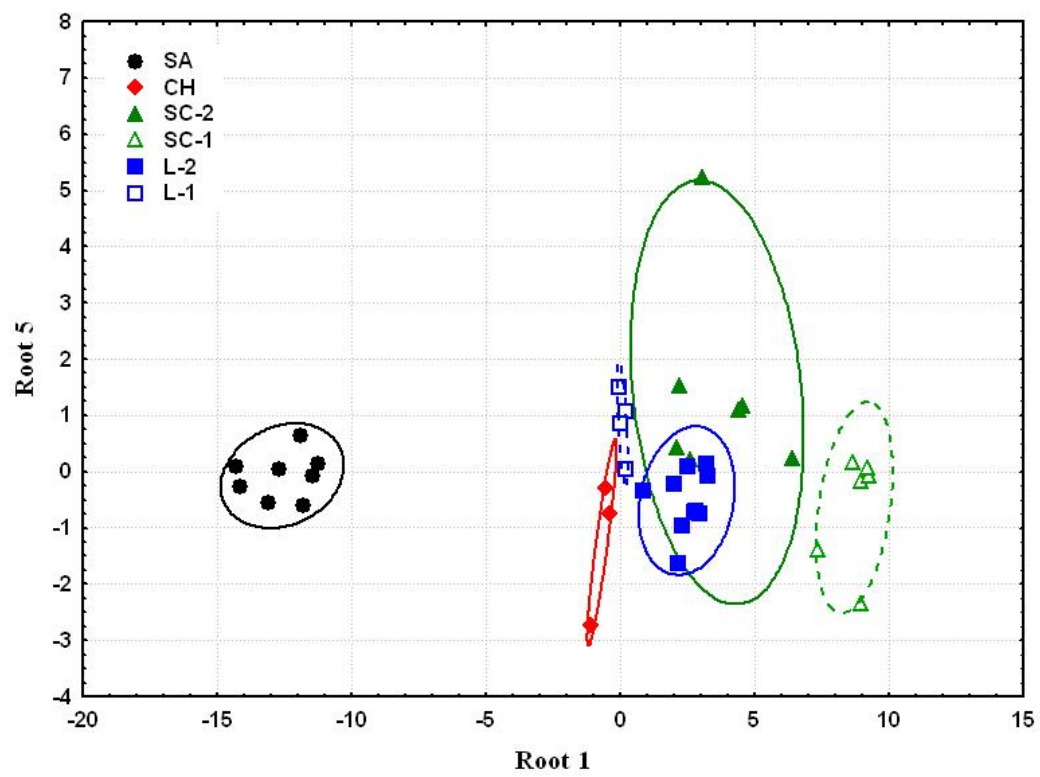
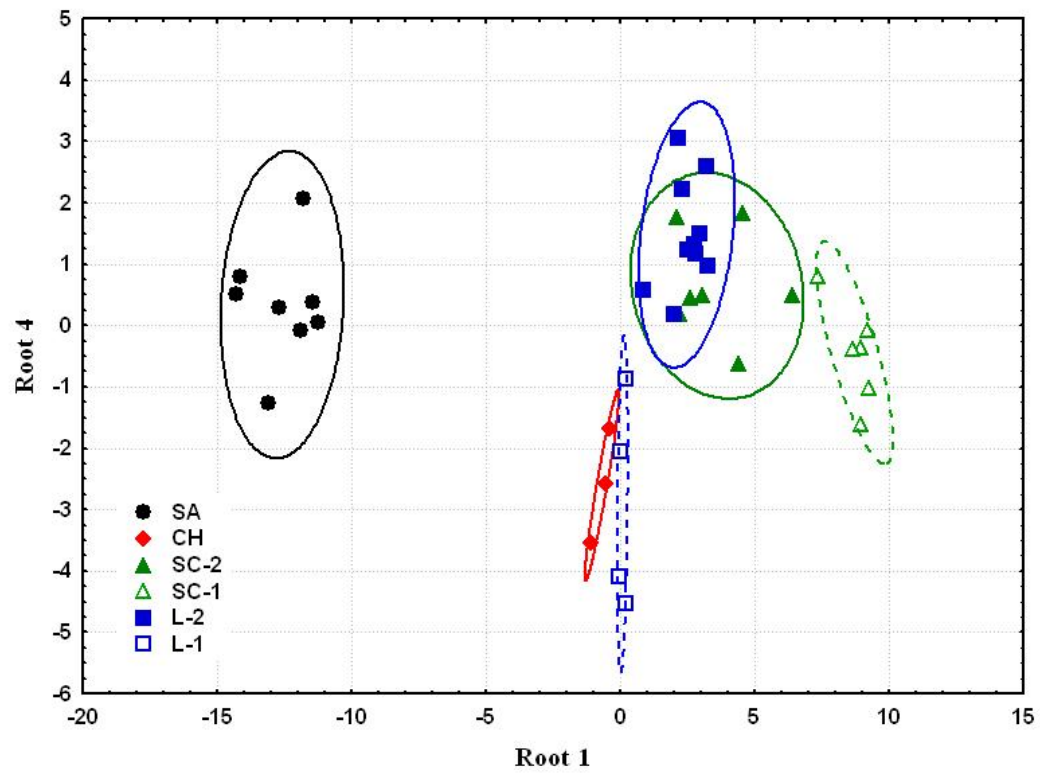
Factor Structure Matrix (fprintrace1n1) Correlations Variables - Canonical Roots (Pooled-within-groups correlations)					
Variable	Root 1	Root 2	Root 3	Root 4	Root 5
V	0.377078	-0.179061	-0.248818	-0.140956	-0.410535
Sm	-0.004912	-0.526233	-0.042542	-0.343798	0.285191
Sc	0.147497	0.048552	0.305251	-0.206487	0.023416
Ni	-0.037456	0.089301	-0.583399	0.146702	0.526307
Ga	0.059373	-0.118678	0.012484	0.605639	-0.340613
K <sub>2</sub> O	-0.025735	0.064046	0.057144	0.172418	-0.730053
Ba	0.153340	0.071486	0.167291	0.202112	-0.662494
Zn	0.074140	-0.386620	-0.440691	0.223530	-0.290623
CaO	0.001828	-0.114730	0.147170	-0.707670	0.284313

Means of Canonical Variables (fprintrace1n1)
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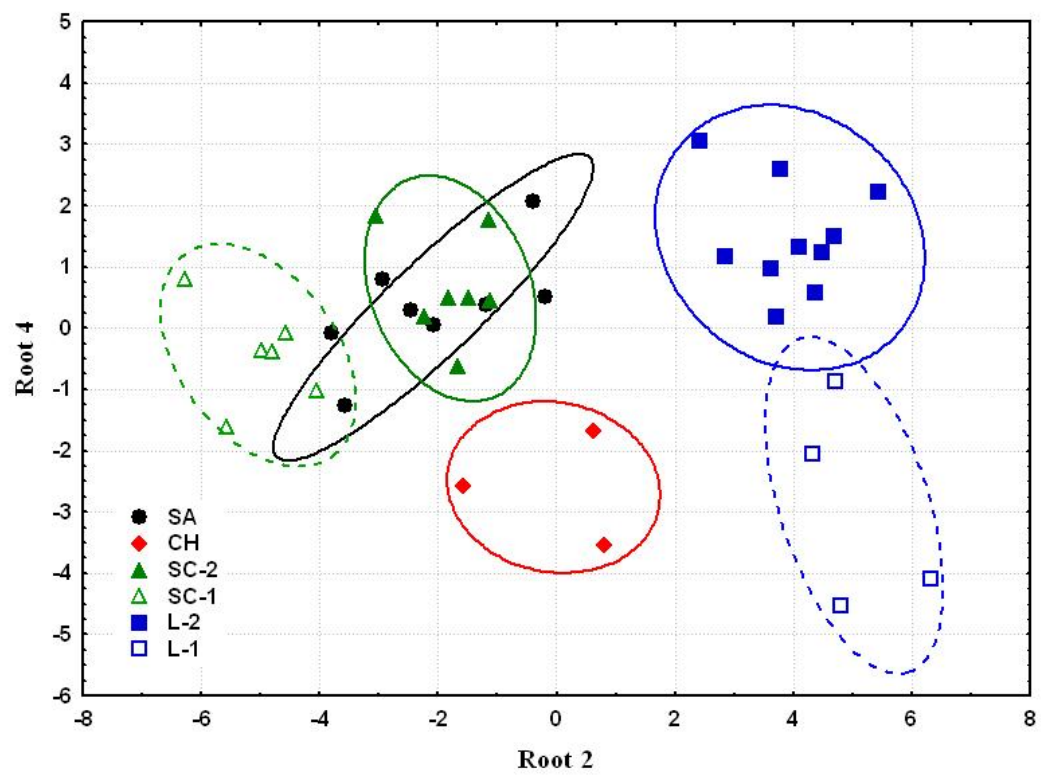
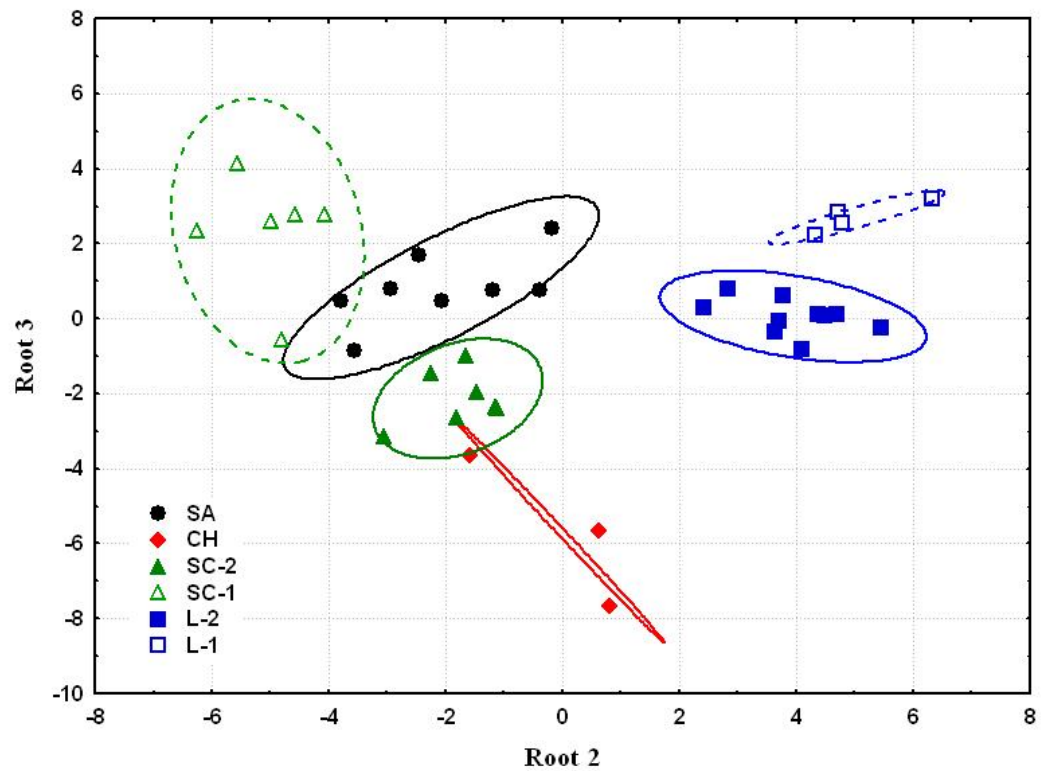
Category	Root 1	Root 2	Root 3	Root 4	Root 5
<b>CH</b>	-0.6811	-0.04856	-5.65414	-2.59790	-1.25928
<b>L1</b>	0.0886	5.02963	2.70262	-2.89144	0.86851
<b>L2</b>	2.4770	3.94294	0.05650	1.47627	-0.51707
<b>SA</b>	-12.5793	-2.07951	0.81641	0.33901	-0.07442
<b>SC1</b>	8.7193	-5.03958	2.33519	-0.44094	-0.63373
<b>SC2</b>	3.6053	-1.78982	-2.13650	0.64719	1.41031

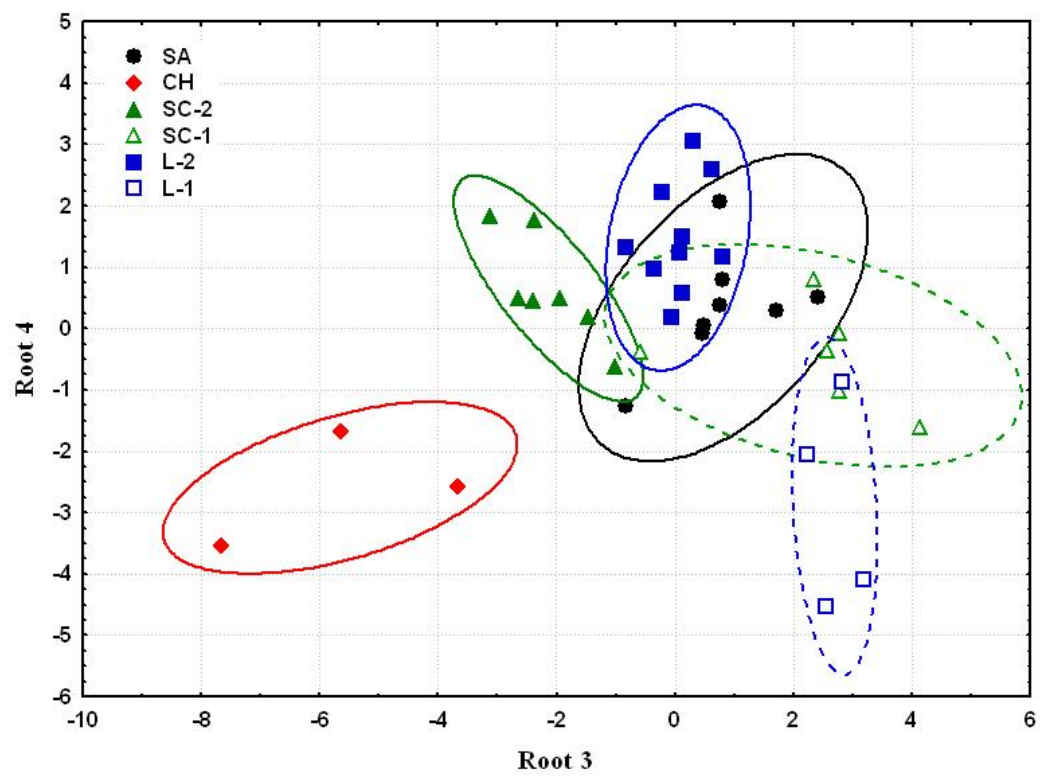
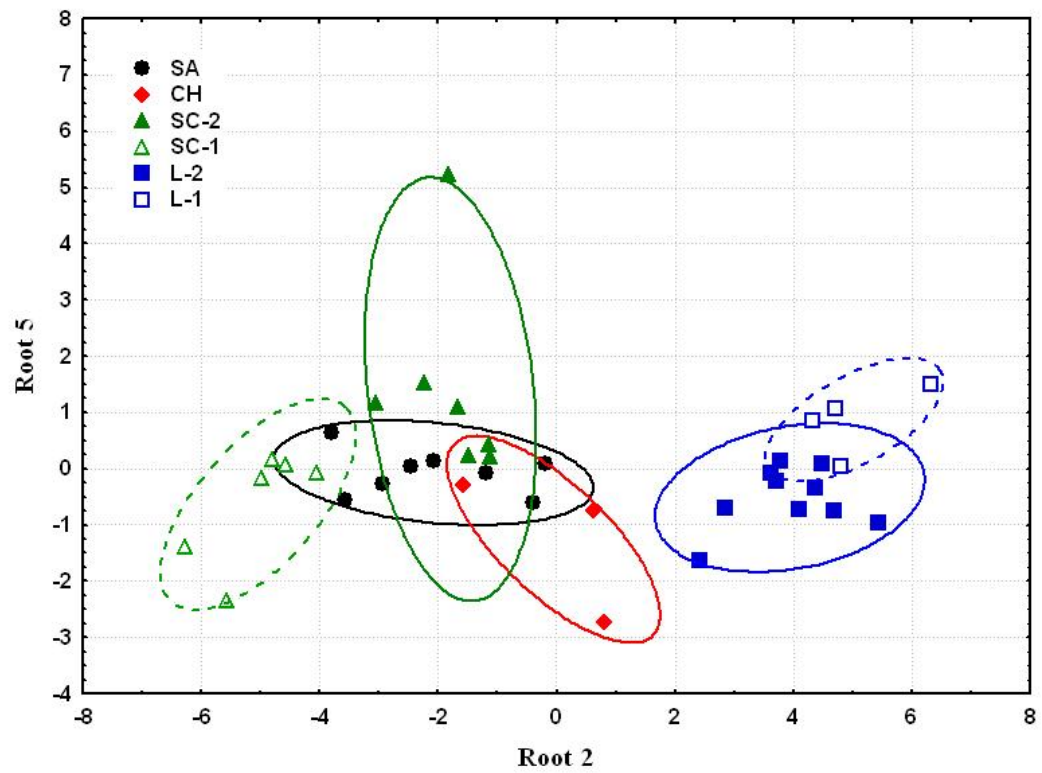
		Unstandardized Canonical Scores (fprinttraceln1)				
Samples	Group	Root 1	Root 2	Root 3	Root 4	Root 5
CH GS-7	CH	-0.5428	-1.58531	-3.66130	-2.58704	-0.28982
CH GS-12	CH	-1.1147	0.81311	-7.65247	-3.53614	-2.74085
CH GS-16	CH	-0.3857	0.62652	-5.64867	-1.67050	-0.74716
L 1A	L1	-0.0359	6.31523	3.19117	-4.09879	1.50281
L 4A	L1	0.2236	4.70230	2.83195	-0.87080	1.06546
L 9A	L1	-0.0105	4.31265	2.23268	-2.05889	0.86624
L10A	L1	0.1773	4.78834	2.55470	-4.53726	0.03955
L GS-2	L2	2.0263	3.69937	-0.06796	0.17093	-0.20982
L GS-4	L2	3.2429	3.62852	-0.34811	0.95982	-0.08517
L GS-10	L2	2.5178	4.48920	0.08326	1.23128	0.09266
L A	L2	2.9359	4.69064	0.11391	1.49627	-0.75134
L 3A	L2	3.2063	3.77980	0.62458	2.59010	0.13607
L 5A	L2	2.2878	5.44215	-0.22702	2.21851	-0.95389
L 6A	L2	2.1371	2.40532	0.29222	3.05595	-1.62602
L 7A	L2	0.8605	4.36895	0.11449	0.56487	-0.34483
L 8A	L2	2.8230	2.83769	0.80305	1.15769	-0.70431
L 11A	L2	2.7327	4.08780	-0.82339	1.31723	-0.72404
SA GS-3	SA	-13.0950	-3.57129	-0.84089	-1.27461	-0.56897
SA GS-13	SA	-11.9113	-3.78968	0.47200	-0.07788	0.64004
SA GS-17	SA	-11.2619	-2.07683	0.47543	0.03972	0.14342
SA GS-18	SA	-11.4313	-1.19602	0.75204	0.37260	-0.07435
SA PVC1	SA	-14.3132	-0.19138	2.41065	0.50677	0.09238
SA PVC2	SA	-12.7206	-2.46427	1.70839	0.28174	0.04452
SA PVC3	SA	-14.1274	-2.94895	0.80469	0.80249	-0.27239
SA GS-1	SA	-11.7740	-0.39766	0.74899	2.06124	-0.60001
SC4	SC1	8.9342	-4.97689	2.57221	-0.35943	-0.18377
SC5	SC1	8.6298	-4.79381	-0.57142	-0.37879	0.15197
SC6	SC1	9.2135	-4.57157	2.77642	-0.07367	0.06036
SC9	SC1	7.3454	-6.26310	2.33583	0.80198	-1.40102
SC10	SC1	9.2463	-4.06217	2.77044	-1.01748	-0.07853
SC11	SC1	8.9467	-5.56992	4.12764	-1.61822	-2.35139
SC GS-11	SC2	2.0974	-1.14638	-2.36593	1.75017	0.42307
SC GS-15	SC2	2.5886	-1.12744	-2.38910	0.44208	0.19945
SC2	SC2	3.0277	-1.81748	-2.64204	0.48769	5.22099
SC3	SC2	2.2104	-2.24284	-1.46553	0.17085	1.52425
SC7	SC2	4.5364	-3.05888	-3.13460	1.81773	1.16149
SC8	SC2	4.3867	-1.65925	-1.00559	-0.63702	1.10427
SC PVC5	SC2	6.3899	-1.47645	-1.95272	0.49881	0.23868
P1	----	2.4991	4.28483	0.15162	1.53633	0.16601
P2	----	-11.9636	-3.96666	3.10057	-0.78431	0.72931
P3	----	3.9735	-2.07418	-2.77997	2.58448	0.94604
P4	----	2.1232	3.87929	1.07762	0.45880	0.15829
P5	----	2.3797	5.08886	-0.77394	1.60491	-0.45293
P6	----	-1.4281	0.28811	1.92544	0.70924	1.64305

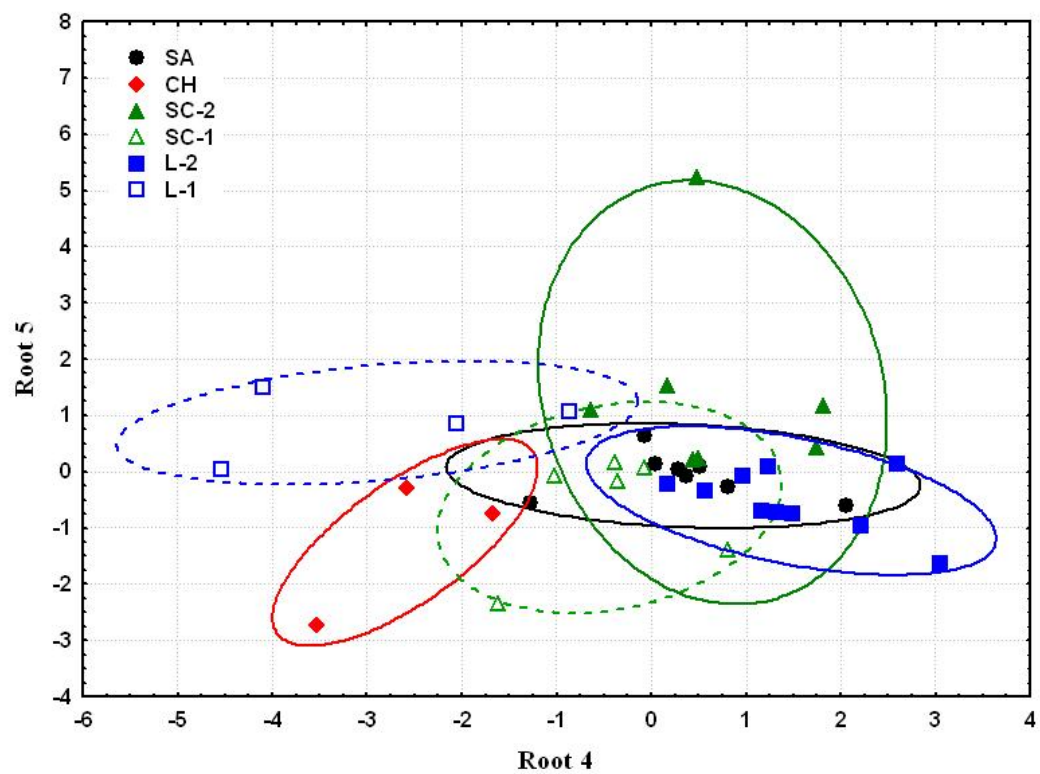
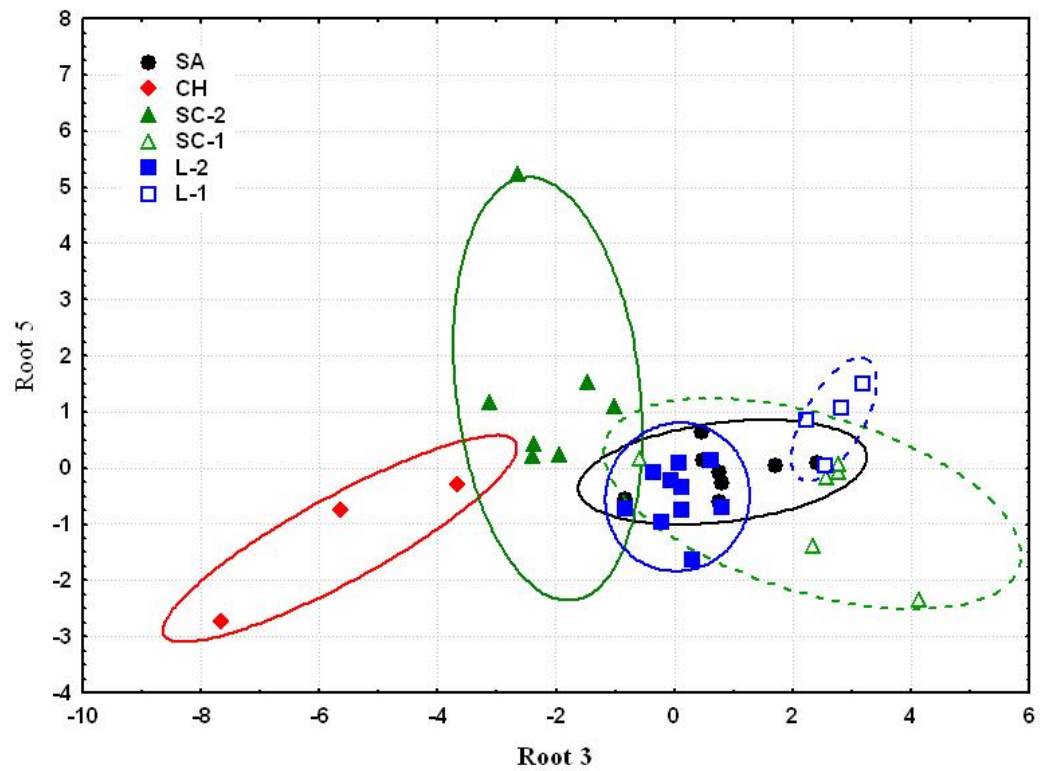












## Classification

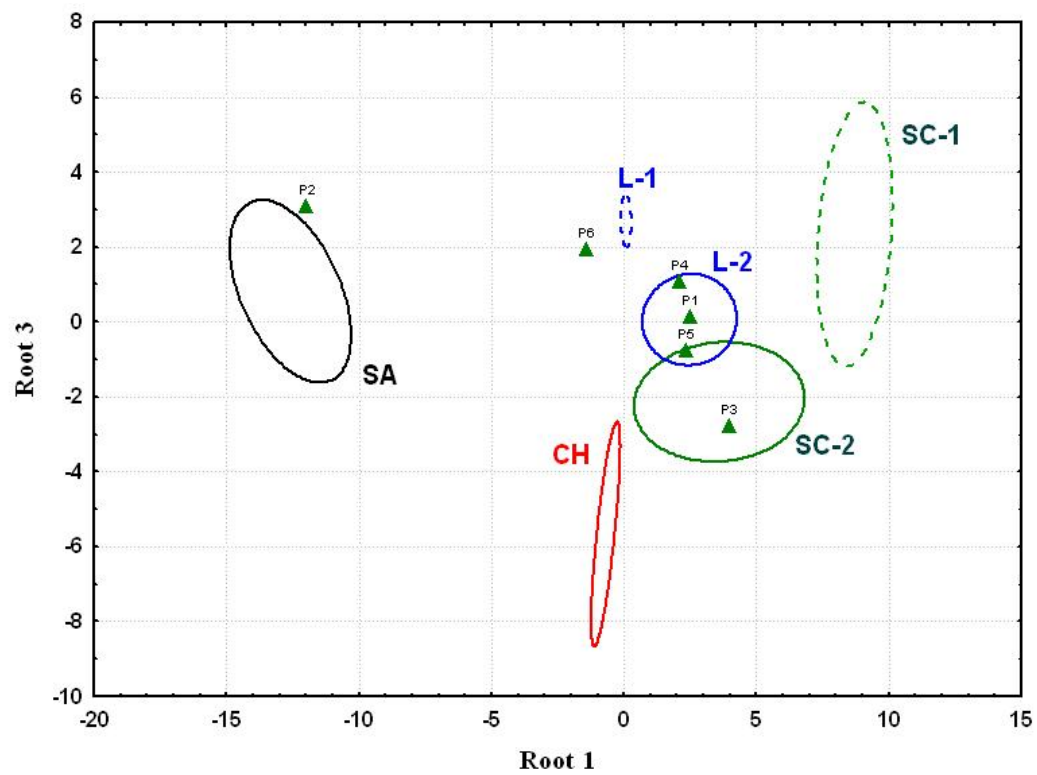
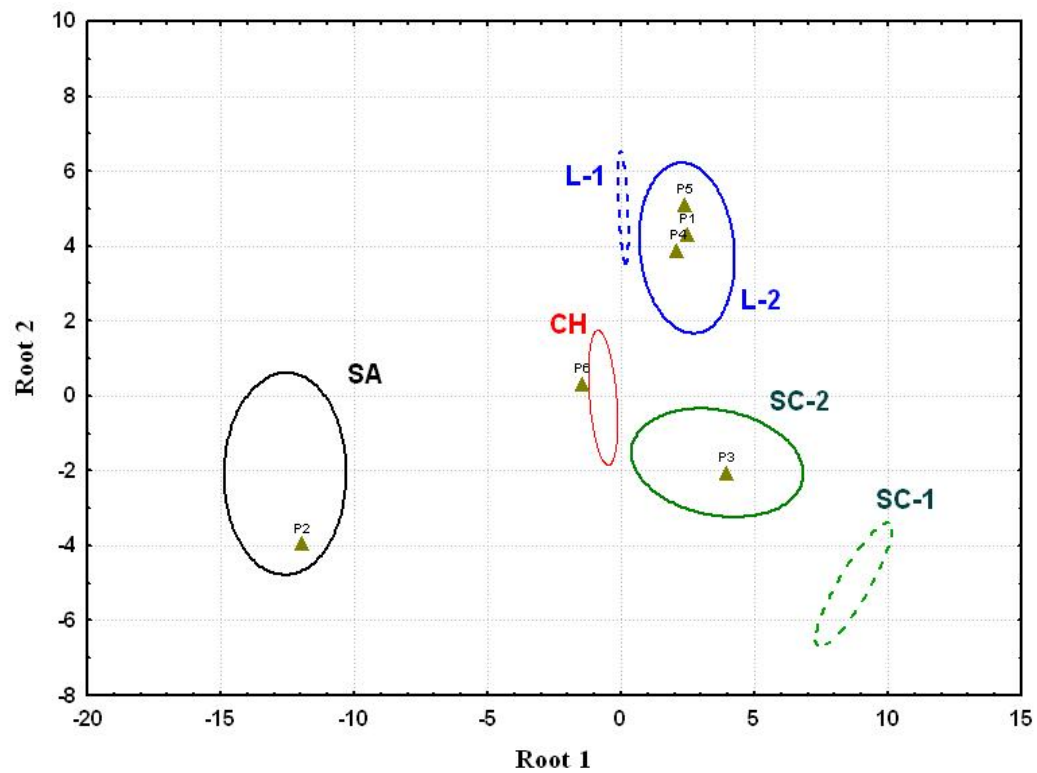
Classification Functions; grouping: Category (fprintraceIn1)						
	<b>CH</b> p=.07895	<b>L1</b> p=.10526	<b>L2</b> p=.26316	<b>SA</b> p=.21053	<b>SC1</b> p=.15789	<b>SC2</b> p=.18421
<b>V</b>	183.47	175.41	180.63	128.17	197.76	185.69
<b>Sm</b>	-51.57	-58.86	-59.11	-40.14	-44.53	-49.28
<b>Sc</b>	302.87	323.98	331.81	256.31	329.99	319.79
<b>Ni</b>	132.60	128.13	133.71	106.25	123.01	132.11
<b>Ga</b>	64.02	103.78	110.78	97.86	123.44	108.68
<b>K<sub>2</sub>O</b>	-230.72	-239.38	-249.28	-199.61	-285.53	-260.67
<b>Ba</b>	241.25	243.96	254.64	205.23	294.33	265.47
<b>Zn</b>	238.29	205.64	224.41	197.56	231.91	232.72
<b>CaO</b>	13.67	21.17	15.54	11.73	12.56	12.38
<b>Constant</b>	-2486.32	-2456.60	-2677.25	-1732.97	-3037.80	-2752.38

Classification Matrix (fprintraceIn1) Rows: Observed classifications Columns: Predicted classifications							
	<b>Percent Correct</b>	<b>CH</b> p=.07895	<b>L1</b> p=.10526	<b>L2</b> p=.26316	<b>SA</b> p=.21053	<b>SC1</b> p=.15789	<b>SC2</b> p=.18421
<b>CH</b>	100.00	3	0	0	0	0	0
<b>L1</b>	100.00	0	4	0	0	0	0
<b>L2</b>	100.00	0	0	10	0	0	0
<b>SA</b>	100.00	0	0	0	8	0	0
<b>SC1</b>	100.00	0	0	0	0	6	0
<b>SC2</b>	100.00	0	0	0	0	0	7
<b>Total</b>	100.00	3	4	10	8	6	7

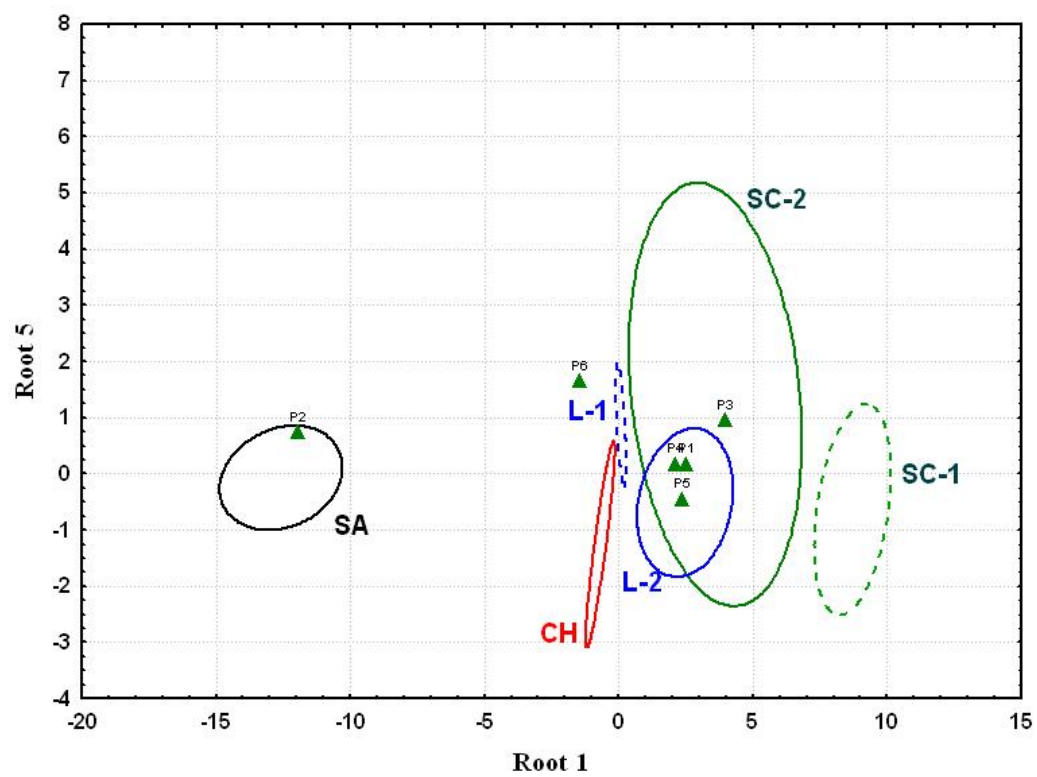
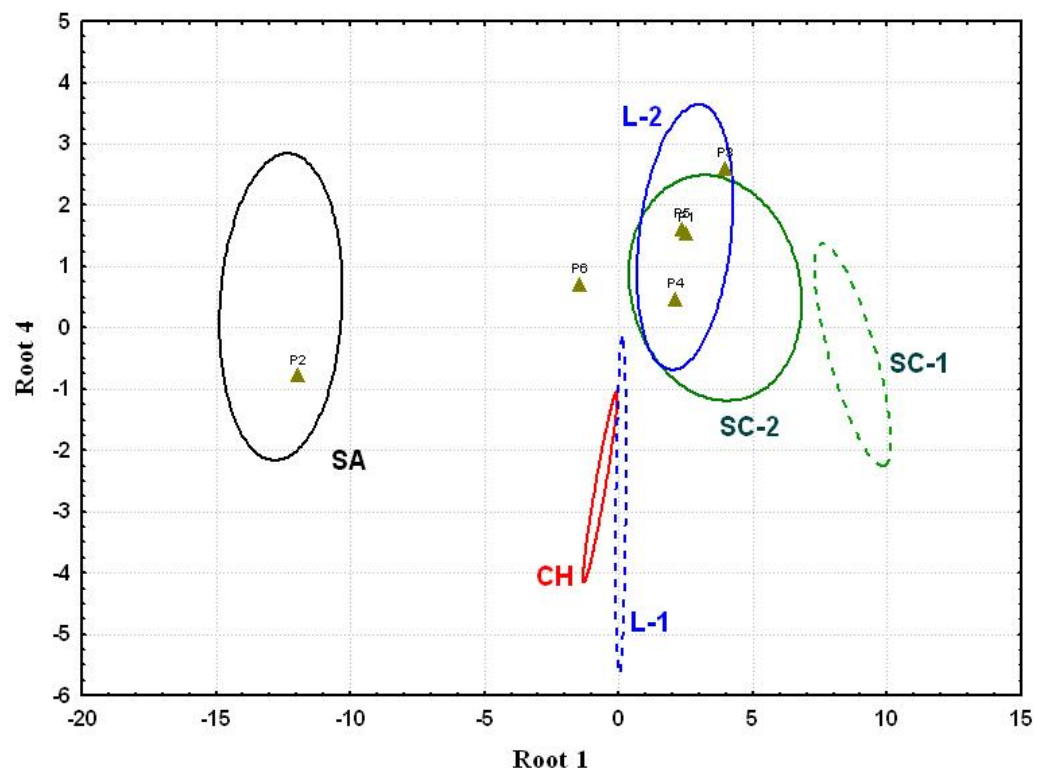
Classification of Cases (fprinttraceln1)							
Incorrect classifications are marked with *							
Samples	Observed Classif.	1 p=.07895	2 p=.10526	3 p=.26316	4 p=.21053	5 p=.15789	6 p=.18421
CH GS-7	CH	CH	SC2	L2	L1	SC1	SA
CH GS-12	CH	CH	SC2	L2	L1	SA	SC1
CH GS-16	CH	CH	SC2	L2	L1	SA	SC1
L 1A	L1	L1	L2	SC2	CH	SA	SC1
L 4A	L1	L1	L2	SC2	CH	SA	SC1
L 9A	L1	L1	L2	SC2	CH	SA	SC1
L10A	L1	L1	L2	CH	SC2	SA	SC1
L GS-2	L2	L2	L1	SC2	CH	SC1	SA
L GS-4	L2	L2	SC2	L1	CH	SC1	SA
L GS-10	L2	L2	L1	SC2	CH	SC1	SA
L A	L2	L2	L1	SC2	CH	SC1	SA
L 3A	L2	L2	SC2	L1	CH	SC1	SA
L 5A	L2	L2	L1	SC2	CH	SA	SC1
L 6A	L2	L2	SC2	L1	CH	SC1	SA
L 7A	L2	L2	L1	SC2	CH	SA	SC1
L 8A	L2	L2	SC2	L1	CH	SC1	SA
L 11A	L2	L2	L1	SC2	CH	SC1	SA
SA GS-3	SA	SA	CH	L1	L2	SC1	SC2
SA GS-13	SA	SA	CH	L1	L2	SC1	SC2
SA GS-17	SA	SA	CH	L1	L2	SC1	SC2
SA GS-18	SA	SA	CH	L1	L2	SC1	SC2
SA PVC1	SA	SA	CH	L1	L2	SC1	SC2
SA PVC2	SA	SA	CH	L1	L2	SC1	SC2
SA PVC3	SA	SA	CH	L1	L2	SC1	SC2
SA GS-1	SA	SA	CH	L1	L2	SC1	SC2
SC4	SC1	SC1	SC2	L2	CH	L1	SA
SC5	SC1	SC1	SC2	L2	CH	L1	SA
SC6	SC1	SC1	SC2	L2	CH	L1	SA
SC9	SC1	SC1	SC2	L2	CH	L1	SA
SC10	SC1	SC1	SC2	L2	CH	L1	SA
SC11	SC1	SC1	SC2	CH	L1	L2	SA
SC GS-11	SC2	SC2	L2	CH	SC1	L1	SA
SC GS-15	SC2	SC2	L2	CH	SC1	L1	SA
SC2	SC2	SC2	L2	CH	SC1	L1	SA
SC3	SC2	SC2	L2	CH	SC1	L1	SA
SC7	SC2	SC2	SC1	L2	CH	L1	SA
SC8	SC2	SC2	L2	SC1	CH	L1	SA
SC PVC5	SC2	SC2	SC1	L2	CH	L1	SA
P1	---	L2	L1	SC2	CH	SC1	SA
P2	---	SA	CH	L1	L2	SC1	SC2
P3	---	SC2	L2	CH	SC1	L1	SA
P4	---	L2	L1	SC2	CH	SC1	SA
P5	---	L2	L1	SC2	CH	SA	SC1
P6	---	L2	L1	SC2	CH	SA	SC1

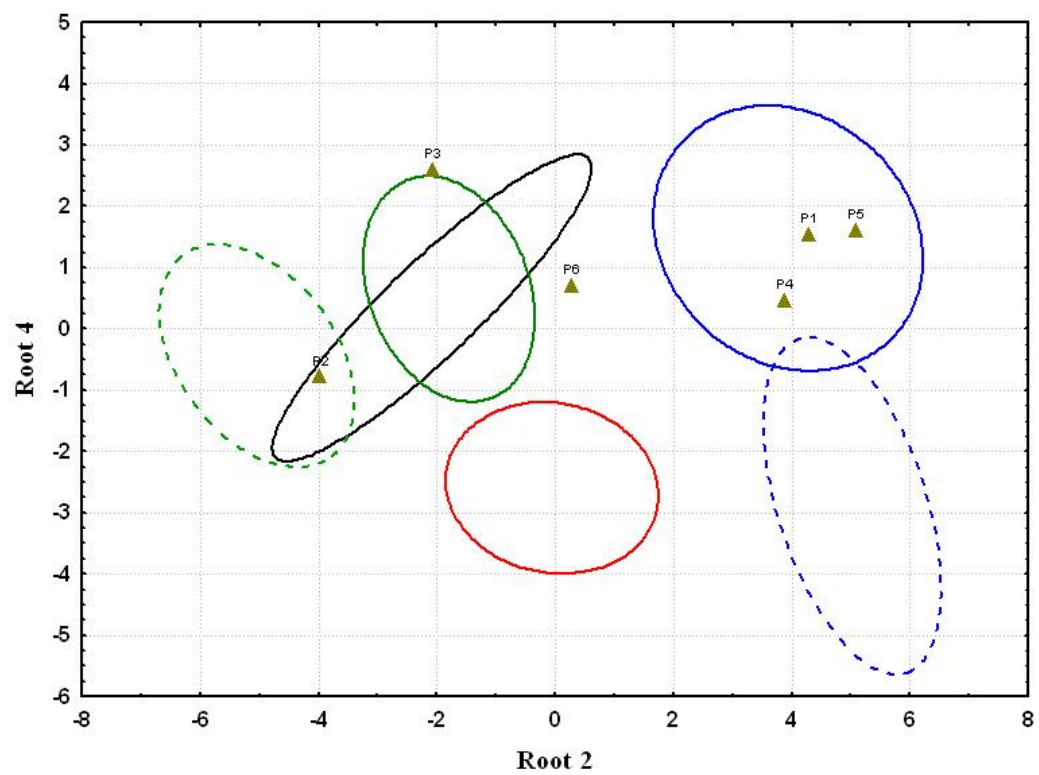
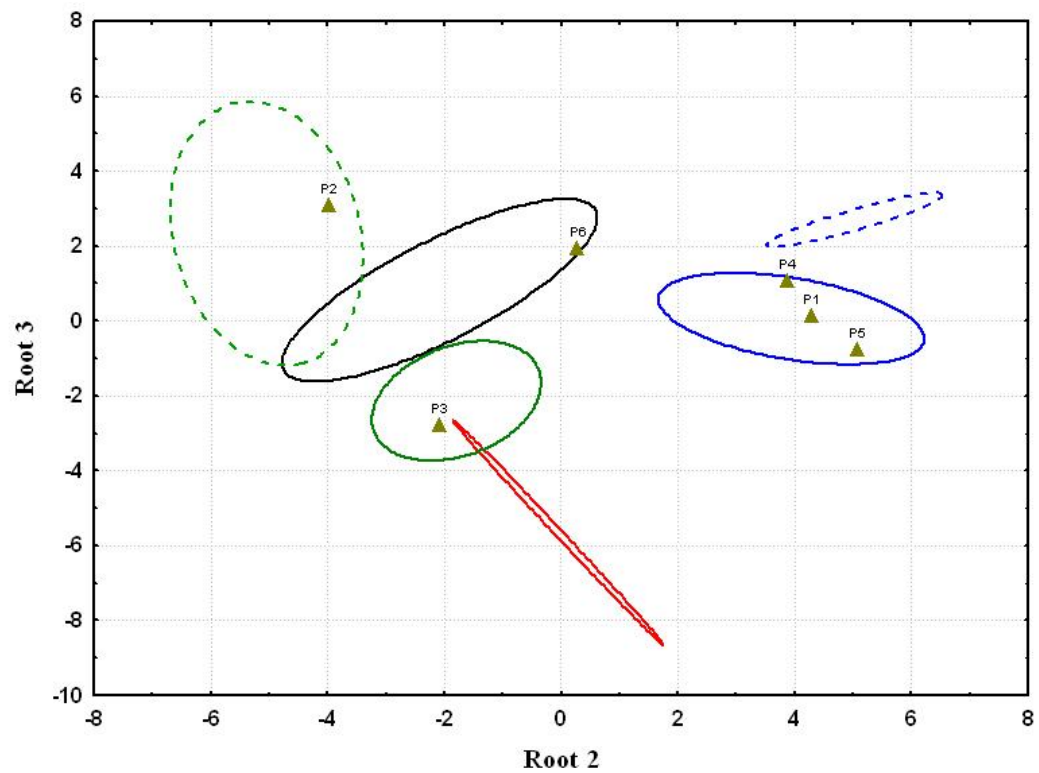
Squared Mahalanobis Distances from Group Centroids (fprinttraceln1)							
Incorrect classifications are marked with *							
Samples	Observed Classif.	CH p=.07895	L1 p=.10526	L2 p=.26316	SA p=.21053	SC1 p=.15789	SC2 p=.18421
CH GS-7	CH	8.7956	87.5936	71.5689	175.2830	139.9053	34.4280
CH GS-12	CH	11.9332	143.8322	116.1284	237.5868	248.6707	98.1462
CH GS-16	CH	4.0040	95.7962	64.0365	204.6343	182.6115	46.4669
L 1A	L1	135.6047	10.3361	63.4991	262.1994	230.8328	136.4363
L 4A	L1	108.4353	8.9091	26.0163	221.3775	175.0468	85.3378
L 9A	L1	87.8464	2.8041	26.8372	208.8424	169.9184	78.3887
L10A	L1	98.7677	5.2859	50.5188	238.5141	188.6378	107.5901
L GS-2	L2	64.2776	26.6687	5.0037	250.4765	130.4225	42.6823
L GS-4	L2	72.7080	38.5609	2.9019	286.2660	116.1901	36.6218
L GS-10	L2	82.5597	32.9802	3.0597	274.7612	139.9786	49.9405
L A	L2	86.4430	37.4091	1.4378	289.4589	137.4353	53.5119
L 3A	L2	105.9844	54.1157	10.4803	296.5969	128.8135	52.1340
L 5A	L2	93.4323	44.7476	4.8311	284.7266	166.6975	67.4674
L 6A	L2	89.6182	66.6823	14.4555	254.9458	124.3339	48.8698
L 7A	L2	67.0460	22.1878	4.6971	223.8671	157.3472	54.6573
L 8A	L2	79.8599	37.9020	5.1779	265.6221	104.8685	38.5417
L 11A	L2	76.7795	49.6253	9.9940	285.6264	141.2906	51.1019
SA GS-3	SA	193.5837	266.6944	309.0008	9.7612	490.4813	293.0364
SA GS-13	SA	188.9918	236.1130	272.1352	5.5644	433.8024	254.0781
SA GS-17	SA	164.3292	195.1795	229.4371	3.7534	414.0881	231.6779
SA GS-18	SA	169.0552	187.7194	222.6530	3.0079	425.2073	237.9795
SA PVC1	SA	267.4216	251.9666	310.9178	14.2282	560.4944	351.1206
SA PVC2	SA	217.8263	234.8128	279.3331	3.8230	470.5202	286.6134
SA PVC3	SA	244.6221	285.4550	325.4320	4.5610	531.5160	328.4511
SA GS-1	SA	189.5095	203.8720	225.9538	9.9171	453.4949	256.0202
SC4	SC1	191.7693	187.0969	132.2547	475.9973	1.5014	65.4675
SC5	SC1	146.3816	191.4194	122.8919	464.1097	13.5588	43.7764
SC6	SC1	198.2207	184.7175	128.6830	485.8417	1.9464	66.3412
SC9	SC1	179.6620	200.3114	135.4938	419.9784	6.7164	63.1261
SC10	SC1	190.8465	172.2562	125.0002	487.2673	3.3814	67.3691
SC11	SC1	231.7977	215.6179	172.6480	506.3154	18.6640	112.1238
SC GS-11	SC2	46.0571	94.1951	37.4559	233.2271	91.6090	9.5152
SC GS-15	SC2	36.2064	83.9668	35.6071	243.6542	79.0061	5.3665
SC2	SC2	86.5762	123.5538	83.7753	292.7762	111.7948	24.2415
SC3	SC2	49.0676	87.5054	49.4560	229.4888	72.5975	5.7743
SC7	SC2	73.2396	146.7915	71.6333	318.4813	64.9127	10.1553
SC8	SC2	60.2595	83.0458	44.1967	294.6260	45.3614	4.5921
SC PVC5	SC2	82.1547	120.1810	54.8269	372.5707	42.7353	13.8636
P1	---	82.7863	34.0357	1.6606	270.8628	136.0167	46.7633
P2	---	232.3754	236.6472	292.8638	16.9064	437.3367	282.9089
P3	---	70.7196	130.5770	54.8298	297.9830	74.1010	9.5694
P4	---	81.5674	21.4377	4.2689	253.4153	127.6789	47.8745
P5	---	79.5402	40.9509	3.6814	281.0815	158.3097	56.7122
P6	---	90.3263	51.7954	50.1997	147.1153	150.8745	59.0548

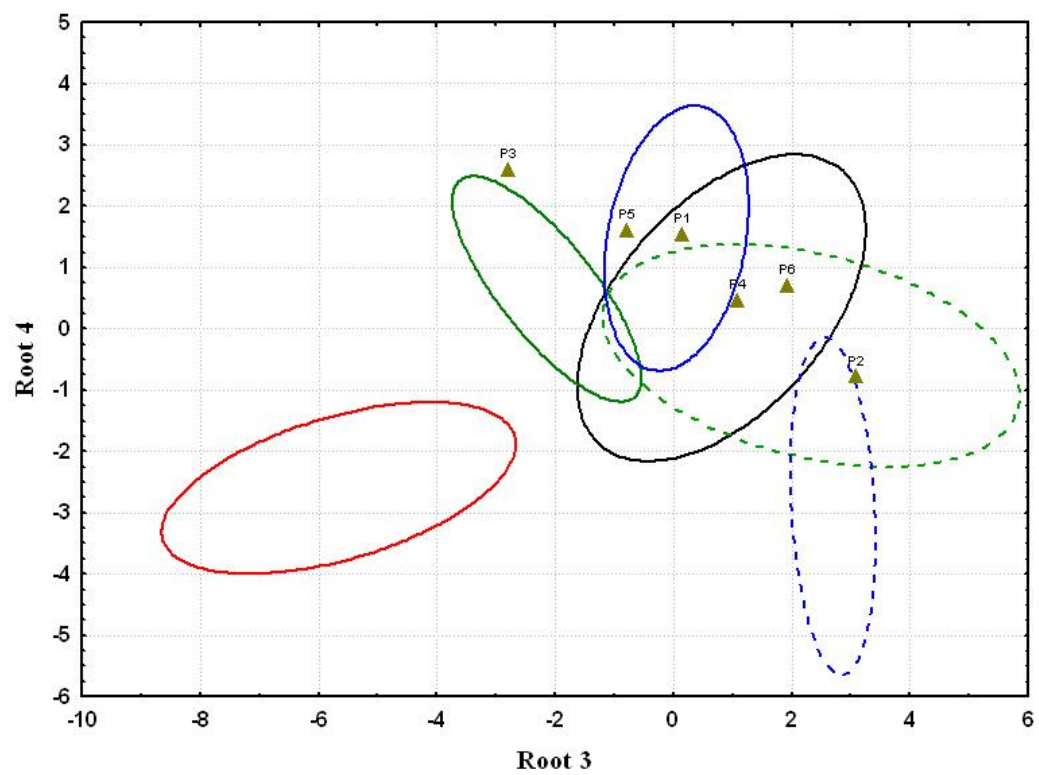
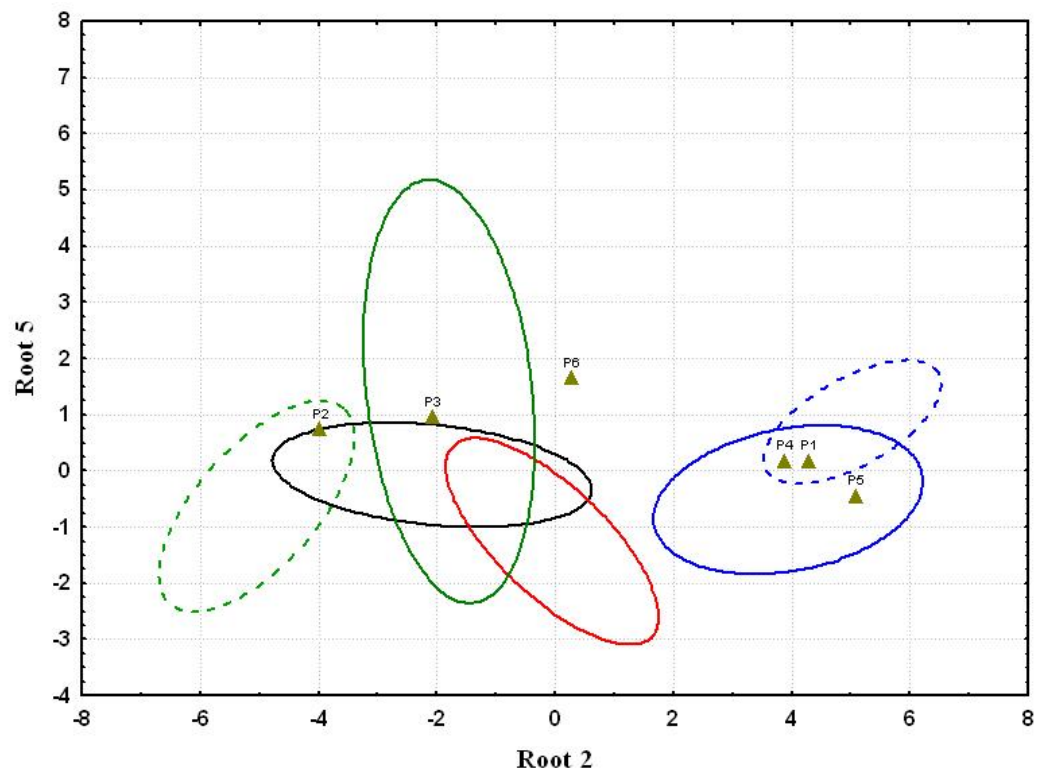
Posterior Probabilities (fprinttraceln1)							
Incorrect classifications are marked with *							
Samples	Observed Classif.	CH p=.07895	L1 p=.10526	L2 p=.26316	SA p=.21053	SC1 p=.15789	SC2 p=.18421
CH GS-7	CH	0.99999	0.00000	0.00000	0.00000	0.00000	0.00001
CH GS-12	CH	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
CH GS-16	CH	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
L 1A	L1	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000
L 4A	L1	0.00000	0.99952	0.00048	0.00000	0.00000	0.00000
L 9A	L1	0.00000	0.99998	0.00002	0.00000	0.00000	0.00000
L10A	L1	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000
L GS-2	L2	0.00000	0.00001	0.99999	0.00000	0.00000	0.00000
L GS-4	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L GS-10	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 3A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 5A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 6A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 7A	L2	0.00000	0.00006	0.99994	0.00000	0.00000	0.00000
L 8A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 11A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
SA GS-3	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-13	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-17	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-18	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC1	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC2	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC3	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-1	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SC4	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC5	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC6	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC9	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC10	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC11	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC GS-11	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC GS-15	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC2	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC3	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC7	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC8	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC PVC5	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
P1	---	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
P2	---	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
P3	---	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
P4	---	0.00000	0.00007	0.99993	0.00000	0.00000	0.00000
P5	---	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
P6	---	0.00000	0.15155	0.84141	0.00000	0.00000	0.00703

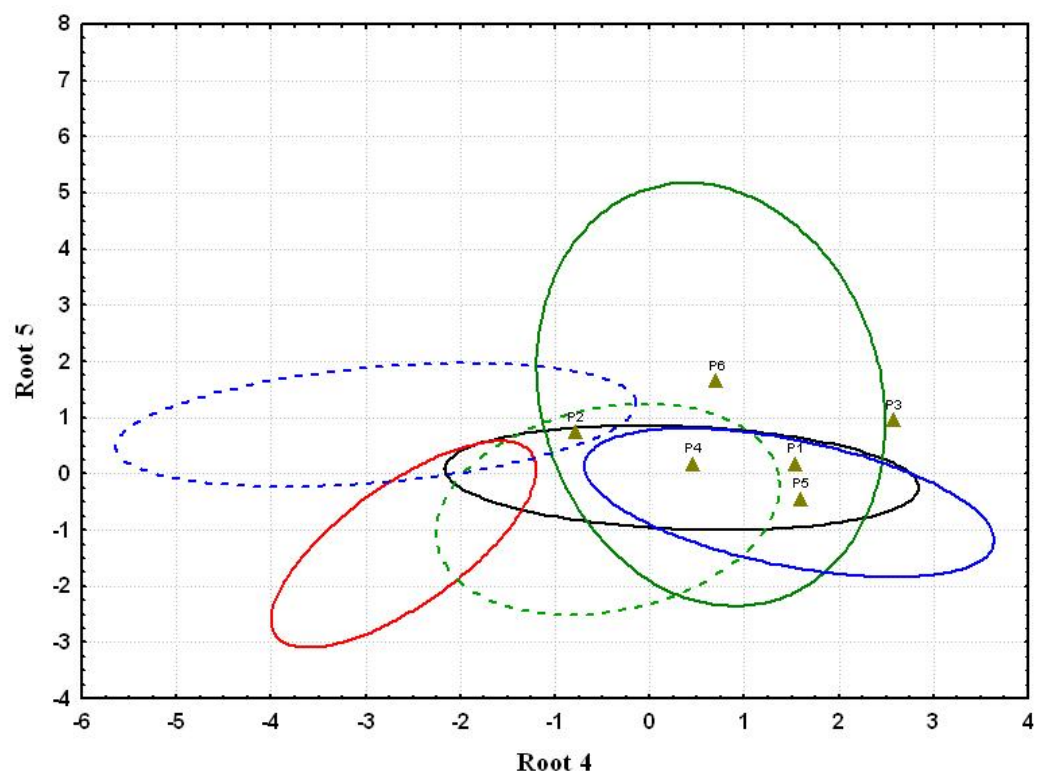
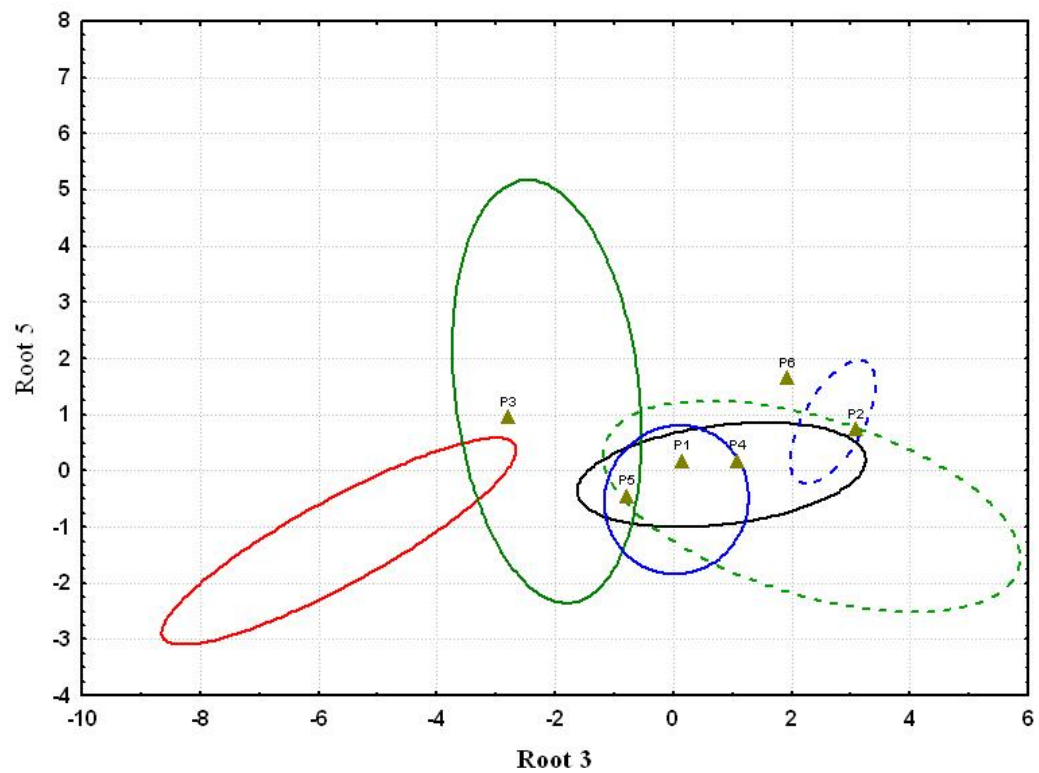












## Six-Category Model, Gunter Element Set

### Element Set:

SiO<sub>2</sub>, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, FeO, MnO, CaO, K<sub>2</sub>O, Ba, Cr, Cu, Ga, Ni, Sc, V, Zn

### Categories:

Category	Abbreviation	Sample IDs	No. Samples
South Africa	SA	GS-1, GS-3, GS-13, GS-17, GS-18,PVC1, PVC2, PVC3	8
China	CH	GS-7, GS-12, GS-16	3
South Carolina Group 1	SC-1	SC4, SC5, SC6, SC9, SC10, SC11	6
South Carolina Group 2	SC-2	GS-11, SG-15, SC2, SC3, SC7, SC8, PVC5	7
Libby Group 1	L-1	1A, 4A, 9A, 10A	4
Libby Group 2	L-2	GS-2, GS-4, GS-10, 2A, 3A, 5A, 6A, 7A, 8A, 11A	10
Total Number of Samples =			38

### Discriminant Function Analysis.

N=38	Discriminant Function Analysis Summary (fprintrace1n1) Step 8, N of vars in model: 8; Grouping: Category (6 grps) Wilks' Lambda: .00022 approx. F (40,111)=16.424 p<0.0000						
	Step	Element Added	Wilks' $\Lambda$	Partial $\Lambda$	F-Remove	p-Level	Tolerance
1	V	0.000561	0.3988	7.54	1.9616E-04	0.6983	TiO <sub>2</sub> , MnO
2	Zn	0.000637	0.3509	9.25	4.3912E-05	0.3720	Cu
3	Ni	0.000811	0.2757	13.14	2.4952E-06	0.5589	none
4	SiO <sub>2</sub>	0.000541	0.4135	7.09	2.9849E-04	0.4953	none
5	Ga	0.000511	0.4381	6.41	5.8133E-04	0.3700	Al <sub>2</sub> O <sub>3</sub>
6	FeO	0.000538	0.4155	7.03	3.1633E-04	0.2097	none
7	Sc	0.000402	0.5564	3.99	8.5234E-03	0.5594	none
8	Ba	0.000351	0.6379	2.84	3.6556E-02	0.5464	none

n = 38	Variables currently not in the model (fprintrace1n1) Df for all F-tests: 5,24					
Element	Wilks' $\Lambda$	Wilks' Partial $\Lambda$	F to Enter	p-level	Toler.	1-Toler.
TiO <sub>2</sub>	0.000063	0.2833	12.1432	0.00001	0.2007	0.7993
Al <sub>2</sub> O <sub>3</sub>	0.000194	0.8668	0.7376	0.60268	0.1950	0.8050
MnO	0.000135	0.6042	3.1448	0.02532	0.7385	0.2615

<b>CaO</b>	0.000165	0.7391	1.6947	0.17425	0.7789	0.2211
<b>K<sub>2</sub>O</b>	0.000076	0.3393	9.3470	0.00005	0.0788	0.9212
<b>Cr</b>	0.000156	0.6992	2.0648	0.10532	0.4049	0.5951
<b>Cu</b>	0.000182	0.8152	1.0884	0.39208	0.4875	0.5125

Squared Mahalanobis Distances (fprintraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
<b>CH</b>	0.0000	76.6194	60.1577	125.8475	62.8720	25.3773
<b>L1</b>	76.6194	0.0000	24.3657	78.7953	64.8621	54.9905
<b>L2</b>	60.1577	24.3657	0.0000	93.7229	47.1244	34.8303
<b>SA</b>	125.8475	78.7953	93.7229	0.0000	183.3785	121.4912
<b>SC1</b>	62.8720	64.8621	47.1244	183.3785	0.0000	30.1417
<b>SC2</b>	25.3773	54.9905	34.8303	121.4912	30.1417	0.0000

F-values; df = 8,26 (fprintraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
<b>CH</b>		12.827	13.557	26.814	12.280	5.204
<b>L1</b>	12.827		6.798	20.520	15.202	13.670
<b>L2</b>	13.557	6.798		40.678	17.257	14.006
<b>SA</b>	26.814	20.520	40.678		61.399	44.294
<b>SC1</b>	12.280	15.202	17.257	61.399		9.510
<b>SC2</b>	5.204	13.670	14.006	44.294	9.510	

p-levels (fprintraceln1)						
Category	CH	L1	L2	SA	SC1	SC2
<b>CH</b>		2.8283E-07	1.6357E-07	1.1228E-10	4.3294E-07	5.9904E-04
<b>L1</b>	2.8283E-07		8.1529E-05	2.1758E-09	5.1483E-08	1.5066E-07
<b>L2</b>	1.6357E-07	8.1529E-05		9.0011E-13	1.3824E-08	1.1813E-07
<b>SA</b>	1.1228E-10	2.1758E-09	9.0011E-13		6.3547E-15	3.2738E-13
<b>SC1</b>	4.3294E-07	5.1483E-08	1.3824E-08	6.3547E-15		4.7395E-06
<b>SC2</b>	5.9904E-04	1.5066E-07	1.1813E-07	3.2738E-13	4.7395E-06	

Summary of Stepwise Analysis (fprintraceln1)											
Variable Enter/ Remove	Step	F to enter/ remove	df 1	df 2	p-level	No. of vars. In Model	Lambda	F-value	df 1	df 2	p-level
<b>V-(E)</b>	1	59.974	5	32	2.609E-15	1	0.096424	59.974	5	32	2.609E-15
<b>Zn-(E)</b>	2	18.892	5	31	1.347E-08	2	0.023825	33.967	10	62	1.914E-21
<b>Ni-(E)</b>	3	14.464	5	30	3.094E-07	3	0.006985	27.953	15	83	3.486E-26
<b>SiO<sub>2</sub>-(E)</b>	4	13.946	5	29	5.601E-07	4	0.002052	26.531	20	97	1.046E-30
<b>Ga-(E)</b>	5	5.726	5	28	9.252E-04	5	0.001015	22.773	25	106	0.000E+00
<b>FeO-(E)</b>	6	3.665	5	27	1.166E-02	6	0.000604	19.719	30	110	0.000E+00
<b>Sc-(E)</b>	7	3.763	5	26	1.072E-02	7	0.000351	17.977	35	112	0.000E+00
<b>Ba-(E)</b>	8	2.838	5	25	3.656E-02	8	0.000224	16.424	40	112	0.000E+00

Chi-Square Tests with Successive Roots Removed (fprintraceln1)						
Roots Removed	Eigen-value	Canonical R	Wilks' Lambda	Chi-Sqr.	df	p-level
0	23.48	0.9794	0.000224	252.16	40	0.000E+00
1	6.45	0.9304	0.005474	156.23	28	0.000E+00
2	2.91	0.8628	0.040770	95.99	18	1.193E-12
3	1.85	0.8057	0.159561	55.06	10	3.078E-08
4	1.20	0.7384	0.454804	23.64	4	9.445E-05

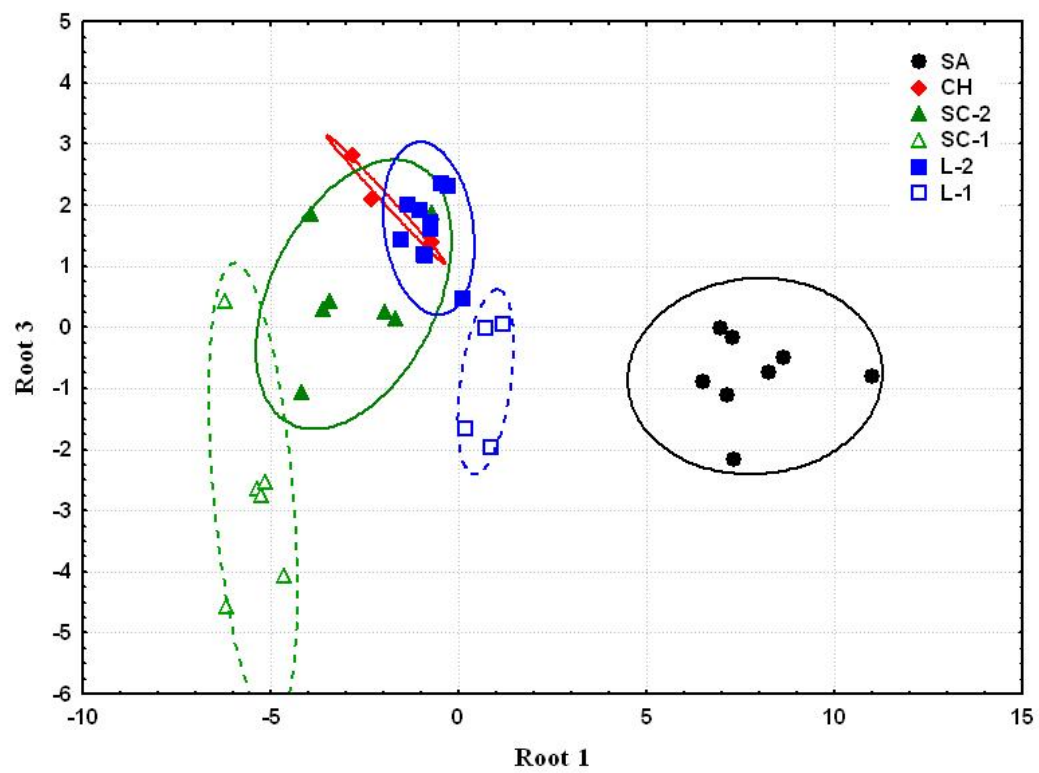
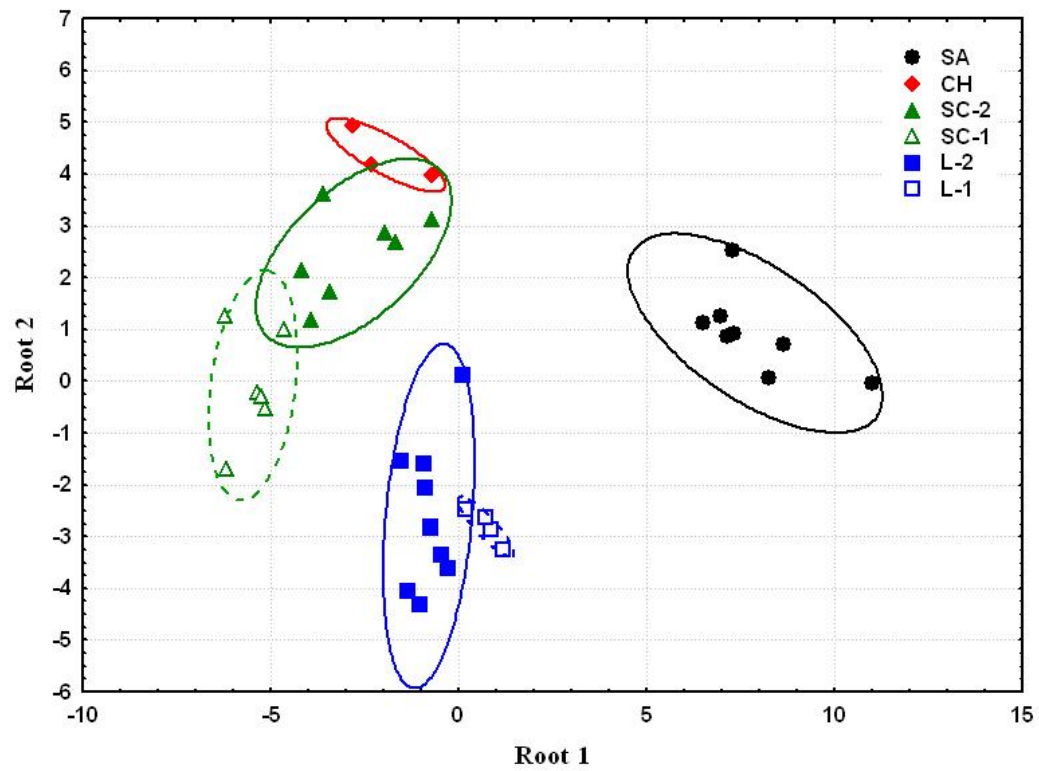
Raw Coefficients (fprintraceln1) for Canonical Variables						Standardized Coefficients (fprintraceln1) for Canonical Variables					
Variable	Root 1	Root 2	Root 3	Root 4	Root 5	Variable	Root 1	Root 2	Root 3	Root 4	Root 5
V	-3.0005	0.94179	0.0305	-1.6200	-0.6251	V	-0.82750	0.259732	0.008418	-0.446783	-0.172395
Zn	1.0354	5.43680	-1.2514	0.3439	-3.7684	Zn	0.22920	1.203546	-0.277030	0.076131	-0.834210
Ni	-1.4189	0.05000	2.3094	0.3458	1.0827	Ni	-0.62775	0.022123	1.021733	0.153004	0.479012
SiO <sub>2</sub>	-15.4169	1.46894	-10.1807	3.3974	20.4904	SiO <sub>2</sub>	-0.71967	0.068571	-0.475241	0.158596	0.956506
Ga	1.3905	-2.16669	-1.3302	5.3064	4.1907	Ga	0.28930	-0.450781	-0.276746	1.104000	0.871885
FeO	-7.3102	-5.72939	3.0481	3.6431	5.3916	FeO	-1.15808	-0.907649	0.482871	0.577138	0.854139
Sc	-1.9730	-1.14155	-0.0772	0.9523	0.8710	Sc	-0.72521	-0.419607	-0.028364	0.350045	0.320154
Ba	-1.1189	-0.54769	0.2874	-0.1724	-0.8703	Ba	-0.65158	-0.318948	0.167371	-0.100414	-0.506839
Constant	101.2293	-8.63200	27.0807	-32.2415	-83.6087	Constant	---	---	---	---	---
Eigenval	23.4750	6.44765	2.9136	1.8503	1.1988	Eigenval	23.47503	6.447648	2.913639	1.850345	1.198750
Cum.Prob.	0.6542	0.83384	0.9150	0.9666	1.0000	Cum.Prob.	0.65417	0.833840	0.915032	0.966595	1.000000

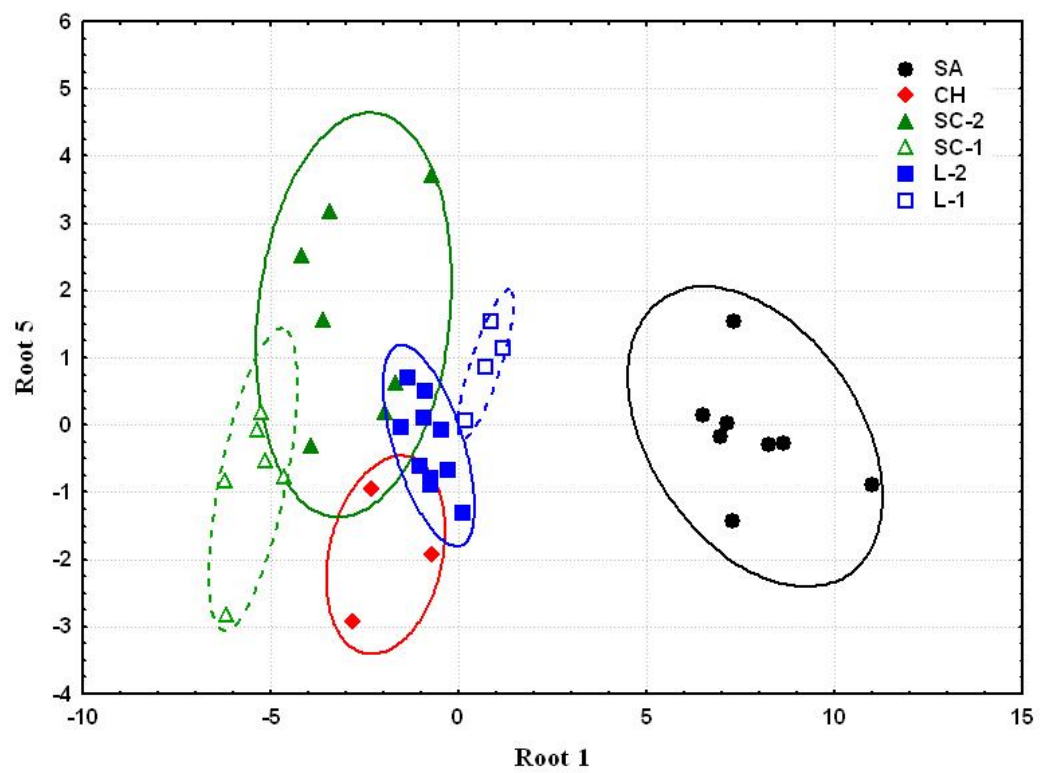
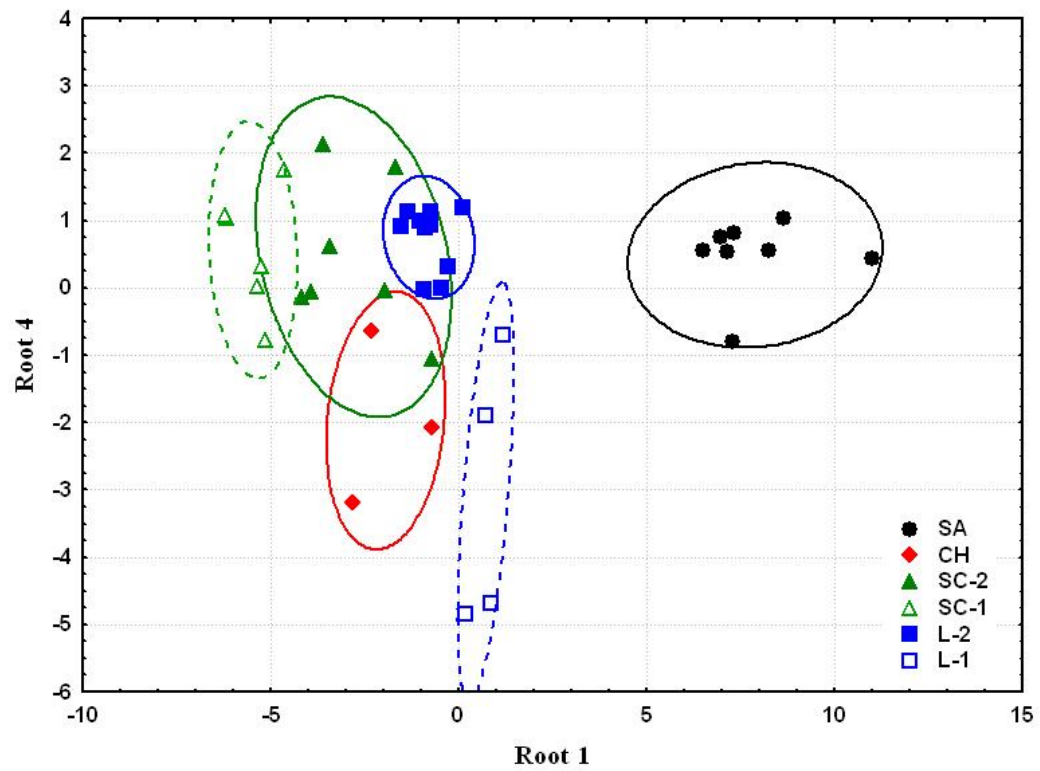
Factor Structure Matrix (fprintraceln1) Correlations Variables - Canonical Roots (Pooled-within-groups correlations)					
Variable	Root 1	Root 2	Root 3	Root 4	Root 5
V	-0.621279	0.137904	-0.072936	0.134725	-0.339426
Zn	-0.171077	0.592799	-0.041899	0.645423	-0.355453
Ni	0.025481	0.332356	0.749675	-0.041160	0.400600
SiO <sub>2</sub>	-0.068055	0.303966	-0.373995	-0.356532	0.673089
Ga	-0.085906	-0.018041	-0.013742	0.814828	-0.153645
FeO	-0.194238	0.018507	0.070432	0.473489	-0.330083
Sc	-0.207276	-0.303071	-0.317602	-0.212157	0.079862
Ba	-0.217496	-0.326132	-0.009081	0.245747	-0.393787

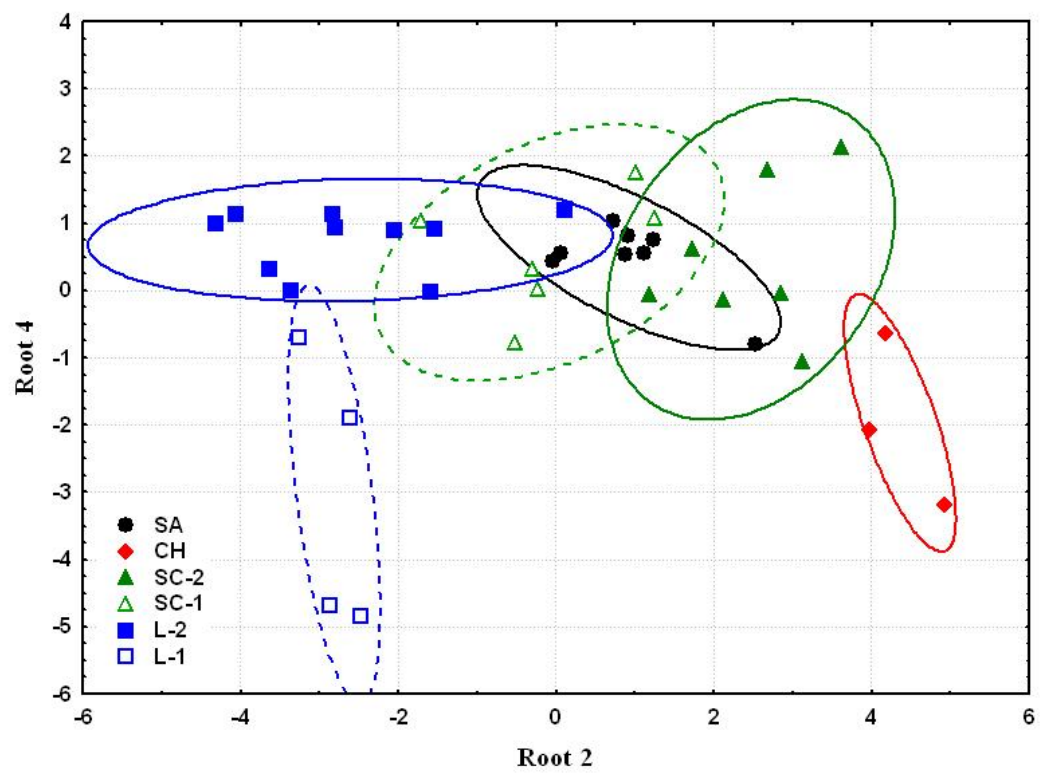
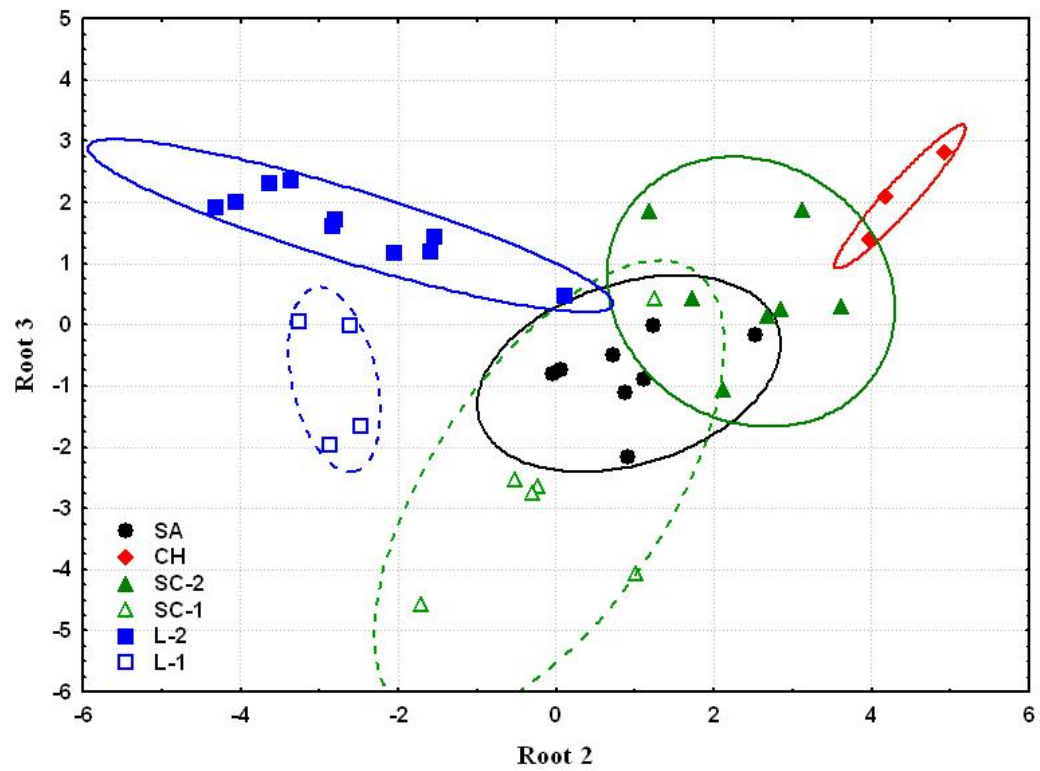
Means of Canonical Variables (fprintraceln1)					
Category	Root 1	Root 2	Root 3	Root 4	Root 5
CH	-1.93534	4.36356	2.09126	-1.97015	-1.93182
L1	0.72577	-2.80911	-0.89469	-3.02662	0.90696
L2	-0.78920	-2.60670	1.61550	0.74891	-0.30707
SA	7.88935	0.92612	-0.80078	0.48831	-0.17141
SC1	-5.46528	-0.08371	-2.69821	0.56622	-0.80886
SC2	-2.78974	2.47229	0.53507	0.46057	1.63753

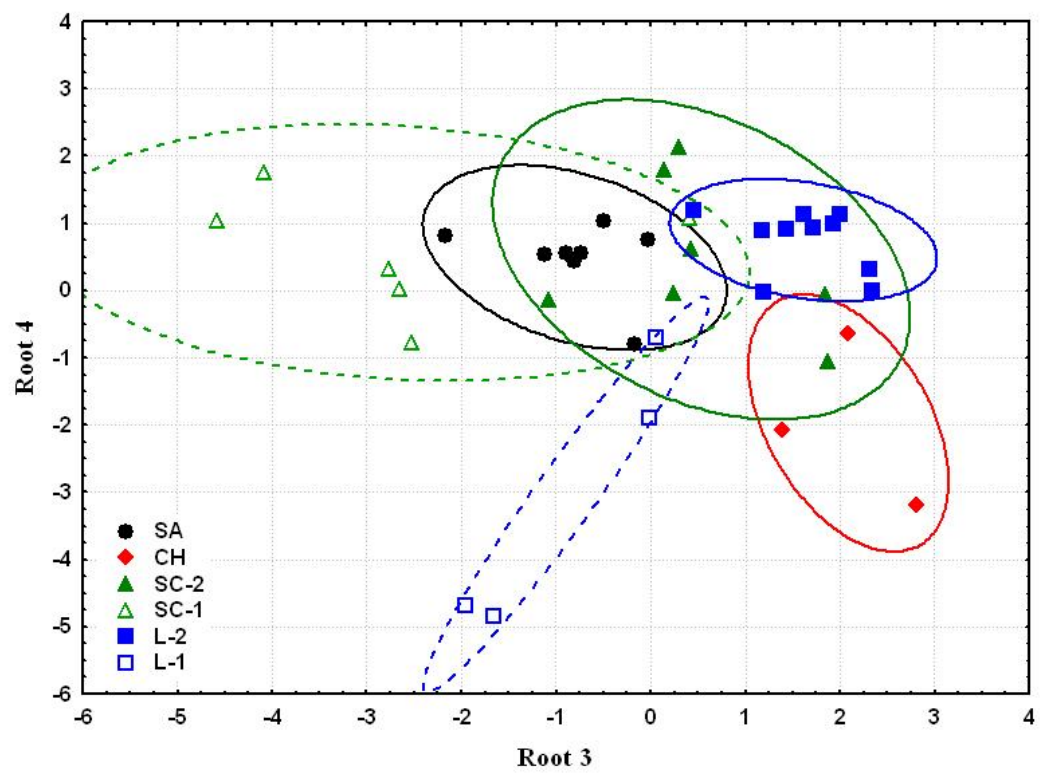
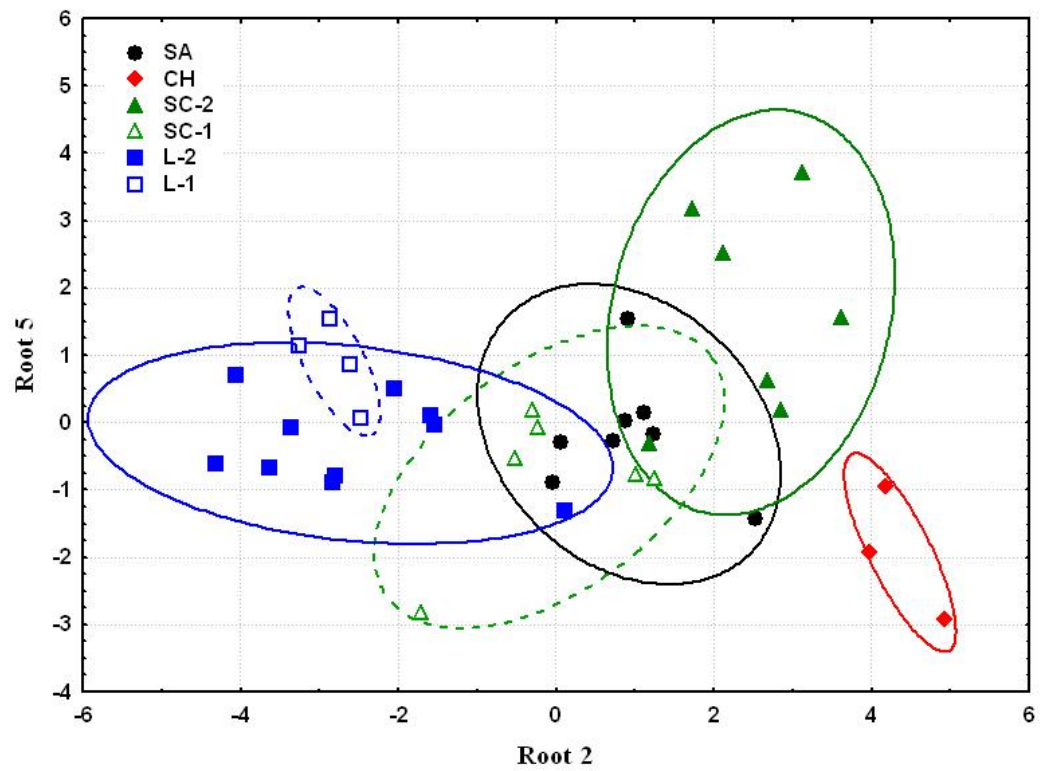
		Unstandardized Canonical Scores (fprintraceln1)				
Samples	Group	Root 1	Root 2	Root 3	Root 4	Root 5
CH GS-7	CH	-0.69734	3.98224	1.39224	-2.07189	-1.92511
CH GS-12	CH	-2.80544	4.92707	2.80143	-3.19405	-2.91827
CH GS-16	CH	-2.30323	4.18138	2.08011	-0.64452	-0.95208
L 1A	L1	0.85059	-2.87445	-1.95986	-4.67919	1.54702
L 4A	L1	1.17111	-3.25860	0.05479	-0.69237	1.14501
L 9A	L1	0.70191	-2.62043	-0.01995	-1.89223	0.86325
L10A	L1	0.17947	-2.48295	-1.65371	-4.84269	0.07255
L GS-2	L2	-0.94188	-1.59254	1.19176	-0.01734	0.11015
L GS-4	L2	-1.52059	-1.54017	1.42653	0.91023	-0.02837
L GS-10	L2	-0.87796	-2.05106	1.17132	0.89274	0.50140
L A	L2	-0.73677	-2.80387	1.71755	0.93435	-0.79515
L 3A	L2	-1.35322	-4.05594	2.00395	1.13280	0.69341
L 5A	L2	-0.29344	-3.62641	2.30719	0.31979	-0.66847
L 6A	L2	-1.05007	-4.32207	1.92202	0.98967	-0.61465
L 7A	L2	-0.47145	-3.35683	2.34289	-0.00796	-0.07080
L 8A	L2	-0.74640	-2.83106	1.61214	1.13961	-0.89684
L 11A	L2	0.09975	0.11293	0.45964	1.19524	-1.30135
SA GS-3	SA	7.30295	2.52794	-0.17177	-0.80514	-1.43410
SA GS-13	SA	6.95744	1.24195	-0.02376	0.76089	-0.18323
SA GS-17	SA	6.49393	1.11144	-0.89435	0.56262	0.15325
SA GS-18	SA	7.14187	0.87245	-1.12079	0.53287	0.01747
SA PVC1	SA	11.01202	-0.03535	-0.80872	0.43928	-0.89277
SA PVC2	SA	8.25028	0.06056	-0.73314	0.56461	-0.30206
SA PVC3	SA	8.62654	0.71439	-0.49080	1.02777	-0.26944
SA GS-1	SA	7.32978	0.91561	-2.16291	0.82360	1.53961
SC4	SC1	-5.37064	-0.23251	-2.65243	0.02531	-0.08111
SC5	SC1	-6.20986	1.25413	0.41387	1.06703	-0.84149
SC6	SC1	-5.25736	-0.29902	-2.75931	0.30825	0.17735
SC9	SC1	-4.64061	1.00553	-4.07946	1.75760	-0.76264
SC10	SC1	-5.12602	-0.52455	-2.53132	-0.78757	-0.53038
SC11	SC1	-6.18717	-1.70584	-4.58060	1.02671	-2.81488
SC GS-11	SC2	-1.67956	2.67393	0.13726	1.79500	0.62335
SC GS-15	SC2	-1.95203	2.85275	0.24562	-0.05134	0.18703
SC2	SC2	-0.72003	3.12718	1.86623	-1.04801	3.70678
SC3	SC2	-3.43839	1.72441	0.43102	0.60946	3.17130
SC7	SC2	-3.61187	3.61608	0.29468	2.12462	1.55684
SC8	SC2	-4.18832	2.12654	-1.07308	-0.13417	2.52124
SC PVC5	SC2	-3.93799	1.18515	1.84376	-0.07158	-0.30384
P1	----	-0.26156	-2.06513	1.58420	1.06594	0.05964
P2	----	8.69043	-0.25506	-1.03886	-0.21658	-1.18127
P3	----	-2.35661	2.57015	1.54772	2.88720	0.16677
P4	----	-0.74208	-2.67744	0.97110	0.73297	0.45596
P5	----	-0.04076	-1.10566	1.32261	0.90582	-0.58316
P6	----	3.99954	-1.83539	1.88625	-1.43582	-0.55584
Li1UI	----	-0.38495	-2.46971	1.53000	1.08665	-0.32298
Li2UI	----	1.04362	-1.50970	1.36524	0.73850	-0.68913
at1UI	----	0.04871	-0.95002	1.20062	1.32636	-0.91847
at2UI	----	0.03762	0.00522	1.27250	1.06674	-1.89720
at3UI	----	-0.27443	2.14973	0.38175	1.36762	-2.56342
at4UI	----	0.06580	-2.04231	1.08686	2.77583	1.01735
ga1UI	----	7.96570	1.02080	-0.56321	0.97440	-1.56908
ga2UI	----	7.71552	1.02703	-0.46388	1.43043	-1.67518
ga3UI	----	7.44537	1.16499	-0.03826	-0.08954	-2.48223
ga4UI	----	8.75772	1.07537	0.11699	0.54800	-2.42305
ga5UI	----	12.40554	2.42181	0.88815	-3.81538	-7.95268
ga6UI	----	6.89926	0.97805	-0.31438	0.23141	-2.23335
ga7UI	----	8.62803	0.40633	-0.26734	2.03683	-1.00923
SA1UI	----	19.57583	-0.98666	-0.91356	5.68981	0.78531
SC1UI	----	-0.99506	2.02520	0.70848	2.46516	-0.16237
SC2UI	----	-0.91236	2.63376	0.54410	2.37406	-0.20080
SC3UI	----	-2.21119	2.19307	1.33394	-0.32004	1.00816
SC4UI	----	-3.15287	1.72153	1.22443	0.67283	2.10052
SC5UI	----	-5.14475	0.29400	-2.56300	5.57109	3.87239
SC6UI	----	-3.90157	2.57179	1.36239	2.50417	1.58709

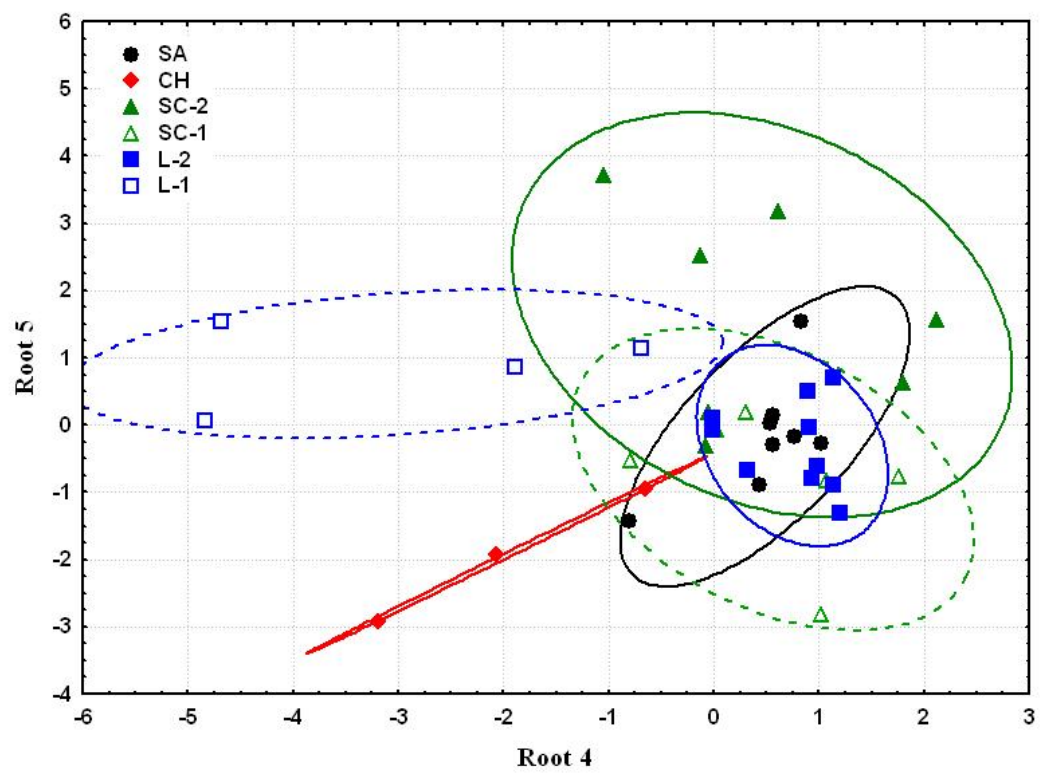
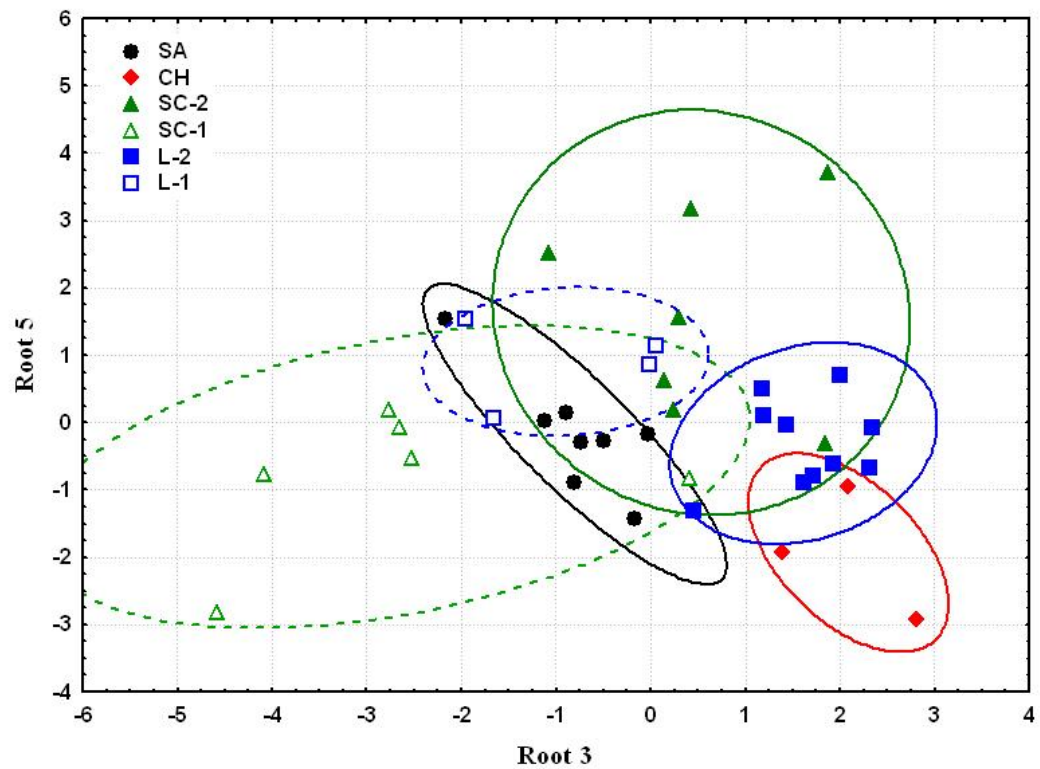














## Classification

Classification Functions; grouping: Category (fprinttraceln1)						
	<b>CH</b> p=.07895	<b>L1</b> p=.10526	<b>L2</b> p=.26316	<b>SA</b> p=.21053	<b>SC1</b> p=.15789	<b>SC2</b> p=.18421
<b>V</b>	126.4	111.5	111.0	88.5	127.9	121.0
<b>Zn</b>	-273.6	-317.1	-314.9	-284.3	-298.8	-295.4
<b>Ni</b>	197.6	189.3	197.2	179.5	193.4	199.8
<b>SiO<sub>2</sub></b>	4273.9	4307.3	4293.4	4191.3	4402.2	4381.5
<b>Ga</b>	231.0	260.5	269.6	276.4	260.3	263.9
<b>FeO</b>	1407.2	1431.2	1456.0	1364.7	1459.2	1447.7
<b>Sc</b>	260.3	264.9	270.0	248.9	276.1	269.7
<b>Ba</b>	104.6	102.4	105.1	92.7	108.2	102.6
<b>Constant</b>	-10947.2	-10891.9	-10994.1	-10241.3	-11572.5	-11426.6

Classification Matrix (fprinttraceln1) Rows: Observed classifications Columns: Predicted classifications							
	<b>Percent Correct</b>	<b>CH</b> p=.07895	<b>L1</b> p=.10526	<b>L2</b> p=.26316	<b>SA</b> p=.21053	<b>SC1</b> p=.15789	<b>SC2</b> p=.18421
<b>CH</b>	100.00	3	0	0	0	0	0
<b>L1</b>	100.00	0	4	0	0	0	0
<b>L2</b>	100.00	0	0	10	0	0	0
<b>SA</b>	100.00	0	0	0	8	0	0
<b>SC1</b>	100.00	0	0	0	0	6	0
<b>SC2</b>	100.00	0	0	0	0	0	7
<b>Total</b>	100.00	3	4	10	8	6	7

Classification of Cases (fprinttraceln1)							
Incorrect classifications are marked with *							
Samples	Observed Classif.	1 p=.07895	2 p=.10526	3 p=.26316	4 p=.21053	5 p=.15789	6 p=.18421
CH GS-7	CH	CH	SC2	L2	L1	SC1	SA
CH GS-12	CH	CH	SC2	SC1	L2	L1	SA
CH GS-16	CH	CH	SC2	L2	SC1	L1	SA
L 1A	L1	L1	L2	SC2	SC1	SA	CH
L 4A	L1	L1	L2	SC2	SA	SC1	CH
L 9A	L1	L1	L2	SC2	SC1	CH	SA
L10A	L1	L1	L2	SC2	SC1	CH	SA
L GS-2	L2	L2	L1	SC2	SC1	CH	SA
L GS-4	L2	L2	SC2	L1	SC1	CH	SA
L GS-10	L2	L2	L1	SC2	SC1	CH	SA
L A	L2	L2	L1	SC2	SC1	CH	SA
L 3A	L2	L2	L1	SC2	SC1	CH	SA
L 5A	L2	L2	L1	SC2	SC1	CH	SA
L 6A	L2	L2	L1	SC2	SC1	CH	SA
L 7A	L2	L2	L1	SC2	SC1	CH	SA
L 8A	L2	L2	L1	SC2	SC1	CH	SA
L 11A	L2	L2	SC2	L1	CH	SC1	SA
SA GS-3	SA	SA	L1	CH	L2	SC2	SC1
SA GS-13	SA	SA	L1	L2	SC2	CH	SC1
SA GS-17	SA	SA	L1	L2	SC2	CH	SC1
SA GS-18	SA	SA	L1	L2	SC2	CH	SC1
SA PVC1	SA	SA	L1	L2	CH	SC1	SC2
SA PVC2	SA	SA	L1	L2	SC2	CH	SC1
SA PVC3	SA	SA	L1	L2	SC2	CH	SC1
SA GS-1	SA	SA	L1	L2	SC2	CH	SC1
SC4	SC1	SC1	SC2	L2	L1	CH	SA
SC5	SC1	SC1	SC2	CH	L2	L1	SA
SC6	SC1	SC1	SC2	L2	L1	CH	SA
SC9	SC1	SC1	SC2	L2	CH	L1	SA
SC10	SC1	SC1	SC2	L2	L1	CH	SA
SC11	SC1	SC1	L2	SC2	L1	CH	SA
SC GS-11	SC2	SC2	CH	L2	SC1	L1	SA
SC GS-15	SC2	SC2	CH	SC1	L2	L1	SA
SC2	SC2	SC2	CH	L2	L1	SC1	SA
SC3	SC2	SC2	SC1	L2	CH	L1	SA
SC7	SC2	SC2	SC1	CH	L2	L1	SA
SC8	SC2	SC2	SC1	CH	L2	L1	SA
SC PVC5	SC2	SC2	CH	L2	SC1	L1	SA
P1	---	L2	L1	SC2	SC1	CH	SA
P2	---	SA	L1	L2	CH	SC2	SC1
P3	---	SC2	L2	CH	SC1	L1	SA
P4	---	L2	L1	SC2	SC1	CH	SA
P5	---	L2	SC2	L1	CH	SC1	SA
P6	---	L1	L2	SA	SC2	CH	SC1
Li1UI	----	L2	L1	SC2	SC1	CH	SA
Li2UI	----	L2	L1	SC2	CH	SA	SC1
at1UI	----	L2	SC2	L1	CH	SC1	SA
at2UI	----	L2	SC2	CH	L1	SC1	SA
at3UI	----	CH	SC2	L2	SC1	L1	SA
at4UI	----	L2	SC2	L1	SC1	CH	SA
ga1UI	----	SA	L1	L2	CH	SC2	SC1
ga2UI	----	SA	L1	L2	CH	SC2	SC1
ga3UI	----	SA	L1	L2	CH	SC2	SC1
ga4UI	----	SA	L1	L2	CH	SC2	SC1
ga5UI	----	SA	L1	CH	L2	SC1	SC2
ga6UI	----	SA	L1	L2	CH	SC2	SC1
ga7UI	----	SA	L2	L1	SC2	CH	SC1
SA1UI	----	SA	CH	L1	L2	SC1	SC2
SC1UI	----	SC2	L2	CH	SC1	L1	SA
SC2UI	----	SC2	CH	L2	SC1	L1	SA
SC3UI	----	SC2	CH	L2	SC1	L1	SA
SC4UI	----	SC2	L2	SC1	CH	L1	SA
SC5UI	----	SC1	SC2	L2	L1	CH	SA
SC6UI	----	SC2	SC1	CH	L2	L1	SA

Squared Mahalanobis Distances from Group Centroids (fprinttraceln1)							
Incorrect classifications are marked with *							
Samples	Observed Classif.	CH p=.07895	L1 p=.10526	L2 p=.26316	SA p=.21053	SC1 p=.15789	SC2 p=.18421
CH GS-7	CH	4.3108	64.4435	56.1811	99.6442	66.3362	28.6322
CH GS-12	CH	7.9282	104.5179	88.4730	168.3455	84.8962	49.1524
CH GS-16	CH	3.6639	76.7996	51.7219	125.4557	53.2863	14.2501
L 1A	L1	102.4727	10.7648	54.9154	101.4578	87.7577	80.9594
L 4A	L1	86.1852	9.9982	14.0804	69.6966	70.2932	53.5250
L 9A	L1	68.8692	2.9524	14.1060	72.4469	61.3445	45.4330
L10A	L1	78.1214	5.4557	43.5319	100.7511	69.2241	69.2156
L GS-2	L2	47.9616	21.0123	4.6999	91.3468	41.7619	25.6373
L GS-4	L2	47.9221	28.9525	2.3462	100.3234	35.9572	22.0171
L GS-10	L2	58.7036	24.4161	2.6634	91.7101	43.1858	27.4732
L A	L2	62.9517	27.8245	0.5999	95.5281	49.6680	39.8685
L 3A	L2	93.6771	37.5476	9.6400	125.1983	63.2981	54.1046
L 5A	L2	73.7995	26.0167	2.4563	97.9961	64.8106	52.2811
L 6A	L2	87.2762	32.3505	3.7852	115.8464	59.5495	56.9939
L 7A	L2	69.2390	22.3982	1.9360	98.5004	62.0534	45.8739
L 8A	L2	66.9733	31.8889	3.3785	98.2910	51.5571	43.1708
L 11A	L2	37.1313	35.3076	12.5529	66.5469	43.4609	24.9407
SA GS-3	SA	96.8068	84.0465	100.0922	7.9378	179.8678	114.7671
SA GS-13	SA	105.2012	72.9229	78.9111	3.0323	165.0490	101.6247
SA GS-17	SA	102.6133	63.4013	74.7227	3.4093	149.9388	93.6034
SA GS-18	SA	115.6095	68.8763	83.2895	1.3454	163.6704	107.2119
SA PVC1	SA	204.2938	130.7741	154.2110	13.2127	277.1115	207.0001
SA PVC2	SA	142.2899	82.2069	97.3455	3.8758	195.2245	136.0486
SA PVC3	SA	144.7827	94.3172	105.6964	2.4799	206.0878	139.9272
SA GS-1	SA	140.2686	78.9033	100.6008	9.7925	175.1642	116.8324
SC4	SC1	63.5385	57.8889	46.1199	181.5245	1.5590	27.9840
SC5	SC1	43.1001	88.0733	48.0533	203.0878	14.2142	21.6422
SC6	SC1	66.5669	57.8526	45.4813	178.9507	1.7561	27.4012
SC9	SC1	73.2269	80.4598	62.8303	171.0078	6.4870	35.6052
SC10	SC1	59.3892	49.8048	43.3344	176.8401	2.8319	30.6840
SC11	SC1	119.3455	103.0284	84.8625	236.7955	21.0870	85.4688
SC GS-11	SC2	31.6378	64.4369	37.0172	102.0321	37.7314	8.4348
SC GS-15	SC2	19.2423	55.2798	39.3006	107.4611	36.3867	8.6780
SC2	SC2	46.4466	67.4526	63.0311	114.2258	87.4042	23.7883
SC3	SC2	46.3718	59.6938	40.9932	143.3556	34.7082	5.0605
SC7	SC2	39.8318	92.7670	58.0984	150.6750	38.4028	9.1142
SC8	SC2	45.3474	61.5666	52.0188	157.0768	22.7906	7.8500
SC PVC5	SC2	22.6098	57.5830	27.1991	149.4586	27.4147	10.9207
P1	---	57.6269	25.1966	0.8641	81.5165	50.4043	30.9933
P2	---	154.3428	88.9051	110.8040	10.2786	210.5865	156.7825
P3	---	36.2642	84.5011	38.6351	123.6442	45.6506	13.8514
P4	---	65.9220	20.6503	1.6642	91.7395	44.7868	33.0324
P5	---	44.4313	26.3389	3.2486	72.1160	47.0517	26.3572
P6	---	83.2521	31.4512	35.8167	41.2082	125.1220	82.2648
Li1UI	----	66.6478	30.9627	5.6068	91.1122	55.1904	40.7349
Li2UI	----	56.1864	27.0291	8.1801	61.2284	64.3636	40.1395
at1UI	----	47.3089	33.0367	6.7772	72.7127	49.3964	29.9458
at2UI	----	38.2004	43.1280	15.6713	75.5275	52.9097	32.9044
at3UI	----	28.4250	64.8759	36.1879	82.3411	51.4432	31.2283
at4UI	----	78.4316	39.7917	8.3526	81.3898	58.1341	35.7464
ga1UI	----	130.8547	95.1354	101.9992	8.0629	192.7156	135.3408
ga2UI	----	124.8178	92.6786	94.5820	5.6905	183.8445	127.7496
ga3UI	----	108.4265	83.6164	92.0265	8.3328	180.3730	125.8937
ga4UI	----	144.8526	113.7104	120.6834	15.9108	223.3870	161.1847
ga5UI	----	270.9758	266.5149	299.6459	124.9970	429.1799	361.7213
ga6UI	----	103.3171	76.3481	82.7462	8.6177	164.9134	114.9477
ga7UI	----	152.6537	105.4279	106.3987	7.1395	209.9126	147.7067
SA1UI	----	585.7692	453.9646	468.6883	187.5434	679.1771	561.6802
SC1UI	----	40.7916	69.9297	35.0106	96.0512	49.7843	20.4334
SC2UI	----	35.5487	72.0206	38.5256	92.9995	49.5120	17.8393
SC3UI	----	19.0754	48.2985	30.3600	112.5763	38.4665	4.4048
SC4UI	----	33.6483	56.3608	31.4648	133.0495	33.6582	2.6192
SC5UI	----	147.9581	138.4651	94.4410	224.4548	56.1016	59.8762
SC6UI	----	53.9656	100.4686	57.1921	167.5251	49.4372	20.0650



Posterior Probabilities (fprintraceIn1) Incorrect classifications are marked with *							
Samples	Observed Classif.	CH p=.07895	L1 p=.10526	L2 p=.26316	SA p=.21053	SC1 p=.15789	SC2 p=.18421
CH GS-7	CH	0.99999	0.00000	0.00000	0.00000	0.00000	0.00001
CH GS-12	CH	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
CH GS-16	CH	0.98841	0.00000	0.00000	0.00000	0.00000	0.01159
L 1A	L1	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000
L 4A	L1	0.00000	0.75488	0.24512	0.00000	0.00000	0.00000
L 9A	L1	0.00000	0.99063	0.00937	0.00000	0.00000	0.00000
L10A	L1	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000
L GS-2	L2	0.00000	0.00011	0.99987	0.00000	0.00000	0.00002
L GS-4	L2	0.00000	0.00000	0.99996	0.00000	0.00000	0.00004
L GS-10	L2	0.00000	0.00001	0.99999	0.00000	0.00000	0.00000
L A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 3A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 5A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 6A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 7A	L2	0.00000	0.00001	0.99999	0.00000	0.00000	0.00000
L 8A	L2	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
L 11A	L2	0.00000	0.00000	0.99857	0.00000	0.00000	0.00143
SA GS-3	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-13	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-17	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-18	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC1	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC2	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA PVC3	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA GS-1	SA	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SC4	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC5	SC1	0.00000	0.00000	0.00000	0.00000	0.97234	0.02766
SC6	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC9	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC10	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC11	SC1	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000
SC GS-11	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	0.99999
SC GS-15	SC2	0.00217	0.00000	0.00000	0.00000	0.00000	0.99783
SC2	SC2	0.00001	0.00000	0.00000	0.00000	0.00000	0.99999
SC3	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC7	SC2	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC8	SC2	0.00000	0.00000	0.00000	0.00000	0.00049	0.99951
SC PVC5	SC2	0.00124	0.00000	0.00042	0.00000	0.00022	0.99812
P1	---	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
P2	---	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
P3	---	0.00001	0.00000	0.00001	0.00000	0.00000	0.99999
P4	---	0.00000	0.00003	0.99997	0.00000	0.00000	0.00000
P5	---	0.00000	0.00000	0.99999	0.00000	0.00000	0.00001
P6	---	0.00000	0.77098	0.21728	0.01173	0.00000	0.00000
Li1UI	----	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
Li2UI	----	0.00000	0.00003	0.99997	0.00000	0.00000	0.00000
at1UI	----	0.00000	0.00000	0.99999	0.00000	0.00000	0.00001
at2UI	----	0.00000	0.00000	0.99987	0.00000	0.00000	0.00013
at3UI	----	0.60857	0.00000	0.04183	0.00000	0.00001	0.34959
at4UI	----	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
ga1UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga2UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga3UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga4UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga5UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga6UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
ga7UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SA1UI	----	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
SC1UI	----	0.00002	0.00000	0.00098	0.00000	0.00000	0.99901
SC2UI	----	0.00006	0.00000	0.00005	0.00000	0.00000	0.99989
SC3UI	----	0.00028	0.00000	0.00000	0.00000	0.00000	0.99972
SC4UI	----	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000
SC5UI	----	0.00000	0.00000	0.00000	0.00000	0.84981	0.15019
SC6UI	----	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

