INL/CON-11-21104
PREPRINT

Contingency Analysis of Cascading Line Outage Events

Power Systems Conference 2011

Thomas L. Baldwin
Magdy S. Tawfik
Miles McQueen

March 2011

The INL is a
U.S. Department of Energy
National Laboratory
operated by
Battelle Energy Alliance

Contingency Analysis of Cascading Line Outage Events

Thomas L. Baldwin, Magdy S. Tawfik, and Miles McQueen

***Abstract*—As the US power systems continue to increase in size and complexity, including the growth of smart grids, larger blackouts due to cascading outages become more likely. Grid congestion is often associated with a cascading collapse leading to a major blackout. Such a collapse is characterized by a self-sustaining sequence of line outages followed by a topology breakup of the network. This paper addresses the implementation and testing of a process for *N-k* contingency analysis and sequential cascading outage simulation in order to identify potential cascading modes. A modeling approach described in this paper offers a unique capability to identify initiating events that may lead to cascading outages. It predicts the development of cascading events by identifying and visualizing potential cascading tiers. The proposed approach was implemented using a 328-bus simplified SERC power system network. The results of the study indicate that initiating events and possible cascading chains may be identified, ranked and visualized. This approach may be used to improve the reliability of a transmission grid and reduce its vulnerability to cascading outages.**

Index Terms— **Transmission reliability, cascading outages, initiating events**.

I. Introduction

A highly interconnected power grid provides economic benefits and a greater level of service reliability. Unfortunately, interconnectivity gives rise to increased probability of cascading failures. The first major cascading failure of the North American power grid occurred in 1965. This failure prompted a number of changes to grid operations and network protection to prevent reoccurrence. Nevertheless, a steady increase in the number of large blackouts has been observed over the past 40 years. The number of blackouts that result in a loss of over 1000 MW of demand doubles every 10 years [1]. Recent events include the August 14, 2003 blackout when upwards of 30 million people in the U.S. Northeast and Southeastern Canada lost power. Major blackouts occur rarely, but when they occur, they can have serious economic and social impact.

Major blackouts are usually the outcome of cascading outages. Power grids suffer sporadic contingencies, some of which are sufficiently large as to impact a considerable fraction of the grid infrastructure. The catastrophic disruption of a power grid is often triggered due to a set of cascading failures while the grid is generally operating near critical loading. During a system wide disturbance, line outages cause topology changes and the redistribution of power flows. In the extreme, cascading line and generator outages can result in wide-area grid failure.

The NERC defines a blackout "as the uncontrolled loss of any system facilities or load, whether because of thermal overload, voltage collapse, or loss of synchronism, except those occurring as a result of fault isolation" [2]. Cascading outages can have a wide-spread effect and take extensive time and effort to recover. Under its transmission planning standards [3], NERC requires analysis of the following categories of contingencies:

- A loss of a single element (Category B)
- A loss of two or more elements (Category C)
- Extreme events resulting in two or more elements removed or cascading out of service (Category D)

Category D is the highest level of severity. The NERC notes in [3] that such contingencies may involve substantial loss of loading demand and generation in a wide area. Portions or all of the interconnected systems may or may not achieve a new and stable operating point, meaning that the system may collapse in a major blackout.

With the push to develop the next generation network and smart grid, the power industry sees the need for a digital upgrade to enable optimization of current operations and to open up new markets for alternative energy resources. The smart grid is likely to implement enhanced two-way communications, advanced sensors, and distributed controls and computing technologies that aim to improve the efficiency and reliability of power delivery [4]. The U.S. Department of Energy states that the smart grid ought to possess seven characteristics; these include [5]:

- Ability to self-heal
- Motivate and include the energy consumer
- Resist attacks
- Provide power quality for the needs of the 21st century
- Accommodate all generation and storage options
- Enable energy markets
- Optimize assets and operate efficiently

Consequently, the power system continues to increase in importance and complexity. One aim of a smarter grid is the ability to recognize vulnerable operating states and potential contingencies that may lead to cascading outages.

This work was carried out at Idaho National Laboratory for the U.S. Department of Energy under Contract DE-AC07-05ID14517.

T. L. Baldwin (e-mail: tom.baldwin@ieee.org), M. S. Tawfik (e-mail: magdy.tawfik@inl.gov), and M. McQueen (e-mail: miles.mcqueen@inl.gov) are with the Idaho National Laboratory, Idaho Falls, ID 83415 USA.

A. Network Theory Applied to the Study of Major Blackouts

The resilience of real-world networks subjected to random events has been an intensively studied topic in network theory. The electric power grid is robust to most random events. Empirical evidence has indicated that even though random failures emerge very locally, the entire network can be largely affected, even resulting in wide area collapse [6].

The system wide failure can be attributed to the limited capacity of each transmission line to support additional power loading. On the other hand, for sufficiently large spare capacity values, no cascading failure occurs and the system maintains a normal function. Thus with the increase of spare capacity, there should be some crossover behavior of the system from wide area collapse to only local loading impacts. This crossover behavior can be used to quantify the robustness of the grid.

In an effort to understand the global dynamics of wide area blackouts a wide area analysis approach is required to capture the overall response of the system. The approach must describe the sequence of rare events that propagate into large-scale blackouts. An approach to the analysis is to utilize simple, but representative, models for the key components of the power system

A variety of research efforts have examined the dynamics of wide area disturbances and blackouts. Much of this work focuses on the underlying weaknesses of the interconnected grid and protection system. Both DC and AC power flow methods have been proposed to analyze the dynamic topologies of cascading wide area disturbances. Probabilistic power flow methods have explored other aspects, such as hidden failures caused by erroneous protective relay settings.

In this approach, a contingency analysis approach and sequential cascading analysis are coupled to identify critical wide area events. The objective of the approach is to identify contingency events that induce cascading line outages leading to major blackouts. Contingency cases are studied with an iterative power flow analysis in which subsequent overloaded lines are taken out of service. Due to the large number of power flow calculations that are required, quick and efficient computational methods such a line shift factors, distribution factors, DC power flows, and P1-Q1 power flows must be considered [7]. To reduce the effort, calculating the fast dynamics of the power system is intentionally neglected.

Fault events, generation failures, and other events which may initiate a wide area disturbance are modeled as contingency analysis events. As recent blackout events over the last decade have indicated, utility control centers may not be aware of the occurrence of these events and make appropriate corrective actions. The developed approach considers $N-3$ and $N-k$ contingency events for triggering an extreme event. A set of indices are utilized in the contingency analysis to measure a progressive weakening system that is likely to result in a cascading failure. A priority list of cases to be studied is derived from these indices [8].

B. Overview of the Paper

An approach for modeling cascading outages is developed in this paper and initial simulation results are provided. To achieve reasonable solution times, the model is currently based on the DC power flow equations. As such it is related to other power grid cascading collapse models in the literature. In [9] a DC power flow based collapse model is used to provide an explanation for the power law distribution in the historical record of power grid blackout sizes. A similar DC power flow based collapse model is used in [10] to better understand the impacts of hidden failures in protection system equipment on cascading power grid failure. The contribution of this work relative to this other work is its integration of the various techniques and models and applying them to a more realistic representation of a power system.

In this work, the power system modeling includes load shedding, generation redispatching, and generation tripping as power imbalances and voltage collapse occur in various areas. Base load nuclear power plants are modeled to respond to various grid events, such as loss of external power. Outages of transmission lines and other grid assets beyond basic fault isolation can be forced by either thermal overload or voltage collapse. A loss-of-synchronism can be partially modeled by a static test for excessive phase angles between key points in the power network. The cascading outage contingency analysis method is tested on the on notional models of the Carolinas sub-region of the SERC Reliability Corporation (SERC).

II. Cascading Grid Modeling

It is desirable to capture many of the general features of a cascading failure in a model simple enough to allow analysis. A key feature in the modeling is the capability of transferring some of the loading to the other grid components when an element overloads and trips offline. An initial overloading condition leads to additional loading of other components, some of which may also overload. This property is necessary for propagating a cascading failure.

Due to their exposure, transmission lines are the components most vulnerable to outage [11]. Most cascading blackouts are triggered with one or more line outages on a heavily loaded grid. Generally, the event goes through the following steps: (i) a cascading sequence of line outages; (ii) the grid breaking into a number of disconnected islands; and (iii) insufficient generation-load balance in the islands causes the loss of some or all of their loads. The August 14, 2003 wide-area blackout event in the Northeastern U.S. and Southeastern Canada experienced these phases [12].

This section develops a simplified model for the first phase of a blackout event, the cascading sequence of line outages. The next sections describe the component parts of the model.

A. Estimating the Line Power Flows

The assumption is made that the transmission system operates in quasi-steady-state conditions. This assumption holds even during the evolution of major disturbances in the system.

The power flow equations model the power injections and provide the solutions to the power flows on the lines. The AC power flow equations are non-linear equations, modeling the flows of both the active and reactive powers. The DC power flow equations are a linear approximation modeling active power flows only [7]. The DC power flow equations have the advantage of faster computation times compared with the AC power flow equations, and are therefore used in this study.

The basic DC power flow equations are:

$$\mathbf{P}_{inj} = -\mathbf{B}\,\boldsymbol{\delta} \tag{1}$$

where $\mathbf{P}_{inj}$ is the vector of active power injections at each bus; $\mathbf{B}$ is the susceptance matrix of the power system, and $\boldsymbol{\delta}$ is the vector of bus angles. The flows in each line can be found by solving for the bus angles using (1) and computing the angular difference across each line weighted by the line susceptance. The angular difference can be computed in matrix form by using the line-bus incidence matrix for the network. This results into the following:

$$\mathbf{P}_{line} = \mathbf{b}\ \mathbf{A}\ \mathbf{B}^{-1}\ \mathbf{P}_{inj} \tag{2}$$

where $\mathbf{P}_{line}$ is the vector of active power flows on each line; $\mathbf{b}$ is a diagonal matrix of the line susceptances, diag($1/x_i$); and $\mathbf{A}$ is the line-bus incidence matrix. The incidence matrix is oriented such that each row represents a line and has two non-zero entries ($a_{ij} = +1$ and a_{ik} -1 for the line i leaving bus j and arriving to bus k.)

A number of assumptions are made by the use of the DC power flow equations. Because the DC power flow equations represent a power flow solution for a system in equilibrium, the possibility of component outages due to line switching transients are ignored. Because the DC power flow equations assume small voltage angle deviations (generally less than 30 degrees between buses) and constant voltage magnitudes, it is assumed that the system is able to maintain voltage levels. Hence, the possibility of a voltage collapse is ignored.

B. Power Injection Modeling and Generator Loading Levels

At the beginning of the cascading outage analysis, the initial generation allocation may be determined by an economic dispatch or a system security solution, which are generally available from an optimal power flow analysis. During the cascading analysis period, generation outages, islanding, and load rejection will dynamically impact the power balance in the grid. The slack bus, a generator bus used computationally as a floating power injection, can be used to rebalance the load and generator power injections, but this approach is often not realistic with utility operating practices.

Another modeling approach is to provide generation allocation based on the load-frequency control scheme, which aims to maintain frequency and interchange flows at desired levels. The allocation of individual generator output is accomplished using base points and participation factors. For example, the results of an economic dispatch can be used as the base-point loading for each generator. As the total system loading changes, the participation factors provide the rate of change of the output for each generating unit [8]. The base point and participation factors are used in the following fashion:

$$P_{i-gen}(t) = P_{i-base} + pf_i\ \Delta P_{total}(t) \tag{3}$$

where

$$\Delta P_{total}(t) = \sum_{j \in loads} P_j(t) - \sum_{i \in gen} P_{i-base} \tag{4}$$

and

$$\sum_{i \in gen} pf_i = 1 \tag{5}$$

After generator outputs have been adjusted to balance the power flows, load shedding and generation tripping conditions are examined. In a quasi-steady state analysis, the power frequency is not available, and the use of a DC power flow model does not provide voltage sag data. Modeling the behavior of load shedding relays must implement an indirect approach. Due to the mechanical limits of the prime movers, generators operating in excess of 110% will have a power mismatch, causing the rotors to decelerate and the power frequency to droop. Other generators in the control area can increase output levels according to updated participation factors to shift generation away from the overloaded unit. If such action causes all generation in the area to exceed their limits, then it is assumed that the power frequency will drop sufficiently to initiate load shedding. Loads in the control area are reduced until generation is operating at or below 100%.

Control areas that are generation rich may reduce unit output levels below the minimum generation levels, causing flameouts. In these cases, the generation is tripped out of service.

$$P_{i-gen} < P_{i-\min} \tag{6}$$

An optimal distribution of generation and load is then calculated. Linear programming is used to minimize the amount of load shed subject to the constraints of the generation and load lying within their upper and lower limits, the line flows not exceeding the maximum flow, and overall generation matching the overall load.

The injected power vector is the combination of the adjusted generation unit outputs and the load demand.

$$\mathbf{P}_{inj}(t) = \mathbf{P}_{gen}(t) - \mathbf{P}_{load}(t) \tag{7}$$

C. Line Capacities and the Overloaded Failure Model

The amount of power that can be sent over a transmission line is limited. The origins of the limits vary, generally depending on the length of the line and the line design. For short lines, the heating of conductors due to line losses establishes a thermal limit. For intermediate-length lines on the order of 100 km (62 mi), the limit is determined by the voltage drop in the line. For longer lines, system (angular) stability sets the limit to the power that can be transferred.

Transmission line congestion, leading to cascading line outages, may be determined by the line flows relative to the capacity of the lines. When actual capacity limits are unknown, loading limits of a line can be estimated from line parameter data [13]. Of the three limits, the voltage drop threshold is strongly influenced by the flows of reactive power and requires a full AC power flow analysis. The thermal limit and the bus angle range for steady-state stability can be approximated using the DC power flow analysis.

An overload violation occurs in a transmission line when the total power flowing exceeds the rated power limit;

$$\left|S_l\right| > S_{l-\max} \tag{8}$$

for full AC modeling, or for DC modeling

$$\left|P_l\right| > P_{l-\max} \tag{9}$$

D. Modeling the Line Protection

Together, circuit breakers, protective relaying, and SCADA provide some measure of overload safe-guarding for transmission lines. Distance relaying, one of the most widely used forms of line protection, can also contribute to outage events as increased loading encroaches into the longest reaching trip region. Zone 3 is most likely to be impacted by load encroachment. Additionally, network protection exhibits stochastic behaviors above the expected deterministic characteristics. This behavior can be modeled using "hidden failure" techniques.

For basic overload conditions, the minimum of the three maximum power ratings (thermal, voltage drop, and angular stability) is used to trip a line when the loading level exceeds the rating by 50%. In particular, the line is taken out of service when

$$\left|P_l\right| > 1.50\, P_{l-\max} \tag{10}$$

Zone 3 distance protection tends to be prone to load encroachment condition and causing possible undesired tripping conditions under heavy loading conditions. Due to uncertainty in the actual fault impedance and the conditions at the remote end of a transmission line, several step-level protections zones are implemented. Each step zone has a different reach impedance value and a corresponding coordinating delay time to trip the breaker.

It is typical in the U.S. to find three zones looking in the forward direction with progressively further reach distances. The first zone protects roughly 85 to 90% of the line from close-in faults with an instantaneous tripping. Zone 2 addresses the problem of fault location uncertainty that occurs beyond 95% of the line. The second zone generally has a minimum 120% reach of the line impedance to ensure coverage to the distant bus. It is a common practice to set the reach to be equal to the protected line impedance plus 50% of the shortest adjacent line at the remote bus. Zone 2 tripping is time-delayed by 20 to 30 cycles to provide coordination with the primary relaying of the adjacent circuits that fall within the reach of Zone 2.

The Zone 3 element typically provides back-up protection for phase faults associated with breaker or relay equipment failure at remote buses. Zone 3 backup protection is time delayed by 0.5 to 30 seconds to coordinate with the Zone 2 protection and circuit breaker tripping timing of the adjacent lines. Generally, the Zone 3 reach is set to 200% of the line impedance. Zone 3 settings are the most challenging to establish and verify that the element will not trip under extreme loading conditions [14].

For zone 3 distance protection conditions, an equivalent line power rating is developed to trip a line when the loading level exceeds this rating. The line is taken out of service when

$$P_l > P_{l-zone3} = \frac{V_i^2}{2x_l} = \frac{1}{2x_l} \tag{11}$$

When a line trips, there is a small but significant probability that the lines connected to either end of the tripped circuit may incorrectly trip due to improper relay operation. These additional line tripping events are called hidden failures because they do not become apparent until the first line tripping "exposes" the adjacent lines to the possibility of an improper relay operation. A hidden failure is associated with a relay, which fails to trip when it should have tripped, or when a relay trips when it shouldn't have tripped. Research has shown that hidden failures in protection systems played a significant role in cascading disturbances [14].

The probability of a line tripping incorrectly can be modeled as an increasing function of the line flow seen by the protective relay. The cumulative probability distribution function is zero below 50% of the line's rated limit, and increases quadraticly to one at ten times the line rating. During a cascading outage simulation, a line may become overloaded and trip, then the adjacent lines connected at either ends are tested for possible overloading. If none of the adjacent lines have reached their respective rated limits, a line protection hidden failure mechanism is applied to each of these lines.

A simulation run can examine the combined deterministic and stochastic line protection behaviors or just the deterministic behavior. A probability threshold, such as 10% can be selected to mark those lines that might be impacted by a hidden fault mechanism, without removing the line from service.

E. Modeling Line Outages

Contingency analysis procedures provide various models for single and multiple failure events such as line and generator outages. These procedures must evaluate a significant number of outage configurations to the point that comprehensive coverage becomes a difficult problem. One way to gain solution speed is to use an approximate model of the power system. From the simplified models, linear sensitivity factors may be derived. These factors calculate an approximate change in line flows for changes in generation and line operation [8].

Line outage distribution factors and generator shift factors are sensitivity factors based on a specific grid topology and operating condition prior to an outage. The factors are well understood, but are limited to the outage of a few lines. In contingency analysis studies, generally one to three outages occur followed by the application of the network sensitivity factors. However, in a cascading outage analysis, four or more outage events generally take place. As a cascading sequence unfolds, the sensitivity factors must be recomputed as the network topology evolves.

An alternative approach to recalculating the sensitivity factors is the use of the DC power flow to approximate the changes in power flows. After each line trip, the power flows in the lines are recalculated and checked for violations of line and generation limits and load shedding operations. The process is repeated until the cascading event stops.

When using the DC power flow, there are several ways to include the effects of line outages. An effective approach is to modify the diagonal line susceptance matrix, **b**, by setting the diagonal entry of the impacted line to zero. In (2) this diagonal matrix appears twice, the second occurrence is within the bus susceptance matrix, **B**.

$$\mathbf{B} = \mathbf{A}^{\mathrm{T}}\, \mathbf{b}\, \mathbf{A} \tag{12}$$

Setting the diagonal entry for the l line forms the updated diagonal matrix $\mathbf{b}'$. Assuming the inverse exists, the power flow equation is solved as.

$$\mathbf{P}'_{line} = \mathbf{b}'\, \mathbf{A} \left(\mathbf{A}^{\mathrm{T}}\, \mathbf{b}'\, \mathbf{A}\right)^{-1} \mathbf{P}_{inj} \tag{13}$$

This method leaves the matrix structure for line l in place, but forces its flow to zero by setting its impedance to infinity. The method is particularly simple to implement in a software tool like MATLAB since all vectors and matrices will have the same size before and after the line outage. The method is equivalent to the procedure of subtracting $1/x_l$ from the **B** matrix entries b_{ij}, b_{ji}, b_{ii}, and b_{jj}.

III. Analysis Methodology

A. Cascading Contingency Analysis Approaches

Two simple approaches have been defined for assessing the robustness of the power system against cascading failures.

The first approach is to examine the set of lines and generators with the highest loading and largest shift factors. This approach is similar to the selection process used in many classical contingency analysis methods. A short list of elements is arranged with descending order of load shifting impact. Elements from this list are removed one by one starting from the element with the highest impact level. The elements are removed until a cascading sequence is initiated. The underlying assumption is that the removal of elements with higher impact is more likely to trigger cascading failures in general.

The second approach is to start with the elements with the smallest loading levels and minimum shift factors. This approach, contrary to the previous strategy, selects the elements in the ascending order of impact to the grid. Again, elements are removed one by one until a cascading sequence is initiated.

B. Grid Breakup

Once a cascading sequence of line outages initiates, the outcome is either a sustained progression to the point where the network separates into a set of disconnected islands or a self-limiting sequence, ending prior to any major formation of islands. The line-to-bus incidence matrix **A** completely defines the network topology and thus contains all the information needed to check for any islanding. Graphical search algorithms, such as the MATLAB tree function, can examine the connectivity of a network topology [16]. These algorithms build graph trees of interconnected bus nodes. Each tree is a collection of buses that form an island.

Knowing which island each node belongs to allows us to add up the nodal power injections into each island, and to thereby determine the power imbalance in each island. This imbalance can be used as a simple predictor of whether or not the island will be able to avoid blackout. For example, one could define a threshold such that, islands with injection imbalance less than the threshold stay lit, while islands with injection imbalance greater than the threshold blackout. However, because we are only interested in what makes a grid robust to cascading line outages, in this work we stop our simulator when the grid breaks apart.

C. Simulator Steps

A simulation for modeling the cascading line outages in power grids was developed. The model is summarized below.

1. Conduct a contingency analysis for a system operating at a summer or winter peak conditions. Identify contingencies that lead to either a number of overloaded circuit conditions or the overloading of a major/critical circuit.
2. Update the $\mathbf{b}'$ diagonal matrix with current list of circuits that are out of service.
3. Check the bus susceptance matrix **B** for full rank and report the number of islands.
4. Solve the DC power flow equations for the power flows on the lines and to obtain the state of the power system. While solving the power flow, adjust generation levels as needed according to their participation factors.
5. Identify generation assets that are operating out of limits and mark those generation assets operating below their minimum power limits as out-of-service.
6. Check for the power balance in each area or island and conduct a load shedding operation as needed.
7. Compare the transmission line and transformer flows to their protection trip points for overloading, zone 3 distance relaying, or relay hidden failures. Mark those circuits that exceed a trip point as out-of-service and remove them from the $\mathbf{b}'$ matrix.
8. Stop if all circuits are below their protection trip points and report that the collapse was self-limited.
9. Repeat from Step 3.

IV. Computational Experiments

Using the simulator developed in the previous section this section makes some preliminary observations about cascading line outages in power grids.

The INL has developed several notional test cases for use in power system studies. The simplified SERC Carolinas test case is representative of actual grid topologies that are found in practice at the 230 kV and 500 kV levels. Table 1 gives the statistics for this case and Figure 1 shows a map of the network.

A. *Cascading Outage Simulation*

An *N*−3 contingency analysis was performed with the simplified SERC-Carolinas network model. Contingency results which caused overloading conditions on four or more other transmission lines were passed onto the cascade analysis. One of the several cascading events which resulted in the breakup of the power grid is presented in Table 2.

The initial contingency consisted of the loss of a major 500 kV/230 kV transformer, one of the 500 kV lines serving the same substation, and another line located 70 miles to the east. The loss of the transformer effectively removed access to a 500 kV tie-line from the north. The outage of these three circuits caused four 230 kV lines that run approximately parallel to the 500 kV line to become overloaded. The sequential tripping of the four overloaded lines forced 14 additional circuits in the area to become overloaded. The loss of the 18 overloaded lines and the three contingencies created three initial islands.

The first and largest area is the local grid connected to the Eastern Interconnect. The generation level was within normal operating ranges. The generation level within the first island that separated from the grid was very close to the total of the maximum power ratings. From the power balance perspective, this island appears to be stable. The generation in the next island had overloaded, and load shedding had begun with the loss of 2027 MW of loading. The line outages had also isolated a single load bus with the loss of 170 MW of loading.

TABLE I
NETWORK STATISTICS FOR THE SIMPLIFIED SERC-CAROLINAS REGION

Buses	328
Lines	520
Generators	91
Load Centers	124

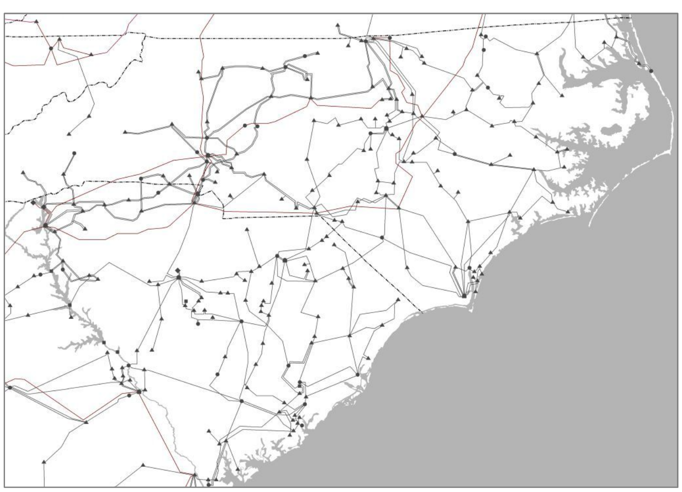


Fig. 1. Simplified model of the EHV network of the SERC-Carolina region used for the cascaded outage study.

TABLE II
CASCADING ANALYSIS RESULTS OF A N-3 CONTINGENCY

Event Stage	Outages	Areas/Islands	Total Unserved Load
Contingency	1, 4, 26	1	0
Circuit Overloads	159, 160, 171 172	1	0
Circuit Overloads	34, 35, 52, 53, 54, 55, 56, 57, 77, 78, 147, 148, 167, 168	3	2197 MW
Circuit Overloads	58, 59, 60, 61	3	4507 MW

A third round of transmission line overloads caused an additional four circuits in the last island to trip offline isolating the only generation station. This resulted in the loss of 2250 MW of generation and forcing a blackout of this island.

In total, 25 circuits were removed from service and approximately 4500 MW of load was dropped from this *N*□3 contingency.

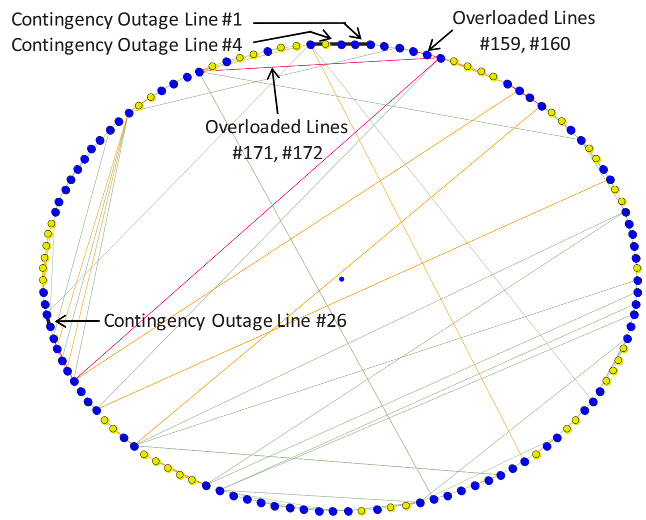
B. *Graphical Representation of a Simulation*

In order to make quick visual comparisons of topologies and the impact on cascading events, bus-line topology maps are drawn as circle graphs. The circular graphs have been optimized to minimize the number of crossover lines [17]. Figure 2 shows the sequence of the cascading line outage scenario for the previous test case as a set of circle plots. To improve clarity, a 226-bus subset of the simplified SERC system is plotted that focuses on the network section breaking apart.

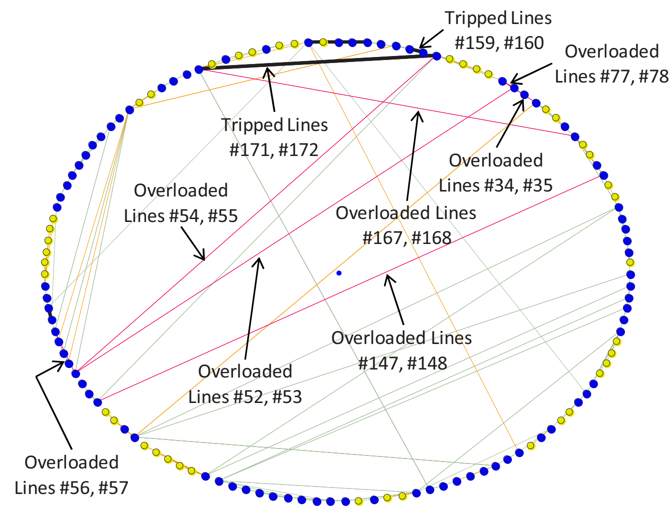
The circle plots are depicted using the following rules. Each bus is represented by a vertex placed on the circumference of the circle. Each circuit or line is represented by an edge. Generation buses are colored yellow and the load buses are colored blue. Transmission lines are colored gray when carrying a flow below 80% of the rated power, orange when operating between 80% and 120% of rated power, and red when operating above 120%. An out-of-service transmission line or transformer is colored in a bold black line.

The set of plots show how outages propagate from line to line. Lines that are colored red and reach a tripping threshold at step *k* are tripped out of service at step *k*+1. The plots show how congestion moves through the grid as line outages occur. A congested line transitions from orange to red over several steps as it is steadily moving towards its overload trip point.

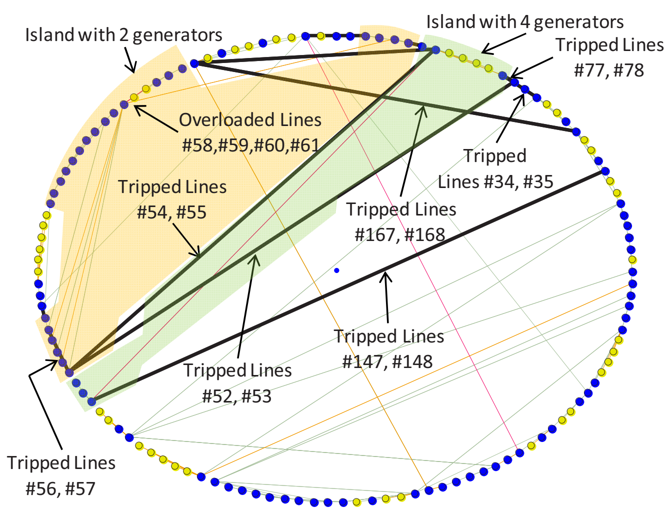
The figures illustrate some important characteristics of cascading line outages in power grids. As power grids are very sparsely connected, the ratio of the number of lines relative to the number of buses generally varies around 2 to 3, which is small from a network theory perspective. Being sparse, the power grids tend to break apart after only a few line outages. An outage of a line causes the magnitude of line flow to increase on at least one of the lines still in service. Recalling that it is the magnitude flow that can trigger the network protection, this implies that line outages may become self-sustaining. Load rejection then becomes an important tool for arresting a cascading outage sequence.



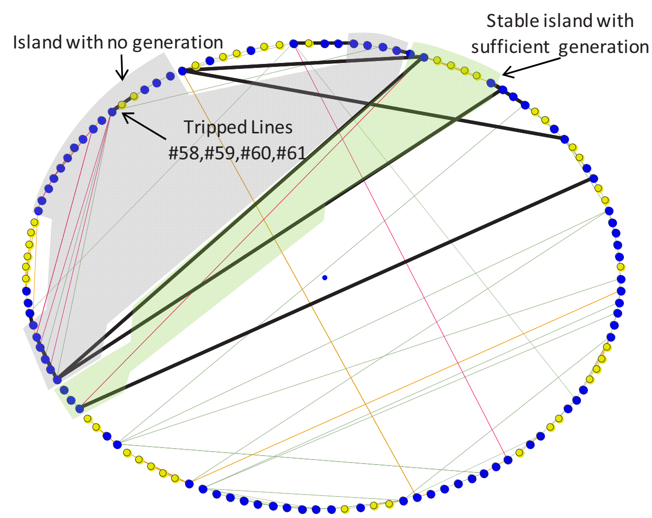
(a) Initial contingency resulting in four heavily overloaded lines.



(b) First cascading step with the four lines tripping, causing the load to transfer to other lines and resulting in 14 additional heavily overloaded lines.



(c) Second step with 14 lines (7 double circuits) tripping, forming two islands and resulting in two heavily overloaded lines in the first island.



(d) Final step with the tripping of the two overloaded lines, separating the generation from the load resulting in a complete blackout for the first island.

Fig. 2. A sequence of circle plots illustrating the cascading outage events starting from a three circuit contingency and ending with the blackout of 27 load buses.

V. Summary and Conclusions

This paper describes a methodology for simulating and analyzing cascading outages in a power grid. The methodology incorporates an *N*□*k* contingency analysis, where *k* is selectable (usually 3 or 4), and provides input to a cascading analysis model. The cascading model checks several limits associated with generator and line operation, including generation minimum active power and line loading limits. Power flow analysis is conducted in each step of the cascade analysis to compute line flows, rebalance generation using participation factors, and solve for any load shedding.

Although a DC power flow analysis is utilized in this implementation of the contingency and cascading outage analysis, more AC power flow algorithms such as P1Q1 and fast decoupled may be used. The P1Q1 provides voltage and reactive power estimates at a relative low computational cost relative to other AC algorithms.

Similar studies need to be conducted to understand relationships between topology and voltage collapse. This will require the use of AC power flow analysis and more complex power grid simulation models.

The study of cascading outages reveals the relationships between cascading line outages and the grid topology. Studies of utility systems may indicate any underlying weaknesses, which can be addressed with improved operating practices or investment of new assets. Electricity demand continues to increase while at the same time it is becoming increasingly more difficult to site new transmission lines. This stressing of the grid drives the need to identify the most important components of a network and then focus preferentially on those components which are most critical.

VI. REFERENCES

[1] P. Pourbeik, P. S. Kundur, and C. W. Taylor, “The anatomy of a power grid blackout-Root causes and dynamics of recent major blackouts”, *IEEE Power and Energy Magazine*, vol. 4, pp. 22-29, Sep.-Oct. 2006.

[2] NERC, “Appendix C: Evaluation of Criteria, Methods, and Practices Used for System Design, Planning, and Analysis Response to NERC Blackout Recommendation 13c,” Online Report, April 2005, http://www.nerc.com/docs/pc/tis/AppC_Regional_Summaries_Recom_13c.pdf

[3] *Transmission Planning (TPL) Standards*: *System Performance Following Loss of a Single Bulk Electric System Element (Category B)*, NERC Standard TPL-002-0, April 2005 [Online: http://www.nerc.com/files/TPL-002-0.pdf].

[4] *Transmission Planning (TPL) Standards: System Performance Following Loss of Two or More Bulk Electric System Elements (Category C)*, NERC Standard TPL-003-0, April 2005 [Online: http://www.nerc.com/files/ TPL-003-0.pdf].

[5] *Transmission Planning (TPL) Standards: System Performance Following Extreme Events Resulting in the Loss of Two or More Bulk Electric System Elements (Category D)*, NERC Standard TPL-002-0, April 2005 [Online: http://www.nerc.com/files/TPL-004-0.pdf].

[6] M. Olken, “Beyond the gridlock: visions, milestones, and trends,” *IEEE Power and Energy Magazine*, vol. 7, pp. 4-6, March/April 2009.

[7] National Energy Technology Laboratory, *The Transmission Smart Grid Imperative, The Modern Grid Strategy*, U.S. Department of Energy, Sept. 2009 [Online: http://www.netl.doe.gov/smartgrid/referenceshelf/whitepapers/The Transmission Smart Grid Imperative_2009_09 _29 .pdf].

[8] Task Force on Understanding, Prediction, Mitigation, and Restoration of Cascading Failures, “Vulnerability assessment for cascading failures in electric power systems,” 2009 IEEE PES Power Systems Conference and Exposition, Seattle WA, March 2009.

[9] M. Li, Q. Zhao, and P. B. Luh, “Decoupled load flow and its feasibility in systems with dynamic topology,” 2009 IEEE PES General Meeting, Calgary, AB, July 2009.

[10] A. J. Wood and B. F. Wollenberg, *Power Generation Operation and Control*, New York: Wiley-Interscience, 1996.

[11] B. A. Carreras, V. E. Lynch, I. Dobson, and D. E. Newman, “Dynamical and probabilistic approaches to the study of blackout vulnerability of power transmission grid,” Proceedings of the 37th Hawaii International Conference on System Sciences, 2004.

[12] J. Chen, J. S. Thorp, and I. Dobson, “Cascading dynamics and mitigation assessment in power system disturbances via a hidden failure model,” *International Journal of Electrical Power and Energy Systems*, vol. 27, pp.: 318-326, 2003.

[13] P. M. Anderson, *Power System Protection*, New York: IEEE Press, McGraw-Hill, 1999.

[14] U.S.-Canada Power System Outage Task Force, *Interim Report: Causes of the August 14th Blackout in the United States and Canada*, November, 2003

[15] Westinghouse, *Electric Transmission and Distribution Reference Book*, Westinghouse, 1964.

[16] M. Lucia, R. Cezari, D. Erwin, J. C. Theron, and M. Thakur, “Perfecting performance of distance protective relays and it's associated pilot protection schemes in extra high voltage (EHV) transmission line applications,” *59th Annual Conference for Protective Relay Engineers*, College Station, TX, 19 June 2006.

[17] K. Bae, J.S. Thorp, “A stochastic study of hidden failures in power system protection,” *Decision Support Systems*, vol. 24, pp. 259-268, 1999.

[18] S. Tamronglak, S. H. Horowitz, A. G. Phadke, and J. S. Thorp, “Anatomy of power system blackouts: preventive relaying strategies,” *IEEE Transactions on Power Delivery*, vol. 11, pp. 708-715, 1996.

[19] G. Strang, *Introduction to Applied Mathematics*. Wellesley Cambridge Press, Wellesley, MA 1986.

[20] M. Baur and U. Brandes, “Crossing reduction in circular layouts,” *Lecture Notes in Computer Science*, Springer, Berlin, DE, vol. 3353, pp. 332-343, 2005.