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A FISSION MATRIX BASED VALIDATION PROTOCOL FOR COMPUTED POWER DISTRIBUTIONS IN THE ADVANCED TEST REACTOR

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ABSTRACT

The Idaho National Laboratory (INL) has been engaged in a significant multiyear effort to modernize the computational reactor physics tools and validation procedures used to support operations of the Advanced Test Reactor (ATR) and its companion critical facility (ATRC). Several new protocols for validation of computed neutron flux distributions and spectra as well as for validation of computed fission power distributions, based on new experiments and well-recognized least-squares statistical analysis techniques, have been under development. In the case of power distributions, estimates of the *a priori* ATR-specific fuel element-to-element fission power correlation and covariance matrices are required for validation analysis. A practical method for generating these matrices using the element-to-element fission matrix is presented, along with a high-order scheme for estimating the underlying fission matrix itself. The proposed methodology is illustrated using the MCNP5 neutron transport code for the required neutronics calculations. The general approach is readily adaptable for implementation using any multidimensional stochastic or deterministic transport code that offers the required level of spatial, angular, and energy resolution in the computed solution for the neutron flux and fission source.

Key Words: Fission Matrix, Covariance, Least-Squares, Neutron Transport, Validation.

1. INTRODUCTION

The Idaho National Laboratory (INL) has initiated a focused effort to upgrade legacy computational reactor physics software tools and protocols used for support of Advanced Test Reactor (ATR) core fuel management, experiment management, and safety analysis. This is being accomplished through the introduction of modern high-fidelity computational software and protocols, with appropriate verification and validation (V&V) according to applicable national standards. A suite of well-recognized stochastic and deterministic transport theory based reactor physics codes and their supporting nuclear data libraries (HELIOS [1], NEWT [2], ATTILA [3], KENO6 [4] and MCNP5 [5]) is in place at the INL for this purpose, and corresponding baseline models of the ATR and its companion critical facility (ATRC) are operational. Furthermore, a capability for rigorous sensitivity analysis and uncertainty quantification based on the TSUNAMI [6] system has been implemented and initial computational results have been obtained. Finally, we are also incorporating the MC21 [7] and SERPENT [8] stochastic simulation and depletion codes into the new suite as additional tools for V&V in the near term and possibly as advanced platforms for full 3-dimensional Monte Carlo based fuel cycle analysis and fuel management in the longer term.

On the experimental side of the effort, several new benchmark-quality code validation measurements based on neutron activation spectrometry have been conducted at the ATRC. Results for the first three experiments, focused on detailed neutron spectrum measurements within the Northwest Large In-Pile Tube (NW LIPT) were recently reported [9] as were some selected results for the fourth experiment, featuring neutron flux spectra within the core fuel elements surrounding the NW LIPT and the diametrically opposite Southeast IPT [10]. In the current paper we focus on computation and validation of the fuel element-to-element power distribution in the ATRC (and by extension the ATR) using data from an additional, recently completed, ATRC experiment. In particular we present a method developed for estimating the covariance matrix for the fission power distribution using the corresponding fission matrix computed for the experimental configuration of interest. This covariance matrix is a key input parameter that is required for the least-squares adjustment validation methodology employed for assessment of the bias and uncertainty of the various modeling codes and techniques.

2. FACILITY DESCRIPTION

The ATR (Figure 1) is a light-water and beryllium moderated, beryllium reflected, light-water cooled system with 40 fully-enriched (93 wt% $^{235}U/U_{Total}$) plate-type fuel elements, each with 19 curved fuel plates separated by water channels. The fuel elements are arranged in a serpentine pattern as shown, creating 5 separate 8-element “lobes”. Gross reactivity and power distribution control during operation are achieved through the use of rotating control drums with hafnium neutron absorber plates on one side. The ATR can operate at powers as high as 250 MW with corresponding thermal neutron fluxes in the flux traps that approach 5.0×10^{14} n/cm^2-s. Typical operating cycle lengths are in the range of 45-60 days.

The ATRC is a nearly-identical open-pool nuclear mockup of the ATR that typically operates at powers in the range of several hundred watts. It is most often used with prototype experiments to characterize the expected changes in core reactivity and power distribution for the same experiments in the ATR itself. Useful physics data can also be obtained for evaluating the worth and calibration of control elements as well as thermal and fast neutron distributions.

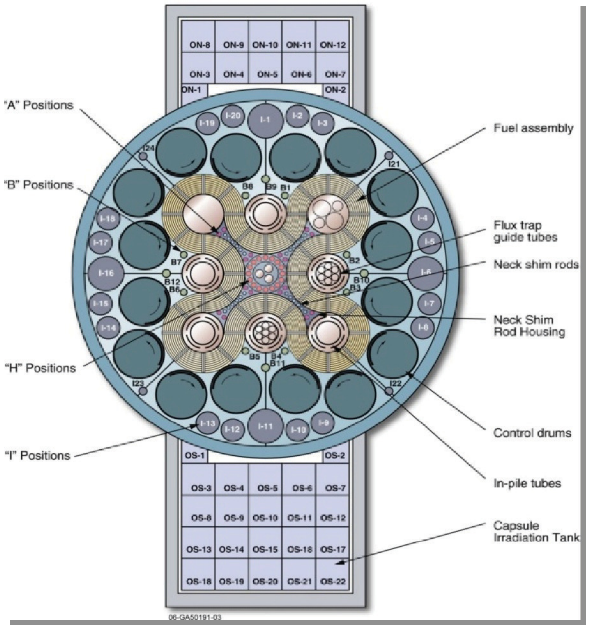

Figure 1. Core and reflector geometry of the Advanced Test Reactor. References to core lobes and in-pile tubes are with respect to reactor north, at the top of the figure.

3. COMPUTATIONAL METHODS AND MODELS

Computational reactor physics modeling is used extensively to support ATR experiment design, operations and fuel cycle management, core and experiment safety analysis, and many other applications. Experiment design and analysis for the ATR has been supported for a number of years by very detailed and sophisticated three-dimensional Monte Carlo analysis, typically using the MCNP5 code, coupled to extensive fuel isotope buildup and depletion analysis where appropriate. On the other hand, the computational reactor physics software tools and protocols currently used for ATR core fuel cycle analysis and operational support are largely based on four-group diffusion theory in Cartesian geometry [11] with heavy reliance on "tuned" nuclear parameter input data. The latter approach is no longer consistent with the state of modern nuclear engineering practice, having been superseded in the general reactor physics community by high-fidelity multidimensional transport-theory-based methods. Furthermore, some aspects of the legacy ATR core analysis process are highly empirical in nature, with many "correction factors" and approximations that require very specialized experience to apply. But the staff knowledge from the 1960s and 1970s that is essential for the successful application of these various approximations and outdated computational processes is rapidly being depleted due to personnel turnover and retirements.

Figure 2 shows the suite of new tools mentioned earlier, how they generally relate to one another, and how they will be applied to ATR. This illustration is not a computational flow chart or procedure *per se*. Specific computational protocols using the tools shown in Figure 2 for routine ATR support applications will be promulgated in approved procedures and other operational documentation. The most recent release of the Evaluated Nuclear Data Files (ENDF/B Version 7) is generally used to provide the basic cross section data and other nuclear parameters required for all of the modeling codes. The ENDF physical nuclear data files are processed into computationally-useful formats using the NJOY or AMPX [12] codes as applicable to a particular module, as shown at the top of Figure 2.

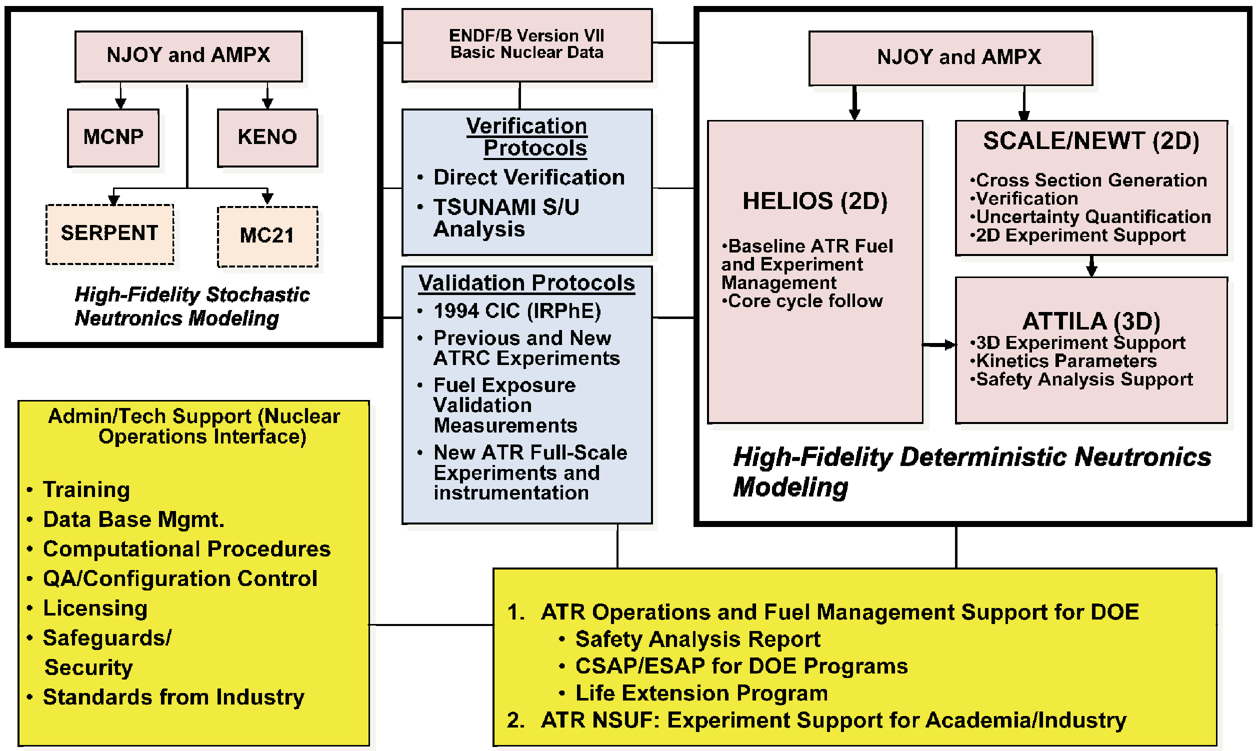

Figure 2. Advanced computational tool suite for the ATR and ATRC, with supporting verification, validation and administrative infrastructure.

4. VALIDATION MEASUREMENTS

In the new validation experiment of interest here, activation measurements that can be related to the total fission power of each of the 40 ATRC fuel elements were made with fission wires composed of 10% by weight ^{235}U in aluminum. The wires were 1 mm in diameter and approximately 0.635 cm (0.25”) in length and were placed in various locations within the cooling channels of each fuel element as shown in Figure 3, at the core axial midplane. The total measured fission powers for the fuel elements are estimated using appropriately-weighted sums of the measured fission rates in the U/Al wires located in each element [13].

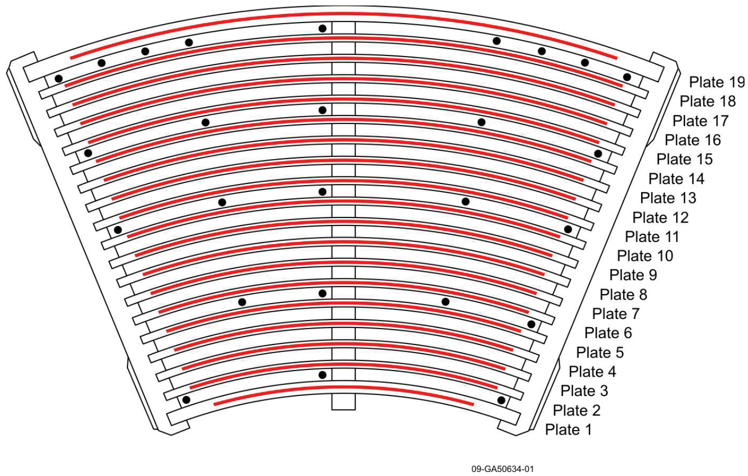

Figure 3. ATR Fuel element geometry, showing standard fission wire positions used for intra-element power distribution measurements.

Figure 4 shows the computed *a priori* (MCNP5) fission powers for the 40 ATRC fuel elements, along with the measured element powers based on the fission wire measurements. The top number (black) in the center of each element is the *a priori* element power (W) calculated by MCNP5. The bottom number (red) is the measurement. Total measured power was 875.5 W. Uncertainties associated with the measured element powers are approximately 5% (1σ). The powers for the five 8-element ATR core "lobes" are also key operating parameters and are formed by summing the powers of Elements 2-9 for the Northeast Lobe, Elements 12-19 for the Southeast Lobe, Elements 22-29 for the Southwest Lobe, Elements 32-39 for the Northwest Lobe and Elements 1, 10, 11, 20, 21, 30, 31, and 40 for the Center Lobe. The significance of the lobe powers will be discussed in more detail later.

5. POWER DISTRIBUTION ADJUSTMENT PROTOCOL

Analysis of the computed and measured power distribution for code validation purposes is accomplished by an adaptation of standard least-squares adjustment techniques that are widely used in the reactor physics community [14]. The least-square methodology is quite general, and can be used to adjust any vector of *a priori* computed parameters against a vector of measured data points that can be related to the parameters of interest through a matrix transform. This produces a "best estimate" of the parameters and their uncertainties, which can then be used to estimate the bias, if any, and the uncertainty of the computational model, and as a tool for improving the model as appropriate.

In the following description of the adjustment equations used in this work, matrix and vector quantities will generally be indicated by **bold** typeface. In some cases, matrices and vectors will be enclosed in square brackets for clarity. The superscripts, "**-1**" and "**T**" respectively, indicate matrix inversion and transposition respectively.

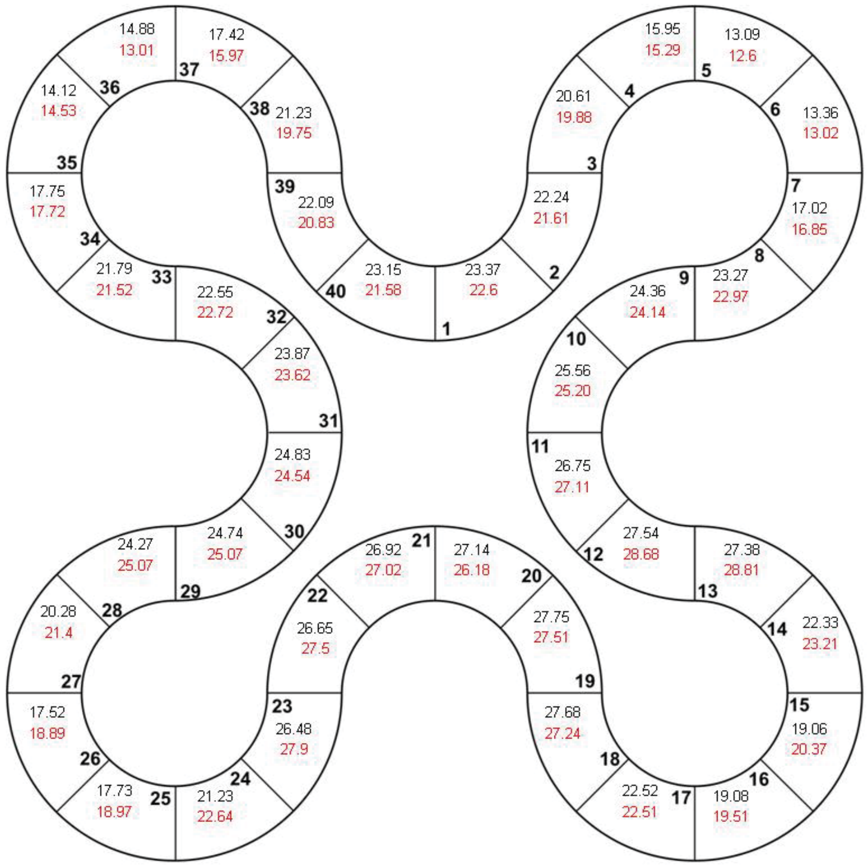

Figure 4. Calculated (black) and measured (red) fuel element powers (W) for ATRC Depressurized Run Support Test 12-5. The fuel element numbers are in bold type.

We begin the mathematical development by constructing the following overdetermined set of linear equations:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & \cdots & a_{1,NE} \\ a_{21} & a_{22} & a_{23} & \cdots & \cdots & a_{2,NE} \\ \vdots & \vdots & \vdots & & & \vdots \\ a_{NM,1} & a_{NM,2} & a_{NM,3} & \cdots & \cdots & a_{NM,NE} \\ 1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & & & \vdots \\ \vdots & \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 1 \end{bmatrix} \bullet \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ \vdots \\ \vdots \\ P_{NE} \end{bmatrix} = \begin{bmatrix} Pm_1 \\ Pm_2 \\ \vdots \\ Pm_{NM} \\ P_{01} \\ P_{02} \\ P_{03} \\ \vdots \\ \vdots \\ P_{0NE} \end{bmatrix} = [A][P] = [Z] \tag{1}$$

and the supporting definition

$$[\mathbf{Cov}(\mathbf{Z})] = \begin{bmatrix} [\mathbf{Cov}(\mathbf{Pm})] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{Cov}(\mathbf{P_0})] \end{bmatrix}, \tag{2}$$

where NE is the total number of fuel elements (i.e. 40 for ATR) and NM is the number of these elements for which element power measurements have been made. NM is typically a number between 1 and NE although multiple power measurements for the same fuel elements may optionally be included if available, possibly causing NM to be greater than NE. The vector **P** is the desired best least-squares estimate for the powers of all 40 fuel elements, the vector **Pm** (containing first NM entries in [Z]) contains the NM measured powers and the vector $\mathbf{P_0}$ (last 40 entries in [Z]) contains the 40 *a priori* estimates, P_{0i} for the element powers, extracted from the computational model of the validation experiment configuration. The top NM rows of the matrix **A** each contain entries that are equal to zero except for the column corresponding to the element for which the measurement on the right-hand side in that row was made, where the entry would be 1.0. The bottom 40 rows of the matrix **A** correspond to the rows of a 40 x 40 identity matrix.

Equation 2 includes the NM x NM and NE x NE covariance matrices for the measured power vector and for the *a priori* power vector respectively. The covariance matrix entries for **[Cov(Pm)]** are based on the uncertainties of the experimental data values in the usual manner. The covariance matrix $[\mathbf{Cov}(\mathbf{P_0})]$ for the *a priori* power vector may be computed explicitly under certain circumstances, or it may be approximated based on the assumption of an element to element fission power correlation function that decreases exponentially with distance between any two elements.

Note that even if only a single measurement (i.e. NM=1) is available for inclusion in Equation 1, the augmented system will still be overdetermined (albeit by only one row) and some useful adjustment and reduction of *a priori* uncertainty may occur in the solution for the power distribution. However, if there are no measurements included in Equation 1, then one simply obtains the uninteresting solution where the adjusted power distribution is identical to the *a priori* power distribution, with the same uncertainty.

At this point it is important to recognize that in the case of ATR and ATRC a simple exponential approximation is not ideally suited for computing the fuel element power correlation matrix needed to construct $[\mathbf{Cov}(\mathbf{P_0})]$ in Equation 2. This is because the serpentine fuel element arrangement places several elements in close proximity even though they do not share faces (e.g. Elements 1 and 10). Thus the element power correlation matrix for these two facilities has a more complex structure than the simple diagonally-dominant arrangement that an exponential formula provides. Furthermore, the availability of an accurate, realistic power correlation matrix is a crucial prerequisite for the successful application of the least-squares methodology [15]. In recognition of these factors, we propose a covariance matrix estimation procedure for ATR applications based on the fission matrix concept, further described below.

5.1. Calculation of the ATR/ATRC Fission Matrix

Each entry, $f_{i,j}$, of the so-called "Fission Matrix", **F** for a critical system composed of a specified number of discrete fissioning regions is defined as the number of first-generation fission neutrons born in region i due to a parent fission neutron born in region j [16]. The index i corresponds to a row of the fission matrix and the index j corresponds to a column. In the case of the ATR and the ATRC application of interest here the fissioning regions are defined to correspond to the fuel elements, so the fission matrix has dimensions of 40 x 40.

Assume now that the exact space, angular and energy distribution of the parent fission source neutrons within each fuel element is known from a detailed high-fidelity transport calculation and that this information is incorporated into the formation of **F.** Then construct the following eigenvalue equation:

$$\mathbf{S} = (1/k)\,\mathbf{FS}, \tag{3}$$

where **S** is the suitably-normalized 40-element fundamental mode vector of total fission source neutrons produced in each of the 40 fuel elements and k is the fundamental mode multiplication factor. Under these conditions the solution to Equation 3 will be the same as is obtained by performing the corresponding high-fidelity transport calculation for the same configuration and integrating the resulting fission source over each fuel element. Of course, if one already has the solution for the detailed high-fidelity transport model then Equation 3 does not provide any new information, but the fission matrix concept can still be very useful and instructive. In particular, there has been a great deal of effort over the years focused on acceleration of Monte Carlo calculations using fission matrix based techniques, with certain assumptions to simplify the estimation of the fission matrix elements as the calculation proceeds, without fully solving the high-fidelity problem explicitly beforehand [16-20].

In the ATR application presented here we employ a fission matrix based approach to determine the fuel element to element fission power correlation matrix and thereby the associated covariance matrix **[Cov(P_0)]** that is required in Equation 2. The example uses the MCNP5 code for the required computations, but in principal the idea should be amenable to implementation using any multidimensional deterministic or stochastic transport solution method, provided that a sufficient level of spatial, angular, and energy resolution can be achieved in the detailed transport solution needed for an accurate calculation of the fission matrix.

In the case of the ATR and ATRC, the fuel element geometry (Figure 3) is represented essentially exactly in MCNP5. Each fuel plate has a separate region for the homogeneous uranium-aluminum fissile subregion and the adjacent aluminum cladding subregions on each side of the fueled layer. Burnable boron poison is also explicitly represented in the fuel plates where it is present. Coolant channels between the plates are explicitly represented, as are the aluminum side plate structures. The active fuel height is 1.2192 m (48") and the elements have essentially the same transverse geometric structure at all axial levels within the active height. Each fuel element contains 1075 grams of ^{235}U.

High-fidelity computation of the fission matrix with MCNP5 (or with any other Monte Carlo code that features similar capabilities) for this particular application is accomplished in two easily-automated steps as follows:

First, run a well-converged fundamental-mode eigenvalue ("K-Code" in MCNP5 parlance) calculation for the ATR or ATRC configuration of interest. Save the detailed volumetric fission neutron source information that includes all fission neutrons starting from within each fuel element. The absolute spatial, angular, and energy distribution of the fission neutrons born in each fuel element must be fully specified in the source file data for that element.

Second, using the fission neutron source file information created as described above, run a set of 40 corresponding fixed-source MCNP5 calculations for the same reactor configuration of interest, one separate well-converged calculation for each fuel element fission neutron source separately. These calculations are run with fission neutron production turned off using the "NONU" input parameter. Fissions induced by the original fission source neutrons sampled from the source file are thereby treated as capture in the sense that no additional fission neutrons are produced to be followed in subsequent histories. The "fission" rate that is tallied in this manner for each fuel element in a given MCNP fixed-source calculation thus includes only the first-generation fissions induced in that element by the original source neutrons emitted by the source fuel element that was active for that calculation. Multiplying this quantity for each fuel element in a given MCNP calculation by the average number of neutrons per fission and then dividing the result by the absolute magnitude of the original fission neutron source associated with the active fuel element then yields the column of the fission matrix corresponding to that source fuel element.

Substitution of the fission matrix from the above process into Equation 3 should reproduce (within the applicable statistical uncertainties) the eigenvalue and the fuel element-to-element fission neutron production distribution of the original MCNP K-Code calculation. Once this is verified, the fission matrix is ready for use in generating the required fuel element fission correlation matrix as described below.

5.2. Construction of the Fission Covariance Matrix

To begin the fission covariance matrix development, we make a key facilitating assumption that the average number of neutrons produced per fission is the same for all of the fissioning regions in the model. This is reasonable for the ATRC experiment of interest here because all 40 fuel elements were identical and unirradiated. Furthermore, MCNP calculations show that the neutron spectrum does not vary from one ATRC fuel element to the next in a manner that significantly affects the ratio of ^{238}U fissions to ^{235}U fissions. Therefore in this case each entry, $f_{i,j}$, of the fission matrix also can be interpreted as the number of first-generation daughter <u>fissions</u> induced (or corresponding fission energy released) in each region i due to a parent <u>fission</u> occurring in region j.

Turning now to the actual computation of the fission power covariance matrix needed in Equation 2, it is important to note that the 40-element fundamental mode vector of fission powers (or fission neutron sources) for each of the 40 ATR or ATRC fuel elements may be viewed as a vector of random variables that are correlated because fission neutrons born in one fuel element can induce new fissions not only in the same element, but in any other fuel element as well, although the probability that a neutron born in one element will induce a fission in another element generally decreases with physical separation of the two fuel elements.

Referring to Equation 3, it can be seen that if the fundamental mode fission source (or power) vector is premultiplied by the fission matrix the resulting vector is, by definition, simply the original vector with all entries multiplied by k-effective. Furthermore if the fundamental mode source or power vector is arbitrarily perturbed in some manner, then premultiplication of the perturbed vector by the fission matrix will force it back toward the original fundamental mode shape, although a number of iterations may be required to converge back to the original vector in applications such as ATR, where the dominance ratio is fairly large. The above observations suggest the following stochastic estimation procedure for constructing the required fission correlation matrix:

1. Generate a vector of 40 normally-distributed random numbers whose mean is 1.0 and whose standard deviation is some nominal small fraction of the mean, e.g. 10%. The fraction specified for the standard deviation is arbitrary, but it should be small enough such that essentially no negative random numbers are ever produced and at the same time it should be large enough to avoid round-off errors in the process described below.

2. Multiply each of the 40 elements of the fundamental mode fission power vector by the corresponding element of the random number vector from Step 1. On the average, half of the fission power entries that are randomly perturbed in this manner will increase and half will decrease.

3. Premultiply the perturbed fundamental-mode fission power vector from Step 2 by the fission matrix and store the resulting perturbed "first-generation" fission power vector.

4. Repeat Steps 1-3 a statistically useful number of times, N (e.g. N=1000), to produce a batch of N 40-element perturbed "first-generation" fission power vectors.

5. Compute the 40x40 covariance matrix for the elements of the N 40-element perturbed "first-generation" fission power vectors using the fundamental definition of covariance. This completes an "inner iteration", producing a statistical estimate of the fission power covariance matrix.

6. Repeat Steps 1-5 a suitable number of times, tallying a running average of the covariance matrices that are produced until satisfactory convergence is obtained. Then compute the correlation matrix associated with the converged covariance matrix.

7. Construct the covariance matrix for the *a priori* powers computed by the modeling code by combining the correlation matrix from Step 6 with a vector of assumed *a priori* uncertainties that are to be associated with the *a priori* power vector.

5.3. Solution of the Adjustment Equations

With the fission power covariance matrix now available, Equations 1 and 2 can be combined in the usual manner to construct the covariance-weighted "Normal Equations" [21] for the system, yielding:

$$\mathbf{BP = A^T [Cov(Z)]^{-1} Z} \tag{4}$$

with

$$\mathbf{B = A^T [Cov(Z)]^{-1} A.} \tag{5}$$

Equation 4 can be solved by any suitable method to yield the adjusted element power vector **P.** The difference between the adjusted power vector and the *a priori* power vector then gives an estimate of the bias of the model, if any, relative to the validation measurements.

Also, since the solution to Equation 4 is:

$$\mathbf{P = B^{-1} A^T [Cov(Z)]^{-1} Z} \tag{6}$$

the covariance matrix for the adjusted powers may be computed by the standard uncertainty propagation formula:

$$\mathbf{Cov(P) = D\ Cov(Z)\ D^T} \tag{7}$$

where

$$\mathbf{D = B^{-1} A^T [Cov(Z)]^{-1}\ .} \tag{8}$$

The diagonal elements of the covariance matrix for the adjusted powers can then be used to estimate the uncertainty of the modeling bias. It may be noted in passing that the covariance matrix for the adjusted power vector is also simply the inverse of **B.**

6.0 RESULTS AND DISCUSSION

The *a priori* and measured power distributions from Figure 4 are plotted in Figure 5, along with the adjusted power distribution corresponding to the measured powers of all 40 elements. The covariance matrix for the *a priori* power vector was computed as described above and normalized to an estimated *a priori* uncertainty of 10% (1σ) for the diagonal entries, based on historical experience. The covariance matrix for the measured powers was assumed to have only diagonal entries of 5% (1σ) for this example. It is a simple matter to include appropriate off-diagonal elements in the latter matrix to account for correlations, for example from a common calibration of the detector used to measure the activity of the fission wires, if desired. The reduced uncertainties for the adjusted element powers in Figure 5, computed using Equation 7, ranged from 3.1% to 3.7%. The correlation matrix associated with the fission power covariance matrix used to compute the adjusted power vector is shown as a contour plot in Figure 6. Key off-diagonal structural features, such as the correlations between nearby, but non-adjacent, Elements 1 and 10, or Elements 11 and 20, etc. are readily apparent. The underlying fission matrix for this example is shown in Figure 7. The same general structure is apparent. Note also that the fission matrix is not necessarily symmetric, while the fission correlation matrix is symmetric by definition.

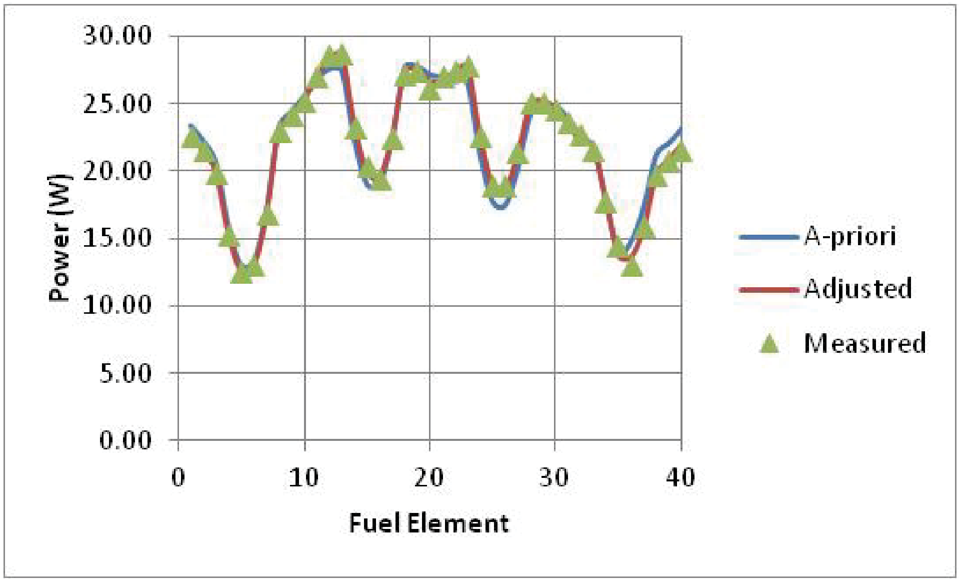

Figure 5. Fuel element power distributions for ATRC Depressurized Run Support Test 12-5. The adjusted power is computed using the measured powers of all 40 fuel elements.

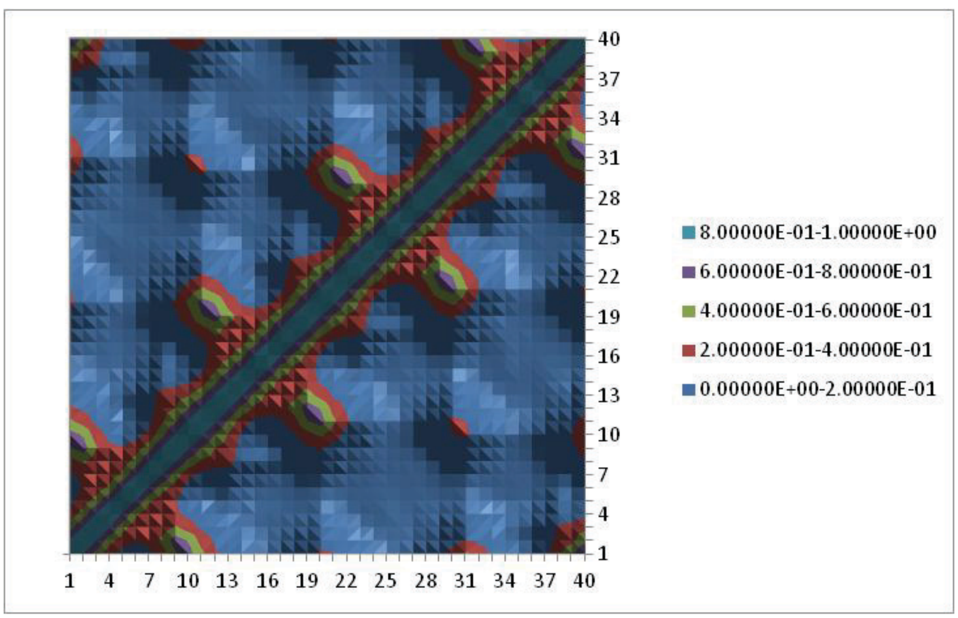

Figure 6. Fission power correlation matrix for the ATRC. The axis numbering corresponds to the fuel element numbers shown in Figure 4.

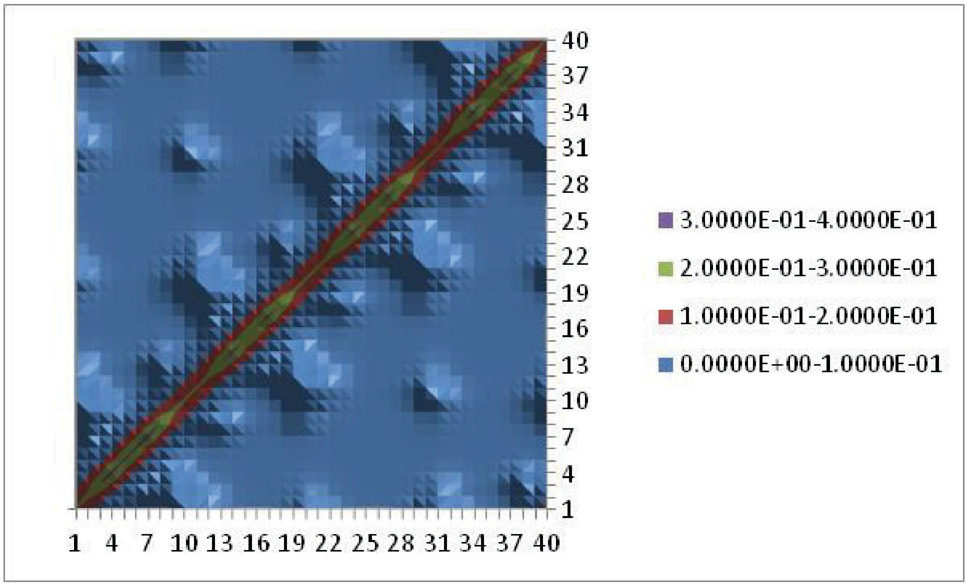

Figure 7. Fission matrix for the ATRC. The axis numbering corresponds to the fuel element numbers shown in Figure 4.

Figure 8 shows the result of an adjustment of the MCNP *a priori flux* where only the powers of the odd-numbered fuel elements in Test 12-5 were included in the analysis. This simulates the relatively common ATR practice where only the odd-numbered fuel element powers are actually measured, and the power for each even-numbered element is assumed to be equal to the measured power in the odd-numbered element on the opposite side of the same lobe. For example, the power in Element 2 is assumed equal to the power in Element 9, the power in Element 4 is assumed equal to the power in Element 7, and so forth around the core. The often-questionable validity of this assumption depends on the overall symmetry of the reactor configuration. In the future the assumption of symmetry will be replaced by the least-square adjustment procedure described here to estimate the powers in the even-numbered elements. The reduced uncertainties for the adjusted element powers in Figure 8 ranged from 3.9% to 4.3% for the odd-numbered elements and from 4.0% to 5.2% for the even-numbered elements, demonstrating how significant uncertainty reduction can occur in the adjusted powers even for elements for which no measurement is included. This is a result of the weighted interpolation effect provided by the element power covariance matrix.

Economizing on the number of measurements even further, Figure 9 shows an adjustment where only the measured powers for Elements 8, 18, 28, and 38 were included in the analysis. This arrangement simulates another ATR protocol that is sometimes used because these elements are representative of the highest-powered elements in each outer lobe. In this case the reduced uncertainties for the adjusted element powers ranged from 4.4% to 4.5% for Elements 8, 18, 28 and 38, from 6.6% to 7% for the immediately adjacent elements and up to 9.9% for the elements that were the most distant from the elements for which measurements were made. It is notable here that some uncertainty reduction occurs even for the most remote fuel elements.

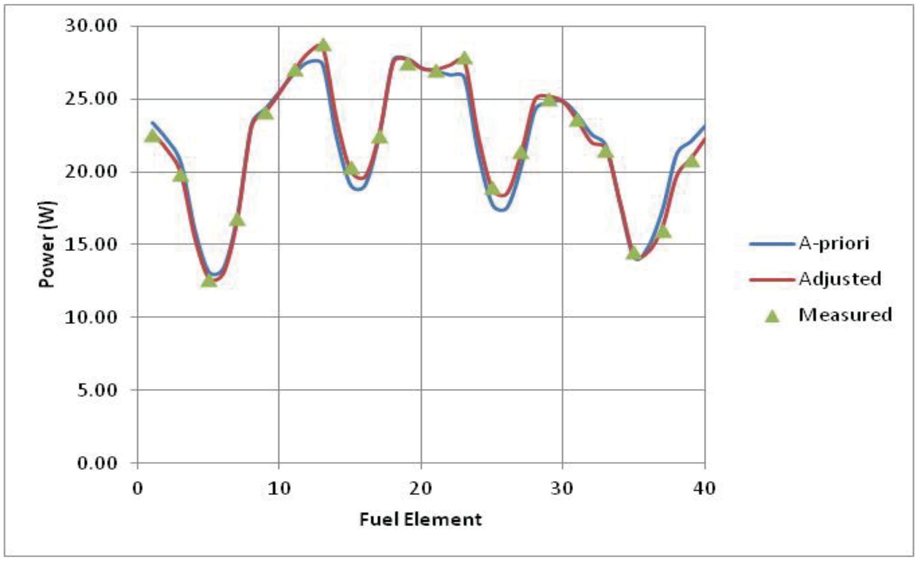

Figure 8. Fuel element power distributions for ATRC Depressurized Run Support Test 12-5. The adjusted power is computed using the measured powers of only the 20 odd-numbered fuel elements.

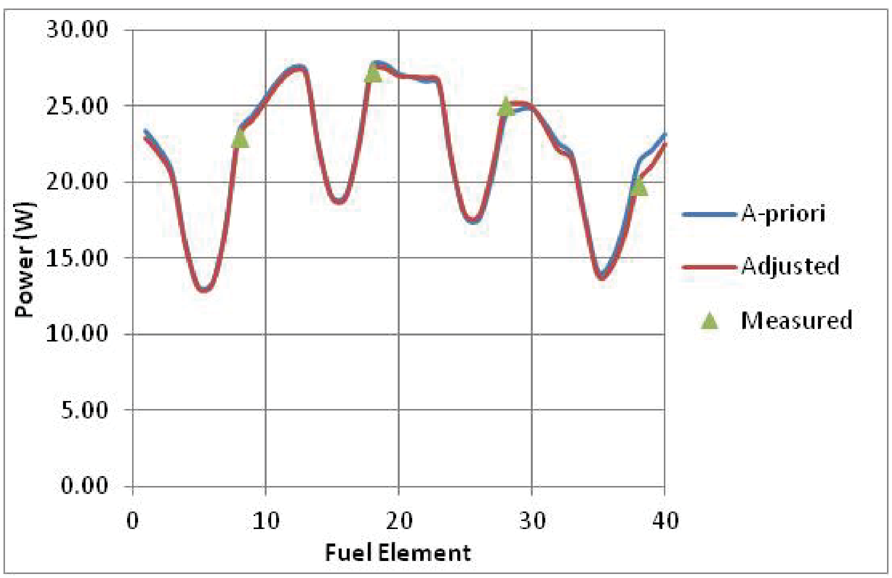

Figure 9. Fuel element power distributions for ATRC Depressurized Run Support Test 12-5. The adjusted power is computed using the measured powers of elements 8, 18, 28 and 38 only.

Finally, Figure 10 illustrates another possible use of the techniques developed in this work. The ATR has an online lobe power measurement system but it does not have an online system for measurement of individual fuel element powers. Measurements of individual element powers currently can only be done by the rather tedious fission wire technique described earlier. The least-squares methodology outlined here also offers a simple, but mathematically rigorous, approach for estimating the fission powers of all 40 fuel ATR fuel elements and their uncertainties using the online lobe power measurements as follows:

In the case of Figure 10 the online lobe power measurements are simulated by the fission wire measurements used for the previous examples. The first five rows of the matrix on the left-hand side of Equation 1 describe the five simulated online lobe power measurements. These rows each contain entries of 0.125 on the left-hand side for the elements included in the lobe corresponding to that row and entries of zero elsewhere. The right hand side of each of these first five rows contains the average of the measured powers from the fission wires for the lobe represented by that row. For example the first row (Lobe 1) contains entries of 0.125 for elements 2 through 9, and the average of the measured powers for elements 2 through 9 appears on the right hand side, and so forth for the other lobes. The reduced uncertainties for the adjusted powers shown in Figure 10 for the 40 elements range from 6.4% to 8.3%.

The results shown in Figure 10 thus illustrate a practical application where the powers for each ATR lobe that are measured online could be entered into Equation 1 each time they are updated (every few seconds), and a corresponding estimate for all of the individual element powers could be immediately produced. Of course the *a priori* power vector would need to be recalculated regularly as the core depletes, control drums rotate, and neck shims are pulled during a cycle. This could however be automated to a large extent, and it should ultimately be quite practical, for example, to update the *a priori* power vector from the model at least daily.

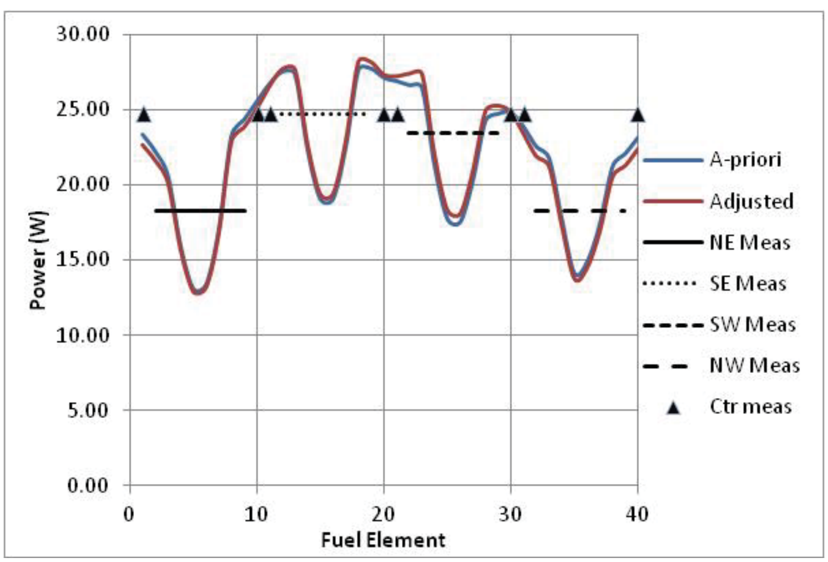

Figure 10. Fuel element power distributions for ATRC Depressurized Run Support Test 12-5. The adjusted power is computed using the measured powers of the five core lobes.

7.0 CONCLUSIONS

In summary, this paper presents a relatively simple but effective fission-matrix-based method for generating the required fuel element covariance information needed for detailed statistical validation and best-estimate adjustment analysis of fission power distributions produced by computational reactor physics models of the ATR (or for that matter, any other type of reactor). The method has been demonstrated using the MCNP5 neutronics code but it can be used with any other Monte Carlo neutronics simulation code as well as with any deterministic neutron transport code that provides a sufficient level of spatial, angular, and energy resolution within each fissioning region of interest. Analyses of this type are useful not only for quantifying the bias and uncertainty of computational models for a specific measured reactor configuration of interest, but they also can serve as guides for model improvement and for estimation of *a priori* modeling uncertainties for related reactor configurations for which no measurements are available.

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