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POINT KINETICS CALCULATIONS WITH FULLY COUPLED THERMAL FLUIDS REACTIVITY FEEDBACK

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ABSTRACT

The point kinetics model has been widely used in the analysis of the transient behavior of a nuclear reactor. In the traditional nuclear reactor system safety analysis codes such as RELAP5, the reactivity feedback effects are calculated in a loosely coupled fashion through operator splitting approach. This paper discusses the point kinetics calculations with the fully coupled thermal fluids and fuel temperature feedback implemented into the RELAP-7 code currently being developed with the MOOSE framework.

Key Words: Point kinetics, Fully coupled, RELAP-7

1. INTRODUCTION

The analysis of the transient behavior of a nuclear power reactor requires reactor system safety analysis codes such as RELAP5 [1]. The analysis often requires the coupled simulation of reactor kinetics and thermal-hydraulics of the reactor core. The widely used models for coupled kinetics and thermal-hydraulics calculations consider a lumped parameter approach for the kinetics, hence the point kinetics model. This is a result of its simplicity to compute transient behavior of a nuclear reactor. The thermal-hydraulics and its feedback effects to the reactivity are normally computed on the basis of a one-, or multi-dimensional core description. In the point kinetics model, it is assumed that reactor power can be separated into space and time functions. The assumption is adequate for cases in which the space distribution remains nearly constant. The point kinetics model has been used in RELAP5 and will remain a viable option for future reactor system safety analysis codes such as RELAP-7 [2]. RELAP-7, a MOOSE-based application [3], is the next generation system safety analysis code being developed at Idaho National Laboratory.

The ordinary differential equations (ODEs) that are used in the point kinetics model are stiff and difficult to obtain accurate solutions. Researchers have developed various time-integration schemes in the past several decades. How to accurately and efficiently solve these equations still remains an active topic of research. Ganapol [4] has an excellent summary of the advantages and disadvantages of the various schemes developed in the past several decades. In this paper, we will not study any new schemes to solve the stiff ODEs. Instead we will focus our efforts on the

implementation of the point kinetics model with thermal hydraulic reactivity feedback in a fully coupled fashion.

2. POINT KINETICS MODEL WITH THERMAL HYDRAULIC FEEDBACK

The point reactor kinetics model computes both the immediate (prompt and delayed neutrons) fission power and the power from decay of fission products. The immediate fission power is the power that is released at the time of fission. Decay power is generated as the fission products and actinides undergo radioactive decay.

The governing equations for the point kinetics model are the following:

$$\frac{dn(t)}{\partial t} = \frac{r(t)-\beta}{\Lambda} n(t) + \sum_{i=1}^N \lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i(t)}{\partial t} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad i = 1, 2, \dots, N \quad (2)$$

where t is time, n is neutron density, $\beta = \sum_{i=1}^N \beta_i$ is the effective delayed neutron fraction, Λ is the prompt neutron generation time, $r(t)$ is the reactivity, $C_i(t)$ is the delayed neutron precursor concentration in group i , β_i is the effective delayed neutron precursor yield of group i , λ_i is the decay constant of group i . N is the number of delayed neutron precursor groups. Only the ODEs for the prompt and delayed neutrons are shown in this paper. For brevity of the paper, the ODEs for decay power are omitted here and they can be found in Reference [1].

The reactivity model implemented in RELAP-7 is the same as the separable model used for RELAP5 [1]. In the separable model, each effect is assumed to be independent of the other effects. The model assumes nonlinear feedback effects from moderator (thermal fluids) density and fuel temperature changes and linear feedback from moderator and fuel temperature changes. Note that in this paper the terms ‘moderator’ and ‘thermal fluids’ are used interchangeably.

The separable model defining reactivity is defined as:

$$r(t) = \sum_{i=1}^{n_s} r_{si}(t) + \sum_{i=1}^{n_\rho} [W_{\rho i} R_\rho (\rho_i(t)) + a_{Mi} \Delta T_{Mi}(t)] + \sum_{i=1}^{n_F} [W_{Fi} R_F (T_{Fi}(t)) + a_{Fi} \Delta T_{Fi}(t)]. \quad (3)$$

The quantities $r_{si}(t)$ are obtained from input tables defining n_s reactivity (or scram) curves as a function of time. R_ρ is a table defining reactivity as a function of the current moderator density of fluid $\rho_i(t)$ in the thermal fluids volume i (density reactivity table). $W_{\rho i}$ is the density volume weighting factor for volume i ; $\Delta T_{Mi}(t)$ is the spatially averaged moderator fluid temperature difference between the current time t and the start of the transient for volume i ; a_{Mi} is the volume fluid temperature coefficient (not including density changes) for volume i ; and n_ρ is the number of thermal fluids volumes in the reactor core. The quantity R_F is a table defining the Doppler reactivity as a function of the heat structure volume average fuel temperature $T_{Fi}(t)$ in heat structure i ; $\Delta T_{Fi}(t)$ is the difference between the current time t and the start of the transient; W_{Fi} and a_{Fi} are the fuel temperature heat structure weighting factor and the heat structure fuel temperature coefficient, respectively, for heat structure i ; and n_F is the number of fuel volumes in a reactor core.

The required thermal fluids temperature and density to calculate feedback reactivity are obtained by solving the following one-dimensional single phase conservation equations of mass, momentum and energy in a pipe:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \quad (4)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial(\rho u^2 + P)}{\partial x} + f \frac{\rho}{2D_h} u|u| - \rho g_x = 0 \quad (5)$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial(\rho u(E + \frac{P}{\rho}))}{\partial x} + h_w a_w (T_M - T_w) + u \left(f \frac{\rho}{2D_h} u|u| - \rho g_x \right) + q_M''' = 0 \quad (6)$$

Where ρ , ρu , ρE represent density, momentum and total energy. P is the pressure, D_h is hydraulic diameter of a pipe, f is the wall friction factor, g_x is the gravity component in the pipe direction. h_w is the wall heat transfer coefficient, a_w is the heating wall area density per unit fluid volume, T_w is the wall temperature and T_M is the fluids temperature, and q_M''' is the external heat source.

The fuel temperatures required to calculate feedback reactivity are obtained by solving the following heat conduction equation of solid materials such as fuel and clad,

$$\rho_s C_p \frac{\partial T_s}{\partial t} - \nabla \cdot (k \nabla T_s) - q''' = 0, \quad (7)$$

where ρ_s , C_p and k are the density, specific heat capacity and thermal conductivity of the solid materials respectively. Energy is deposited into the fuel using the reactor power calculated by the point kinetics model through the heat generation rate per unit volume q''' . The heat is transferred from the heat structure to the fluid by a heat flux boundary condition [5].

3. SOLUTION ALGORITHMS

Eqs. (1) through (7) present a coupled set of multi-physics problems on heterogeneous geometries. The thermal fluid equations are coupled to the heat conduction equation through the heat flux boundary condition. The point kinetics equations are coupled to the heat conduction equation through the specific volume heat generation rate in the fuel (q''') as well as coupled to the thermal fluids equations through energy directly deposited into the fluids (q_M'''). The equations are nonlinear with wide range of time scales and spatial scales and special techniques are required to solve them. Fig. 1 schematically shows how the information exchange happens in coupled calculations with point kinetics.

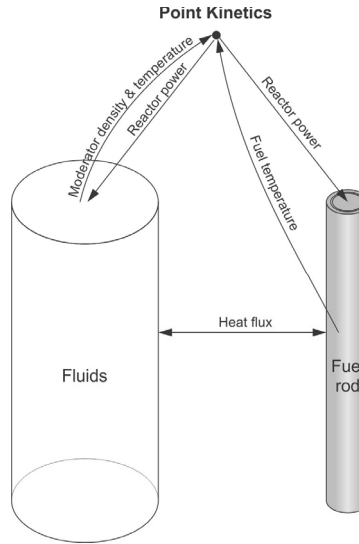


Figure 1. Schematics of simultaneous information exchange in coupled point kinetics calculations.

In the RELAP5 code, operator splitting approach was used in solving the thermal fluids, heat conduction and the point kinetics equations. This approach is often referred to as a loosely coupling approach. In RELAP5, the thermal fluids equations are solved using the semi-implicit time integration scheme. The spatial discretization uses a staggered mesh, finite volume method. In this method, the thermodynamic state variables such as density, pressure, and temperatures, are evaluated at the volume center while the fluid velocities are evaluated at the volume surfaces. The time step size is limited by the Courant limit due to the stabilization limit of the scheme. The heat conduction equation is solved using a finite difference scheme. The point kinetics ODEs are solved using the modified Runge-Kutta method of Cohen [1]. The fifth-order integration scheme is used in RELAP5. The point kinetics equations are advanced with the same time step as the thermal fluids and heat conduction equations, and reactivity is assumed to vary linearly between time step values. The maximum time step for the reactor kinetics advancement is the thermal fluids and heat conduction time step. The data exchange between the point kinetics calculation and the other calculations are explicit. The point kinetics calculations lag the thermal fluids, heat conduction and heat transfer calculations. The reactor power used in thermal fluids and heat conduction is the value at the beginning of the time step. The end of time step values from thermal fluids and heat conduction calculations are used to compute the reactivity used in the point kinetics calculations [1].

The RELAP-7 code, on the other hand, is developed using the MOOSE [2] framework with the continuous finite element method as the spatial discretization method. The MOOSE framework allows all the equations (Eqs. 1 to 7) to be solved implicitly to achieve nonlinear multi-physics tight coupling. The single vector, which includes all the unknown variables from the thermal fluids equations, the heat conduction equation and the point kinetics equations for a specific problem, is solved iteratively with the Jacobian-free Newton Krylov (JFNK) method [5]. All of the participating and interacting physics are solved in a fully coupled fashion, and the simulations consequently represent more closely the physical phenomena. An accompanying paper by Zou et al. [6] describes how the multi-mesh and multi-physics simulations of the thermal fluids, heat conduction, and conjugate heat transfer are implemented in RELAP-7 to

analyze the interconnected one- and zero-dimensional flow networks.

4. NUMERICAL RESULTS

Some results from two test cases are presented in this section. The first test case presents the results of benchmarking the time integration scheme used in RELAP-7 versus the benchmark results generated by Ganapol [4]. The second test case includes the reactivity feedback from thermal fluids and fuel temperatures. In this case, one flow channel with heat structure is used to represent a reactor core.

4.1 Benchmarking the time-integration scheme for the point kinetics model

The MOOSE framework implemented several different time-integration schemes, such as backward Euler, BDF2 [7] and Crank-Nicolson. The backward Euler is first order accurate in time, while BDF2 and Crank-Nicolson are second order accurate in time. These two second-order integration schemes are used in the solving the point kinetics ODEs. As is well known, the point kinetics equations are very stiff. In order to build the confidence that the second order time integration schemes are adequate, the results from certain standalone calculations without thermal fluids and fuel temperature feedback are benchmarked with Ganapol's latest benchmark results [4]. The solutions from various reactivity step insertion are compared with Ganapol's benchmark results. The step reactivity insertion cases include -0.5\$, 0.5\$ and 1.0\$ reactivity insertion. Table I shows the kinetic parameters used in the benchmark calculations. Tables II and III present the comparison between the RELAP-7 calculated results and the benchmark results.

Table I. Kinetic parameters with $\Lambda=5.0E-4$ s

Group	β_i	$\lambda_i(1/s)$
1	0.000285	0.0127
2	0.0015975	0.0317
3	0.001410	0.115
4	0.0030525	0.311
5	0.00096	1.40
6	0.000195	3.87

Table II. Comparison of neutron density for step reactivity insertion of 0.5\$ and -0.5\$

Time	Step insertion of 0.5\$		Step insertion of -0.5\$	
	Benchmark [4]	RELAP-7	Benchmark [4]	RELAP-7
1.0000E-01	1.533113E+00	1.532988E+00	6.989252E-01	6.990126E-01
1.0000E+00	2.511494E+00	2.511492E+00	6.070536E-01	6.070546E-01
1.0000E+01	1.421502E+01	1.421524E+01	3.960777E-01	3.960783E-01

Table III. Comparison of neutron density for step reactivity insertion of 1.0\$

Time	Step insertion of 1.0\$		
	Benchmark [4]	RELAP-7	% Difference
1.0000E-01	2.515766E+00	2.515804E+00	0.0015
5.0000E-01	1.036253E+01	1.036367E+01	0.0110
1.0000E+00	3.218354E+01	3.219039E+01	0.02128
1.0000E+01	3.246979E+09	3.253768E+09	0.2091

The time step size of 1.0E-4 s was used in the RELAP-7 calculations. The comparison results from Table II and III indicate that the second order time integration scheme gives accurate results when compared to the benchmark results. However, for a large reactivity insertion case such as 1.0\$, the difference is noticeable as the time gets larger. This indicates that higher order time integration schemes would be required to accurately solve the point kinetics ODEs. As has been pointed out in the introduction section, higher order integration schemes are left for future work. This work only focuses on demonstrating the coupling of point kinetics with reactivity feedback in a fully coupled fashion.

4.2. Fully coupled calculations for one flow channel test case

The second test case is a simple core model with only one flow channel and heat structure to represent a reactor core. Some key parameters used for this model problem is shown in Table IV.

Table IV. Model parameters for one channel test case

Core model parameters		Fuel rod geometry data	
Core power (MWth)	2772	Fuel pellet diameter (cm)	0.9391
Core height (m)	3.6576	Gap thickness (cm)	0.00955
Core mass flow rate (kg/s)	16052.4	Clad outer diameter (cm)	1.0928
Core inlet temperature (K)	564.17	Heat transfer surface area per unit flow volume	276.574
Core inlet pressure (MPa)	15.17		
Core flow area (m ²)	4.57		

Both RELAP5 and RELAP-7 input files were built for this test problem. Fig. 2 shows the nodalization diagram of the RELAP5 and RELAP-7 model. In the RELAP5 model, the time-dependent volumes are used to set the boundary conditions. A pipe component is used to represent the core thermal fluids. A heat structure attached to the pipe represents the heat conduction in the fuel rods. A single junction and time-dependent junction are used to connect the core model with the boundaries. In the RELAP-7 model, a core channel component is used to represent the core thermal fluids, heat conduction and conjugate heat transfer. Time-dependent mass flow rate and time-dependent volume components are used to set the boundary conditions and connect with the core channel. Both the RELAP5 and RELAP-7 models were first run to steady-state without the consideration of reactivity feedback. The reactor power of 2772 MWth was used for steady-state calculations. The transient cases were subsequently run using both RELAP5 and RELAP-7 with a step reactivity insertion of 1.0\$ to initiate the transient. The mass flow rate is fixed at 16052.4 kg/s during transient runs. The weighting factors $W_{\rho i}$, $W_{F i}$ and the

temperature coefficients a_{Mi} , a_{Fi} are input into RELAP5 and RELAP-7 input files. In this test case, the nonlinear feedback effects from moderator density and fuel temperature changes are not considered. Only the linear feedback from moderator and fuel temperature changes are considered. Fig. 3 shows the calculated feedback reactivity following the initiation of the transients. Fig. 4 shows the reactor power responses following the initiation of the transients. The y-axis on Fig. 4 uses logarithmic scale to represent the reactor thermal power in Watts. Fig. 4 shows that RELAP5 and RELAP-7 produced similar results but with noticeable differences. RELAP5 predicted higher peak power than RELAP-7. The differences can be mainly attributed to the operator splitting approach versus fully coupled approach in those two codes.

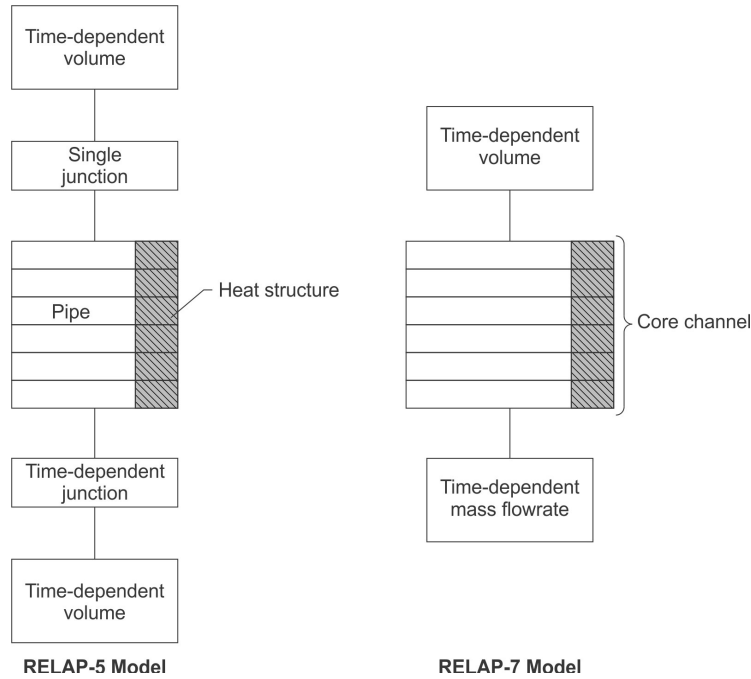


Figure 2. RELAP5 and RELAP-7 nodalization

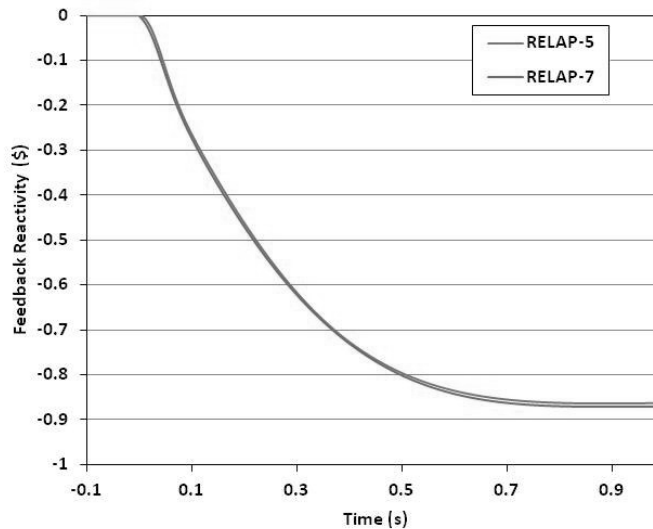


Figure 3. Calculated feedback reactivity (in \$) following 1.0\$ step reactivity insertion

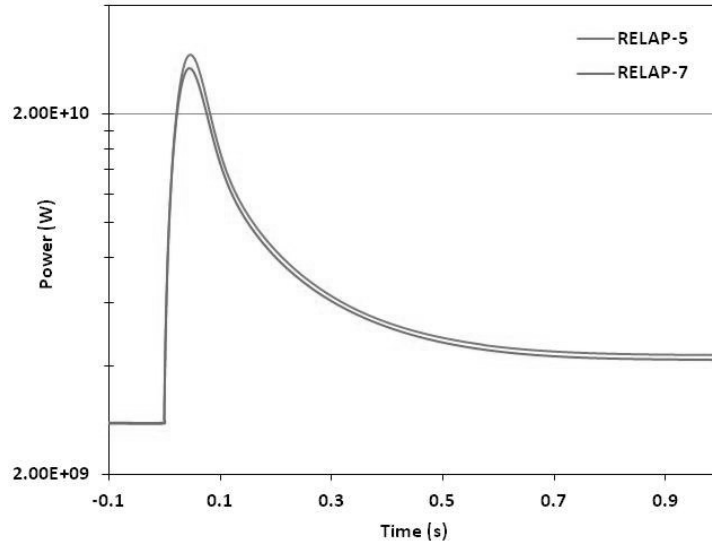


Figure 4. Calculated reactor power response following 1.0\$ step reactivity insertion

5. CONCLUSIONS AND FUTURE WORK

Due to its simplicity, the point kinetics model is widely used in the reactor dynamics analysis. In the traditional nuclear power plant system safety analysis codes such as RELAP5, the point kinetics model is implemented with reactivity feedback effects in a loosely coupled fashion. In RELAP-7, the point kinetics model with reactivity feedback effects is implemented in a fully coupled fashion. Results from test cases indicate that the fully coupled calculations worked very well.

Since RELAP-7 now only has second order time integration schemes, a higher order scheme should be implemented in the future. In addition, the underlying difficulties with obtaining the kinetic parameters for the point kinetics model warrant further investigations. The kinetic parameters are calculated from the static solution of the reactor in the initial steady state but not the actual state of the reactor during the transient. Therefore, the kinetic parameters are only the approximate for transient applications. This needs to be improved going forward to fully realize the benefits of fully coupled multi-physics simulations.

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