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C++ Implementation of IAPWS Water/Steam Properties

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SUMMARY

For the calculations of water-involved systems, such as safety analysis of light water reactors, it is essential to provide accurate water properties. The International Association for the Properties of Water and Steam is an international non-profit association of national organizations concerned with the properties of water and steam [1]. It provides internationally accepted formulations of water/steam properties for scientific and industrial applications. The purpose of this work is to provide a stand-alone software package in C++ programming language to provide accurate and efficient water/steam properties evaluation, based on the latest IAPWS releases. The discussion on related IAPWS releases, code implementations and verifications are provided in details.

C++ Implementation of IAPWS Water Properties

1. INTRODUCTION

For the calculations of water-involved systems, such as safety analysis of light water reactors (LWRs), it is essential to provide accurate water properties. The International Association for the Properties of Water and Steam (IAPWS) is an international non-profit association of national organizations concerned with the properties of water and steam [1]. It provides internationally accepted formulations of water/steam properties for scientific and industrial applications. For example, the water/steam properties provided by National Institute of Standards and Technology (NIST) are mostly relied on IAPWS water/steam properties releases [2]. The IAPWS water/steam properties have also been adopted in reactor system codes, such as the latest version of RELAP5 [3]. The main purpose of this work is to implement the latest version of water/steam properties provided by IAPWS in C++ programming language. It is aimed to provide a stand-alone software package for water/steam properties, which can be used in reactor system analysis codes and other similar codes written in C++. The work includes the implementation of fundamental equation of states functions, transport properties, and necessary interface functions.

In nuclear engineering field, the safety analysis of reactor systems are generally performed using reactor system analysis codes. In such codes, the solutions of flow in pipe network are obtained by means of solving one-dimensional flow equations. From physics point of view, water properties functions are ways to calculate thermodynamic and transport properties. For example, one wants to calculate water densities at given temperatures and pressures. From mathematical point of view, water properties, including both equation of states (EOS) and transport properties, are necessary to close the equation system. In general, system analysis codes cover a wide range of temperature and pressure conditions. For typical steady state analysis for pressurized water reactors (PWRs), the pressure ranges from atmospheric pressure to the primary system pressure (~15.5 MPa), and the temperature ranges from room temperature to ~350 °C. For accident analyses, the pressure can go up to 20 MPa and the temperature rises to over 2000K. For super critical water reactors, both pressure and temperature could be even beyond the critical point. Some simplified equation of states, such as stiffened gas equation of states, are easy to implement and having relatively low computational cost. However, such simplified EOS has large errors or covers a small temperature and pressure range, as such EOS is generally curve fitted within limit region. It is in general incapable to handle the wide temperature and pressure ranges encountered for reactor safety analysis. On the other hand, in the process of solving flow equations, the number of function calls on water/steam properties could be enormous. This could be a potential code efficiency issue if an iterative method, such as Newton's method, is used as the solving scheme. Thus, it is essential to provide a software package to calculate water/steam properties not only accurately, but also efficiently.

2. IAPWS WATER/STEAM PROPERTIES

This work is based on the latest IAPWS releases on water/steam properties. For reactor safety analysis, water/steam properties in both single-phase region and saturation region, as well as transport properties will be all needed. This work will rely on several latest IAPWS releases. Namely, the IAPWS releases include the 1995 version for the water/steam thermodynamic properties for general and scientific use [4], and its latest reversion published in 2009 [5], the latest water/steam properties based on the industrial formulation [9], IAPWS releases on thermal conductivity [6] and viscosity [7] for ordinary water substance, and the IAPWS release on surface tension [8]. In the remaining of this document, as commonly used, we will refer reference [4] as IAPWS-95, reference [5] as IAPWS-95Rev, reference [9] as IAPWS-IF97, respectively.

Reference [4] and [5] are the basis of IAPWS-95 water/steam properties in this work, which covers the stable single-phase and saturation region. Reference [6], [7], and [8] are used to develop transport properties, including thermal conductivity, viscosity and surface tension. Reference [9] is also the source for implementing IAPWS-IF97 basic equations. However it will be used for metastable vapor region properties calculation only. It is not intended to repeat the IAPWS document, but in the following sections, we will include the most fundamental and important concepts and functions in IAPWS releases.

2.1 IAPWS-95 and IAPWS-95Rev

The fundamental formulation for the IAPWS-95 is the specific Helmholtz free energy f, in non-dimensional form,

$$\frac{f(\rho,T)}{RT} = \phi(\delta,\tau) = \phi^{\circ}(\delta,\tau) + \phi^{r}(\delta,\tau) \tag{1}$$

where, $\delta = \rho/\rho_c$, $\tau = T_c/T$, and T_c, ρ_c are temperature and density at critical point, respectively. ϕ° and ϕ^{r} are the ideal-gas part and the residual part of the Helmholtz free energy, respectively. All thermodynamic properties then will be expressed as functions of ϕ°, ϕ^{r}, and their derivatives. Thus, all thermodynamic properties are eventually provided as functions of density and temperature. It should be noted that the residual part, ϕ^{r}, includes 56 complicated polynomial and exponential terms. It would be an expensive call if using this function directly in a production code. We will discuss strategies to avoid the expensive function call in later sections of this document.

Besides the single-phase region, the IAPWS-95 also provides the properties along the phase boundary, which is based on the phase equilibrium condition. This equilibrium condition is determined by three non-linear equations based on the equality of pressure, temperature and Gibbs free energy on the vapor-liquid saturation boundary (See equations 6.9a, 6.9b and 6.9c in reference [4]). This set of equations has to be solved with iterative method, such as Newton's method. However, when approaching the critical point, the Newton's method becomes inefficient and eventually fails as the Jacobian matrix becomes singular. In reference [10], a different concept, fugacity, was used to calculate the phase boundary. It however also experiences difficulty as approaching the critical point. As an alternative, the auxiliary equations will be used to calculate the phase boundary (See equations 2.5, 2.5(a), 2.6 and 2.7 in reference [4]). These set of auxiliary equations could be traced back to reference [11], and the verification is performed based this document. It should be noted that, as pointed in reference [4], these auxiliary equations have extremely small difference compared to the IAPWS95 non-linear equilibrium equations, however they are not thermodynamically consistent with IAPWS-95. This set of equations will still be used for the saturation line properties calculation, and details such as accuracy analysis will be discussed in the verification part of this document.

As stated in reference [4], the IAPWS-95 formulation is validated in the entire stable fluid region from the melting curve to 1273 K for temperature, and up to 1000 MPa for pressure (See section 6.3.1, item (1) in reference [4] for details). Figure 1 shows the validity region with associated uncertainties at different regions in a p-T diagram. This formulation can also be reasonably extrapolated to temperature and pressure ranges beyond the validated region shown above. As stated in the same reference, the formulation behaves reasonably for temperature up to 5000 K, and pressure up to 100 GPa (See section 6.3.1, item (2) in reference [4] for details). The IAPWS-95 formulation can also be used to calculate thermodynamic properties at metastable regions, including subcooled liquid, superheated liquid and subcooled vapor regions. The later two metastable regions are of interest in reactor safety analysis like applications. However, when later developing the IAPWS-IF97 formulation, it was found that the IAPWS-95 is not suitable for the metastable subcooled vapor region properties (See section 6.3.1, item

(3) in reference [4] for details). Therefore, it is recommended to use the IAPWS-IF97 formulation for this specific region, which will be described in the next section.

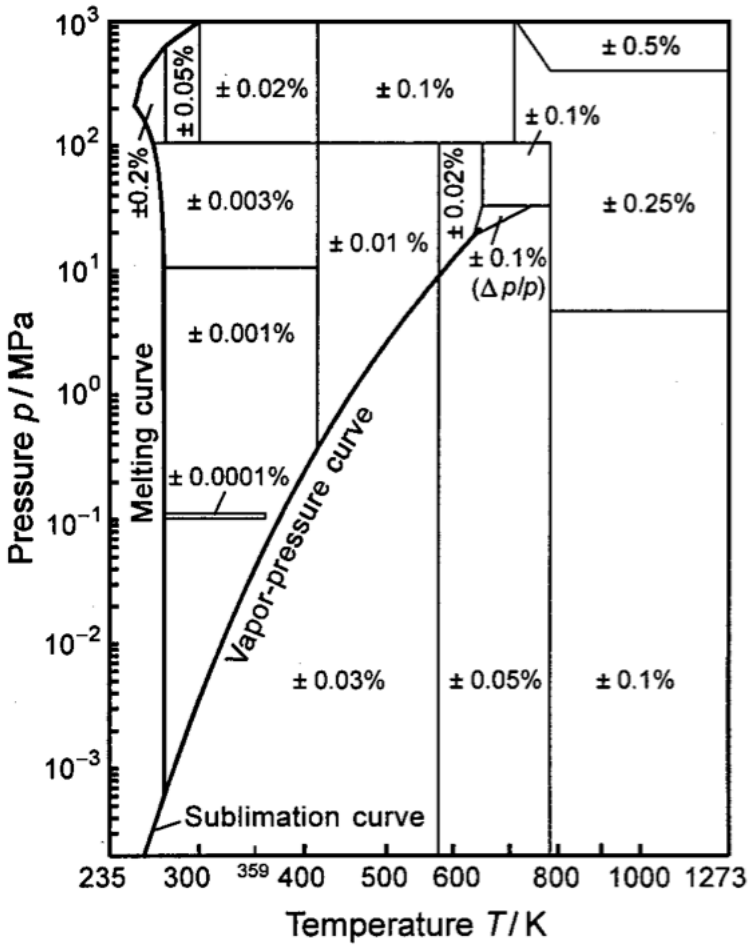

FIG. 6.1. Percentage uncertainties in density estimated for IAPWS-95, Eq. (6.4). In the enlarged critical region (triangle), the uncertainty is given as percentage uncertainty in pressure. This region is bordered by the two isochores 527 and 144 kg m^{-3} and by the 30 MPa isobar. The positions of the lines separating the uncertainty regions are approximate.

Figure 1, Validity region of IAPWS-95 formulation in a *p-T* diagram [4]

In the revised IAPWS-95Rev released in 2009 [5], two coefficients used in the calculation of triple point pressure were improved to match more accurately the arbitrary conventions for the zeros of energy and entropy. Other than this minor change, all remaining formulations and coefficients were kept identical.

2.2 IAPWS-IF97

In 1997, the "IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" was proposed for industrial use, called IAPWS-IF97 for short [9]. Compared to IAPWS95, the industrial formula version provides a complete set of backward equations such that iterations could be

avoided when backward calculations are needed. Different than the IAPWS-95 approach, the IAPWS-IF97 divides its entire validated range into 5 different regions (shown in Figure 2). In each region, different basis equation is used based on either the Gibs free energy equation (g) or the Helmholtz free energy (f). The saturation line is labeled as region 4 and its basic equation is a $p(T_s)$ correlation. The main purpose to introduce the IAPWS-IF97 in this work is to calculate the metastable vapor region properties, which is a supplementary region of the region 2. Nevertheless, the equations of all 5 regions are provided in this work and code verification work has also been done accordingly. Since IAPWS-IF97 provides the advantages of backward equations, it is also potentially to develop the IAPWS-IF97 as a stand-alone software package. This will require additional assessment on the feasibility to use IAPWS-IF97 formulation for reactor safety analysis.

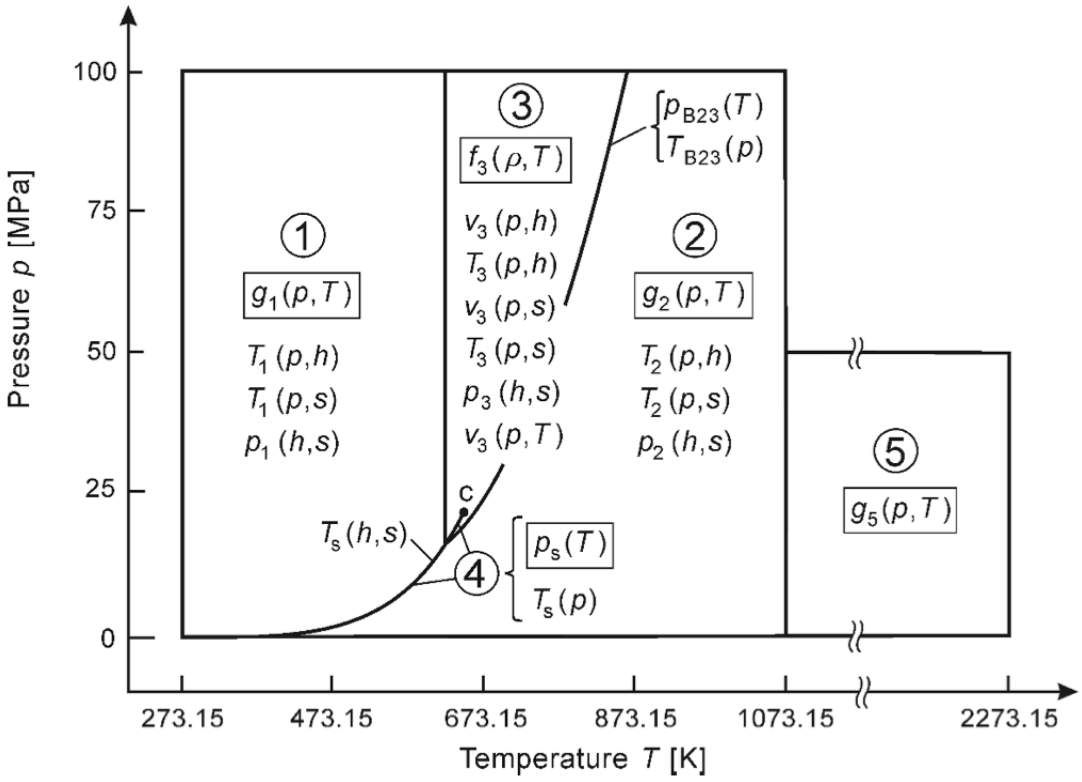

Figure 2, Regions and equations of the industrial formulation IAPWS-IF97 [9]

2.3 IAPWS Transport Properties

The water/steam transport properties, such as thermal conductivity, viscosity and surface tension, are also based on IAPWS releases [6][7][8]. The validity region for the thermal conductivity and viscosity are from 0 to 1000 MPa for pressure, 273.16 to 1173.15 K for temperature, respectively. The surface tension validity region covers the entire vapor-liquid saturation line, from the triple point up to the critical point.

3. IMPLEMENTATION AND VERIFICATION

In this section, we will provide details on the implementation of the IAPWS water/steam thermodynamic properties and transport properties. This includes the fundamental functions provided in

IAPWS releases, as well as necessary intermediate interface functions. The purpose of this work is to provide a portable stand-alone software package, which can be easily maintained and embedded into any computer code. The software package is designed to write with C++, using namespaces to group functions for IAPWS-95, IAPWS-IF97, IAPWS transport properties and interfaces. The package is maintained and controlled by using software tools CMake and git. The tool, git, provides a perfect way to track and control the source codes. The tool, CMake, along its testing capability, provides an environment to verify the code development.

3.1 Code implementation

The IAPWS-95Rev namespace implements the IAPWS-95 reversion released in 2009. However, since this reversion only had minor changes, i.e., two coefficients were improved. Almost all code implementations are based on the IAPWS-95 document.

The code implementation of this namespace is divided into 4 parts:

Part 1, formulations of Helmholtz free energy and its derivatives;

Part 2, formulations of thermodynamic properties based on IAPWS-95Rev formulation;

Part 3, formulations of thermodynamic properties on the saturation line, temperature as input parameter;

Part 4, formulations of thermodynamic properties on the saturation line, pressure as input parameter.

Part 1 includes the implementation of Helmholtz free energy and its derivatives. Part 2 includes the implementation of thermodynamic properties, such as pressure, entropy, internal energy and etc., using functions provided in part 1. For both parts, readers are referred to sections 6.1 and 6.2 of reference [4] for details. Parts 3 and 4 provide the vapor-liquid saturation line functions, as well as necessary derivatives along the saturation line. As mentioned previously, auxiliary equations were used for the calculation for the saturation line, instead of the three non-linear equilibrium equations. As stated earlier, these auxiliary equations have extremely small difference compared to the IAPWS95 non-linear equilibrium equations, however they are not thermodynamically consistent with IAPWS-95 formulation. The validity and uncertainty of this set of equations were provided in reference [11], however some of the validations were based on early data in 1980s. To confirm its validity, a comparison was done between the accuracy of these auxiliary equations and the formulations provided in the IAPWS-IF97, using the IAPWS-95 phase equilibrium results as reference 'exact' values. The results are shown in Figure 3. In most of the region, the auxiliary equations show smaller magnitude of relative error compared to the IAPWS-IF97 formulation. As stated in IAPWS-IF97 document, its saturation properties are within measurement uncertainties. Combined with the results observed in Figure 3 and statement made in IAPWS-IF97, it is safe to use this set of auxiliary equations for saturation line properties.

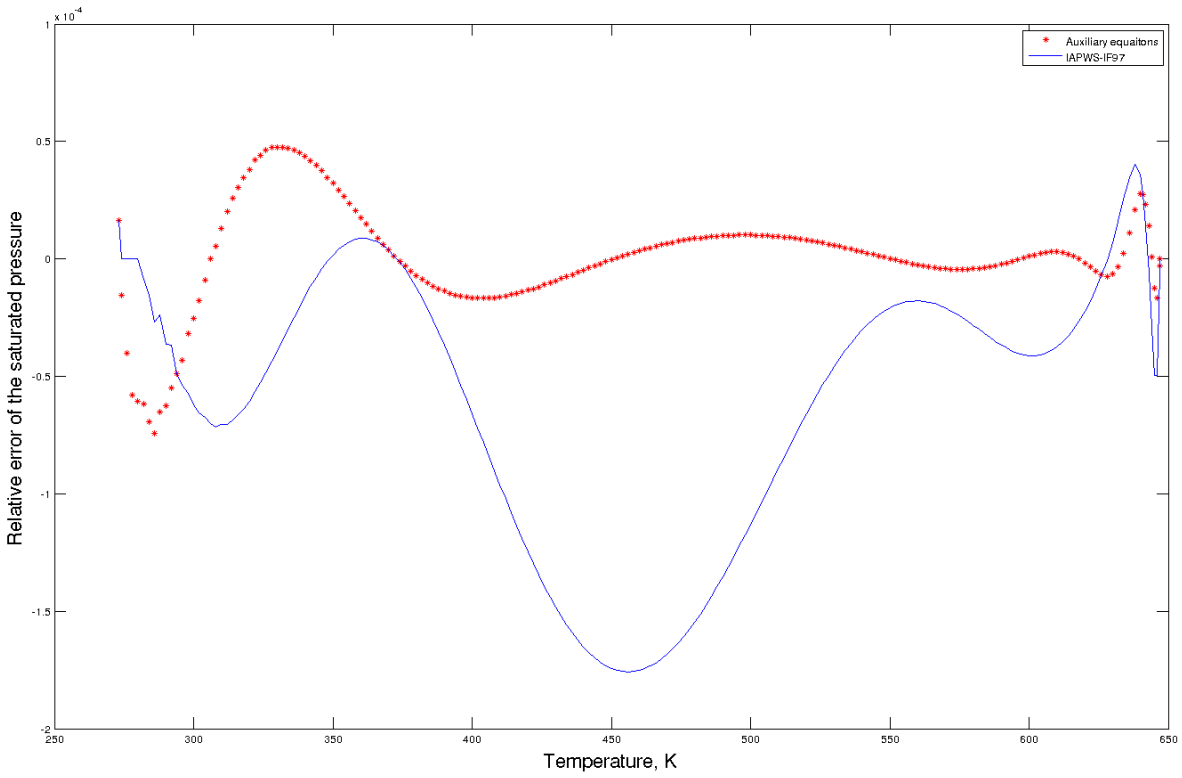

Figure 3, Relative errors of the saturated pressure, calculated from auxiliary equations and IAPWS-IF97 formulation, compared with IAPWS-95 "exact" values

The IAPWS-IF97 namespace implements the 1997 industrial formulation release. The code implementation of this namespace is divided into 7 parts:

Part 1, boundary line between region 2 and 3;

Part 2, region 1, using Gibbs free energy $g_1(p, T)$ as the basis function;

Part 3, region 2, using Gibbs free energy $g_2(p, T)$ as the basis function;

Part 4, supplemental metastable vapor region;

Part 5, region 3, using Helmholtz free energy $f_3(p, T)$ as the basis function;

Part 6, region 4, the saturation line;

Part 7, region 5, using Gibbs free energy $g_5(p, T)$ as the basis function.

The implementations are straightforward, thus interested readers are referred to Chapter 2 of Part A in reference [9] for details. The implementations of transport properties are included in the IAPWS-TRANS namespace, and interested readers are referred to references [6], [7], and [8] for details.

The direct call on IAPWS water properties will be expensive as it is explained previously. For application use, an efficient interface has to be designed to reduce computational cost on water properties evaluations. In RELAP5, the IAPWS water properties are prepared in tabulated form during its installation [3]. A same strategy will be used. This requires the implementations of iterative 'backward'

equations and tabulated 'backward' equations, which is implemented in the WaterPropertiesInterface namespace. However, it should be noted that these interfaces are application specific. For example, in fluid flow system, an equation system with unknown set {density, momentum and total energy} will require a different interface than an equation system with unknown set {pressure, velocity and temperature}. Therefore, we are not intended to cover every possible interface in the WaterPropertiesInterface namespace. Only several commonly used interfaces will be covered in order to test the interface design, numerical design, and interface efficiency. More specific interfaces are expected to implement according to user's specific requirement. We will use density as an example to illustrate the implementation of iterative and tabulated 'backward' equation, which will calculate the density from given pressure and temperature. As the basic IAPWS formulation calculates pressure from given density and temperature, an iterative method (an optimized hybrid Secant and bisection method in this work) is used to find the density if pressure and temperature are given. For tabulated method, a pre-generated p-T-ρ is prepared and an interpolation method (such as bilinear and bicubic) could be used to calculate the density from given pressure and temperature quickly. The same method could be used for interface functions, such as (p, h) or (p, s) pair is given.

Appendix A summarizes the properties function interfaces currently available in the package. It includes the straightforward implementations of available IAPWS functions, as well as limited user-friendly 'backward' functions. More such user-friendly functions will be necessary for specific application. However, in this package we are not intended to cover every possible function interface, which we would like to leave for future work.

3.2 Code verification

The code verification in this work includes three main parts, code verification of IAPWS water/steam properties, accuracy verification of tabulated 'backward' equations, and code efficiency of tabulated 'backward' equations. The accuracy of the iterative 'backward' equation is not necessary as its accuracy is controlled as input. The code verification of IAPWS water/steam properties includes 12 tests, which is straightforward as all IAPWS releases provide verification guidelines. These 12 tests are summarized in Table 1, and more detailed information is provided in the Appendix B.

A pre-generated p-T-ρ matrix was prepared using the iterative method, which is then used perform verification task for the accuracy and efficiency of the tabulated 'backward' equations. This pre-generated matrix uses non-uniform grid size for both temperature and pressure. For temperature, the entire validity region is divided into two regions with grid points at {273.16, 1200, 2000} K, and with grid sizes {1, 5} K in these two regions. For pressure, the entire validity region is divided into five regions with grid points at {611.655 Pa, 0.5 MPa, 20 MPa, 25 MPa, 40 MPa, 100 MPa}, and with grid sizes {0.001 MPa, 0.05 MPa, 0.01 MPa, 0.1 MPa, 1E MPa} in these five regions. A total of 10^5 (p, T) pairs were used to verify the accuracy and efficiency of the tabulated 'backward' equations. The pressure has 100 points, starting from 0.01 MPa with an increasing step of 0.5 MPa. The temperature has 1000 points, starting from 273.16 K with an increasing step of 0.7 K. Therefore, in total 10^5 (p, T) pairs were obtained.

Figure 4 and Figure 5 show the relative error of the tabulated 'backward' equation using bilinear interpolation and bicubic interpolation, compared to the values calculated from iterative method, respectively. The results show that both bilinear and bicubic method give reasonable accuracy in the order of 10^{-4}. The bicubic method shows much better accuracy compared to the bilinear method. It should also be noted that, there are several noticeable peaks in the error plot, suggesting future grid refinement in these regions might be necessary. The efficiency of the interpolation methods have also been tested and verified. A random (p, T) pair, (4.56789 MPa, 400.23456 K), was picked for density evaluation. The density evaluation was repeated 10^5 times to record the CPU time cost. Table 2 shows the CPU time costs of three methods, namely the iterative method, bilinear interpolation and bicubic interpolation. (Note, the

absolute speed is not of interest when comparing the relative efficiency, so that the hardware is irrelevant and not listed). It is clear that the interpolation method is ~3 orders faster and the results verified the efficiency of interpolation methods. However, special treatment will be necessary close to the saturation line region, which will require more sophisticated interpolation scheme.

Table 1, List of 12 verification tests for IAPWS water/steam properties

Name	Description	Verified From
Test 01	IAPWS-95Rev Helmholtz free energy and its derivatives	Table 6 of reference [5]
Test 02	IAPWS-95Rev single-phase water/steam thermodynamic properties	Table 7 of reference [5]
Test 03	Auxiliary equations for saturation line	Table 1 of reference [11]
Test 04	Thermal conductivity	Table 4 and 5 of reference [6]
Test 05	Viscosity	Table 4 of reference [7]
Test 06	Surface tension	Table 1 of reference [8]
Test 07	IAPWS-IF97 region 1	Table 2.5 of reference [9]
Test 08	IAPWS-IF97 region 2	Table 2.11 of reference [9]
Test 09	IAPWS-IF97 metastable vapor region	Table 2.14 of reference [9]
Test 10	IAPWS-IF97 region 3	Table 2.18 of reference [9]
Test 11	IAPWS-IF97 region 4	Table 2.20 and 2.21 of reference [9]
Test 12	IAPWS-IF97 region 5	Table 2.27 of reference [9]

Table 2, CPU time cost of different 'backward' equations

Method	CPU Time (s)
Iterative method	3.451×10^{1}
Bilinear interpolation	2.241×10^{-2}
Bicubic interpolation	8.896×10^{-2}

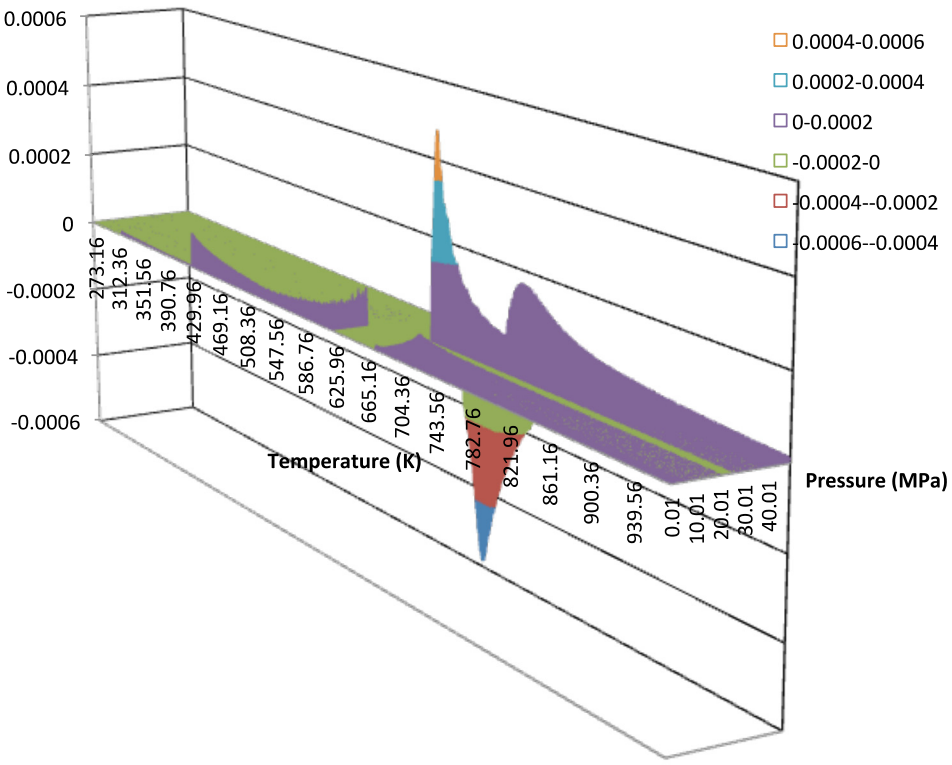

Figure 4, Relative error using bilinear interpolation

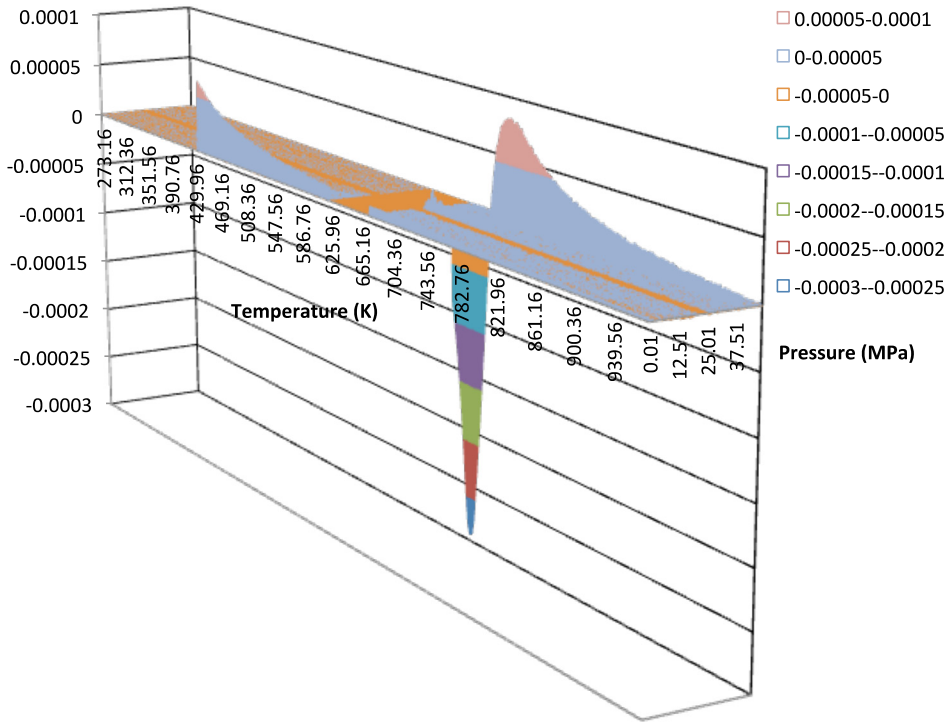

Figure 5, Relative error using bicubic interpolation

4. SUMMARY

The purpose of this work is to provide a stand-alone C++ software package for accurate and efficient water/steam properties evaluation. The implementations and code verification of IAPWS water/steam properties were accomplished. Interface functions, such as the 'backward' equations, were also designed and verified in terms of accuracy and efficiency. This software package will be a useful support for codes development, such as reactor safety analysis codes.

5. REFERENCE

1. The International Association for the Properties of Water and Steam, http://www.iapws.org

2. A. H. Harvey and E. W. Lemmon, "NIST/ASME Steam Properties—STEAM, Version 3.0, User's Guide", National Institute of Standards and Technology, 2013

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6. "Release on the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Substance", International Association for the Properties of Water and Steam, 2011

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11. W. Wagner and A. Pruss, "International Equations for the Saturation Properties of Ordinary Water Substance. Revised According to the International Temperature Scale of 1990", *J. Phys. Chem. Ref. Data*, **Vol.** 22, **No.** 3, 1993

12. Thermophysical Properties of Fluid Systems, http://webbook.nist.gov/chemistry/fluid/

Appendix A

Available Functions List

IAPWS-95Rev

$p(\rho, T)$	Pressure
$s(\rho, T)$	Entropy
$h(\rho, T)$	Enthalpy
$u(\rho, T)$	Internal energy
$g(\rho, T)$	Gibbs free energy
$c_v(\rho, T)$	Isochoric heat capacity
$c_p(\rho, T)$	Isobaric heat capacity
$c_\sigma(\rho, T)$	Saturated liquid heat capacity
$w(\rho, T)$	Speed of sound
$\mu(\rho, T)$	Joule–Thomson coefficient
$\delta_T(\rho, T)$	Isothermal throttling coefficient
$\beta_s(\rho, T)$	Isentropic temperature-pressure
$F(\rho, T)$	Fugacity
$f(\rho, T)$	Helmholtz free energy

IAPWS-95Rev Auxiliary Equations

$p_s(T)$	Saturation pressure
$T_s(p)$	Saturation temperature
$\frac{dp_s}{dT}(T); \frac{dp_s}{dT}(p)$	The derivative of saturation pressure with respect to temperature
$\rho_{l,s}(T); \rho_{l,s}(p)$	Liquid phase saturation density
$\rho_{v,s}(T); \rho_{v,s}(p)$	Vapor phase saturation density
$h_{l,s}(T); h_{l,s}(p)$	Liquid phase saturation enthalpy
$h_{v,s}(T); h_{v,s}(p)$	Vapor phase saturation enthalpy
$u_{l,s}(T); u_{l,s}(p)$	Liquid phase saturation internal energy
$u_{v,s}(T); u_{v,s}(p)$	Vapor phase saturation internal energy
$s_{l,s}(T); s_{l,s}(p)$	Liquid phase saturation entropy
$s_{v,s}(T); s_{v,s}(p)$	Vapor phase saturation entropy
$h_{lv}(T); h_{lv}(p)$	Latent heat

Transport Properties

$k(\rho, T)$	Thermal conductivity
$\mu(\rho, T)$	Viscosity
$\sigma(T)$	Surface tension

IAPWS-IF97 Regions 1,2,5

$v(p, T)$	Specific volume
$h(p, T)$	Enthalpy
$u(p, T)$	Internal energy
$s(p, T)$	Entropy
$c_p(p, T)$	Isobaric heat capacity
$c_v(p, T)$	Isochoric heat capacity

$w(p, T)$	Speed of sound
$\alpha_v(p, T)$	Isobaric cubic expansion coefficient
$\kappa_T(p, T)$	Isothermal compressibility

IAPWS-IF97 Region 3

$p(\rho, T)$	Pressure
$h(\rho, T)$	Enthalpy
$u(\rho, T)$	Internal energy
$s(\rho, T)$	Entropy
$c_p(\rho, T)$	Isobaric heat capacity
$c_v(p, T)$	Isochoric heat capacity
$w(\rho, T)$	Speed of sound
$\alpha_v(\rho, T)$	Isobaric cubic expansion coefficient
$\kappa_T(\rho, T)$	Isothermal compressibility
$\alpha_p(\rho, T)$	Relative pressure coefficient
$\kappa_p(\rho, T)$	Isothermal stress coefficient

IAPWS-IF97 Region 4

$p_s(T)$	Saturation pressure
$T_s(p)$	Saturation temperature

Backward Equations

$\rho(p, T)$	Density
$T(p, h); T(p, s); T(p, u)$	Temperature
$p(T, s)$	Pressure

* Helper functions, such as intermediate derivatives, are not listed.

Appendix B

Verification Tests Outputs

Test 01, IAPWS-95Rev Helmholtz free energy and its derivatives

Reference data [5]

Table 6. Values for the ideal-gas part ϕ^o, Eq. (5), and for the residual part ϕ^r, Eq. (6), of the dimensionless Helmholtz free energy together with the corresponding derivatives[a] for $T = 500$ K and $\rho = 838.025$ kg m^{-3}

ϕ^o	$= 0.204\,797\,733 \times 10^1$	ϕ^r	$= -0.342\,693\,206 \times 10^1$
ϕ^o_δ	$= 0.384\,236\,747$	ϕ^r_δ	$= -0.364\,366\,650$
$\phi^o_{\delta\delta}$	$= -0.147\,637\,878$	$\phi^r_{\delta\delta}$	$= 0.856\,063\,701$
ϕ^o_τ	$= 0.904\,611\,106 \times 10^1$	ϕ^r_τ	$= -0.581\,403\,435 \times 10^1$
$\phi^o_{\tau\tau}$	$= -0.193\,249\,185 \times 10^1$	$\phi^r_{\tau\tau}$	$= -0.223\,440\,737 \times 10^1$
$\phi^o_{\delta\tau}$	$= 0$	$\phi^r_{\delta\tau}$	$= -0.112\,176\,915 \times 10^1$

[a] For the abbreviated notation of the derivatives of ϕ^o and ϕ^r see the footnotes of Tables 4 and 5, respectively.

Code output:

```
      The following 12 quantities are (using Latex syntax):
      (1), \phi^0; (2), \phi^0_\delta; (3), \phi^0_{\delta\delta}; (4),  \phi^0_\tau; (5),
\phi^0_{\tau\tau}; (6),  \phi^0_{\delta\tau}
      (7), \phi^r; (8), \phi^r_\delta; (9), \phi^r_{\delta\delta}; (10), \phi^r_\tau; (11),
\phi^r_{\tau\tau}; (12), \phi^r_{\delta\tau}
      #
            2.04797733e+00
            3.84236747e-01
           -1.47637878e-01
            9.04611106e+00
           -1.93249185e+00
            0.00000000e+00
      #
           -3.42693206e+00
           -3.64366650e-01
            8.56063701e-01
           -5.81403435e+00
           -2.23440737e+00
           -1.12176915e+00
```

Test 02, IAPWS-95Rev single-phase water/steam thermodynamic properties

Reference data [5]

Table 7. Thermodynamic property values in the single-phase region for selected values of T and ρ

T/K	ρ/ (kg m^{-3})	p/MPa	c_v /(kJ kg^{-1} K^{-1})	w/(m s^{-1})	s/(kJ kg^{-1} K^{-1})
300	$0.996\,556\,0 \times 10^{3}$	$0.992\,418\,352 \times 10^{-1}$ [a]	$0.413\,018\,112 \times 10^{1}$	$0.150\,151\,914 \times 10^{4}$	0.393 062 643
	$0.100\,530\,8 \times 10^{4}$	$0.200\,022\,515 \times 10^{2}$	$0.406\,798\,347 \times 10^{1}$	$0.153\,492\,501 \times 10^{4}$	0.387 405 401
	$0.118\,820\,2 \times 10^{4}$	$0.700\,004\,704 \times 10^{3}$	$0.346\,135\,580 \times 10^{1}$	$0.244\,357\,992 \times 10^{4}$	0.132 609 616
500	0.435 000 0	$0.999\,679\,423 \times 10^{-1}$	$0.150\,817\,541 \times 10^{1}$	$0.548\,314\,253 \times 10^{3}$	$0.794\,488\,271 \times 10^{1}$
	$0.453\,200\,0 \times 10^{1}$	0.999 938 125	$0.166\,991\,025 \times 10^{1}$	$0.535\,739\,001 \times 10^{3}$	$0.682\,502\,725 \times 10^{1}$
	$0.838\,025\,0 \times 10^{3}$	$0.100\,003\,858 \times 10^{2}$	$0.322\,106\,219 \times 10^{1}$	$0.127\,128\,441 \times 10^{4}$	$0.256\,690\,919 \times 10^{1}$
	$0.108\,456\,4 \times 10^{4}$	$0.700\,000\,405 \times 10^{3}$	$0.307\,437\,693 \times 10^{1}$	$0.241\,200\,877 \times 10^{4}$	$0.203\,237\,509 \times 10^{1}$
647	$0.358\,000\,0 \times 10^{3}$	$0.220\,384\,756 \times 10^{2}$	$0.618\,315\,728 \times 10^{1}$	$0.252\,145\,078 \times 10^{3}$	$0.432\,092\,307 \times 10^{1}$
900	0.241 000 0	0.100 062 559	$0.175\,890\,657 \times 10^{1}$	$0.724\,027\,147 \times 10^{3}$	$0.916\,653\,194 \times 10^{1}$
	$0.526\,150\,0 \times 10^{2}$	$0.200\,000\,690 \times 10^{2}$	$0.193\,510\,526 \times 10^{1}$	$0.698\,445\,674 \times 10^{3}$	$0.659\,070\,225 \times 10^{1}$
	$0.870\,769\,0 \times 10^{3}$	$0.700\,000\,006 \times 10^{3}$	$0.266\,422\,350 \times 10^{1}$	$0.201\,933\,608 \times 10^{4}$	$0.417\,223\,802 \times 10^{1}$

[a] In the liquid-water region at low pressures small changes in density along an isotherm cause large changes in pressure. For this reason, due to an accumulation of small errors, a particular computer code or a particular PC may fail to reproduce the pressure value with nine significant figures.

Code output:

```
      This test verifies the IAPWS95Rev single phase thermodynamic properties.
      The Results can be verified from Reference [5], page 14, Table 7.
      #
T/(K)          rho/(kg/m^3)   p/(MPa)        Cv/(kJ/kg-K)   w/(m/s)        s/(kJ/kg-K)
3.00000000e+02 9.96556000e+02 9.92418352e-02 4.13018112e+00 1.50151914e-03 3.93062643e-01
3.00000000e+02 1.00530800e+03 2.00022515e+01 4.06798347e+00 1.53492501e-03 3.87405401e-01
3.00000000e+02 1.18820200e+03 7.00004704e+02 3.46135580e+00 2.44357992e-03 1.32609616e-01
5.00000000e+02 4.35000000e-01 9.99679423e-02 1.50817541e+00 5.48314253e-04 7.94488271e+00
5.00000000e+02 4.53200000e+00 9.99938125e-01 1.66991025e+00 5.35739001e-04 6.82502725e+00
5.00000000e+02 8.38025000e+02 1.00003858e+01 3.22106219e+00 1.27128441e-03 2.56690919e+00
5.00000000e+02 1.08456400e+03 7.00000405e+02 3.07437693e+00 2.41200877e-03 2.03237509e+00
6.47000000e+02 3.58000000e+02 2.20384756e+01 6.18315728e+00 2.52145078e-04 4.32092307e+00
9.00000000e+02 2.41000000e-01 1.00062559e-01 1.75890657e+00 7.24027147e-04 9.16653194e+00
9.00000000e+02 5.26150000e+01 2.00000690e+01 1.93510526e+00 6.98445674e-04 6.59070225e+00
9.00000000e+02 8.70769000e+02 7.00000006e+02 2.66422350e+00 2.01933608e-03 4.17223802e+00
```

Test 03, Auxiliary equations for saturation line

Reference data [11]

Table 1. Thermodynamic property values calculated at three selected temperatures

	T = 273.16 K	T = 373.1243 K	T = 647.096
p/Pa	611.657	0.101325×10^6	22.064×10^6
(dp/dT)/(Pa K^{-1})	44.436693	3.616×10^3	268×10^3
ρ'/(kg m^{-3})	999.789	958.365	322
ρ''/(kg m^{-3})	0.00485426	0.597586	322
α/(J kg^{-1})	−11.529101	417.65×10^3	1548×10^3
h'/(J kg^{-1})	0.611786	419.05×10^3	2086.6×10^3
h''/(J kg^{-1})	2500.5×10^3	2675.7×10^3	2086.6×10^3
ϕ/(J kg^{-1}K^{-1})	− 0.04	1.303×10^3	3.578×10^3
s'/(J kg^{-1}K^{-1})	0	1.307×10^3	4.410×10^3
s''/(J kg^{-1}K^{-1})	9.154×10^3	7.355×10^3	4.410×10^3

Code output:

```
The listed data are:
p [Pa]
dp/dT [Pa / K]
rho_liquid_sat [kg / m^3]
rho_vapor_sat [kg / m^3]
alpha [J / kg]
enthalpy_liquid_sat [J / kg]
enthalpy_vapor_sat [J / kg]
phi [J / kg-K]
entropy_liquid_sat [J / kg-K]
entropy_vapor_sat [J / kg-K]
#
T = 273.16 [K]               T = 3.7312430e+02 [K]        T = 6.4709600e+02 [K]
 6.1165707E+02               1.0132502E+05                2.2064000E+07
 4.4436693E+01               3.6164725E+03                2.6798559E+05
 9.9978913E+02               9.5836523E+02                3.2200000E+02
 4.8542626E-03               5.9758632E-01                3.2200000E+02
-1.1529101E+01               4.1764641E+05                1.5480264E+06
 6.1178616E-01               4.1905443E+05                2.0865742E+06
 2.5005386E+06               2.6757198E+06                2.0865742E+06
-4.4453067E-02               1.3031386E+03                3.5781794E+03
-7.0017867E-06               1.3069122E+03                4.4104328E+03
 9.1541148E+03               7.3549380E+03                4.4104328E+03
```

Test 04, Thermal conductivity

Reference data [6]

Table 4. Sample points for computer-program verification of the correlating equation, Eq. (15). At these points, $\bar{\lambda}_2 = 0$.

T (K)	ρ (kg·m^{-3})	λ (mW·m^{-1}·K^{-1})
298.15	0	18.434 188 3
298.15	998	607.712 868
298.15	1200	799.038 144
873.15	0	79.103 465 9

Note: Some derivatives from the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use [3] diverge at $\rho = 0$; for those points in Table 4 $\bar{\lambda}_2$ must be set to zero. For the liquid points at 298.15 K, $\Delta\bar{\chi}$ calculated from Eq. (23) is less than zero, so (as stated in Sec. 2.7) it must be set to zero for calculations to proceed and $\bar{\lambda}_2 = 0$.

Table 5. Sample points for computer-program verification of the correlating equation, Eq. (15), including the critical-enhancement contribution $\bar{\lambda}_2$. For all points, $\bar{\lambda}_0$ (647.35 K) = 51.576 479 7.

T (K)	ρ (kg·m^{-3})	$\bar{\lambda}_1$	$\bar{\lambda}_2$	λ (mW·m^{-1}·K^{-1})
647.35	1	1.006 849 7	0.000 130 0	51.929 892 4
647.35	122	2.144 517 3	20.316 232 0	130.922 885
647.35	222	3.484 073 6	188.091 206	367.787 459
647.35	272	4.223 370 8	540.133 176	757.959 776
647.35	322	4.968 195 3	1187.513 54	1443.755 56
647.35	372	5.696 125 0	356.533 33	650.319 402
647.35	422	6.397 342 9	118.931 062	448.883 487
647.35	750	11.587 053 2	3.341 930 3	600.961 346

Code output:

```
This test verifies the thermal conductivity.
The Results can be verified from Reference [6], page 10, Table 4 and 5.
#
              T/(K)         rho/(kg/m^3)          k/(mW/m-K)
      2.98150000e+02       0.00000000e+00       1.84341883e+01
      2.98150000e+02       9.98000000e+02       6.07712868e+02
      2.98150000e+02       1.20000000e+03       7.99038144e+02
      8.73150000e+02       0.00000000e+00       7.91034659e+01
      6.47350000e+02       1.00000000e+00       5.19298924e+01
      6.47350000e+02       1.22000000e+02       1.30922885e+02
      6.47350000e+02       2.22000000e+02       3.67787459e+02
      6.47350000e+02       2.72000000e+02       7.57959776e+02
      6.47350000e+02       3.72000000e+02       6.50319402e+02
      6.47350000e+02       4.22000000e+02       4.48883487e+02
      6.47350000e+02       7.50000000e+02       6.00961346e+02
```

Test 05, Viscosity

Reference data [7]

Table 4. Sample points for computer-program verification of the correlating equation, Eq. (10), with $\bar{\mu}_2$=1.

T (K)	ρ (kg·m^{-3})	μ (μPa·s)
298.15	998	889.735100
298.15	1200	1437.649467
373.15	1000	307.883622
433.15	1	14.538324
433.15	1000	217.685358
873.15	1	32.619287
873.15	100	35.802262
873.15	600	77.430195
1173.15	1	44.217245
1173.15	100	47.640433
1173.15	400	64.154608

Code output:

```
This test verifies the viscosity.
The Results can be verified from Reference [7], page 8, Table 4.
#
     2.981500000e+02     9.980000000e+02     8.897351001e+02
     2.981500000e+02     1.200000000e+03     1.437649467e+03
     3.731500000e+02     1.000000000e+03     3.078836223e+02
     4.331500000e+02     1.000000000e+00     1.453832449e+01
     4.331500000e+02     1.000000000e+03     2.176853583e+02
     8.731500000e+02     1.000000000e+00     3.261928697e+01
     8.731500000e+02     1.000000000e+02     3.580226172e+01
     8.731500000e+02     6.000000000e+02     7.743019529e+01
     1.173150000e+03     1.000000000e+00     4.421724451e+01
     1.173150000e+03     1.000000000e+02     4.764043308e+01
     1.173150000e+03     4.000000000e+02     6.415460785e+01
```

Test 06, Surface tension

Reference data [8]

Table 1 Surface Tension of Ordinary Water Substance

(1) Temp. t, °C	(2) Surf. Tension experimental σ mN/m	(3) Uncertainty Δσ mN/m	(4) Surf. Tension calculated σ_{calc} mN/m	(5) difference $\sigma_{calc}-\sigma$ mN/m
0.01	75.64	0.38	75.65	0.01
5	74.94	0.37	74.94	0.00
10	74.23	0.37	74.22	- 0.01
15	73.49	0.37	73.49	- 0.00
20	72.74	0.36	72.74	- 0.00
25	71.98	0.36	71.97	- 0.01
30	71.19	0.36	71.19	0.00
35	70.41	0.35	70.40	- 0.01
40	69.59	0.35	69.60	0.01
45	68.78	0.34	68.78	- 0.00
50	67.93	0.34	67.94	0.01
55	67.09	0.34	67.10	0.01
60	66.24	0.33	66.24	- 0.00
65	65.36	0.33	65.37	0.01
70	64.47	0.32	64.48	0.01
75	63.57	0.32	63.58	0.01
80	62.68	0.31	62.67	- 0.01
85	61.76	0.31	61.75	- 0.01
90	60.82	0.30	60.82	- 0.00
95	59.88	0.30	59.87	- 0.01
100	58.92	0.29	58.91	- 0.01
105	57.95	0.29	57.94	- 0.01
110	56.97	0.28	56.96	- 0.01
115	55.98	0.28	55.97	- 0.01
120	54.97	0.27	54.97	- 0.00
125	53.96	0.27	53.96	- 0.00
130	52.94	0.26	52.93	- 0.01
135	51.90	0.26	51.90	- 0.00
140	50.86	0.25	50.86	- 0.00
145	49.81	0.25	49.80	- 0.01
150	48.75	0.24	48.74	- 0.01
155	47.67	0.24	47.67	0.00
160	46.58	0.23	46.59	0.01
165	45.49	0.23	45.50	0.01
170	44.40	0.22	44.41	0.01
175	43.30	0.22	43.30	0.00
180	42.19	0.22	42.19	0.00
185	41.07	0.22	41.07	0.00
190	39.95	0.22	39.95	- 0.00
195	38.82	0.22	38.81	- 0.01
200	37.68	0.22	37.67	- 0.01

Table 1 continued

(1) Temp. t, °C	(2) Surf. Tension experimental σ mN/m	(3) Uncertainty Δσ mN/m	(4) Surf. Tension calculated σ_{calc} mN/m	(5) difference $\sigma_{calc}-\sigma$ mN/m
205	36.54	0.22	36.53	- 0.01
210	35.40	0.22	35.38	- 0.02
215	34.24	0.22	34.23	- 0.01
220	33.09	0.22	33.07	- 0.02
225	31.92	0.22	31.90	- 0.02
230	30.76	0.22	30.74	- 0.02
235	29.58	0.22	29.57	- 0.01
240	28.40	0.22	28.39	- 0.01
245	27.22	0.22	27.22	- 0.00
250	26.05	0.22	26.04	- 0.01
255	24.86	0.21	24.87	0.01
260	23.66	0.21	23.69	0.03
265	22.46	0.21	22.51	0.05
270	21.29	0.20	21.34	0.05
275	20.14	0.20	20.16	0.02
280	18.93	0.20	18.99	0.06
285	17.76	0.19	17.83	0.07
290	16.60	0.19	16.66	0.06
295	15.45	0.19	15.51	0.06
300	14.30	0.18	14.36	0.06
305	13.18	0.18	13.22	0.04
310	12.04	0.17	12.09	0.05
315	10.92	0.16	10.97	0.05
320	9.81	0.16	9.86	0.05
325	8.73	0.15	8.77	0.04
330	7.66	0.14	7.70	0.04
335	6.61	0.13	6.65	0.04
340	5.59	0.12	5.63	0.04
345	4.60	0.11	4.63	0.03
350	3.64	0.10	3.67	0.03
355	2.74	0.10	2.74	0.00
360	1.89	0.10	1.88	- 0.01
365	1.12	0.10	1.08	- 0.04
370	0.45	0.10	0.39	- 0.06

Code output:

```
This test verifies the surface tension.
The Results can be verified from Reference [x], page 3-4, Table 1.
#
              T/(C) surf tension/(mN/m)
             0.0101           7.565e+01
        5.00000e+00           7.494e+01
        1.00000e+01           7.422e+01
        1.50000e+01           7.349e+01
        2.00000e+01           7.274e+01
        2.50000e+01           7.197e+01
        3.00000e+01           7.119e+01
        3.50000e+01           7.040e+01
        4.00000e+01           6.960e+01
        4.50000e+01           6.878e+01
        5.00000e+01           6.794e+01
        5.50000e+01           6.710e+01
        6.00000e+01           6.624e+01
        6.50000e+01           6.537e+01
        7.00000e+01           6.448e+01
        7.50000e+01           6.358e+01
```

8.00000e+01	6.267e+01
8.50000e+01	6.175e+01
9.00000e+01	6.082e+01
9.50000e+01	5.987e+01
1.00000e+02	5.891e+01
1.05000e+02	5.794e+01
1.10000e+02	5.696e+01
1.15000e+02	5.597e+01
1.20000e+02	5.497e+01
1.25000e+02	5.396e+01
1.30000e+02	5.293e+01
1.35000e+02	5.190e+01
1.40000e+02	5.086e+01
1.45000e+02	4.980e+01
1.50000e+02	4.874e+01
1.55000e+02	4.767e+01
1.60000e+02	4.659e+01
1.65000e+02	4.550e+01
1.70000e+02	4.441e+01
1.75000e+02	4.330e+01
1.80000e+02	4.219e+01
1.85000e+02	4.107e+01
1.90000e+02	3.995e+01
1.95000e+02	3.881e+01
2.00000e+02	3.767e+01
2.05000e+02	3.653e+01
2.10000e+02	3.538e+01
2.15000e+02	3.423e+01
2.20000e+02	3.307e+01
2.25000e+02	3.190e+01
2.30000e+02	3.074e+01
2.35000e+02	2.957e+01
2.40000e+02	2.839e+01
2.45000e+02	2.722e+01
2.50000e+02	2.604e+01
2.55000e+02	2.487e+01
2.60000e+02	2.369e+01
2.65000e+02	2.251e+01
2.70000e+02	2.134e+01
2.75000e+02	2.016e+01
2.80000e+02	1.899e+01
2.85000e+02	1.783e+01
2.90000e+02	1.666e+01
2.95000e+02	1.551e+01
3.00000e+02	1.436e+01
3.05000e+02	1.322e+01
3.10000e+02	1.209e+01
3.15000e+02	1.097e+01
3.20000e+02	9.864e+00
3.25000e+02	8.774e+00
3.30000e+02	7.703e+00
3.35000e+02	6.652e+00
3.40000e+02	5.625e+00
3.45000e+02	4.628e+00
3.50000e+02	3.665e+00
3.55000e+02	2.745e+00
3.60000e+02	1.877e+00
3.65000e+02	1.080e+00
3.70000e+02	3.882e-01

Test 07, IAPWS-IF97 region 1

Reference data [9]

Table 2.5 Thermodynamic property values calculated from the basic equation $g_1(p,T)$, Eq. (2.3), for selected temperatures and pressures [a]

Property	$T = 300$ K $p = 3$ MPa	$T = 300$ K $p = 80$ MPa	$T = 500$ K $p = 3$ MPa
v [m^3 kg^{-1}]	$0.100\,215\,168 \times 10^{-2}$	$0.971\,180\,894 \times 10^{-3}$	$0.120\,241\,800 \times 10^{-2}$
h [kJ kg^{-1}]	$0.115\,331\,273 \times 10^{3}$	$0.184\,142\,828 \times 10^{3}$	$0.975\,542\,239 \times 10^{3}$
u [kJ kg^{-1}]	$0.112\,324\,818 \times 10^{3}$	$0.106\,448\,356 \times 10^{3}$	$0.971\,934\,985 \times 10^{3}$
s [kJ kg^{-1} K^{-1}]	0.392 294 792	0.368 563 852	$0.258\,041\,912 \times 10^{1}$
c_p [kJ kg^{-1} K^{-1}]	$0.417\,301\,218 \times 10^{1}$	$0.401\,008\,987 \times 10^{1}$	$0.465\,580\,682 \times 10^{1}$
c_v [kJ kg^{-1} K^{-1}]	$0.412\,120\,160 \times 10^{1}$	$0.391\,736\,606 \times 10^{1}$	$0.322\,139\,223 \times 10^{1}$
w [m s^{-1}]	$0.150\,773\,921 \times 10^{4}$	$0.163\,469\,054 \times 10^{4}$	$0.124\,071\,337 \times 10^{4}$
α_v [K^{-1}]	$0.277\,354\,533 \times 10^{-3}$	$0.344\,095\,843 \times 10^{-3}$	$0.164\,118\,128 \times 10^{-2}$
κ_T [MPa^{-1}]	$0.446\,382\,123 \times 10^{-3}$	$0.372\,039\,437 \times 10^{-3}$	$0.112\,892\,188 \times 10^{-2}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
#
The listed data are:
specific volume [m^3 / kg]
specific enthalpy [kJ / kg]
specific internal energy [kJ / kg]
specific entropy [kJ / kg-K]
Cp [kJ / kg-K]
Cv [kJ / kg-K]
sound speed [m / s]
alpha_v [1/K]
K_T [1/MPa]
#
T = 300 [K]           T = 3.00000000e+02 [K]       T = 5.00000000e+02 [K]
p = 3e+06             p = 8.00000000e+07           p = 3.00000000e+06
1.00215168E-03        9.71180894E-04               1.20241800E-03
1.15331273E+02        1.84142828E+02               9.75542239E+02
1.12324818E+02        1.06448356E+02               9.71934985E+02
3.92294792E-01        3.68563852E-01               2.58041912E+00
4.17301218E+00        4.01008987E+00               4.65580682E+00
4.12120160E+00        3.91736606E+00               3.22139223E+00
1.50773921E+03        1.63469054E+03               1.24071337E+03
2.77354533E-04        3.44095843E-04               1.64118128E-03
4.46382123E-04        3.72039437E-04               1.12892188E-03
```

Test 08, IAPWS-IF97 region 2

Reference data [9]

Table 2.11 Thermodynamic property values calculated from the basic equation $g_2(p,T)$, Eq. (2.6), for selected temperatures and pressures [a]

Property	$T = 300$ K $p = 0.0035$ MPa	$T = 700$ K $p = 0.0035$ MPa	$T = 700$ K $p = 30$ MPa
v [m^3 kg^{-1}]	$0.394\,913\,866 \times 10^{2}$	$0.923\,015\,898 \times 10^{2}$	$0.542\,946\,619 \times 10^{-2}$
h [kJ kg^{-1}]	$0.254\,991\,145 \times 10^{4}$	$0.333\,568\,375 \times 10^{4}$	$0.263\,149\,474 \times 10^{4}$
u [kJ kg^{-1}]	$0.241\,169\,160 \times 10^{4}$	$0.301\,262\,819 \times 10^{4}$	$0.246\,861\,076 \times 10^{4}$
s [kJ kg^{-1} K^{-1}]	$0.852\,238\,967 \times 10^{1}$	$0.101\,749\,996 \times 10^{2}$	$0.517\,540\,298 \times 10^{1}$
c_p [kJ kg^{-1} K^{-1}]	$0.191\,300\,162 \times 10^{1}$	$0.208\,141\,274 \times 10^{1}$	$0.103\,505\,092 \times 10^{2}$
c_v [kJ kg^{-1} K^{-1}]	$0.144\,132\,662 \times 10^{1}$	$0.161\,978\,333 \times 10^{1}$	$0.297\,553\,837 \times 10^{1}$
w [m s^{-1}]	$0.427\,920\,172 \times 10^{3}$	$0.644\,289\,068 \times 10^{3}$	$0.480\,386\,523 \times 10^{3}$
α_v [K^{-1}]	$0.337\,578\,289 \times 10^{-2}$	$0.142\,878\,736 \times 10^{-2}$	$0.126\,019\,688 \times 10^{-1}$
κ_T [MPa^{-1}]	$0.286\,239\,651 \times 10^{3}$	$0.285\,725\,461 \times 10^{3}$	$0.818\,411\,389 \times 10^{-1}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
#
The listed data are:
specific volume [m^3 / kg]
specific enthalpy [kJ / kg]
specific internal energy [kJ / kg]
specific entropy [kJ / kg-K]
Cp [kJ / kg-K]
Cv [kJ / kg-K]
sound speed [m / s]
alpha_v [1/K]
K_T [1/MPa]
#
T = 300 [K]           T = 7.00000000e+02[K]        T = 7.00000000e+02 [K]
p = 3500              p = 3.50000000e+03           p = 3.00000000e+07
3.94913866E+01        9.23015898E+01               5.42946619E-03
2.54991145E+03        3.33568375E+03               2.63149474E+03
2.41169160E+03        3.01262819E+03               2.46861076E+03
8.52238967E+00        1.01749996E+01               5.17540298E+00
1.91300162E+00        2.08141274E+00               1.03505092E+01
1.44132662E+00        1.61978333E+00               2.97553837E+00
4.27920172E+02        6.44289068E+02               4.80386523E+02
3.37578289E-03        1.42878736E-03               1.26019688E-02
2.86239651E+02        2.85725461E+02               8.18411389E-02
```

Test 09, IAPWS-IF97 metastable vapor region

Reference data [9]

Table 2.14 Thermodynamic property values calculated from the $g_{2,\mathrm{meta}}(p,T)$ equation, Eq. (2.9), for selected values of temperature and pressure [a]

Property	$T = 450$ K $p = 1$ MPa	$T = 440$ K $p = 1$ MPa	$T = 450$ K $p = 1.5$ MPa
v [m^3 kg^{-1}]	0.192 516 540	0.186 212 297	0.121 685 206
h [kJ kg^{-1}]	$0.276\,881\,115 \times 10^4$	$0.274\,015\,123 \times 10^4$	$0.272\,134\,539 \times 10^4$
u [kJ kg^{-1}]	$0.257\,629\,461 \times 10^4$	$0.255\,393\,894 \times 10^4$	$0.253\,881\,758 \times 10^4$
s [kJ kg^{-1} K^{-1}]	$0.656\,660\,377 \times 10^1$	$0.650\,218\,759 \times 10^1$	$0.629\,170\,440 \times 10^1$
c_p [kJ kg^{-1} K^{-1}]	$0.276\,349\,265 \times 10^1$	$0.298\,166\,443 \times 10^1$	$0.362\,795\,578 \times 10^1$
c_v [kJ kg^{-1} K^{-1}]	$0.195\,830\,730 \times 10^1$	$0.208\,622\,142 \times 10^1$	$0.241\,213\,708 \times 10^1$
w [m s^{-1}]	$0.498\,408\,101 \times 10^3$	$0.489\,363\,295 \times 10^3$	$0.481\,941\,819 \times 10^3$
α_v [K^{-1}]	$0.318\,819\,824 \times 10^{-2}$	$0.348\,506\,136 \times 10^{-2}$	$0.418\,276\,571 \times 10^{-2}$
κ_T [MPa^{-1}]	$0.109\,364\,239 \times 10^1$	$0.111\,133\,230 \times 10^1$	0.787 967 952

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
#
The listed data are:
specific volume [m^3 / kg]
specific enthalpy [kJ / kg]
specific internal energy [kJ / kg]
specific entropy [kJ / kg-K]
Cp [kJ / kg-K]
Cv [kJ / kg-K]
sound speed [m / s]
alpha_v [1/K]
K_T [1/MPa]
#
T = 450 [K]           T = 4.40000000e+02 [K]       T = 4.50000000e+02 [K]
p = 1e+06             p = 1.00000000e+06           p = 1.50000000e+06
1.92516540E-01        1.86212297E-01               1.21685206E-01
2.76881115E+03        2.74015123E+03               2.72134539E+03
2.57629461E+03        2.55393894E+03               2.53881758E+03
6.56660377E+00        6.50218759E+00               6.29170440E+00
2.76349265E+00        2.98166443E+00               3.62795578E+00
1.95830730E+00        2.08622142E+00               2.41213708E+00
4.98408101E+02        4.89363295E+02               4.81941819E+02
3.18819824E-03        3.48506136E-03               4.18276571E-03
1.09364239E+00        1.11133230E+00               7.87967952E-01
```

Test 10, IAPWS-IF97 region 3

Reference data [9]

Table 2.18 Thermodynamic property values calculated from the basic equation $f_3(\rho, T)$, Eq. (2.11), for selected temperatures and densities [a]

Property	$T = 650$ K $\rho = 500$ kg m^{-3}	$T = 650$ K $\rho = 200$ kg m^{-3}	$T = 750$ K $\rho = 500$ kg m^{-3}
p [MPa]	$0.255\,837\,018 \times 10^{2}$	$0.222\,930\,643 \times 10^{2}$	$0.783\,095\,639 \times 10^{2}$
h [kJ kg^{-1}]	$0.186\,343\,019 \times 10^{4}$	$0.237\,512\,401 \times 10^{4}$	$0.225\,868\,845 \times 10^{4}$
u [kJ kg^{-1}]	$0.181\,226\,279 \times 10^{4}$	$0.226\,365\,868 \times 10^{4}$	$0.210\,206\,932 \times 10^{4}$
s [kJ kg^{-1} K^{-1}]	$0.405\,427\,273 \times 10^{1}$	$0.485\,438\,792 \times 10^{1}$	$0.446\,971\,906 \times 10^{1}$
c_p [kJ kg^{-1} K^{-1}]	$0.138\,935\,717 \times 10^{2}$	$0.446\,579\,342 \times 10^{2}$	$0.634\,165\,359 \times 10^{1}$
c_v [kJ kg^{-1} K^{-1}]	$0.319\,131\,787 \times 10^{1}$	$0.404\,118\,076 \times 10^{1}$	$0.271\,701\,677 \times 10^{1}$
w [m s^{-1}]	$0.502\,005\,554 \times 10^{3}$	$0.383\,444\,594 \times 10^{3}$	$0.760\,696\,041 \times 10^{3}$
α_v [K^{-1}]	$0.168\,653\,107 \times 10^{-1}$	$0.685\,312\,229 \times 10^{-1}$	$0.441\,515\,098 \times 10^{-2}$
κ_T [MPa^{-1}]	$0.345\,506\,956 \times 10^{-1}$	$0.375\,798\,565$	$0.806\,710\,817 \times 10^{-2}$
α_p [K^{-1}]	$0.190\,798\,153 \times 10^{-1}$	$0.818\,019\,386 \times 10^{-2}$	$0.698\,896\,514 \times 10^{-2}$
β_p [kg m^{-3}]	$0.565\,652\,647 \times 10^{3}$	$0.238\,728\,962 \times 10^{2}$	$0.791\,475\,213 \times 10^{3}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
#
The listed data are:
specific volume [m^3 / kg]
specific enthalpy [kJ / kg]
specific internal energy [kJ / kg]
specific entropy [kJ / kg-K]
Cp [kJ / kg-K]
Cv [kJ / kg-K]
sound speed [m / s]
alpha_v [1/K]
K_T [1/MPa]
#
T = 650 [K]            T = 6.50000000e+02 [K]        T = 7.50000000e+02 [K]
rho = 500 [kg/m^3]     rho = 2.00000000e+02 [kg/m^3] rho = 5.00000000e+02 [kg/m^3]
2.55837018E+01         2.22930643E+01                7.83095639E+01
1.86343019E+03         2.37512401E+03                2.25868845E+03
1.81226279E+03         2.26365868E+03                2.10206932E+03
4.05427273E+00         4.85438792E+00                4.46971906E+00
1.38935717E+01         4.46579342E+01                6.34165359E+00
3.19131787E+00         4.04118076E+00                2.71701677E+00
5.02005554E+02         3.83444594E+02                7.60696041E+02
1.68653107E-02         6.85312229E-02                4.41515098E-03
3.45506956E-02         3.75798565E-01                8.06710817E-03
1.90798153E-02         8.18019386E-03                6.98896514E-03
5.65652647E+02         2.38728962E+01                7.91475213E+02
```

Test 11, IAPWS-IF97 region 4

Reference data [9]

Table 2.20 Saturation-pressure values calculated from Eq. (2.13) for selected temperatures [a]

T [K]	p_s [MPa]
300	$0.353\,658\,941 \times 10^{-2}$
500	$0.263\,889\,776 \times 10^{1}$
600	$0.123\,443\,146 \times 10^{2}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Table 2.21 Saturation-temperature values calculated from Eq. (2.14) for selected pressures [a]

p [MPa]	T_s [K]
0.1	$0.372\,755\,919 \times 10^{3}$
1	$0.453\,035\,632 \times 10^{3}$
10	$0.584\,149\,488 \times 10^{3}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
This test verifies the IAPWS-IF97 region 4, saturation line properties.
The Results can be verified from Reference [9], page 26, Table 2.20 and 2.21.
#
               T [K]         p_sat [MPa]
                 300      3.53658941e-03
                 500      2.63889776e+00
                 600      1.23443146e+01
           p [MPa]           T_sat [K]
     1.00000000e-01      3.72755919e+02
     1.00000000e+00      4.53035632e+02
     1.00000000e+01      5.84149488e+02
```

Test 12, IAPWS-IF97 region 5

Reference data [9]

Table 2.27 Thermodynamic property values calculated from the basic equation $g_5(p,T)$, Eq.(2.15), for selected temperatures and pressures [a]

Property	$T = 1500$ K $p = 0.5$ MPa	$T = 1500$ K $p = 30$ MPa	$T = 2000$ K $p = 30$ MPa
v [m^3 kg^{-1}]	$0.138\,455\,090 \times 10^{1}$	$0.230\,761\,299 \times 10^{-1}$	$0.311\,385\,219 \times 10^{-1}$
h [kJ kg^{-1}]	$0.521\,976\,855 \times 10^{4}$	$0.516\,723\,514 \times 10^{4}$	$0.657\,122\,604 \times 10^{4}$
u [kJ kg^{-1}]	$0.452\,749\,310 \times 10^{4}$	$0.447\,495\,124 \times 10^{4}$	$0.563\,707\,038 \times 10^{4}$
s [kJ kg^{-1} K^{-1}]	$0.965\,408\,875 \times 10^{1}$	$0.772\,970\,133 \times 10^{1}$	$0.853\,640\,523 \times 10^{1}$
c_p [kJ kg^{-1} K^{-1}]	$0.261\,609\,445 \times 10^{1}$	$0.272\,724\,317 \times 10^{1}$	$0.288\,569\,882 \times 10^{1}$
c_v [kJ kg^{-1} K^{-1}]	$0.215\,337\,784 \times 10^{1}$	$0.219\,274\,829 \times 10^{1}$	$0.239\,589\,436 \times 10^{1}$
w [m s^{-1}]	$0.917\,068\,690 \times 10^{3}$	$0.928\,548\,002 \times 10^{3}$	$0.106\,736\,948 \times 10^{4}$
α_v [K^{-1}]	$0.667\,539\,000 \times 10^{-3}$	$0.716\,950\,754 \times 10^{-3}$	$0.508\,830\,641 \times 10^{-3}$
κ_T [MPa^{-1}]	$0.200\,003\,859 \times 10^{1}$	$0.332\,881\,253 \times 10^{-1}$	$0.329\,193\,892 \times 10^{-1}$

[a] Programmed functions should be verified using 8 byte real values for all variables.

Code output:

```
#
The listed data are:
specific volume [m^3 / kg]
specific enthalpy [kJ / kg]
specific internal energy [kJ / kg]
specific entropy [kJ / kg-K]
Cp [kJ / kg-K]
Cv [kJ / kg-K]
sound speed [m / s]
alpha_v [1/K]
K_T [1/MPa]
#
T = 1500 [K]          T = 1.50000000e+03 [K]        T = 2.00000000e+03 [K]
p = 500000            p = 3.00000000e+07            p = 3.00000000e+07
1.38455090E+00        2.30761299E-02                3.11385219E-02
5.21976855E+03        5.16723514E+03                6.57122604E+03
4.52749310E+03        4.47495124E+03                5.63707038E+03
9.65408875E+00        7.72970133E+00                8.53640523E+00
2.61609445E+00        2.72724317E+00                2.88569882E+00
2.15337784E+00        2.19274829E+00                2.39589436E+00
9.17068690E+02        9.28548002E+02                1.06736948E+03
6.67539000E-04        7.16950754E-04                5.08830641E-04
2.00003859E+00        3.32881253E-02                3.29193892E-02
```