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ABSTRACT

Oregon State University (OSU) and the Idaho National Laboratory (INL) are currently collaborating on a test program which entails hydro-mechanical testing of a generic plate type fuel element, or generic test plate assembly (GTPA), for the purpose of qualitatively demonstrating mechanical integrity of uranium-molybdenum monolithic plates as compared to that of uranium aluminum dispersion, and aluminum fuel plates onset by hydraulic forces. This test program supports ongoing work conducted for/by the Global Threat Reduction Initiative (GTRI) Fuels Development Program. This study's focus supports the ongoing collaborative effort by detailing the derivation of an analytic solution for deflection of a heterogeneous plate under a uniform, distributed load in order to predict the deflection of test plates in the GTPA. The resulting analytical solutions for three specific boundary condition sets are then presented against several test cases of a homogeneous plate. In all test cases considered, the results for both homogeneous and heterogeneous plates are numerically identical to one another, demonstrating correct derivation of the heterogeneous solution. Two additional problems are presents herein that provide a representative deflection profile for the plates under consideration within the GTPA. Furthermore, qualitative observations are made about the influence of a more-rigid internal fuel-meat region and its influence on the overall deflection profile of a plate. Present work is being directed to experimentally confirm the analytical solution's results using select materials.

KEYWORDS
Deflection, Pressure, Sandwich Plate

1 INTRODUCTION

The Global Threat Reduction Initiative's (GTRI's) Fuels Development program is presenting working to qualify an ultra-high density low enriched uranium fuel to enable conversion of the present highly enriched fuel that is found in five reaming U.S. research and test reactors. A part of the qualification of such fuel necessitates the demonstration that the integrity of the proposed fuel meets or exceeds the limits of the existing fuel. Numerous studies are presently underway investigating microstructural topics such as fuel-cladding blistering threshold, burnup limits, etc. Several studies are being conducted from a macro-structural perspective as well with the objective of providing sufficient evidence that the proposed fuel is acceptably safe for operation under select conditions. One such study includes hydro-mechanical testing of fuel plates under representative flow conditions that would likely be experienced in-pile; this work is being performed at Oregon State University (OSU) in collaboration with the Idaho National Laboratory (INL).

Oregon State University and INL are currently collaborating on a test program which entails hydro-mechanical testing of a generic plate type fuel element, or generic test plate assembly (GTPA), for the purpose of qualitatively demonstrating mechanical integrity of uranium-molybdenum monolithic plates as compared to that of uranium aluminum dispersion, and aluminum fuel plates onset by hydraulic forces [1].

Herein, an analytical solution is developed for the purpose of predicting the deflection profile that the heterogeneous test plate(s) will undergo when subject to hydraulic forces at OSU. Results from this solution form are presented against several test cases of a homogeneous plate for the purpose of qualitatively validating the derivation of the solution of a heterogeneous plate. Two additional problems are presents herein that provide a representative deflection profile for the plates under consideration within the GTPA. Furthermore, qualitative observations are made about the influence of a more-rigid internal fuel-meat region and its influence on the overall deflection profile of a plate. Present work is being directed to experimentally confirm the analytical solution's results using select materials. The final outcome of this study yields an analytic solution for heterogeneous 'wide-beam' deflection given three explicit edge boundary condition sets; these solution forms may be compared against experimental data to lend additional credibility toward the experimental outcomes.

2 BRIEF SURVEY OF LITERATURE

Numerous studies have been conducted on the topic of deflection of plates (homogeneous and heterogeneous) under static and dynamic conditions. These studies focus on topics including nonlinearity in numerical modeling, methods for convergence of stiff problems, spectral characterization, and many others. The most applicable literature for this study is authored by Bedford [2], which provides an approach from first principles for the analytical solution to this problem.

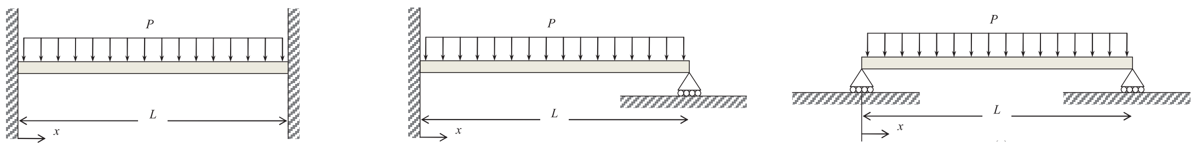
Interest in the deflection of metal plates, both experimentally and analytically, is varied, with most of the focus pertaining to vibrational analysis or other industrial applications. Rama Bhat [3] relates deflection to total energy (strain energy plus potential energy of load distributed) and normalized plate deflection is examined between several modes of the particular solution on a finite length.

Wang [4] represents a normal thin plate (similar to wide beam) theory with an extension for rotary inertia for vibrating plates. In addition, Wang gives an exact relationship between deflection of normal thin plate with and without the included rotary inertia term. Most similar to this study, Wang [4] predicts the two dimensional solution to the problems that are presented in this study, but requires computational tools to determine deflection in the plates. Wang allows the loads to be non-uniform, in contrast to this study, and investigates only a single set of boundary conditions: supports the plates with simply supported corners, rather than edges.

Most recently, Bish [5] applied the principle or rotation-rate continuity to the plastic distortion of a clamped plate of mild-steel deflected by a flat-ended rigid right solid cylinder. Also, Sapountzakis [6] investigated large deflections in plates stiffened by parallel beams. Though this study into increased material stiffness might be applicable, Sapountzakis addresses the nonlinearity that occurs due to plastic deformation. Several assumptions are made herein that limit the deformation to the elastic regime.

3 MODEL AND METHODOLOGY

In this study, three separate boundary conditions are considered for the plates. The plates are modeled as 'wide-beams' with the following boundary conditions, shown in Figure 1(a) as fixed-fixed (F-F), Figure 1(b) fixed-simply supported (F-SS), and Figure 1(c) simply supported-simply supported (SS-SS), respectively.



(a) (b) (c)

Figure 1: Three edge boundary conditions considered herein

Select plates that will be tested in the HMFTF contain a uranium-molybdenum layer sandwiched between aluminum clad (graphically depicted below in the sketch below). When attempting to theoretically characterize this plate for total beam deflection, three regions must be defined. These three regions are displayed in Figure 2.

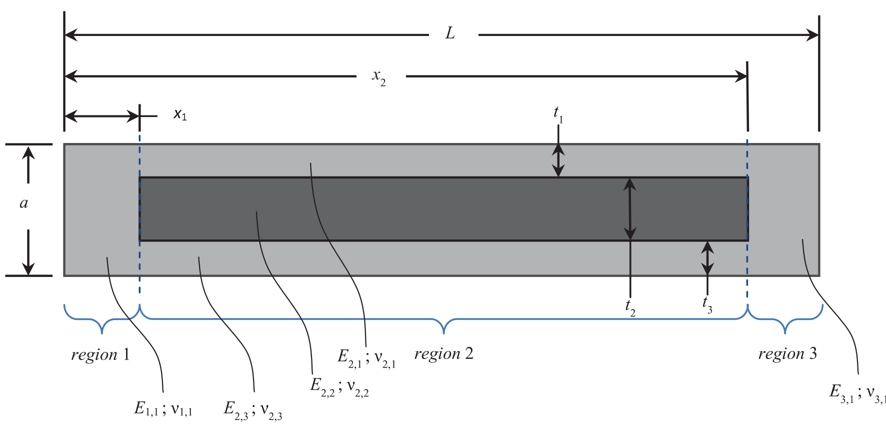


Figure 2: Heterogeneous plate with three distinct regions.

It is necessary to begin the calculation at the most fundamental definition of a uniformly distributed load (w) on a beam, given as:

$$D\frac{d^4y}{dx^4} = w \tag{1}$$

where D is the flexural rigidity of the beam. For this study, flexural rigidity may change between each discrete region of the plate. Using wide-beam theory, flexural rigidity is dependent on a material's modulus of elasticity (E), its thickness (a), and Poisson's ratio (v). For this study, flexural rigidity of the central region of the plate is not homogeneous. From a study performed by Jensen [7, 8], the flexural rigidity of a sandwich structure having three layers may be evaluated by

$$D_2 = \frac{1}{3}\left(\frac{2E_{OR}}{\left(1-\nu_{OR}^2\right)}\left(\frac{3}{4}t_{IR}^2 t_{OR} + \frac{3}{2}t_{OR}^2 t_{IR} + t_{OR}^3\right) + \frac{E_{IR}}{1-\nu_{IR}^2}\frac{t_{IR}^3}{4}\right), \quad (2)$$

where terms for the inner region (subscript *IR*) correspond to subscript 2,2 and terms for outer region (subscript *OR*) correspond to 2,1 and 2,3 in Figure 2. Simplified as a homogeneous plate when $t_{IR} = 0$ and $t_{OR} = a/2$ (half the thickness of the plate per Figure 2), equation (2) reduces to

$$D = \frac{E}{1-\nu^2}\frac{a^3}{12}, \quad (3)$$

which is the fundamentally recognized expression for flexural rigidity of a wide-beam. Additionally, as t_{IR} approaches a and t_{OR} approaches 0, equation (2) reduces to the same equation given in equation (3), however containing material properties appropriate for the inner region.

In order to acquire the analytic relation for out of plane deformation (y) as a function of plate span-width (x), equation (1) must be integrated four times with appropriate boundary conditions applied throughout. By integrating equation (1) once, the following relation is obtained which includes a constant of integration (C_1).

$$D_1 \frac{d^3 y_1}{dx^3} = wx + C_1 \quad (4)$$

This constant, along with all subsequent constants of integration, are solved for through the application of appropriate boundary conditions. The constants, C_1 through C_4 are found based on the specific boundary condition cases considered herein. Integrating (4) once, twice, and three times, yields equations (5), (6), and (7), respectively.

$$D_1 \frac{d^2 y_1}{dx^2} = \frac{wx^2}{2} + C_1 x + C_2 \quad (5)$$

$$D_1 \frac{dy_1}{dx} = \frac{wx^3}{6} + C_1\frac{x^2}{2} + C_2 x + C_3 \quad (6)$$

$$D_1 y_1(x) = \frac{wx^4}{24} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \quad (7)$$

Because there are three discrete regions along the span-wise direction of the wide-beam, (the homogeneous ends and the heterogeneous middle) the displacement equation must be used replicated for each additional region. This is done so as to account for separate properties and boundary conditions for each region, shown in equations (8) and (9).

$$D_2 y_2(x) = \frac{wx^4}{24} + C_5\frac{x^3}{6} + C_6\frac{x^2}{2} + C_7 x + C_8 \quad (8)$$

$$D_3 y_3(x) = \frac{wx^4}{24} + C_9\frac{x^3}{6} + C_{10}\frac{x^2}{2} + C_{11} x + C_{12} \quad (9)$$

3.1 Calculation

The following assumptions were made in support of this calculation:

- The plate is modeled as a beam, with its flexural rigidity defined as a wide beam (taking into account Poisson's ratio).

- All boundary conditions and geometric conditions are ideal (i.e. exactly as input into the boundary values and conditions within a given scenario).
- Beam deflection is small in magnitude.
- The beam does not plastically deform.

3.1.1 Fixed-Fixed Edges

For a homogeneous wide-beam having both edges fixed (i.e. fixed-fixed or F-F boundary conditions), the theoretical solution for displacement of a wide-beam along the span-wise direction is given by [2]:

$$y(x) = \frac{w}{24D} x^2 (L-x)^2 \tag{10}$$

For a heterogeneous wide-beam having both edges fixed (i.e. fixed-fixed or F-F boundary conditions), the following boundary conditions for each region are utilized:

$$\underbrace{\begin{gathered} y_1(0)=0 \\ y_1(x_1)=y_2(x_1) \\ \left.\frac{dy_1}{dx}\right|_{x=0}=0 \\ M_1(0)=D\left.\frac{d^2y_1}{dx^2}\right|_{x=0}=\frac{wL^2}{12} \end{gathered}}_{\text{Region 1}} \qquad \underbrace{\begin{gathered} y_2(x_2)=y_3(x_2) \\ \left.\frac{dy_1}{dx}\right|_{x=x_1}=\left.\frac{dy_2}{dx}\right|_{x=x_1} \\ \left.\frac{dy_2}{dx}\right|_{x=x_2}=\left.\frac{dy_3}{dx}\right|_{x=x_2} \\ M_1(x_1)=D\left.\frac{d^2y_1}{dx^2}\right|_{x=x_1}=M_2(x_1)=D\left.\frac{d^2y_2}{dx^2}\right|_{x=x_1} \end{gathered}}_{\text{Region 2}} \qquad \underbrace{\begin{gathered} y_3(L)=0 \\ \left.\frac{dy_3}{dx}\right|_{x=L}=0 \\ M_3(L)=D\left.\frac{d^2y_3}{dx^2}\right|_{x=L}=\frac{wL^2}{12} \\ M_2(x_2)=D\left.\frac{d^2y_2}{dx^2}\right|_{x=x_2}=M_3(x_2)=D\left.\frac{d^2y_3}{dx^2}\right|_{x=x_2} \end{gathered}}_{\text{Region 3}}$$

Using these boundary conditions, it is possible to create a coefficient matrix and forcing function with equation (4) through (9). The system of equations for the given boundary conditions is presented in (11). Note that rows and columns in the respective coefficient matrix and forcing function have containing all zeros have been omitted for simplicity.

$$\begin{bmatrix} x_1 & -x_1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{x_1^2}{2} & \frac{-D_1}{D_2}\frac{x_1^2}{2} & \frac{-D_1x_1}{D_2} & \frac{-D_1}{D_2} & 0 & 0 & 0 & 0 & 0 \\ \frac{x_1^3}{6} & \frac{-D_1}{D_2}\frac{x_1^3}{6} & \frac{-D_1}{D_2}\frac{x_1^2}{2} & \frac{-D_1x_1}{D_2} & \frac{-D_1}{D_2} & 0 & 0 & 0 & 0 \\ 0 & x_2 & 1 & 0 & 0 & -x_2 & -1 & 0 & 0 \\ 0 & \frac{x_2^2}{2} & x_2 & 1 & 0 & \frac{-D_2}{D_3}\frac{x_2^2}{2} & \frac{-D_2x_2}{D_3} & \frac{-D_2}{D_3} & 0 \\ 0 & \frac{x_2^3}{6} & \frac{x_2^2}{2} & x_2 & 1 & \frac{-D_2}{D_3}\frac{x_2^3}{6} & \frac{-D_2}{D_3}\frac{x_2^2}{2} & \frac{-D_2x_2}{D_3} & \frac{-D_2}{D_3} \\ 0 & 0 & 0 & 0 & 0 & L & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{L^2}{2} & L & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{L^3}{6} & \frac{L^2}{2} & L & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{bmatrix} = \begin{bmatrix} \frac{-wL^2}{12} \\ \frac{wx_1^3}{6}\left(\frac{D_1}{D_2}-1\right)-\frac{wx_1L^2}{12} \\ \frac{wx_1^4}{24}\left(\frac{D_1}{D_2}-1\right)-\frac{wx_1^2L^2}{24} \\ 0 \\ \frac{wx_2^3}{6}\left(\frac{D_2}{D_3}-1\right) \\ \frac{wx_2^4}{24}\left(\frac{D_2}{D_3}-1\right) \\ \frac{-5wL^2}{12} \\ \frac{-wL^3}{6} \\ \frac{-wL^4}{24} \end{bmatrix} \tag{11}$$

3.1.2 Fixed-Simply Supported Edges

For a homogeneous wide-beam having one edge fixed and one edge simply supported (i.e. fixed-simply supported or F-SS boundary conditions), the theoretical solution for displacement of a wide-beam along the span-wise direction is given by [2]:

$$y(x) = \frac{w}{48D} x\left(L^3 - 3Lx^2 + 2x^3\right) \tag{12}$$

For a heterogeneous wide-beam having one edge fixed and one edge simply supported (i.e. fixed-simply supported or F-SS boundary conditions), the following boundary conditions for each region are utilized:

$$\underbrace{\begin{gathered} y_1(0) = 0 \\ y_1(x_1) = y_2(x_1) \\ V_1(0) = D\left.\frac{d^3 y_1}{dx^3}\right|_{x=0} = \frac{-3wL}{8} \\ M_1(0) = D\left.\frac{d^2 y_1}{dx^2}\right|_{x=0} = 0 \end{gathered}}_{\text{Region 1}} \quad \underbrace{\begin{gathered} y_2(x_2) = y_3(x_2) \\ \left.\frac{dy_1}{dx}\right|_{x=x_1} = \left.\frac{dy_2}{dx}\right|_{x=x_1} \\ \left.\frac{dy_2}{dx}\right|_{x=x_2} = \left.\frac{dy_3}{dx}\right|_{x=x_2} \\ M_1(x_1) = D\left.\frac{d^2 y_1}{dx^2}\right|_{x=x_1} = M_2(x_1) = D\left.\frac{d^2 y_2}{dx^2}\right|_{x=x_1} \end{gathered}}_{\text{Region 2}} \quad \underbrace{\begin{gathered} y_3(L) = 0 \\ \left.\frac{dy_3}{dx}\right|_{x=L} = 0 \\ M_3(L) = D\left.\frac{d^2 y_3}{dx^2}\right|_{x=L} = \frac{wL^2}{8} \\ M_2(x_2) = D\left.\frac{d^2 y_2}{dx^2}\right|_{x=x_2} = M_3(x_2) = D\left.\frac{d^2 y_3}{dx^2}\right|_{x=x_2} \end{gathered}}_{\text{Region 3}}$$

Similar to that found in (11) the above boundary conditions may be formulated to create a coefficient matrix and forcing function.

3.1.3 Simply Supported-Simply Supported Edges

For a homogeneous wide-beam having both edges simply supported (i.e. simply supported-simply supported or SS-SS boundary conditions), the theoretical solution for displacement along the span-wise direction is given by [2]:

$$y(x) = \frac{w}{24D} x\left(L^3 - 2Lx^2 + x^3\right) \tag{13}$$

For a heterogeneous wide-beam having both edges simply supported (i.e. simply supported-simply supported or SS-SS boundary conditions), the following boundary conditions for each region:

$$\underbrace{\begin{gathered} y_1(0) = 0 \\ y_1(x_1) = y_2(x_1) \\ V_1(0) = D\left.\frac{d^3 y_1}{dx^3}\right|_{x=0} = \frac{-wL}{2} \\ M_1(0) = D\left.\frac{d^2 y_1}{dx^2}\right|_{x=0} = 0 \end{gathered}}_{\text{Region 1}} \quad \underbrace{\begin{gathered} y_2(x_2) = y_3(x_2) \\ \left.\frac{dy_1}{dx}\right|_{x=x_1} = \left.\frac{dy_2}{dx}\right|_{x=x_1} \\ \left.\frac{dy_2}{dx}\right|_{x=x_2} = \left.\frac{dy_3}{dx}\right|_{x=x_2} \\ M_1(x_1) = D\left.\frac{d^2 y_1}{dx^2}\right|_{x=x_1} = M_2(x_1) = D\left.\frac{d^2 y_2}{dx^2}\right|_{x=x_1} \end{gathered}}_{\text{Region 2}} \quad \underbrace{\begin{gathered} y_3(L) = 0 \\ V_3(0) = D\left.\frac{d^3 y_3}{dx^3}\right|_{x=0} = \frac{-wL}{2} \\ M_3(L) = D\left.\frac{d^2 y_3}{dx^2}\right|_{x=L} = 0 \\ M_2(x_2) = D\left.\frac{d^2 y_2}{dx^2}\right|_{x=x_2} = M_3(x_2) = D\left.\frac{d^2 y_3}{dx^2}\right|_{x=x_2} \end{gathered}}_{\text{Region 3}}$$

Similar to that found in (11) the above boundary conditions may be formulated to create a coefficient matrix and forcing function.

4 RESULTS

4.1 Test Cases

Three test cases are considered herein to qualitatively verify that the analytical solution of the heterogeneous model is correctly computed. This is done through comparison of the resulting Test Case's solution form to the solution form of the homogeneous plate. The boundary values and conditions employed for each test case are presented in Table 1 and are detailed as follows:

- **Test Case 1** – Considers the variance in the regions of the heterogeneous plate by assuming that region 2 occupies the entire span-wise length. If the solution for the heterogeneous plate is derived appropriately, it should approach the homogeneous solution as the lengths of regions 1 and 3 approach nil.

- **Test Case 2** – Considers the variance in the regions of the heterogeneous plate by assuming that region 1, 2, and 3 occupy and equal effective percentage along the span-wise length. If the solution for the heterogeneous plate is analytically derived appropriately, it should converge to the same solution as the homogeneous plate given that the region 1, region 2, and region 3 are of equal length.

  Test Case 3 – Considers the variance in the regions of the heterogeneous plate by assuming that region 2 occupies the no span-wise length. If the solution for the heterogeneous plate is analytically derived appropriately, it should converge to the same solution as the homogeneous plate given that region 2 length equals null.

Table 1: Boundary value parameters for Test Cases 1, 2, and 3

Parameter		Value		
		Test Case 1	Test Case 2	Test Case 3
Pressure [psi]		20	20	20
Dimensions	t_1 [in]	0.0175	0.0175	0.0175
	t_2 [in]	0.015	0.015	0.015
	t_3 [in]	0.0175	0.0175	0.0175
	$a = t_1 + t_2 + t_3$ [in]	0.050	0.050	0.050
	x_1 [in]	0.0	1.333	2.0
	x_2 [in]	4.0	2.666	2.0
	L [in]	4.0	4.0	4.0
Material Properties	$E_{1,1}$ [psi]	1.000E7	1.000E7	1.000E7
	$v_{1,1}$ [#]	0.33	0.33	0.33
	$E_{2,1}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{2,1}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$
	$E_{2,2}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{2,2}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$
	$E_{2,3}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{2,3}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$
	$E_{3,1}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{3,1}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$

From the deformation profiles presented in Figure 3(a-c), it may be qualitatively observed that the solution for the heterogeneous plate is analogous to that of the homogeneous plate. This is confirmed through inspection of the numerical values for this figure. Note that each region's solution for the heterogeneous model is graphically depicted by a unique color in Figure 3. From the deformation profiles presented in Figure 3(d-f), it may be qualitatively observed that the solution for the heterogeneous plate is analogous to that of the homogeneous plate. This is confirmed through inspection of the numerical values for this figure. The heterogeneous solution does in fact numerically mimic that of the homogeneous solution. From the deformation profiles presented in Figure 3(g-i), it may be qualitatively observed that the solution for the heterogeneous plate is analogous to that of the homogeneous plate. This is confirmed through inspection of the numerical values for this figure.

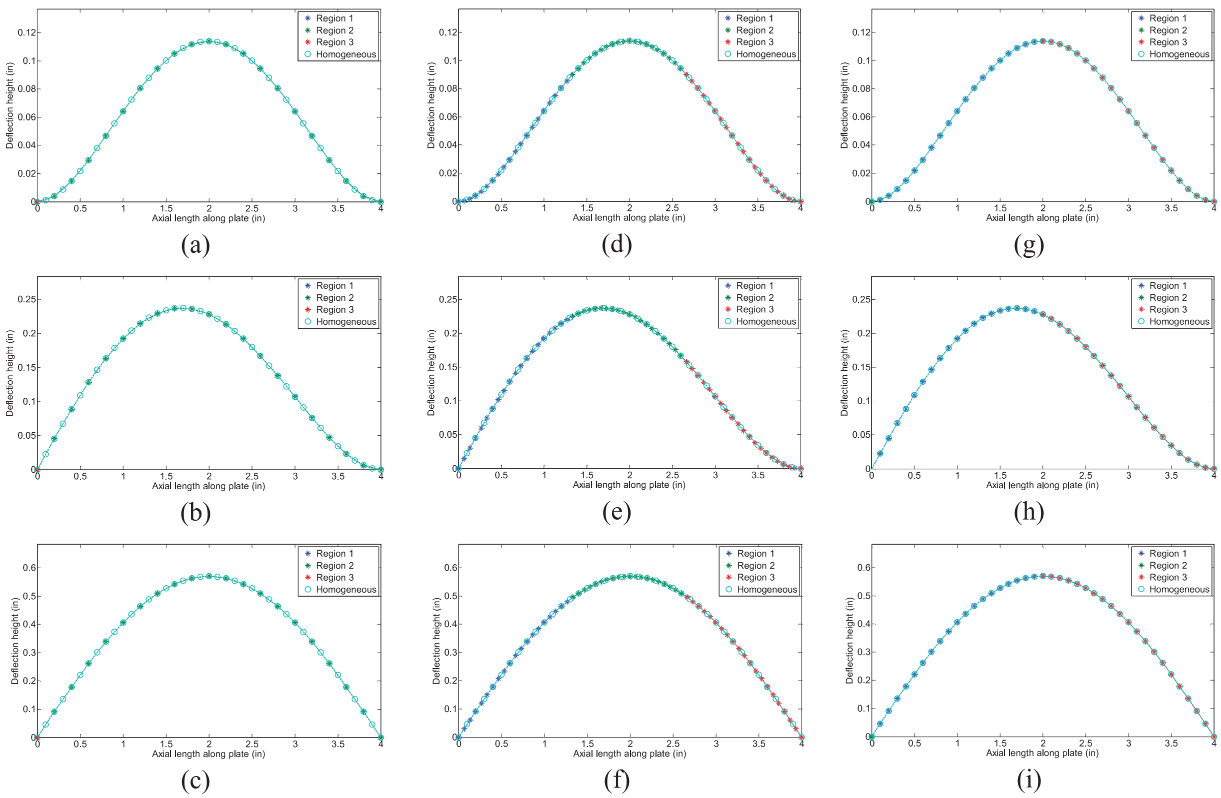


Figure 3: Plate Deflection for (a-c) Test Case 1, (d-f) Test Case 2, and (g-i) Test Case 3

4.2 Problems

Two problems are considered herein that are representative of like geometric boundary values and conditions that are intended for use within the testing program being conducted at OSU. The boundary values and conditions employed for each problem are presented in Table 2 and are detailed as follows:

- **Problem 1** – Intended to represent the maximum values for which the fueled composition is positioned into the fuel plate of the GTPA. Qualitatively, if the fueled material is more rigid than that of the unfueled region, and the fueled region occupies more space than nominally thought, it should, in fact, be mechanically more rigid than when under the nominal dimensions.

- **Problem 2** – Intended to represent the minimum values for which the fueled composition is positioned into the fuel plate of the GTPA. Qualitatively, if the fueled material is more rigid than that of the unfueled region, and the fueled region occupies less space than nominally thought, it should, in fact, be mechanically less rigid than when under the nominal dimensions.

Note that material properties were taken from the most updated reference available to the authors that include U10Mo and Aluminum alloys [9]. Dimensional information was tabulated from specification requirements [10] and as-built drawings [11] resulting from the Generic Test Plate Assembly fabrication process.

Table 2: Boundary value parameters for Problem 1

Parameter		Problem 1		Problem 2	
		Heterogeneous	Homogeneous	Heterogeneous	Homogeneous
Pressure [psi]		20	20	20	20
Dimensions	t_1 [in]	0.019	-	0.0186	-
	t_2 [in]	0.015	-	0.0098	-
	t_3 [in]	0.019	-	0.0186	-
	$a = t_1 + t_2 + t_3$ [in]	0.053	0.053	0.047	0.047
	x_1 [in]	0.125	-	0.250	-
	x_2 [in]	3.875	-	3.750	-
	L [in]	4.0	4.0	4.0	4.0
Material Properties	$E_{1,1}$ [psi]	9.903005E6	9.903005E6	9.903005E6	9.903005E6
	$v_{1,1}$ [#]	0.33	0.33	0.33	0.33
	$E_{2,1}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{2,1}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$
	$E_{2,2}$ [psi]	1.2660932E7	$E_{1,1}$	1.2660932E7	$E_{1,1}$
	$v_{2,2}$ [#]	0.324	$v_{1,1}$	0.324	$v_{1,1}$
	$E_{2,3}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{2,3}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$
	$E_{3,1}$ [psi]	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$	$E_{1,1}$
	$v_{3,1}$ [#]	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$	$v_{1,1}$

From Figure 4(a-c), the deflection profile for the homogeneous case and the heterogeneous plate of Problem 1 (F-F edge boundaries) are visually indistinguishable due to the close, but not identical result. However, the maximum deflection value for the heterogeneous plate was found to be 0.0962 inches. As seen in Table 3, when reducing the flexure of the edge boundary conditions, the discrepancy between the maximum displacements increases.

Comparing the displacement values acquired from Problem 2 to that of Problem 1, the maximum deflection is significantly increased for all edge boundary condition types given a thinner, as would be expected. The comparison of results in Problem 1 and 2 explicitly demonstrate that deflection of a plate under constant load is much more sensitive to the total thickness of the plate (even within fabrication tolerance) than that of the fuel meat region within the plate.

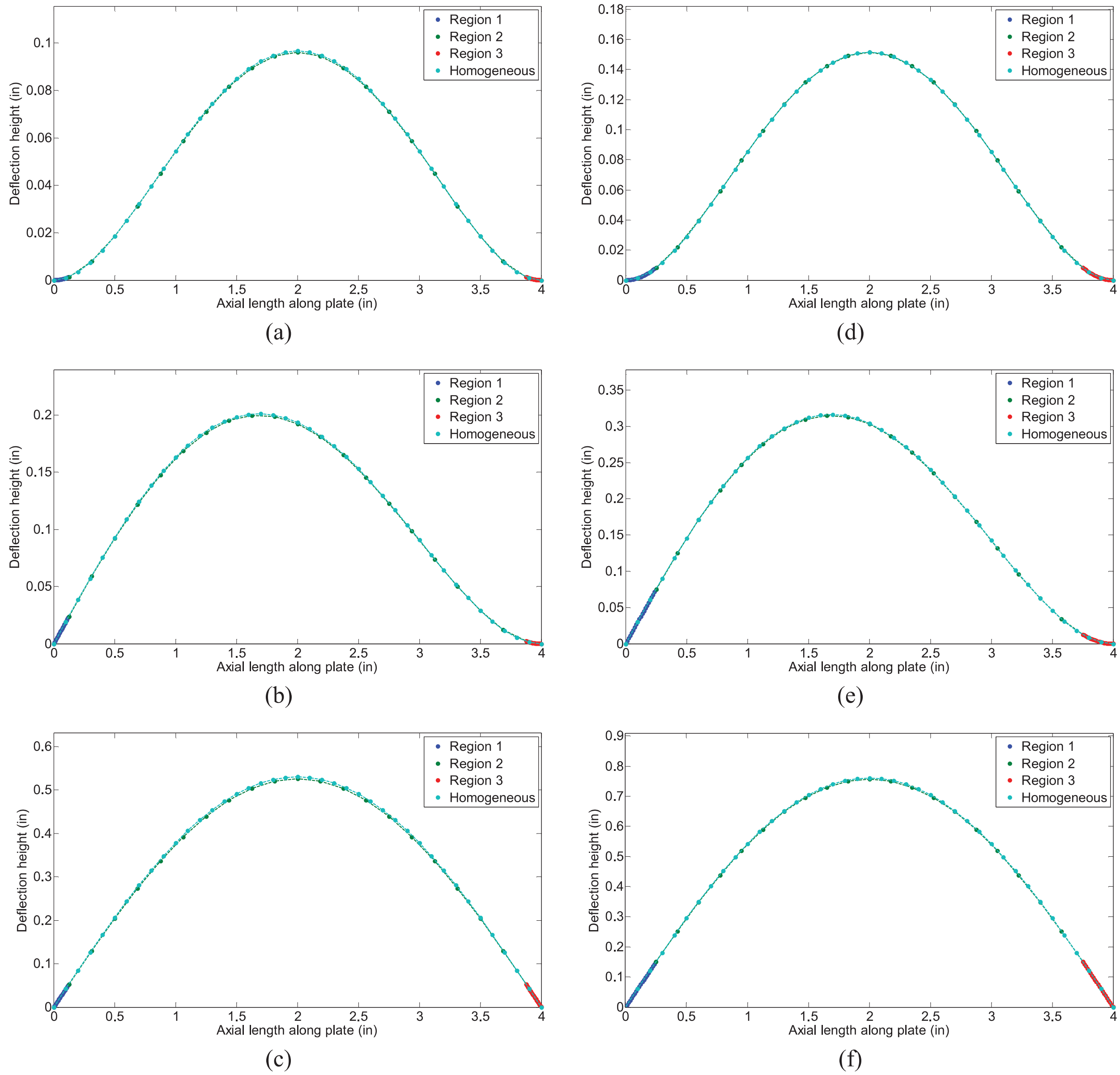


Figure 4: Plate Deflection for (a-c) Problem 1 and (d-f) Problem 2

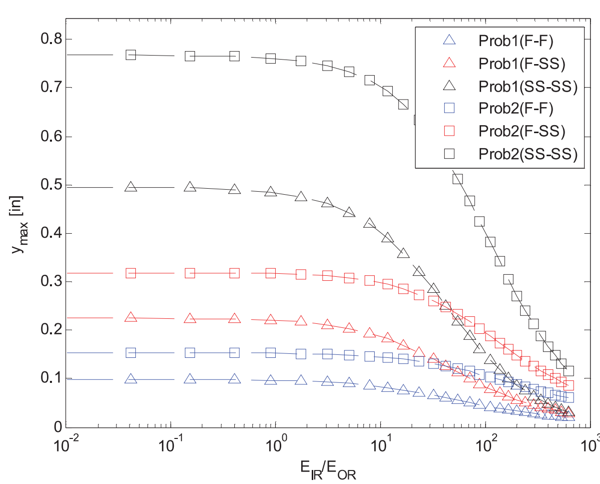
Table 3. Summary of Maximum Deflections.

Boundary Conditions	Maximum Deflection [in]			
	Problem 1		Problem 2	
	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous
F-F	0.0967	0.0962	0.1521	0.1517
F-SS	0.2008	0.1997	0.3161	0.3152
SS-SS	0.5303	0.5256	0.7604	0.7577

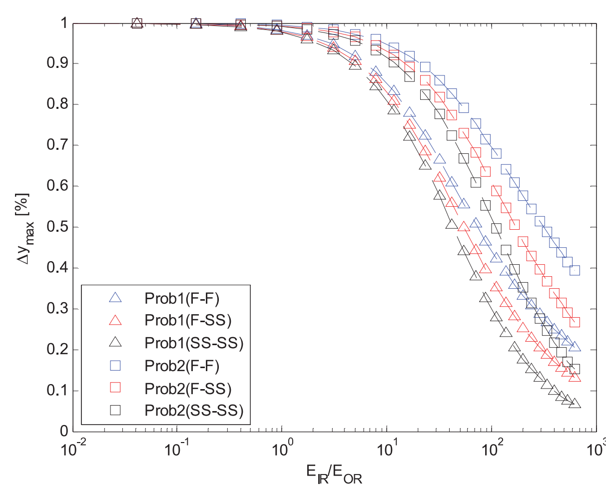
4.2.1 Influence of Flexural Rigidity

From Figure 4(a-c) and Figure 4(d-f), it is difficult to discern the homogeneous case from the heterogeneous case, due to the similarity of the inner region and outer region's modulus of elasticity values. As this ratio is decreased, the maximum deflection of the heterogeneous plate approaches the value of maximum deflection for a homogeneous plate. In contrast, as the ratio is increased, the maximum deflection of the heterogeneous plate begins to diverge from that of the equivalent homogeneous plate. As previously discussed, a plate's 'flexural rigidity' is a characterization of its resistance to change in position under an applied load. This poses the question, "what flexural rigidity for the interior region is sufficient to yield significant change in the deflection of a heterogeneous plate?" As seen in equations (2) and (3), flexural rigidity (D) is proportional to modulus of elasticity (E). In an attempt to address this rhetorically posed question, modulus of elasticity of the interior region (region 2,2 from Figure 2) was varied in Problems 1 and 2, while holding all other boundary values and conditions constant, and the maximum displacement was tabulated. The results of this sensitivity calculation are presented Figure 5.

Several observations may be deduced from the trends shown in Figure 5. Specifically, from Figure 5(b), modulus of elasticity influence the maximum deflection to the greatest degree for the weakest boundary condition set considered (SS-SS). Additionally, for all cases considered, the interior region's modulus of elasticity was required to be approximately 8 times (at a minimum) greater than the exterior region in order to influence the maximum deflection by 10% or greater. This leads to an observation that for sandwich structures (similar to this), the inner material must be approximately one order of magnitude greater in 'strength' than the external region in order to have an appreciable influence on the change in deflection, at which point, it begins to have a significant effect on the result.



(a)



(b)

Figure 5: (a) value of maximum deflection and (b) relative change in max deflection versus change in internal region flexural rigidity

5 CONCLUSIONS

This study developed an analytical solution for a heterogeneous wide beam under uniform loading given three unique boundary conditions. During this study three test cases and two problems were considered.

For the test cases, all results were numerically indistinguishable from the analytic solution for plate deflection with a single material.

For the two problems, the F-F boundary condition permitted the smallest of plate deflection as compared to the F-SS and SS-SS conditions. In addition, Problem 1 contained a greater amount of inner region material, which is more rigid than the material comprising the outer regions of the plate. As a result, Problem 1 had a lower value of deflection when compared to its identical boundary conditions for Problem 2. These results are summarized in Table 3.

A sensitivity calculation was conducted on the influence of material properties (specifically modulus of elasticity) of the inner region relative to the outer region. This calculation demonstrated that within the confines of the conditions considered herein, a material having approximately one order of magnitude greater rigidity than the cladding must occupy the inner region in order for an appreciable influence on the maximum deflection to be observed.

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