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Derivation of GFDM Based on OFDM Principles

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Abstract—**This paper starts with discussing the principle based on which the celebrated orthogonal frequency division multiplexing (OFDM) signals are constructed. It then extends the same principle to construct the newly introduced generalized frequency division multiplexing (GFDM) signals. This novel derivation sheds light on some interesting properties of GFDM. In particular, our derivation seamlessly leads to an implementation of GFDM transmitter which has significantly lower complexity than what has been reported so far. Our derivation also facilitates a trivial understanding of how GFDM (similar to OFDM) can be applied in MIMO channels.**

I. Introduction

While the orthogonal frequency division multiplexing (OFDM) has enjoyed its dominance in the present and past wireless standards for broadband communications, there is a strong tendency among those who explore candidate waveforms for 5G to replace OFDM because of its well-known limitations. OFDM suffers from some bandwidth inefficiency, since a significant portion of each data packet is allocated to the cyclic prefix segments. Also, OFDM has been found to be a poor choice in multiuser applications where any loss of synchronization among users leads to a significant loss in performance, [1], [2].

Generalized frequency division multiplexing (GFDM) is one of the candidate waveforms that has been proposed and currently being explored extensively by the 5GNow group, [3]. The GFDM waveform have already been presented in a number of conferences, e.g., [4]–[11], and its versatility as a 5G candidate has been discussed in a great detail in [12].

The thrust of this paper is to give a novel presentation of GFDM which shows how it can be constructed based on the fundamental principle of OFDM waveform. We remind the reader that in OFDM each symbol waveform is a summation of a number of tones, each modulated by a quadrature amplitude modulated (QAM) symbol. We show that this principle is also applicable to GFDM waveform construction. This derivation of GFDM leads to a number of interesting results. Firstly, we find that there exists a more straightforward and less complex implementation for GFDM transmitter than what has been presented in the previous literature, [9]. Secondly, since each GFDM waveform is constructed by adding a number of tones, its adoption to MIMO channels is found to be as straightforward as is the case for OFDM.

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This paper is organized as follows. In Section II, we highlight the fundamental principle based on which OFDM waveforms are constructed. Next, in Section III, we introduce a less understood, but relevant concept: *frequency spreading design of digital filters*. We combine the results of Sections II and III to introduce a new construction for GFDM waveforms in Section IV. The transmitter and receiver implementations are discussed in Section V and Section VI, respectively. In Section VII, we show that GFDM can be extended to MIMO channels straightforwardly. The concluding remarks are made in Section VIII.

Notations: We found it convenient to give our presentation through a mix of continuous time and discrete time signals. We use (t) to denote the continuous time variable and $[n]$ to denote discrete time index. Subcarrier index is denoted by subscript k. Discrete time index, n, when has to be added to a continuous time function is put as a subscript, e.g., $x_n(t)$. Bold lower case is used for column vectors and bold upper case for matrices. All vectors are in column form. The vector and matrix transpose and Hermitian are indicated by the superscripts 'T' and 'H', respectively. We use $\boldsymbol{\mathcal{F}}_N$ to denote discrete Fourier transform (DFT) matrix of size N. We also assume that $\boldsymbol{\mathcal{F}}_N$ is normalized, such that $\boldsymbol{\mathcal{F}}_N\boldsymbol{\mathcal{F}}_N^{\rm H} = \mathbf{I}_N$, where $\mathbf{I}_N$ denotes the identity matrix of size N. Hence, $\boldsymbol{\mathcal{F}}^{-1} = \boldsymbol{\mathcal{F}}_N^{\rm H}$ both denote inverse DFT (IDFT) and these may be used interchangeably. The terms FFT and IFFT refer to the fast implementations of DFT and IDFT, respectively.

II. OFDM Principles

Each OFDM (super) symbol carriers N QAM symbols $s_k[n]$, $k = 0, 1, \cdots, N-1$ and is constructed as

$$x_n(t) = \sum_{k=0}^{N-1} s_k[n] e^{j\frac{2k\pi}{T}t}. \tag{1}$$

Related to (1), there are a few points that should be noted:

1) $x_n(t)$ is a summation of N tones, weighted by the data symbols $s_k[n]$.
2) These tones are spaced at $F = 1/T$ and located at the frequencies $f = 0$, $1/T$, $2/T$, $\cdots$, $(N-1)/T$.
3) $x_n(t)$ is periodic with a period of T.
4) The Fourier series coefficients of $x_n(t)$ are the data symbols $s_k[n]$, $k = 0, 1, \cdots, N-1$, and the construction of $x_n(t)$ may be viewed as an inverse Fourier series.
5) If $x_n(t)$ is passed through a channel with the transfer function $H(f)$, the channel output, excluding its transient response, is obtained as

$$y_n(t) = \sum_{k=0}^{N-1} H\left(\frac{k}{T}\right) s_k[n] e^{j\frac{2k\pi}{T}t}. \tag{2}$$

6) Obviously, $y_n(t)$ is also period with period of T.
7) The Fourier series coefficients of $y_n(t)$ are $H\left(\frac{k}{T}\right) s_k[n]$, $k = 0, 1, \cdots, N-1$. Hence, the transmitted data symbols $s_k[n]$ can be extracted from samples of $y_n(t)$, by applying a Fourier series analysis and equalizing the results by the inverse of channel gains at the respective frequencies.

In practice, where a digital circuitry or a software radio is used for implementation, (1) is implemented through an inverse discrete Fourier transform, or equivalently and conveniently, through an IFFT. IFFT output delivers only on cycle of the sampled version of $x_n(t)$; say $x_n[m]$, for $m = 0, 1, \cdots, N-1$. Similarly, the data extraction at the receiver is performed by applying an FFT to the samples of a single period of $y_n(t)$, $y_n[m]$, and equalizing the results by the inverse of the channel gains, as noted in item 7), above.

Clearly, it will be resource inefficient to transmit $x_n(t)$ for any period of time beyond the minimum duration that one needs for correct extraction of the data symbols $s_n[k]$ at the receiver. This minimum duration is T (one period of $x_n(t)$) plus the duration of the channel response, $T_{\rm ch}$. The latter is needed to absorb the channel transient response. To serve this purpose, in digital implementation of the transmitter, a number of samples, equivalent to $T_{CP} \geq T_{\rm ch}$, from the end of the FFT output are prefixed to its beginning. The prefixed

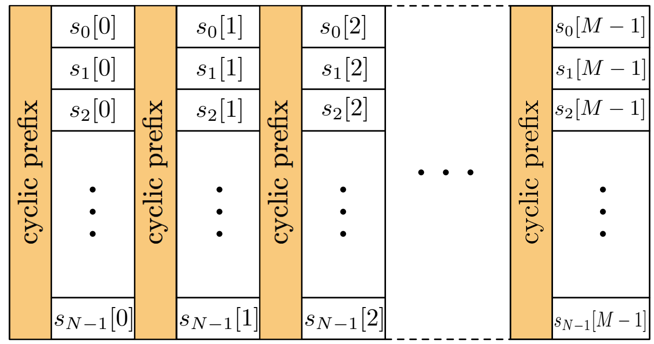

Fig. 1. An OFDM packet.

samples, because of obvious reasons, are called *cyclic prefix*. At the receiver, the FFT is applied after removing the cyclic prefix.

From the above discussion, it follows that each OFDM symbol has a duration of $T + T_{\rm CP}$, and a data packet consisting of M OFDM symbols may be expressed, in continuous time, as

$$\begin{aligned} x(t) &= \sum_{n=0}^{M-1} x_n\left(t - n(T+T_{\rm CP})\right) \\ &= \sum_{n=0}^{M-1} s_k[n] e^{j\frac{2\pi}{T}(t-n(T+T_{\rm CP}))}. \end{aligned} \tag{3}$$

To facilitate our discussions in the subsequent parts of this paper, we have presented in Fig. 1 the structure of an OFDM packet. As follows from the above discussion, each OFDM symbol (consisting of N QAM symbols) needs to be cyclic prefixed.

III. Frequency Spreading Design of Digital Filters

Matrin, [13], and Mirabbasi and Martin, [14], suggested a method of designing a class of finite impulse response (FIR) square-root Nyquist (N) filters whose impulse response is expressed as

$$p[n] = \begin{cases} c_0 + 2\sum_{k=1}^{K-1} c_k \cos\left(\frac{2\pi kn}{L}\right), & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where $L = KN$ is the filter order, N is the spacing between zero-crossings of Nyquist pulse-shape $q[n] = p[n] \star p[-n]$, and the parameter K is referred to as overlapping factor.

We also note that (4) may be rearranged as

$$p[n] = \sum_{k=0}^{L-1} \tilde{c}_k e^{j\frac{2\pi kn}{L}} \tag{5}$$

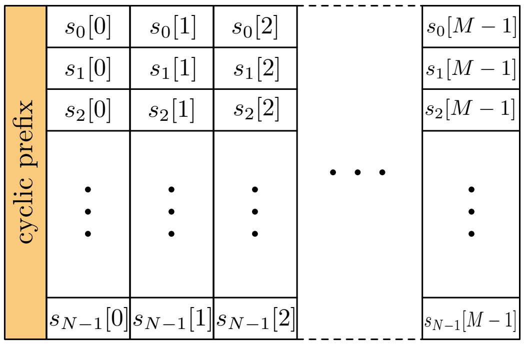

Fig. 2. A GFDM packet.

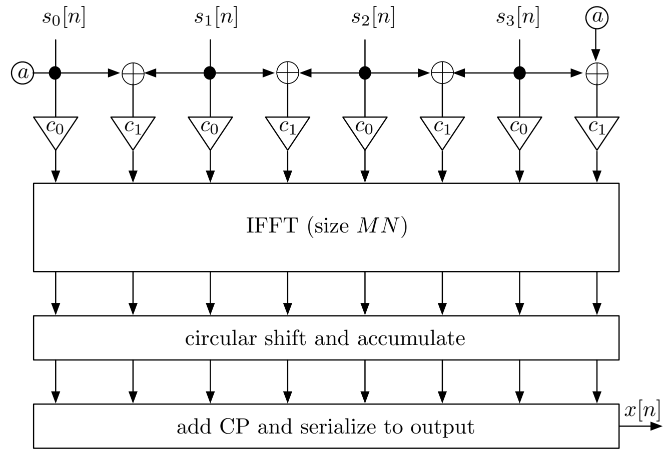

Fig. 3. An implementation of GFDM transmitter, for $N = 4$ and $K = 2$. M can be any value.

where

$$\tilde{c}_k = \begin{cases} c_k, & 0 \le k \le K-1 \\ 0, & K \le k \le L-K \\ c_{L-k}, & L-K+1 \le k \le L-1. \end{cases} \tag{6}$$

This observation implies that in the frequency domain, over a grid of L frequency beans, $p[n]$ is characterized by the $2K-1$ non-zero coefficients $\tilde{c}_k$ or, equivalently, $2K-1$ complex frequency beans/tones. The modulated versions of this filter, when used to realize a filter bank, are similarly characterized by $2K-1$ non-zero coefficients/tones; shifted the corresponding frequency bands.

IV. GFDM Derivation

As opposed to OFDM, packet construction in GFDM is such that only one CP is needed to take care of the channel transient response. In addition, the data symbols over each subcarrier are filtered through a well-localized passband filter that limits the intercarrier interference (ICI) to only adjacent subcarriers. Fig. 2 presents the structure of a GFDM packet. The data symbols are spread across time and frequency, as in OFDM. However, the data stream in each subcarrier is controlled through a filter which confines its frequency response to a limited bandwidth. Similar to OFDM data symbols are transmitted at an interval T and subcarrier spacing is set equal to $F = 1/T$.

From the discussion presented above, for OFDM, one may deduce that to allow the use of a single CP in GFDM, the modulated signal that carries all the data symbols in a GFDM packet should be a single cycle of a periodic signal with the length of MT. Such a periodic signal may be constructed, following the OFDM signal synthesis, according to the following formulas. The contribution from the n data column in the GFDM packet of Fig. 2 is synthesized as

$$x_n(t) = \sum_{l=0}^{N-1} \sum_{k=-K+1}^{K-1} s_l[n]\tilde{c}_k e^{j\frac{2\pi(Ml+k)}{MT}(t-nT)}. \tag{7}$$

Note that this synthsizes the data symbols $s_k[n]$ over N subcarrier bands following the frequency method of the previous section. We also note that $x_n(t)$, effectively, is the sum of MN tones at the frequencies 0, $1/MT$, $2/MT$, $\cdots$. Since all these tones may be considered as periodic with a period of MT, $x_n(t)$ is also periodic with the same period.

The complete packet carrying all the data symbols in the GFDM packet of Fig. 2 is obtained by summing up the result of (7) over $0 \le n \le M-1$. That is

$$x(t) = \sum_{n=0}^{M-1} x_n(t). \tag{8}$$

Obviously, since the components $x_n(t)$ are periodic with a period MT, $x(t)$ is also periodic and has the same period. The GFDM packet is thus obtained by taking a segment of $x(t)$ over the interval $-T_{\rm CP} \le t \le MT$.

Following the above concept, synthesizing a packet of GFDM in discrete time can be performed according to the block diagram shown in Fig. 3. Without any loss of generality, the presentation here is for the case where $N = 4$, $K = 2$, and M. The "circular shift and accumulate" block adds up the results of IFFT for each set of data symbols, after applying a circular shift. The circular shift is to take care of the time delay between the successive data columns in the packet format of Fig. 2. The delay associated with the symbol set time index n in continuous time is nT. In discrete time, this corresponds to nN sample delay.

V. Transmitter Implementation

Direct implementation of GFDM transmitter according to the block diagram of Fig. 3 requires a total of M IFFT operations of size NM, plus additional operations prior and after IFFT. This complexity can be reduced significantly, by some rearrangement of the operations as explained next.

The equation that relates the input data symbols $s_k[n]$ and the IFFT output, denoted by the length MN column vector $\mathbf{x}[n]$, may be expressed as

$$\mathbf{x}[n] = \boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{C}\mathbf{s}_{\rm e}[n] \tag{9}$$

where $\mathbf{s}_{\rm e}[n] = [s_0[n]\ \mathbf{0}\ s_1[n]\ \mathbf{0}\ \cdots\ s_{M-1}[n]\ \mathbf{0}]^{\rm T}$, $\mathbf{0}$ is a row of $M-1$ zeros,

$$\mathbf{C} = \begin{bmatrix} c_0 & c_{-1} & \cdots & c_{K-1} \\ c_1 & c_0 & \cdots & c_{K-2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{K-1} & c_{K-2} & \cdots & 0 \\ 0 & c_{K-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_{-(K-1)} & 0 & \cdots & c_{-(K-2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{-1} & c_{-2} & \cdots & c_0 \end{bmatrix}$$

and $\boldsymbol{\mathcal{F}}_{MN}$ is the DFT matrix of size $MN \times MN$. We note that $\mathbf{C}$ is circular matrix of size $MN \times MN$.

Direct implementation (9) is performed in two steps:

1) The circular convolution of the first column of $\mathbf{C}$ and the vector $\mathbf{s}_{\rm e}[n]$ is performed to obtain $\mathbf{C}\mathbf{s}_{\rm e}[n]$.
2) An IFFT of size MN applied to the result of step 1) to obtain $\mathbf{x}[n]$.

The complexity of this procedure is dominantly determined by Step 2); an IFFT of size MN.

Alternatively, $\mathbf{x}[n]$ may be calculated by arranging (9) as

$$\mathbf{x}[n] = \boldsymbol{\mathcal{F}}_{MN}^{-1}\underbrace{\boldsymbol{\mathcal{F}}_{MN}\left[(\boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{c}_1)\odot(\boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{s}_{\rm e}[n])\right]}_{\mathbf{C}\mathbf{s}_{\rm e}[n]} \tag{10}$$

where $\mathbf{c}_1$ is the first column of $\mathbf{C}$, $\odot$ denotes point-wise multiplication, and the circular convolution of the vectors $\mathbf{c}_1$ and $\mathbf{s}_{\rm e}[n]$ are performed through point-wise multiplication of their respective inverse DFTs and then applying a DFT to the result. Here, we have chosen to use inverse DFT domain instead of the common approach of using DFT domain, because this, as explained next, gets us to the low complexity implementation that we strive for.

Obviously, (10) reduces to

$$\mathbf{x}[n] = \mathbf{h}\odot(\boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{s}_{\rm e}[n]) \tag{11}$$

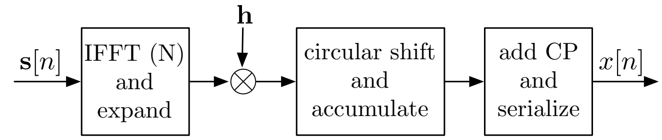

Fig. 4. A simplified implementation of GFDM transmitter.

where $\mathbf{h} = \boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{c}_1$ is the vector of prototype filter coefficients. We note that the computational complexity of (11) is still dominantly determined by the complexity of the operation $\boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{s}_{\rm e}[n]$, i.e., computation of IFFT of $\mathbf{s}_{\rm e}[n]$. Moreover, since $\mathbf{s}_{\rm e}[n]$ is the expended version of the length N column vector $\mathbf{s}[n] = [s_0[n]\ s_1[n]\ \cdots\ s_{N-1}[n]]^{\rm T}$, $\boldsymbol{\mathcal{F}}_{MN}^{-1}\mathbf{s}_{\rm e}[n]$ can be obtained by M repetitions of $\boldsymbol{\mathcal{F}}_{N}^{-1}\mathbf{s}[n]$ in a column. Hence, following (11), $\mathbf{x}[n]$ can be calculated through an IFFT of size N.

Next, we note that the final content of the circular shift and accumulate block in Fig. 3, upon including all the data symbols of the packet, will be the vector

$$\mathbf{x}_{\rm all} = \sum_{n=0}^{M-1}\text{circshift}(\mathbf{x}[n], nN) \tag{12}$$

where circshift$(\cdot)$ means downward circular shift. After adding the CP to $\mathbf{x}_{\rm all}$, we have a complete packet to transmit.

The results presented in (11) and (12) have the following interpretation. According to (11), each set of data symbols modulate the set of carrier tones, added together, and the result is truncated by a well designed window defined by $\mathbf{h}$. The results for different choices of time index n are then (circularly) shifted and added together to obtain the vector $\mathbf{x}_{\rm all}$.

Now considering the results of (11) and (12) and the discussions following these equations, the simplified transmitter block diagram of Fig. 4 is obtained. Following this block diagram,the computation of each packet involves M FFT operations, each of size N, M dot multiplication of $\mathbf{h}$ by the expanded output of IFFT, and M vector additions. For convenience, as a measure of complexity, each complex multiplication followed my a complex addition is referred to one operation. Accordingly, the number of operations for IFFTs will be $(MN/2)\log_2 N$. We note that the elements of $\mathbf{h}$ are real-valued and the number of non-zero entries of it may be smaller than its length MN. We assume this to be KN, where $K \leq M$; see Section III. We count each real by complex multiplication as half a complex multiplication. Hence, multiplications by $\mathbf{h}$ and additions to 'circular shift and add' block are counted as $KMN/2$ operations. Adding our results, we find that the number of operation to calculate each GFDM packet will be $(MN/2)\log_2 N + KMN/2$ operations. With a typical value of $K = 4$, the complexity formula that we get is

$$C_1 = \frac{MN}{2}\log_2 N + 2MN \text{ operations.} \qquad (13)$$

Taking the formula (7) of [9], a previous paper on efficient implementation of transmitter in GFDM, and assuming that the number of active subcarriers is equal to N (i.e., in formula (7) of [9] we have set $K = N$) to match our assumption here, the reported result in [9] is[1]

$$C_2 = \frac{3MN}{2}\log_2 MN + 2MN \text{ operations.} \qquad (14)$$

A quick comparison of (13) and (14) reveals that the implementation proposed in this paper is three time or more less complex than that of [9].

VI. Receiver Implementation

The key idea for development of any receiver implementation is to recall that after removing the CP, the received signal vector $\mathbf{y}$, of size $MN \times 1$ may be thought as one period of a periodic signal whose spectral content are those of the transmit signal vector $\mathbf{x}_{\text{all}}$, scaled by the channel gains at the respective frequencies (the transmitted tones). The channel equalization at the receiver, thus, become a trivial task. An FFT is applied to the received vector $\mathbf{y}$ and each element of the result will be divided by the channel gain at the respective frequency. The result will be the frequency domain equalized vector $\mathbf{y}_{\text{f,e}} = \mathbf{x}_{\text{all,f}} + \mathbf{v}_{\text{f}}$, where $\mathbf{x}_{\text{all,f}}$ and $\mathbf{v}_{\text{f}}$ are the FFTs of $\mathbf{x}_{\text{all}}$ and the channel noise vector $\mathbf{v}$, respectively. The procedure presented in [5] is a computationally efficient method for extracting the data symbols from $\mathbf{y}_{\text{f,e}}$.

VII. Extension to MIMO Channels

The key idea that has made application of OFDM to MIMO channels trivial relies on the fact that each subcarrier signal is a pure tone and thus for each channel link it will be affected by a complex gain. Furthermore, in an $N_{\text{t}} \times N_{\text{r}}$ MIMO channel, a transmit data vector $\mathbf{s}_k$ of the kth subcarrier, results in a received vector

$$\mathbf{u}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{v}_k, \quad \text{for } k = 0, 1, \cdots, N-1 \qquad (15)$$

where $\mathbf{v}_k$ is the channel noise vector and $\mathbf{H}_k$ is the channel gain matrix at the kth subcarrier frequency. Applying a zero-forcing (ZF) or a minimum mean square error (MMSE) equalizer to (15), will allow one to obtain an estimate of $\mathbf{s}_k$.

[1]In [9], the number of operations to perform an FFT of size N is assumed to be $N\log_2 N$. Here, we have replaced with the more accurate figure $(N/2)\log_2 N$.

Noting that each GFDM packet signal is made up of MN tones, the same concept can be applied. An FFT of size MN is applied to the CP stripped received vector $\mathbf{y}$ of each received antenna and the results of the same indexed outputs are collected to obtain an equation similar to (15). Then, a ZF or MMSE equalizer is applied for each tone separately. This process gives estimates of the k element of the signal vector $\mathbf{x}_{\text{all}}$ for all the space multiplexed signals. Collating the elements corresponding to each space multiplexed signal gives the estimate of the corresponding $\mathbf{x}_{\text{all}}$ which then may be processed according to [5], to extract the data symbols.

VIII. Conclusion

This paper presented a novel point of view of GFDM. We showed that GFDM builds based on the same principles as OFDM: information is transmitted by modulating a number of pure sine waves/tones. This point of view led us to discovery of an implementation of the GFDM transmitter whose complexity is about one third of the best that has been reported in the previous literature. Our point of view also allows deeper understanding of GFDM, particularly, its application in MIMO channel becomes obvious.

References

[1] M. Morelli, C. C. J. Kuo, and M. O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review," *Proc. of the IEEE*, vol. 95, no.7, pp. 1394–1427, July 2007.

[2] B. Farhang-Boroujeny, "OFDM Versus Filter Bank Multicarrier," *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92–112, 2011.

[3] "http://www.5gnow.eu."

[4] M. Matthe, L. Mendes, and G. Fettweis, "Generalized frequency division multiplexing in a gabor transform setting," *Communications Letters, IEEE*, vol. 18, no. 8, pp. 1379–1382, Aug 2014.

[5] I. Gaspar, N. Michailow, A. Navarro, E. Ohlmer, S. Krone, and G. Fettweis, "Low complexity gfdm receiver based on sparse frequency domain processing," in *Vehicular Technology Conference (VTC Spring), 2013 IEEE 77th*, June 2013, pp. 1–6.

[6] M. Matthe, L. L. Mendes, and G. Fettweis, "Space-time coding for generalized frequency division multiplexing," in *European Wireless 2014; 20th European Wireless Conference; Proceedings of*, May 2014, pp. 1–5.

[7] G. Fettweis, M. Krondorf, and S. Bittner, "Gfdm - generalized frequency division multiplexing," in *Vehicular Technology Conference, 2009. VTC Spring 2009. IEEE 69th*, April 2009, pp. 1–4.

[8] R. Datta, N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized frequency division multiplexing in cognitive radio," in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*, Aug 2012, pp. 2679–2683.

[9] N. Michailow, I. Gaspar, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized frequency division multiplexing: Analysis of an alternative multi-carrier technique for next generation cellular systems," in *Wireless Communication Systems (ISWCS), 2012 International Symposium on*, Aug 2012, pp. 171–175.

[10] R. Datta, N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized frequency division multiplexing in cognitive radio," in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*, Aug 2012, pp. 2679–2683.

[11] M. Danneberg, R. Datta, A. Festag, and G. Fettweis, "Experimental testbed for 5g cognitive radio access in 4g lte cellular systems," in *Sensor Array and Multichannel Signal Processing Workshop (SAM), 2014 IEEE 8th*, June 2014, pp. 321–324.

[12] N. Michailow, M. Matthe, I. Gaspar, A. Caldevilla, L. Mendes, A. Festag, and G. Fettweis, "Generalized frequency division multiplexing for 5th generation cellular networks (invited paper)," pp. 1–1, 2014.

[13] K. Martin, "Small side-lobe filter design for multitone data-communication applications," *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on*, vol. 45, no. 8, pp. 1155–1161, 1998.

[14] S. Mirabbasi and K. Martin, "Overlapped complex-modulated transmultiplexer filters with simplified design and superior stopbands," *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on*, vol. 50, no. 8, pp. 456–469, 2003.