Wireless Sensor Network Power Profiling Based on IEEE 802.11 and IEEE 802.15.1 Communication Protocols: Modeling and Simulation

M2CA-14-ID-BSU_-0704-013

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October 2015



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INL/EXT-15-37005 Revision 0

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Prepared for the U.S. Department of Energy Office of Nuclear Energy Under DOE Idaho Operations Office Contract DE-NE0008255 (This page intentionally left blank)

ABSTRACT

The goal of this collaborative research between Idaho National Laboratory, Boise State University, and University of Houston under the Nuclear Energy Enabling Technologies Program's Advanced Sensors and Instrumentation Pathway is aimed at developing efficient and reliable thermoelectric (TE) generators for selfpowered wireless sensor nodes (WSNs) for nuclear applications. The power harvesting technology has crosscutting significance to all U.S. Department of Energy Office of Nuclear Energy research and development programs, as it will enable self-powered WSNs in multiple nuclear reactor designs and spent fuel storage facilities using thermal energy available in a nuclear power plant or spent fuel storage facility. This project will address the technology gap that exists in realizing truly WSNs due to the need for cables to connect to external power supplies and develop TE power harvesting devices to deliver sufficient power to drive the WSNs. The outcomes of the project will lead to significant advancement in sensors and instrumentation technology, reducing cost, improving monitoring reliability, and therefore enhancing safety. The selfpowered WSNs could support the long-term safe and economical operation of all reactor designs and fuel cycle concepts, as well as spent fuel storage and many other nuclear science and engineering applications.

Most wireless sensor network (comprising of thousands of WSNs) applications require operation over extended periods of time beginning with their deployment. Network lifetime is extremely critical for most applications and is one of the limiting factors for energy-constrained networks. Based on applications, there are wide ranges of different energy sources suitable for powering WSNs. A battery is traditionally used to power WSNs. The deployed WSN is required to last for a long time. Due to the finite amount of energy present in batteries, it is not feasible to replace batteries. Recently there has been a new surge in the area of energy harvesting where ambient energy in the environment can be utilized to prolong the lifetime of WSNs. Some of the sources of ambient energies are solar power, thermal gradient, human motion and body heat, vibrations, and ambient radio frequency energy.

The design and development of thermoelectric generators to power WSNs that would remain active for a long period of time requires comprehensive understanding of WSN operations. This motivates the research in modeling the lifetime, i.e., power consumption, of a WSN by taking into consideration various node and network level activities. A WSN must perform three essential tasks: sense events, perform quick local information processing of sensed events, and wirelessly exchange locally processed data with the base station or with other WSNs in the network. Each task has a power cost per unit time and an additional cost when switching between tasks. A number of other considerations must also be taken into account when computing the power consumption associated with each task. The considerations include the number of events occurring in a fixed active time period and the duration of each event, event-information processing time, total communication time, and number of retransmissions. Additionally, at the network level the communication of information data packets between WSNs involves collisions, latency, and retransmission, which result in unanticipated power losses.

This report focuses on rigorous stochastic modeling of power demand for a schedule-driven WSN utilizing Institute of Electrical and Electronics Engineers 802.11 and 802.15.4 communication protocols. The model captures the generic operation of a schedule-driven WSN when an external event occurs, i.e., sensing, followed by processing, and followed by communication. The report describes the development of an expression to compute the expected energy consumption per operational cycle of a schedule-driven WSN by taking into consideration the node level activities, i.e., sensing and processing, and the network level activities, i.e., channel access, packet collision, retransmission attempts, and transmission of a data packet.

ACKNOWLEDGMENTS

This report was made possible through funding by the U.S. Department of Energy Nuclear Energy Enabling Technologies Program. We are grateful to Suibel Schuppner at the U.S. Department of Energy and Bruce Hallbert at Idaho National Laboratory for championing this effort. We thank Jodi L. Vollmer and Heather Medema for technical editing and formatting of the document.

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ACRONYMS

ACK	acknowledgement
ADC	analog-to-digital convertor
BE	back-off exponent
BI	beacon interval
BOD	back-off delay
BP	back-off period
CA	collision avoidance
CAP	Contention Access Period
CCA	clear channel assessment
CSMA	carrier sense multiple access
CTS	clear-to-send
CW	contention window
DCF	distributed coordination function
DIFS	Distributed Inter Frame Space
HEX	heat exchanger
IEEE	Institute of Electrical and Electronics Engineers
INL	Idaho National Laboratory
ISM	industrial, science, and medical
MAC	medium access control
NB	number of back-off
NPP	nuclear power plant
PER	packet error rate
RTS	request-to-send
SD	superframe duration
SIFS	Short Inter-Frame Space
SMP	semi-Markov process
TE	thermoelectric
TEG	thermoelectric generator
WSN	wireless sensor node
WPAN	wireless personal area network
WLAN	wireless local area network

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Wireless Sensor Node Power Profiling Based on 802.11 and 802.15.4 Wireless Protocols: Modeling and Simulation

1. INTRODUCTION

Design and technical advancements in sensing, processing, and communication capabilities of small, portable, and battery-powered devices, known as wireless sensor nodes (WSNs), have drawn extensive research attention. These WSNs, when interconnected wirelessly, regardless of topology, form a network known as a wireless sensor network and have found vast application in science, engineering, and new consumer applications (Akyildiz et al. 2002).

A WSN has an operational architecture as shown in Figure 1. From Figure 1, observe that a WSN has (1) a single energy source unit (battery and direct-current converter); (ii) an onboard embedded processing capability (microcontroller unit), (iii) an onboard sensing unit (sensors and analog-to-digital convertor, ADC) that samples the surrounding environment for sensing physical phenomenon and converts the analog signal into a digital signal, (iv) an onboard storage (memory) to temporarily store the locally processed information from the microcontroller unit, and (v) an onboard radio transceiver which wirelessly exchanges locally processed information with the base station or with other nodes in the network. Applications and energy limitation are the two key factors taken into consideration while selecting onboard components; since, WSNs are inherently resource constrained and must usually operate for extended periods of time from their limited and local energy reserves (Culler et al. 2004).



Figure 1. Architecture of a wireless sensor node (Culler et al. 2004).

Most wireless sensor network applications require operation over extended periods of time beginning with their deployment (Raghunathan et al. 2006). Network lifetime is extremely critical for most applications and is one of the limiting factors for energy-constrained networks (Dietrich and Dressler 2009). Based on these applications, wide ranges of different energy sources are suitable for powering WSNs (Rabaey et al. 2000; Beeby et al. 2007). A battery is traditionally used to power WSNs. Battery discharge behavior is nonlinear with time and depends on various factors: discharge rate, temperature,

humidity, discharge/charge cycles, etc. The deployed WSN is required to last for a long time. Due to a finite amount of energy present in batteries, it is not feasible to replace them. Recently there has been a new surge in the area of energy harvesting where ambient energy in the environment can be utilized to prolong the lifetime of WSNs. Some of the sources of ambient energies are solar power, thermal gradient, human motion and body heat, vibrations, and ambient radio-frequency energy. A detailed review of different energy harvesting techniques is presented in (Gilbert and Balouchi 2008; Clayton et al. 2012).

The application of power harvesting technology to energize WSNs has crosscutting significance to all U. S. Department of Energy Office of Nuclear Energy research and development programs, as it will enable self-powered WSNs (ORNL 2006) in multiple nuclear reactor designs and spent fuel storage facilities. Wireless communications enable the elimination of communication wires; a technology gap still exists in realizing truly WSNs due to the need for external power supply cables.

The Advanced Sensors and Instrumentation Pathway under the Nuclear Energy Enabling Technologies Crosscutting Technology Development Program is supporting this research project to address this important technology gap and develop thermoelectric (TE) power harvesting devices to deliver sufficient power to drive the WSNs. Advancements in TE material efficiency over the last decade (Mehta et al. 2012; Yan et al. 2012; Poudel et al. 2008) has sparked a great deal of research in this field when compared to other applications of power harvesting technologies. With the abundance of waste heat energy available in a nuclear facility, thermoelectric generators (TEGs) are one the most promising power harvesting technologies that would benefit the nuclear industry. A collaborative team of researchers from Idaho National Laboratory, Boise State University, and the University of Houston is performing this project. This partnership brings together expertise and resources encompassing three aspects critical to project success—TEs, nuclear science, and wireless sensors.

The project goal is to develop efficient and reliable TEGs based on high-efficiency nanostructured bulk materials that directly convert heat into electricity to power WSNs for nuclear applications, as shown in Figure 2. Thermoelectric devices at their most fundamental layer consist of one n-type and one p-type leg bonded together to form a unicouple. When many unicouples are combined electrically in series, and thermally in parallel the device becomes known as a TE module. Adding a form of heat sink or heat exchanger (HEX) to both the hot and cold sides of the module, the device is then known as a TEG (as shown in Figure 2). The benefit of self-powered WSNs goes beyond the cost savings of eliminating the need for cable installation and maintenance. Self-powered WSNs will offer significant expansion in remote monitoring of nuclear facilities by providing important data on nuclear power plant (NPP) equipment and component status during station blackouts or accident conditions; thus, WSNs will significantly improve reliability and safety in NPPs and spent fuel storage facilities (Clayton et al. 2012).



Figure 2. Thermoelectric materials and devices.

1.1 Problem Statement

The design and development of TEGs to power WSNs that would remain active for a long period of time requires comprehensive understanding of WSN operations. This motivates the research in modeling the lifetime, i.e., power consumption, of a WSN by taking into consideration various node and network level activities. A TEG device must be capable of satisfying varying power demands of a WSN under different operating conditions, including normal or abnormal conditions. A WSN must perform three essential tasks: sense events, perform quick local information processing of sensed events, and wirelessly exchange locally processed data with either the base station or other WSNs in the network. Each task has a power cost per unit time and an additional cost when switching between tasks. A number of other considerations must also be taken into account when computing the power consumption associated with each task. These considerations include the number of events occurring in a fixed active time period and the duration of each event, event-information processing time, total communication time, and number of retransmissions. Additionally, at the network level the communication of information data packets between WSNs involves collisions, latency, and retransmission, which result in unanticipated power losses. Accurate information of power per unit time impacts due to collisions, latency, and retransmission needs to be accounted for in WSN power consumption modeling.

It is essential to obtain a representative WSN power consumption model that accounts for all of the WSN level and network level considerations. For energy-constrained WSN, monitoring all of the WSN functions is an infeasible luxury. Therefore, a stochastic approach is the only reasonable alternative in lieu of complete WSN and network monitoring. Figure 3 shows a schematic of self-powered WSN using TEG.





1.2 Report Layout

This report focuses on rigorous stochastic modeling of power demand for a schedule-driven WSN utilizing Institute of Electrical and Electronics Engineers (IEEE) 802.11 and 802.15.4 communication protocols. The model captures the generic operation of a schedule-driven WSN when an external event occurs, i.e., sensing, followed by processing, and followed by communication. The report describes development of an expression to compute the expected energy consumption per operational cycle of a schedule-driven WSN by taking into consideration the node level activities, i.e., sensing and processing,

and the network level activities, i.e., channel access, packet collision, retransmission attempts, and transmission of a data packet.

The report is laid out as follows:

Section 2 describes schedule-driven operation WSN, its different operation states, and stochastic transition between the states based on modeling assumptions and the frequency of event-occurrence.

Section 3 presents an overview of IEEE 802.11 and IEEE 802.15.4 communication protocols.

Section 4 describes the computation of the expected value and the variance of energy consumption by a WSN based on IEEE 802.11 and IEEE 802.15.4 protocols in each state.

Section 5 describes the modeling and simulation results. The expected value and the variance of energy consumed by a WSN based on IEEE 802.11 and IEEE 802.15.4 protocols in each state and in one cycle are computed. The results are verified via simulation.

Section 6 summarizes the modeling and simulation research effort in the area of WSN power profiling and outlines the research path forward to integrate WSN modeling with TEG model.

2. Stochastic Behavior of a Schedule-Driven Wireless Sensor Node

The operation of a WSN can be broadly categorized as trigger-driven operation and schedule-driven operation. In this report and research, schedule-driven operation, also known as duty-cycle operation, is considered, because in nuclear applications most deployed WSN would be required to periodically send the collected data to the base station under normal operation. In case of abnormal operation, the periodicity of data transmission can be increased as needed. For trigger-driven operation, maintenance or an operator has to explicitly request information, which might be undesirable in an abnormal situation. A WSN will imply schedule-driven WSN hereinafter unless otherwise mentioned.

2.1 Schedule-Driven Wireless Sensor Node and Operational States

A WSN comprises *sleep* and *active* time periods corresponding to SLEEP and ACTIVE states. During the SLEEP state, the WSN is dormant and consumes a very little power. During the ACTIVE state, the WSN senses the environment for the occurrence of an event, processes any sensed event, and transmits (utilizing defined communication protocol) the processed information to the base station or to other WSNs. In addition, it routes information packets received from other WSNs.

In this research, six different operational states of a WSN are identified (Agarwal 2011). Thus, the operational state space of a WSN is $S = \{S_0, S_{1i}, S_{1e}, S_{1r}, S_2, S_3\}$ where

- (i) S_0 represents the SLEEP state during which the WSN is completely disconnected from the base station and from other WSNs;
- (ii) S_{1i} represents the ACTIVE state when no event is detected;
- (iii) S_{1e} represents the ACTIVE state when only a sensing-event occurs;
- (iv) S_{1r} represents the ACTIVE state when only a receiving-event occurs, i.e., the WSN receives a packet of information from other WSNs and forwards or relays the information to the receiving WSN (or to the base station);
- (v) S_2 represents the ACTIVE state during which processing of event information is performed; and
- (vi) S_3 represents the ACTIVE state during which information is transmitted.

Each state in **S** has a certain associated power consumption, which discharges the WSN battery. An additional energy cost is associated with transitions between states. Let $I_S = \{0, 1i, 1e, 1r, 2, 3\}$ denote the index for the possible states of the WSN. Based on modeling assumptions and the frequency of event-occurrence, the WSN transitions to a particular state S_k , $k \in I_S$. The time duration a WSN spends in a particular state is not fixed, except for the sleep state, T_{S_0} which is fixed (for the purpose of this research and can be easily converted to a random variable). Thus the time spent in each of the other states are considered to be a random variable with mean value μ_k , $k \in I_S$. After spending a random amount of time in a particular state, the WSN transitions to the next allowable state with transition probability p_{ij} ,

 $i \neq j$. Thus, to describe the transition of the WSN from one state to another state, the following information is required: (i) the present state of the WSN and (ii) the random amount of time it spends in the present state. This is precisely the structure of a semi-Markov process (SMP). An SMP, a special case of Markov process, accounts for the amount of time that a process spends in each state before making the transition from its present state to the next state. Therefore, an SMP is utilized to describe the operational behavior of a WSN.

The components of WSN (as shown in Figure 1) have different operational modes when the WSN is in the ACTIVE state. The sensor has two operational modes: ON and OFF. A CPU is considered to have three operational modes: OFF, IDLE, and ACTIVE. A transceiver also has three operational modes: OFF, Receive (Rx), and Transmit (Tx). Table 1 displays different operational states of a WSN, the corresponding power costs, and the different operational modes of its components. In addition, it is assumed that the transition energy cost from S_{1i} state to S_{1r} state ($E_{1i,1r}$) or from S_2 state to S_{1r} state ($E_{2,1r}$) is the same, i.e., $E_{2,1r} = E_{1i,1r} \triangleq E_{tr2}$. Similarly, the energy cost associated with the transition from an S_0 to an ACTIVE state is defined as $E_{01} \triangleq E_{tr1}$. Figure 4 shows the power consumption for each state for different commercially available WSNs.

State	Sensor	CPU	Transceiver	Power
S ₀	Off	Off	Off	P_0
S_{1i}	On	Idle	Off	P_{1i}
S _{1e}	On	On	Off	P_{1e}
S _{1r}	Off	On	Rx	P_{1r}
<i>S</i> ₂	Off	On	Off	<i>P</i> ₂
<i>S</i> ₃	Off	On	Тх	<i>P</i> ₃

Table 1. Operational states of a WSN with component states and power consumptions.



Figure 4. Power consumption for each state for different WSNs.

2.2 Modeling Assumptions

To model the transition of a WSN from one state to another, the following assumptions are taken into consideration.

- 1. The time duration a senor node spends in a sleep state is fixed T_{S_0} .
- 2. Two types of events are considered; sensing event and relay event. Both of the events are considered independent.
- 3. The arrival of a sensing event is modeled as a Poisson process whose distribution function is given as $f_{SE} = \frac{\lambda_{SE}^k}{k!} e^{-\lambda_{SE}}$ where λ_{SE} is the average sensing event arrival rate and k is the number of

occurrences of a sensing event.

- 4. Since the event generation has a Poisson model, the inter-event arrival time is exponentially distributed with means $1/\lambda_{\rm SE}$ for sensing event and $1/\lambda_{\rm RE}$ for relay event. In general, the exponential distribution function is given as $f(t) = \lambda e^{-\lambda t}$ where λ is the event arrival rate; in case of sensing event $\lambda = \lambda_{\rm SE}$ and in case of a relay event $\lambda = \lambda_{\rm RE}$.
- 5. Similar to Assumptions 3, the arrival of a relay event is modeled as a Poisson process whose distribution function is given as $f_{\text{RE}} = \frac{\lambda_{\text{RE}}^k}{k!} e^{-\lambda_{\text{RE}}}$ where λ_{RE} is the average relay event arrival rate and k is the number of occurrences of a relay event.
- 6. A sensing event occurs for average time duration of $\overline{T}_e \triangleq \mu_{S_1}$ seconds.
- 7. When an event is detected, the WSN processes it and sends the information to the base station (or to another WSN) with probability 1.
- 8. The WSN radio transmits a packet of information at a fixed transmission power level.

- 1. Each operational cycle of a WSN is independent and identically distributed.
- 2. The communication between WSNs (or between a WSN and the base station) is based on carrier sense multiple access (CSMA) / collision avoidance (CA) medium access control (MAC) protocol. This assumption holds for both IEEE 802.11 and IEEE 802.15.4 communication protocols.
- 3. In this research, the maximum number of retransmissions allowed per packet is 3, i.e., $N_{\text{Re}Tx}^{\text{max}} = 3$. The packet is considered dropped if it is not transmitted successfully within $N_{\text{Re}Tx}^{\text{max}}$ attempts.
- 4. During transmission of a packet, retransmission is observed because of packet collision or packet error rate (PER) at the receiver's end.
- 5. During a back-off process (see Section 3.1 for details), a back-off value is randomly selected as per a uniform distribution, and the number of retransmissions is modeled as a truncated Poisson process.

2.3 Stochastic Operation

Based on modeling assumptions and the frequency of event-occurrence, a WSN transitions to a particular state S_k , $k \in I_S$ where $I_S = \{0, 1i, 1e, 1r, 2, 3\}$ denotes the index for the possible states of the WSN. The time duration a WSN spends in a particular state is not fixed, except for the sleep state duration, T_{S_0} (which is fixed as per our assumption). Thus, we consider the time spent in each of the other states to be a random variable (each governed by a probability distribution) with the expected (mean) value μ_{S_k} and variance $\sigma_{S_k}^2$, $k \in I_S$. After spending a random amount of time in the state of index k, the WSN transitions to the next allowable state, of index j, with the transition probability p_{ki} , $k \neq j$.

The values of μ_{S_k} and $\sigma_{S_k}^2$ depend on the probability distribution used to describe the stay of the WSN in each state. For example, the amount of time a WSN remains idle in the idle state conditionally depends on the occurrence of an event during the maximum active time period. In this research, event occurrence is a random process and is modeled as a Poisson process. By the property of the Poisson process, the time of event occurrence over any time interval is uniformly distributed. Therefore, occurrence of an event during the maximum active time period is governed by a uniform distribution (see Appendix A). Similarly, the amount of time required to process a sensing event in the processing state is described by an exponential distribution.

The initial state of a WSN is S_0 (sleep), which has a fixed duration of T_{S_0} . The transition of the WSN from S_0 to any particular state during an active time period depends on event occurrence. In case of an event (sensing or relay), a WSN can transit to any one of the two states: S_{1e} (sensing event) or S_{1r} (relay event) with probability α and $(1 - \alpha)\beta$ respectively, where β is the probability of a relay event occurring.

In the case of no event, the WSN in the state S_0 transits to the state S_{1i} (idle) with probability $1 - (\alpha + \beta - \alpha\beta)$ (as both sensing and relay events are assumed to be independent). Further, if there is no event, the WSN remains in the state S_{1i} for a pre-defined duration of T_A seconds before returning back to the state S_0 with probability 1 where T_A is the maximum active time period that a WSN can remain idle with no event before transitioning to the sleep state.

If the WSN is in S_{1e} (sensing), it will next transit to the state S_2 (processing) with probability 1. If the WSN is in the state S_{1r} (relaying), it will next transit to the state S_3 (transmitting) with probability 1.

Similarly, if the WSN is in the state S_2 , it will next transit to the state S_3 with probability 1. We note here that the rate at which information is processed will be denoted by κ , for which $\frac{1}{\kappa}$ is the expected processing time. Once the WSN enters the state S_3 , it transitions to the state S_0 with probability 1 upon completion of transmission of data packets either to the base station (or to another WSN).

The stochastic operation of a WSN is illustrated in Figure 5 and is summarized with the following (well established) probability transition matrix \mathbf{P} .

S_0	S_{1i}	S_{1e}	S_{1r} S_{1r}	2 '	<i>S</i> ₃	
0	$1 - \alpha - \beta + \alpha \beta$	α	$(1-\alpha)\beta$	0	0]
1	0	0	0	0	0	
0	0	0	0	1	0	
0	0	0	0	0	1	
0	0	0	0	0	1	
1	0	0	0	0	0	
	$ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} $	$ \begin{bmatrix} 0 & 1 - \alpha - \beta + \alpha \beta \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} $	$ \begin{bmatrix} 0 & 1 - \alpha - \beta + \alpha \beta & \alpha \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$	$ S_0 \qquad S_{1i} \qquad S_{1e} \qquad S_{1r} \qquad S_{$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ S_0 \qquad S_{1i} \qquad S_{1e} \qquad S_{1r} \qquad S_2 \qquad S_3 $ $ \begin{bmatrix} 0 & 1 - \alpha - \beta + \alpha\beta & \alpha & (1 - \alpha)\beta & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} $

(1)

The entries of the matrix **P** must satisfy the following constraints:

$$0 \le p_{ij} \le 1, \ i, j \in \mathbf{S}$$

$$\sum_{j \in \mathbf{S}} p_{ij} = 1, \ i \in \mathbf{S} \text{ and } i \neq j$$
(3)



Figure 5. State transition diagram (Agarwal 2011).

3. Communication Protocols

A number of IEEE standardized protocols have been developed for various types of wireless network applications. These protocols vary in cost to implement, complexity, and data rate. Such variation within the numerous protocols and how they compare can be seen in Figure 6.



Figure 6. Comparison of wireless communication protocols.

In this research, IEEE 802.11 and IEEE 802.15.4 communication protocols are studied, because they are most widely used in many applications and are of interest to the nuclear industry. Most preferred wireless communication mode in the nuclear industry is Wi-Fi (a.k.a. IEEE 802.11). It is known that IEEE 802.11 has high-energy consumption making them less preferred communication protocol for design of self-powered WSNs. However, if a TEG device performance can be maximized to meet the energy (power) requirements of IEEE 802.11, then TEG powered WSN will easily be able to meet the energy requirements of other variant of protocols within the wireless personal area network (WPAN) zone.

3.1 Overview of IEEE 802.11

Wireless local area network (WLAN, also known as Wi-Fi) is a set of low tier, terrestrial network technologies for data communication. The WLAN standard operates on the 2.4 GHz and 5 GHz Industrial, Science and Medical (ISM) frequency bands, as specified by the IEEE 802.11 standard (IEEE Std 802.11 2007). IEEE 802.11 comes in many different variations like IEEE 802.11a/b/g/n. The MAC of IEEE 802.11protocol has two modes: Distributed Coordination Function (DCF) and Point Coordination Function.

The power consumption of a WSN utilizing the IEEE 802.11 MAC protocol (IEEE Std 802.11 2007) based on DCF is evaluated in this research. DCF is a random-access scheme based on the CSMA/CA protocol. DCF employs two techniques for packet transmission:

- (i) The default and basic two-way handshaking technique called the basic access mechanism
- The four-way handshaking technique, known as request-to-send/clear-to-send (RTS/CTS) mechanism.

The basic access mechanism is characterized by a packet transmission and immediate transmission of a positive acknowledgement (ACK) by the destination (receiver) WSN, upon successful reception of a packet transmitted by the sender WSN. Explicit transmission of an ACK is required since, in the wireless medium, a transmitting WSN cannot determine if a packet is successfully received by listening to its own transmission.

In the RTS/CTS mechanism, before transmitting a packet, a WSN "reserves" the channel by sending a special RTS short frame. Some destination (receiver) WSN acknowledges the receipt of the RTS frame by sending back a CTS frame, after which normal packet transmission and ACK response occurs. For a more detailed and complete description of 802.11 protocols, refer to the 802.11 standards (IEEE Std 802.11 2007).

A WSN in the transmission state, before making the decision to transmit a packet (as per any of the above-mentioned transmission mechanism), undergoes a random back-off process^a (this is a feature of the CSMA/CA protocol and is discussed in the following section), in which the WSN monitors the channel to minimize the probability of collision with packets being transmitted by other WSNs. At the end of the back-off process, the WSN makes a transmission attempt. If the transmission attempt is unsuccessful, the WSN repeats the back-off process before making another transmission attempt of the same packet. This procedure is repeated until the packet is successfully transmitted or the maximum number of allowed retransmission attempts is reached. Thus, time in the transmission state (S_3) is a random variable that depends on the number of retransmission attempts per packet. This in turn depends on the maximum number of allowable retransmissions, the total length of the time associated with the back-off process, and the time lost due to unsuccessful transmissions.

A successful transmission of a packet is observed if a sender WSN receives an ACK signal from the destination WSN. Alternatively, an unsuccessful transmission is observed if a sender WSN does not receive an ACK signal from the destination WSN. There are many possible reasons for a packet transmission to be unsuccessful. In this research, unsuccessful transmissions are due to (i) PER, i.e., an error occurring in the communication medium, and (ii) packet collision. An unsuccessful transmission occurring over the channel due to PER or packet collision are assumed to be independent.

During the transmission of a packet over the channel, a packet error rate at the receiver end can occur due to transmission channel noise, interference, distortion, bit synchronization problems, etc. Basically, when the packet is corrupted at the destination WSN, i.e., at the receiver end, the sender WSN will not receive an ACK signal from the destination WSN.

3.1.1 Overview of Back-off Process

Recall that the IEEE 802.11 DCF protocol adopts a discrete random back-off scheme where the back-off time scale is slotted. In IEEE 802.11 the default value of aSlotTime is 20 µs for IEEE 802.11b and 9 µs for IEEE 802.11a/g. If no medium activity is indicated for the duration of a particular back-off slot then the Back-off slot is decreased by aSlotTime. For each packet before the start of the m-th transmission attempt, a WSN uniformly selects the back-off time (in terms of number of slots, as a part of the back-off scheme), denoted by $B_m \in [0,1,...,W_m-1]$, where $W_m = 2^{m-1}CW_{\min}$ b is the size (in terms of slots) of the contention window during the m-th transmission attempt. Here, CW_{\min} is the minimum size of the contention window. A contention window is a time slot during which a WSN will try to access the transmission medium and will attempt to transmit a packet at the end.

The selected value of $B_m \in [0,1,...,W_m - 1]$ is assigned to the back-off time counter. The back-off time counter is decremented by 1 as long as the channel is sensed idle, is temporarily "frozen" when a transmission by neighboring WSNs is detected on the channel, and the decrement of the time counter is

a. In the network setup, more than one WSN tries to access the channel and transmit its own packet over the channel. It may result in packet collision. To minimize such situations, each WSN in the network undergoes a random back-off process before transmitting the packet.

b. In some literature, back-off scheme is assigned a separate index (say) whose value starts at zero and increases by 1 after each unsuccessful transmission attempt. Then, $W_i = 2^i CW_{\min}$. Here i = m - 1.

reactivated when the channel is sensed idle again for more than a Distributed Inter Frame Space (DIFS). The channel is considered "idle" when the WSN from its neighboring WSNs senses no activity. Alternatively, the channel is considered "busy" when there is transmission activity by the neighboring WSNs. Transmission is attempted when the back off time counter reaches zero; this would mean the end of the first contention window has been reached.

As the back-off time counter decrements, the slot time length,^c denoted as t_{slot} , is of importance because it can either be a constant value or a variable value. Recall that the back-off counter is decremented by 1 if the channel remains idle for more than one DIFS. This value is fixed and is known as empty slot time (T_{empty}). On the other hand, the decrement of the back-off time counter is stopped when the channel is sensed busy, and thus the (variable) time interval between two consecutive slot decrements may be much longer than the empty (idle) slot time, as it may involve successful or unsuccessful packet transmissions by neighboring WSNs that have different time lengths.

If the m-th transmission attempt was successful, the back-off timer of the associated WSN will be reset for the next packet. However, if the transmission attempt was unsuccessful, the contention window (CW) will be incremented. This will allow WSN additional time to retry the data packet. The value of CW is within the interval $[CW_{\min}, CW_{\max}]$ and is dependent on retransmission attempts, N_{ReTx} . Once a CW reaches CW_{\max} and $N_{\text{ReTx}} = N_{\text{ReTx}}^{\max}$, if WSN fails to transmit the data packet successfully, the data packet is dropped. Figure 7 illustrates the exponential increase of the CW with retransmission attempts.

Figure 8 shows an example of the basic access mechanism for the back-off process. Let us consider two WSNs, A and B sharing the same wireless channel. At the end of a successful packet transmission by the WSN A (indicated by the receipt of an ACK signal), WSN A waits for a DIFS and randomly selects a back-off time equal to 10 for this example before transmitting its next packet. The back-off counter of WSN A is decremented as the channel is observed to be idle. We observe that WSN B attempts to transmit its own packet when the back-off time counter of WSN A is at 7 (as indicated in Figure 8). As a consequence, the channel is sensed busy by the WSN A, and the back-off time counter of the WSN A is frozen at its value 7, until WSN B has transmitted/dropped its packet and the channel is available again. In Figure 7, a successful transmission of the packet by WSN B is presented. Once the WSN B packet has been successfully transmitted, the back-off time counter of WSN A has to wait for a DIFS before it reinitiates the decrement process (as indicated in Figure 8). Once the back-off time counter of the WSN A decreases to zero, it attempts the transmission of the packet.

c. Slot time length is defined as the time interval between two consecutive back-off timer counter decrement. In this research, slot time length is referred to as medium time length, as the slot can be empty or busy.



Figure 7. Exponential increase of CW.



Figure 8. An illustration of basic access mechanism with back-off process.

3.2 Overview of IEEE 802.15.4

The IEEE 802.15.4-defined physical and MAC layer characteristics are responsible for establishing connectivity between devices with low power consumption, low cost, and low data rate. The IEEE 802.15.4 standard (IEEE Std. 802.15.4 2006) is commonly known as ZigBee, but ZigBee has some additional features to those of IEEE 802.15.4. It operates in the 868-MHz, 915-MHz, and 2.4-GHz ISM bands. The IEEE 802.15.4 MAC layer has two modes (IEEE Std. 802.15.4 2006): non-beacon enabled mode and beacon-enabled mode.

3.2.1 Non-Beacon Enabled Mode

The non-beacon mode employs the unslotted CSMA/CA algorithm. The un-slotted implies that a WSN does not synchronize timing slots with the coordinator; it can simply transmit its packet if the medium is idle. If the medium is busy, it waits for a random period of time before trying to transmit. The unslotted IEEE 802.15.4 CSMA/CA algorithm begins by setting the number of back-offs (NB) to zero and the back-off exponent (BE) to its initial value (3 by default). WSN then delays for a random amount of aUnitBackoffPeriod (320us by default) determined by an exponential system using the BE. At the end of the back-off period, the WSN will perform a clear channel assessment (CCA). If the channel is first sensed busy, the transmitting WSN will increase its BE and wait another random amount of time before

sensing the medium again. Since the BE dictates how long the WSN will wait before sensing the channel, increase in the BE value leads to a long wait time to gain channel access, which in turn leads to a large amount of energy consumption. If instead the channel is sensed idle, the WSN immediately transmits its packet and waits for an ACK from the receiver. A WSN can sense the channel occupied for up to macMaxCSMABackoffs (4 by default) before it will declare the process a failure. If the acknowledgement is not received in a predetermined amount of time, the WSN will attempt to retransmit the packet up to a maximum amount of retransmissions, macMaxFrameRetries (3 by default), before the packet is dropped. Figure 9 illustrates the unslotted and slotted CSMA/CA algorithm used in the IEEE 802.15.4 protocol.

3.2.2 Beacon-Enabled Mode

The beacon-enabled mode employs slotted CSMA/CA algorithm. The slotted CSMA/CA algorithm employs two periods: active (divided into 16 time slots) and optional inactive (device enters a low-power mode), as illustrated in Figure 10. In Figure 10, the beacon interval (BI) and superframe duration (SD) is given as

$$BI = aBaseSuperframeDuration * 2^{BO}$$
(4)

 $SD = aBaseSuperframeDuration * 2^{SO}$

where aBaseSuperframeDuration = 960 symbols, *BO* is beacon order, and *SO* is superframe order. The values of *BO*, *SO*, and other parameters are summarized in Appendix F.

(5)

At the beginning of the active period, the coordinator sends beacon frames with information regarding the period duration so the duty cycle can vary. The contention access period (CAP) follows the beacon, allowing WSNs to gain medium access using slotted CSMA/CA.

The slotted CSMA/CA algorithm is based on a basic time unit called back-off period (BP), which is equal to *aUnitBackoffPeriod* = 80 bits (0.32 ms). The slotted CSMA/CA back-off algorithm mainly depends on three variables: (1) the BE enables the computation of the back-off delay, (2) the CW represents the number of BPs during which the channel must be sensed idle before channel access, (3) the NB represents the number of times the CSMA/CA algorithm was required to back-off while attempting to access the channel.

First, the number of back-offs and the contention window are initialized (NB = 0 and $CW = CW_{init} = 2$). The back-off exponent is also initialized to BE = 2 or $BE = \min(2, macMinBE)$. The default value of macMinBE is equal to 3. The algorithm starts counting down a random number of BPs uniformly generated within $[0, 2^{BE} - 1]$. The countdown must start at the boundary of a BP. When the BP timer expires, the algorithm then performs one CCA operation at the BP boundary to assess the channel activity. If the channel is idle, the algorithm will decrement the CW_{init} value by 1 and will back-off randomly again within $[0, 2^{BE} - 1]$. If the channel is busy, the value of CW_{init} is not decremented; instead, the values of BE and NB are incremented as $BE = \min(BE + 1, macMaxBE)$ and NB = NB + 1, respectively. Incrementing BE increases the probability for having greater back-off delays. If the maximum number of back-offs (NB = macMaxCSMABackoff = 5) is reached, the algorithm reports a failure as shown in Figure 9.

In case the channel is idle and CW = 1, at the end of the BP timer, another CCA is performed. If the channel is sensed Idle again, CW is decremented by 1 again, i.e., CW = 0, and the WSN will attempt to transmit the packet provided that the remaining BPs in the current CAP are sufficient to transmit the frame and the subsequent ACK signal. If not, the CCAs and the frame transmission are both deferred to



Figure 9. Flow diagram illustrating unslotted and slotted CSMA/CA algorithm of the IEEE 802.15.4 (Campbell et al. 2011).



Figure 10. Structure of a beacon interval in beacon-enabled CSMA/CA algorithm.

the next superframe. This is referred to as CCA deference. The flow chart illustrating slotted CSMA/CA algorithm is presented in Figure 9. The slotted or unslotted CSMA/CA algorithm is performed by a WSN at every transmission attempt. If $N_{\text{Re}Tx} = N_{\text{Re}Tx}^{\text{max}}$ and packet are not successfully transmitted, the packet is dropped and the information is lost.

4. Computation of the Expected Value and the Variance of Energy Consumption by a WSN in Each State

As mentioned in Section 2, the operational state space of a WSN is $\mathbf{S} = \{S_0, S_{1i}, S_{1e}, S_{1r}, S_2, S_3\}$ and the amount of time the WSN spends in a state before transitioning to the next state is a random variable (each governed by a probability distribution) with the expected value μ_{S_k} (mean) and variance $\sigma_{S_k}^2$, $k \in I_S$. In this section, the expected value and the variance of the random time that a WSN spends in the sleeping state (S_0), the idle state (S_{1i}), the sensing event state (S_{1e}), the processing state (S_2), the relay state (S_{1r}), and transmission state (S_3) are computed.

4.1 Sleep State (S_0)

As per Assumption 1, a WSN stays in the state S_0 for fixed time duration of T_{S_0} . Therefore, the expected time the WSM spends in S_0 is T_{S_0} and the variance is zero.

$$\mathbf{E}\left[T_{S_0}\right] = T_{S_0} \text{ and } Var\left[T_{S_0}\right] = 0 \tag{6}$$

4.2 Idle State (S_{1i})

After spending T_{S_0} seconds in the sleep state, the WSN enters the active state. In the active state, the WSN spends a certain amount of time in the idle state, S_{1i} , denoted as $T_{S_{1i}}$, until an event occurs. The length of time a WSN stays in the idle state depends on (1) occurrence of event and (2) the time of event occurrence during the active time period.

The occurrence of an event is a random process and is modeled as a Poisson process. An event can occur at any time during the active time period. As per the property of the Poisson process, the time of occurrence of an event within any time interval is uniformly distributed (see Appendix A). Because both the event occurrence and the time of event occurrence are random, the length of $T_{S_{12}}$ is a random variable.

Therefore, the expected value and the variance of the random variable $T_{S_{1i}}$, denoted as $\mu_{S_{1i}}$ and $\sigma_{S_{1i}}^2$,

respectively, conditionally depend on the occurrence of an event during the active time period. In case of no event during the entire active time period, the WSN stays idle for a predefined fixed time duration before checking for a relay event (which is not present by assumption) before transitioning back to the sleep state. The expected value and the variance of $T_{S_{1i}}$ when no event occurs are

$$\mathbf{E}\left[T_{S_{1i}|\text{No Event}}\right] = \mu_{S_{1i}|\text{No Event}} = T_A \text{ and } Var\left[T_{S_{1i}|\text{No Event}}\right] = \sigma_{S_{1i}|\text{No Event}}^2 = 0$$
(7)

where T_A is the maximum length of the active time period when no event occurs.

In this research, it is assumed that at least a single event (sensing or relay) occurs during the interval $[t_A, t_A + T_A]$ where t_A represents the time instance a WSN enters the active state. Since the arrival time of the event is uniformly distributed over $[t_A, t_A + T_A]$, the random variable $T_{S_{1i}}$, conditioned on the occurrence of at least a single event, is described by a uniform probability distribution. Therefore,

$$\mathbf{E}\left[T_{S_{1i}|Event}\right] = \mu_{S_{1i}|Event} = \int_{t_A}^{t_A + T_A} \frac{\tau - t_A}{T_A} d\tau = \frac{T_A}{2}$$
(8)

where τ is the time instance an event occurs over $[t_A, t_A + T_A]$. The variance of the random variable $T_{S_{1i}}$ conditioned on the occurrence of at least a single event in the interval $[t_A, t_A + T_A]$ is

$$\sigma_{S_{1i}|\text{Event}}^2 = \frac{T_A^2}{12} \tag{9}$$

4.3 Sensing State (S_{1e})

When a single sensing event occurs during $[t_A, t_A + T_A]$, the WSN transitions to the sensing state S_{1e} and the amount of time it stays in S_{1e} , denoted as $T_{S_{1e}}$, depend on the duration of the sensing event. Let T_e^{\min} be the minimum time duration of a sensing event and T_e^{\max} be the maximum time duration of a sensing event. Recall that as per Assumption 7 in the modeling assumptions in Section 2.2, the WSN stays in the sensing state for a time duration uniformly distributed between T_e^{\min} and T_e^{\max} . Therefore, the expected value and the variance of the random time $T_{S_{1e}}$ are

$$\mathbf{E}[T_e] = \bar{T}_e = \mu_{S_{1e}} = \frac{(T_e^{\min} + T_e^{\max})}{2}$$
(10)

$$Var[T_e] = \sigma_{S_{1e}}^2 = \frac{(T_e^{\max} - T_e^{\min})^2}{12}$$
(11)

4.4 **Processing State** (S_2)

Once the sensing process of a WSN is complete, it enters the processing state S_2 . The amount of time a WSN stays in the processing state depends on the time taken to process the sensed event. The processor of a WSN takes a random amount of time to process the sensed event because it depends upon the existing number of tasks already being performed by the processor (which in our modeling is unknown and random). Therefore, the processing time, denoted as T_2 , is a random variable and is described by an exponential distribution with the parameter κ where κ is the average processing rate (Simunic et al. 2001; Jung et al. 2009).^d The expected value and the variance of T_2 , denoted as μ_{S_2} and

 $\sigma_{S_2}^2$, respectively have well-known expressions given by equations (12) and (13).

$$\mathbf{E}\left[T_2\right] = \mu_{S_2} = \kappa^{-1} \tag{12}$$

$$Var[T_2] = \sigma_{S_2}^2 = \kappa^{-2}$$
(13)

4.5 Relay State (S_{1r})

The expected time a WSN stays in the relay state is expressed as,

$$\mathbf{E}\left[T_{S_{1r}}\right] = \frac{L}{R} + T_{SIFS} + T_{ACK} = \mu_{S_{1r}}$$
(14)

where

- (i) L is the length of the packet in bits (assumed to be fixed) to be received;
- (ii) R is the channel bandwidth in Kbps;
- (iii) T_{SIFS} is the duration of Short Inter-Frame Space (SIFS)^e; and
- (iv) T_{ACK} is the time taken to acknowledge the receipt of a relay event. Equation (14) is not random; here $\mu_{S_{12}}$ is always a fixed known quantity.

4.6 Transmission State (S_3)

The total time that a WSN spends in the transmission state S_3 , denoted as T_{S_3} , to transmit a packet of L bits (assumed to be fixed) at a fixed transmission rate of R kilobits per second (kbps) is

$$T_{S_3} = T_{MAC} + \frac{L}{R} + T_{SIFS} + T_{ACK}$$
(15)

where

d. Simunic et al. (2001) in their work verified experimentally that the processing time can be well characterized by an exponential distribution.

e. T_{SIFS} is the time duration that a sensor node has to wait before sending the ACK signal indicating the successful reception of the packet.

- (i) T_{SIFS} is the fixed duration of SIFS;
- (ii) T_{ACK} is the ACK transmission time (fixed as per IEEE standards); and
- (iii) T_{MAC} is defined as the time from the instant the WSN starts trying to send a packet until the beginning of its successful transmission. T_{MAC} is a random variable, because it depends on a back-off process of the communication protocol and the number of retransmission attempts. Hence, T_{S_3} is also a random variable. The expected value and the variance of T_{S_3} are given in equations (16) and (17), respectively.

$$\mathbf{E}\left[T_{S_3}\right] = \mathbf{E}\left[T_{MAC}\right] + \mu_{S_{1r}}$$
(16)

$$Var\left[T_{S_3}\right] = Var\left[T_{MAC}\right] \tag{17}$$

At this point, expressions for $\mathbf{E}[T_{MAC}]$ and $Var(T_{MAC})$ are computed in the following section. Table 2 summarizes the expression for the expected value and the variance of the random amount of time T_{S_k} , $k \in I_S$, a WSN spends in each state during a single cycle of operation.

4.6.1 Computation of Expected Value and Variance of Total Back-off Delay for IEEE 802.11 ($\mathbf{E}[T_{MAC}]$ and $Var(T_{MAC})$)

 T_{MAC} is defined as the time from the instant the WSN starts trying to send a packet until the beginning of its successful transmission. T_{MAC} is a random variable as it depends on a back off process of both IEEE 802.11 and IEEE 802.15.4 communication protocols and the number of retransmission attempts. Hence, T_{S_3} is also a random variable. The $\mathbf{E}[T_{MAC}]$ and $Var(T_{MAC})$ for IEEE 802.11 are given by equations (18) and (19) respectively (See Appendix D for details).

$$\mathbf{E}\left[T_{MAC}\right] = \frac{\left(1 - p_u\right)}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}-1} p_u^{N_{\text{Re}Tx}} \mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right]$$
(18)

$$Var[T_{MAC}] = \frac{\left(1 - p_{u}\right)}{1 - p_{u}^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}^{\text{max}}} \left[Var[T_{MAC}(N_{\text{Re}Tx})] + \left(\mathbf{E}[T_{MAC}(N_{\text{Re}Tx}) - \mathbf{E}[T_{MAC}]]\right)^{2}\right]$$
(19)

where

- (i) p_u is the probability that a medium is busy due to an unsuccessful transmission;
- (ii) $N_{\text{Re}Tx} \in \{0, 1, 2, ..., N_{ReTx}^{\text{max}} 1\}$ is the number of retransmission attempts and is truncated on the right at $N_{ReTx}^{\text{max}} 1$ and is represented by a truncated probability mass function derived in Appendix B;

State	Expected time (μ_{S_k})	Variance $(\sigma_{S_k}^2)$
S ₀	T_{S_0}	0
S _{1i}	No Event: T_A	0
	Any Event: $\frac{T_A}{2}$	$\frac{T_A^2}{12}$
S _{1e}	$\frac{(T_e^{\min} + T_e^{\max})}{2}$	$\frac{(T_e^{\max} - T_e^{\min})^2}{12}$
S _{1r}	$\frac{L}{R} + T_{SIFS} + T_{ACK}$	0
<i>S</i> ₂	$\frac{1}{\kappa}$	$\frac{1}{\kappa^2}$
<i>S</i> ₃	$\frac{L}{R} + T_{SIFS} + T_{ACK} + \mathbf{E} \big[T_{MAC} \big]$	$Var(T_{MAC})$

Table 2. The expected value and the variance of the time a WSN spends in each state.

(iii) N_{ReTx}^{\max} is the maximum number of retransmissions allowed before dropping the packet; and

(iv) $\mathbf{E}[T_{MAC}(N_{\text{Re}Tx})]$ and $Var[T_{MAC}(N_{\text{Re}Tx})]$, as expressed in equations (20) and (21), respectively, are the expected value and the variance of the MAC delay for IEEE 802.11 during the retransmission attempt $N_{\text{Re}Tx}$. See Appendix D for details.

$$\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right] = \sum_{m=0}^{N_{\text{Re}Tx}} \mathbf{E}\left[BOD_{m}\right] + N_{\text{Re}Tx}\overline{T}_{unsucc}, \quad N_{\text{Re}Tx} \in \{0, 1, 2, ..., N_{ReTx}^{\text{max}} - 1\}$$
(20)

$$Var[T_{MAC}(N_{\text{Re}Tx})] = \sum_{m=0}^{N_{\text{Re}Tx}} Var[BOD_m], \ N_{\text{Re}Tx} \in \{0, 1, 2, ..., N_{ReTx}^{\text{max}} - 1\}$$
(21)

Here

(i) $\mathbf{E}[BOD_m]$ and $Var[BOD_m]$ are the expected value and the variance of the back-off delay (BOD), denoted as BOD_m , during the m – th transmission attempt respectively; and

(ii) \overline{T}_{unsucc} is the average delay due to each unsuccessful transmission attempt.

The formulas for the expected value and the variance of the back-off delay BOD_m for the m – th transmission attempt are expressed in equations (22) and (23), respectively

$$\mathbf{E}\left[BOD_{m}\right] = \mathbf{E}\left[B_{m}^{Total}\right]\mathbf{E}\left[t_{medium}\right]$$
(22)

$$Var[BOD_{m}] = \mathbf{E} \begin{bmatrix} B_{m}^{Total} \end{bmatrix} Var[t_{medium}] + Var \begin{bmatrix} B_{m}^{Total} \end{bmatrix} \mathbf{E}^{2}[t_{medium}]$$
(23)

where

- (i) $\mathbf{E}\begin{bmatrix} B_m^{Total} \end{bmatrix}$ and $Var\begin{bmatrix} B_m^{Total} \end{bmatrix}$ are the expected value and the variance of the number of mediums (empty or busy) observed during the entire back-off process of the m th transmission attempt; and
- (ii) $\mathbf{E}[t_{medium}]$ and $Var[t_{medium}]$ are the expected value and the variance of the random medium time (t_{medium}) length (empty or busy) during the m th transmission attempt.

The detailed and rigorous derivations of $\mathbf{E}\begin{bmatrix}B_m^{Total}\end{bmatrix}$, $Var\begin{bmatrix}B_m^{Total}\end{bmatrix}$, $\mathbf{E}\begin{bmatrix}t_{medium}\end{bmatrix}$, and $Var\begin{bmatrix}t_{medium}\end{bmatrix}$ refer to Agarwal (2011). The expressions of $\mathbf{E}\begin{bmatrix}B_m^{Total}\end{bmatrix}$, $Var\begin{bmatrix}B_m^{Total}\end{bmatrix}$, $\mathbf{E}\begin{bmatrix}t_{medium}\end{bmatrix}$, and $Var\begin{bmatrix}t_{medium}\end{bmatrix}$ are in equations (24) to (27) respectively.

$$\mathbf{E} \begin{bmatrix} B_m^{Total} \end{bmatrix} = \mathbf{E} \begin{bmatrix} B_m \end{bmatrix} \mathbf{E} \begin{bmatrix} M \end{bmatrix}$$
(24)

$$Var\left[B_{m}^{Total}\right] = \mathbf{E}\left[B_{m}\right]Var\left[M\right] + Var\left[B_{m}\right]\mathbf{E}^{2}\left[M\right]$$
(25)

$$\mathbf{E}[t_{medium}] = (1 - p_t)T_{empty} + p_s\overline{T}_{succ} + p_u\overline{T}_{unsucc}$$
(26)

$$Var[t_{medium}] = (1 - p_t) \left(T_{empty} - \mathbf{E}[t_{medium}] \right)^2 + p_s \left(\overline{T}_{succ} - \mathbf{E}[t_{medium}] \right)^2 + p_u \left(\overline{T}_{unsucc} - \mathbf{E}[t_{medium}] \right)^2$$
(27)

where

- (i) M denotes the number of mediums observed during the k th decrement of the back-off counter for the m th transmission attempt. In this research, it is assumed $M_1, M_2, K M_{B_m}$ is a sequence of independent and identically distributed random variables each of mean $\mathbf{E}[M]$, i.e., $\mathbf{E}[M_k] = \mathbf{E}[M] \forall k \ k = 1, 2, K, B_m$;
- (ii) For each packet before the start of the m th transmission attempt, a WSN uniformly selects the back-off time (in terms of number of slots, as a part of the back-off scheme), denoted by $B_m \in [0,1,...,W_m 1]$;
- (iii) T_{empty} denotes the time period the medium remains unoccupied and is a fixed value;
- (iv) \overline{T}_{succ} denotes the average time length of a busy medium of a WSN because of successful transmissions by neighboring WSNs over the channel;
- (iv) \overline{T}_{unsucc} denotes the average time length of a busy medium of a WSN because of unsuccessful transmissions by neighboring WSNs over the channel;

- (v) p_t denotes the probability that a WSN detects a busy medium which is equal to the probability that at least one of the neighboring WSNs is transmitting;
- (vi) $(1 p_t)$ denotes the probability of detecting an idle medium; and
- (vii) $t_{medium,k}$ denotes the time duration of the k th medium during the m th transmission attempt. It is a random variable, and in this research it is assumed that $t_{medium,1}, t_{medium,2},..., t_{medium,B_m^{Total}}$ is a sequence of independent and identically distributed random variables of the same mean, i.e., $\mathbf{E}[t_{medium,k}] = \mathbf{E}[t_{medium}] \forall k, k = 1,2,K, B_m^{Total}$.

4.6.2 Computation of Expected Value and Variance of Total Back-off Delay for IEEE 802.15.4 ($\mathbf{E}[T_{MAC}]$ and $Var(T_{MAC})$)

The T_{MAC} in case of IEEE 802.15.4 also depends on the frequency of the collisions and the channel access attempts. By using the same definition of p_t and p_u here, the following expression is used to compute the expected time a WSN spends listening before transmitting a packet, i.e., T_{MAC} .

$$\mathbf{E}[T_{MAC}] = \sum_{n=0}^{N_{ReTx}^{max}} (1 - p_{CSMAfail})^n p_c^n \left\{ p_{CSMAfail} \overline{T}_{CSMAfail} + (1 - p_{CSMAfail})(\overline{T}_{CSMAnofail} + T_{TA} + T_{ACK}) \right\}$$
(28)

where

(i) $p_{CSMAfail}$ represents the probability of channel access failure and is given by equation (29)

$$p_{CSMAfail} = p_u^{macMaxCSMABackoff+1}$$
(29)

(ii) $\overline{T}_{CSMAfail}$ describes the expected time of each transmission attempt that concludes in a channel access failure after (*macMaxCSMABackoff* +1) CSMA wait and (*macMaxCSMABackoff* +1) CCA failures.

$$\overline{T}_{CSMAfail} = \sum_{n=0}^{macMaxCSMABackof} \left(0.5 \left(2^{\min(macMinBE+n,\max cMaxBE} - 1 \right) \overline{T}_{backoff} + \overline{T}_{CCA} \right)$$
(30)

The default values of *macMaxCSMABackof*, *macMinBE*, *macMaxBE* are summarized in Appendix F.

(iii) $\overline{T}_{CSMAnofail}$ describes the expected delay introduced by the CSMA/CA algorithm and CCA operations of an attempt that does not lead to channel access

$$\overline{T}_{CSMAnofail} = \frac{\left(1 - p_u\right)}{1 - p_u^{N_{ReTx}^{max}}} \sum_{n=0}^{macMaxCSMABackoff} p_u^{N_{ReTx}} \left\{\sum_{m=0}^{n} \overline{T}_{CSMAfail}\right\}$$
(31)

4.7 Expected Value and Variance of Energy in Each State

Given the constant power consumed by a WSN in each state in Table 1 along with the expected value and variance of the time the WSN spends in each state in Table 2, the expected value and the variance of energy consumed by the WSN in each state is computed. The energy consumed by the WSN in each state,

denoted by E_{S_k} , before transitioning to the next state is also a random variable and can be written as

$$E_{S_k} = T_{S_k} P_k \tag{32}$$

where

- (i) T_{S_k} is the random amount of time a WSN spends in each state, except in the sleep state; and
- (ii) P_k , $k \in I_S$ is the power cost associated with each state. Here, $I_S = \{0, 1i, 1e, 1r, 2, 3\}$ denotes the index for the possible states of the WSN. The expected value and the variance of the random variable E_{S_k} are

$$\mathbf{E}\begin{bmatrix}E_{S_k}\end{bmatrix} = \mathbf{E}\begin{bmatrix}T_{S_k}\end{bmatrix}P_k = \mu_{S_k}P_k$$

$$Var\begin{bmatrix}E_{S_k}\end{bmatrix} = \sigma_{S_k}^2 P_k^2$$
(34)

The expected value μ_{S_k} (mean) and the variance $\sigma_{S_k}^2$ of random variables T_{S_k} , $k \in I_S$ are summarized in Table 2. Using the values from Tables 1 and 2 in equations (33) and (34), the expected value and the variance of the energy consumed by the WSN in each state is computed. Table 3 summarizes the expected value ($\mathbf{E}[E_{S_k}]$) and the variance ($Var[E_{S_k}]$) of random variable E_{S_k} , $k \in I_S$.

4.8 Computation of Expected Value of T_{cvcle} and E_{cvcle}

In this research, six different operational states a WSN can transition through during a single cycle period T_{cycle} are identified. The time duration a WSN spends in a particular state is a random variable with mean value μ_{S_k} , $k \in I_S$ where $I_S = \{0, 1i, 1e, 1r, 2, 3\}$ denotes the index for the possible states of the WSN. The WSN transitions from its present state to the next state based on event occurrence probability, i.e., α and β . Therefore, the cycle period, T_{cycle} , can be presented as a summation of random duration T_S , a WSN spends in each state with event occurrence probabilities as

$$T_{cycle} = T_{S_0} + T_{S_{1i}} + \alpha \left(T_{S_{1e}} + T_{S_2} \right) + \beta T_{S_{1r}} + (\alpha + \beta - \alpha \beta) T_{S_3}$$
(35)

where

(i) $T_{S_{12}}, T_{S_{12}}, T_{S_{12}}, T_{S_{22}}, T_{S_{23}}$ are random time durations a WSN spends in its states;

- (ii) α is the probability that at least one sensing event occurs during an active time period; and
- (iii) β is the probability that at least one relay event occurs during an active time period.

State	Expected Energy $\left(\mathbf{E}\left[E_{S_{k}}\right]\right)$	Variance $\left(Var\left[E_{S_k} \right] \right)$
S ₀	$T_{S_0}P_0$	0
	No Event: $T_A P_{1i}$	0
S _{1i}	Any Event: $\frac{T_A}{2}P_{1i}$	$\frac{T_A^2}{12}P_{li}^2$
S _{1e}	$\frac{(T_e^{\min} + T_e^{\max})}{2} P_{1e}$	$\frac{(T_e^{\max} - T_e^{\min})^2}{12} P_{1e}^2$
S _{1r}	$\left(\frac{L}{R} + T_{SIFS} + T_{ACK}\right) P_{1r}$	0
<i>S</i> ₂	$\frac{1}{\kappa}P_2$	$\frac{1}{\kappa^2}P_2^2$
S ₃	$\left(\frac{L}{R} + T_{SIFS} + T_{ACK}\right) P_3 + \mathbf{E} \left[T_{MAC}\right] P_3$	$Var[T_{MAC}]P_3^2$

Table 3. The expected value and the variance of the energy a WSN spends in each state.

The random variable $T_{S_{1i}}$ can take two values depending on event occurrence: (i) $T_{S_{1i}|No_Event}$ with probability $(1 - (\alpha + \beta - \alpha\beta))$ and (ii) $T_{S_{1i}|Event}$ with probability $(\alpha + \beta - \alpha\beta)$, respectively. $T_{S_{1i}|No_Event}$ corresponds to the maximum time duration a WSN stays in the active state when no event occurs and it is a constant random variable T_A . $T_{S_{1i}|Event}$ corresponds to the time duration a WSN stays in the active state idle when an event occurs and is a random variable. By substituting the two values of $T_{S_{1i}}$ and corresponding probability values into equation (35), equation (36) is obtained $T_{cycle} = T_{S_0} + (1 - (\alpha + \beta - \alpha\beta))T_{S_{1i}|No_Event} + (\alpha + \beta - \alpha\beta)T_{S_{1i}|Event} + \alpha T_{S_{1e}} + \beta T_{S_{1r}} + \alpha T_{S_2} + (\alpha + \beta - \alpha\beta)T_{S_3}$ (36)

Take the expectation on both sides of equation (36) to obtain,

$$\mathbf{E}\left[T_{cycle}\right] = \mu_{S_0} + (1 - \alpha - \beta + \alpha\beta)\mu_{S_{1i}|\mathbf{No_event}} + (\alpha + \beta - \alpha\beta)\mu_{S_{1i}|\mathbf{Event}} + \alpha(\mu_{S_{1e}} + \mu_{S_2}) + \beta(\mu_{S_{1r}}) + (\alpha + \beta - \alpha\beta)\mu_{S_3}$$

(37)

where $\mu_{S_k} \triangleq \mathbf{E}[T_{S_k}]$. Substitute the expected values (μ_{S_k}) of $T_{S_0}, T_{S_{1i}}, T_{S_{1e}}, T_{S_1}, T_{S_2}, T_{S_3}$ summarized in Table 2 and simplify to obtain

$$\mathbf{E}\left[T_{cycle}\right] = T_{S_0} + (1 - \alpha - \beta + \alpha\beta)T_A + (\alpha + \beta - \alpha\beta)\frac{T_A}{2} + \alpha(\mu_{S_{1e}} + \frac{1}{\kappa}) + \beta\left(\frac{L}{R} + T_{SIFS} + T_{ACK}\right) + (\alpha + \beta - \alpha\beta)\mu_{S_3}$$
$$= T_{S_0} + T_A - (\alpha + \beta - \alpha\beta)\frac{T_A}{2} + \alpha\left(\mu_{S_{1e}} + \frac{1}{\kappa}\right) + \beta\left(\frac{L}{R} + T_{SIFS} + T_{ACK}\right) + (\alpha + \beta - \alpha\beta)\mu_{S_3}$$
(38)

Similar, the energy consumed by a WSN per cycle of operation can be expressed as

$$E_{cycle} = T_{S_0}P_0 + (1 - (\alpha + \beta - \alpha\beta))T_{S_{1i}|No_Event}P_{1i} + (\alpha + \beta - \alpha\beta)T_{S_{1i}|Event}P_{1i} + E_{tr1} + \alpha T_{S_{1e}}P_{1e} + \beta T_{S_{1r}}P_{1r} + \alpha T_{S_2}P_2 + (\alpha + \beta - \alpha\beta)T_{S_3}P_3 + E_{tr3}$$
(39)

Take the expectation on both sides of equation (40), to obtain the expected value of the random variable E_{cycle} , denoted as $\mathbf{E} \left[E_{cycle} \right]$,

$$\mathbf{E}\left[E_{cycle}\right] = T_{S_0}P_0 + T_A P_{1i} - (\alpha + \beta - \alpha\beta)\frac{T_A}{2}P_{1i} + E_{tr1} + \alpha\left(\mu_{S_{1e}}P_{1e} + \frac{1}{\kappa}P_2\right) + \beta\left(\frac{L}{R} + T_{SIFS} + T_{ACK}\right)P_{1r} + (\alpha + \beta - \alpha\beta)\mu_{S_3}P_3 + E_{tr3}$$

$$(40)$$

Observe that in equation (40), the transition energy costs E_{tr1} and E_{tr3} are accounted. Recall, that E_{tr1} is the transition energy cost associated with the transition from the state S_0 to the state S_{1i} and E_{tr2} is the transition energy cost associated with the transition from the state S_2 to the state S_3 . The value of α and β are derived in Appendix E

Modeling Verification via Simulation

Following a detailed theoretical formulation of a WSN expected value and the variance of energy consumption in each state and in a single operation cycle, validation of the model via MATLAB[®] simulation is presented in this section. All parameter values used in model and simulation for both the communication protocols are summarized in Tables 6 to 9 in Appendix H. The default parameter values associated with IEEE 802.15.4 and IEEE 802.11 MAC layer are summarized in Appendices F and Appendix G, respectively.

The MATLAB[®]-based simulator consists of four units: (i) event (sensing or relaying) generating unit, (ii) processing unit, (iii) relaying unit, and (iv) transmission unit.

4.9 Event Generating Unit

A WSN spends T_{S_0} seconds in the sleep state S_0 before entering the active state at t_w . At this instance, $\left[T_{cycle,k}^{sim}\right]^1 = T_{S_0}$ and $\left[E_{cycle,k}^{sim}\right]^1 = T_{S_0}P_0 + E_{tr1}$, i.e., $\left[E_{cycle,k}^{sim}\right]^1$, includes the energy consumed by the WSN in the sleep state S_0 and the transition energy cost from S_0 to S_{1i} , i.e., E_{tr1} . Note, the superscript value on $T_{cycle,k}^{sim}$ and $E_{cycle,k}^{sim}$ denotes the number of updates performed per cycle.

Once the WSN enters the active state, the *event generating unit* can generate two types of events: sensing event and relay event, via a Poisson distribution. During the interval $[t_w, t_w + T_A]$, whether an event (sensing or relay) occurs or not is simulated as a Bernoulli trial using a single input MATLAB

function *bern_gen*. In case of sensing event generation, the input to *bern_gen* is the probability α and in case of relaying event generation, the input to *bern_gen* is the probability β . Recall here (i) α is the probability that at least one sensing event occurs during the interval $[t_w, t_w + T_A]$ as per a Poisson distribution and (ii) β is the probability that at least one relay event occurs during the interval $[t_w, t_w + T_A]$ as per a Poisson distribution. The values of α and β are given by equations (E1) and (E2) respectively, in Appendix E.

When an event (sensing or relay) is identified during the interval $[t_w, t_w + T_A]$, the time instant of occurrence is computed using the MATLAB function *uniform_gen*. At this instant, update:

(i) $\left[T_{cycle,k}^{sim}\right]^2 = \left[T_{cycle,k}^{sim}\right]^1 + T_{S_{1i}} + T_{S_{1e}}$, i.e., $\left[T_{cycle,k}^{sim}\right]^1$ plus the time duration the WSN remains in the idle state S_{1i} before an event occurs and the time duration for which an event occurs, i.e., S_{1e} , and

(ii)
$$\left[E_{cycle,k}^{sim}\right]^2 = \left[E_{cycle,k}^{sim}\right]^1 + T_{S_{1i}}P_{1i} + T_{S_{1e}}P_e$$
, i.e., $\left[E_{cycle,k}^{sim}\right]^1$ plus the energy consumed by the WSN when in states S_{1e} and S_{1e} respectively.

WSN when in states S_{1i} and S_{1e} , respectively.

If the event generator is unable to generate any event during the active time period, the WSN stays in the active state for T_A seconds before reentering the sleep state S_0 . Then, $\left[T_{cycle,k}^{sim}\right]^2 = \left[T_{cycle,k}^{sim}\right]^1 + T_A$ and $\left[E_{cycle,k}^{sim}\right]^2 = \left[E_{cycle,k}^{sim}\right]^1 + T_A P_{1i} + E_{tr3}$, i.e., $\left[E_{cycle,k}^{sim}\right]^1$ plus the energy consumed by the WSN during the

active time period T_A .

4.10 Processing Unit

Once the sensing event is observed during the interval $[t_w, t_w + T_A]$, the event is processed by the processing unit as an exponential distribution. The time duration a WSN spends processing a packet is simulated using a single input MATLAB function *exprnd*. The input to the function exprnd is $1/\kappa$ where κ is the average processing rate. At this point then, update: (i) $T_{cycle,k}^{sim}$ to $[T_{cycle,k}^{sim}]^3 = [T_{cycle,k}^{sim}]^2 + T_{S_2}$, i.e., it also include the sensing event processing time duration when the WSN is in the processing state S_2 ;

and (ii) $E_{cycle,k}^{sim}$ to $\left[E_{cycle,k}^{sim}\right]^3 = \left[E_{cycle,k}^{sim}\right]^2 + T_{S_2}P_2$ which now includes the energy consumed by the WSN while processing the sensing event.

4.11 Relaying Unit

The *relaying unit* simulates the routing functionality of a WSN. During an active time period, the WSN may have a relay event to transmit. The time duration the WSN stays in the relay state S_{1r} to

receive a packet of constant length is simulated. At this point then, update: (i) $T_{cycle,k}^{sim}$ to

 $\left[T_{cycle,k}^{sim}\right]^4 = \left[T_{cycle,k}^{sim}\right]^3 + T_{S_{1r}}$ to include the time taken to receive a relay event when the WSN is in the

relay state S_{1r} and (ii) $E_{cycle,k}^{sim}$ to $\left[E_{cycle,k}^{sim}\right]^4 = \left[E_{cycle,k}^{sim}\right]^3 + T_{S_{1r}}P_{1r}$ which now includes the energy consumed by the WSN in the relay state S_{1r} .

4.12 Transmitting Unit

The event (sensing or relay) occurring during an active time period is stored in the transmission buffer and is transmitted by a WSN to the base station (or to another WSN). The transmission of a packet of information is based on IEEE 802.11 CSMA/CA MAC and IEEE 802.15.4 slotted CSMA/CA MAC communication protocols. The transmitting unit simulates the communication protocol to gain access to the channel and then successfully transmit the packet. The transmitting unit transmits the packet once it has channel access. At this point, update (i) $T_{cycle,k}^{sim}$ to $\left[T_{cycle,k}^{sim}\right]^5 = \left[T_{cycle,k}^{sim}\right]^4 + T_{S_3}$ to include the time taken by the WSN to gain access to the channel and to transmit the packet successfully (or the time taken to reach N_{ReTx}^{max} retransmissions) and (ii) $E_{cycle,k}^{sim}$ to $\left[E_{cycle,k}^{sim}\right]^5 = \left[E_{cycle,k}^{sim}\right]^4 + T_{S_{1r}}P_{1r} + E_{tr3}$, which now includes the energy consumed by the WSN while gaining access of the channel and successfully transmitting the packet and the transition energy cost E_{tr2} . A WSN reenters the sleep state S_0 after completing a single cycle of operation when (i) the *event generating unit* is unable to generate an event during the interval $[t_w, t_w + T_A]$, or (ii) the transmission buffer is empty after successful transmission of the packet, or (iii) N_{ReTx}^{max} is reached. By following the above simulation procedure, the cycle period $T_{cycle,k}^{sim} \triangleq \left[T_{cycle,k}^{sim}\right]^5$ and the per cycle energy consumption $E_{cycle,k}^{sim} \triangleq \left[E_{cycle,k}^{sim}\right]^5$ corresponding to the k - th cycle of operation of a WSN are determined by simulation.

4.13 Results

The MATLAB[®] simulation routines are used to verify models discussed in Section 4. A large number of simulations are performed to compute the expected value and the variance of energy consumptions. Figure 11 shows the expected energy consumption per cycle of a WSN based on IEEE 802.11 communication protocol. In Figure 2, observe that model and simulation results exhibit good concurrence. Also, the expected energy consumption per cycle increases as the event occurrence probability increases. This is supports the understanding that as the number of events (sensing or relay) increases, the WSN has to spend more time processing and transmitting the data packet, thereby increasing the energy consumption.

Figure 12 shows the expected energy consumption per cycle for different duration of ACTIVE state. As expected the average energy consumption per cycle increases with the increase in ACTIVE state time period. The simulation results matches the model performance for different ACTIVE time period.

Figure 13 shows the expected power consumption of a WSN based on IEEE 802.11 communication protocol per cycle for different duty cycle and event occurrence probability values. Duty cycle (dc) is defined as a ration of ACTIVE time period over total cycle time period, i.e.,

$$Duty \ Cycle \ (dc) = \frac{T_A}{T_A + T_{S_0}}$$
(41)

Similar performance is evaluated for a WSN based on the IEEE 802.15.4 communication protocol in Figure 14. Observe that the average power consumption is significantly low for a WSN based on IEEE

802.15.4 communication protocol. This supports the increasing application of IEEE 802.15.4 based WSNs in different applications.



Figure 11. Expected energy consumption per cycle of a WSN based on IEEE 802.11 communication protocol (Agarwal 2011).



Figure 12. Expected energy consumption per cycle of a WSN for different ACTIVE time period based on IEEE 802.11 communication protocol (Agarwal 2011).



Figure 13. Expected power consumption per cycle of a WSN for different event occurrence probability based on IEEE 802.11 communication protocol.



Figure 14. Expected power consumption per cycle of a WSN for different event occurrence probability based on IEEE 802.15.4 communication protocol.

Figure 15 and Figure 16 compare expected power consumption per cycle of a WSN for different dc values based on IEEE 802.11 and IEEE 802.15.4 communication protocols respectively. The significant different exists in the average power consumptions of two protocols because of the low expected time spend by a WSN in the transmission state (S_3) state. Figure 17 and Figure 18 shows (model and

simulation) the expected time a WSN based on IEEE 802.15.4 protocol spends listening for channel availability, which in turn depends on PER and channel occupied probability.



Figure 15. Expected power consumption per cycle of a WSN for different duty cycle based on IEEE 802.11 communication protocol.



Figure 16. Expected power consumption per cycle of a WSN for different duty cycle based on IEEE 802.15.4 communication protocol.



Figure 17. Expected listening time per cycle of a WSN for different channel occupied probability based on IEEE 802.15.4 communication protocol.



Figure 18. Expected listening time per cycle of a WSN for different PER probability based on IEEE 802.15.4 communication protocol.

5. CONCLUSIONS AND FUTURE WORK

The report described rigorous stochastic modeling of power demand for a WSN utilizing IEEE 802.11 and 802.15.4 communication protocols. The model captures the generic operation of a WSN when an

external event occurs, i.e., sensing, followed by processing, and followed by communication. The report describes the development of an expression to compute the expected energy consumption per operational cycle of a WSN by taking into consideration the node level activities, i.e., sensing and processing, and the network level activities, i.e., channel access, packet collision, retransmission attempts, and transmission of a data packet.

In this research, six different operational states of a WSN were identified and each state had a certain associated power consumption, which discharges the WSN battery. An additional energy cost is associated with transitions between states. After spending a random amount of time in a particular state, the WSN transitions to the next allowable state with transition probability p_{ii} , $i \neq j$. Thus, the transition

of the WSN from one state to another state was described precisely by a SMP. An SMP, a special case of Markov process, accounts for the amount of time that a process spends in each state before making the transition from its present state to the next state. Therefore, an SMP was utilized to describe the operational behavior of a WSN. The complex network level activities were modeled to compute the expected value and the variance of time a WSN spends in the transmission state.

The difference between IEEE 802.11 and IEEE 802.15.4 back-off process and channel access scheme were summarized and rigorously modeled. This allowed computation of the expected value and the variance of the energy consumption of a WSN in each state. Utilizing the state transition probabilities and computed expected value and the variance of energy consumption in each state, the expected value of the energy consumption of the WSN per cycle of operation was also modeled.

As part of future research, the focus is (i) to perform detailed analysis comparing the expected value and the variance of energy consumption per cycle for a WSN based on IEEE 802.11 and IEEE 802.15.4 communication protocols under different operating conditions, (ii) to perform detailed analysis of network metrics like throughput and latency for both IEEE 802.11 and IEEE 802.15.4, and (iii) to integrate TEG and WSN models to perform self-powered WSN modeling and study the performance under different operating conditions.

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Appendix A

Relationship between Poisson Process and Uniform Distribution

Consider a Poisson process with rate parameter λ . In the interval (0,t], if one event occurs as per the Poisson process, then by the property of the Poisson process, the time of that occurrence is uniformly distributed over (0,t] (Ross 2005).

In a Poisson process, the probability that the number of events, say n, occurring in an interval of length t is given as

$$\Pr(N(t) - n) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$

To see that this property of the Poisson process is true, the formal derivation is as follows: We split the interval (0,t] into two non-overlapping subintervals: (0,s] and (s,t]. Now what happens in two nonoverlapping intervals is independent. If one event occurs during (0,s], then no events occur during the interval (s,t] of length t-s, given that only one event occurs in the interval (0,t]. Let t_1 be the time instant an event occurs where $t_1 \le s$. Here, n = 1.

$$Pr(t_1 \le s \mid N(t) = 1) = \frac{Pr(\text{one event in } (0, s], \text{ no event in } (s, t])}{Pr(N(t) = 1)}$$
$$= \frac{Pr(N(s) = 1)Pr(N(t - s) = 0)}{Pr(N(t) = 1)}$$
$$= \frac{\lambda s e^{-\lambda s} \cdot e^{-\lambda(t - s)}}{\lambda e^{-\lambda t}} = \frac{s}{t}$$

The second step is due to independence of two non-overlapping intervals. Therefore, the probability that the event occurred during the subinterval $s, 0 \le s \le t$ is proportional to the size of the interval s. That is, the event is uniformly distributed. The above result generalizes to any number of arrivals in the interval (0,t] by order statistic.

Appendix B

Number of Retransmissions as truncated Probability Mass Function

A packet undergoes retransmission if its transmission was unsuccessful. By assumption, each transmission of a packet is independent and has a fixed probability value p_s of being successful and p_u of being unsuccessful. As the number of retransmissions observed by a packet is random and is truncated (bounded) at $N_{\text{Re}Tx}^{\text{max}} - 1$, it is given by a probability mass function

$$\Pr^{trunc} \left[N_{\text{Re}Tx} = n \right] = \frac{(1 - p_u) p_u^n}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}}, \quad n \in \{0, 1, ..., N_{\text{Re}Tx}^{\text{max}} - 1\}$$
(B1)

 $N_{\text{Re}Tx}$ is the number of retransmissions experienced by a packet before successful transmission with probability p_s . If $N_{\text{Re}Tx}$ is not truncated, then

$$f(n) = \Pr[N_{\text{Re}Tx} = n] = p_u^n p_s$$
(B2)

In this research, $N_{\text{Re}Tx}$ is truncated at $N_{\text{Re}Tx}^{\text{max}} - 1$ and has a truncated probability mass function expressed as

$$\Pr^{trunc} \left[N_{\operatorname{Re}Tx} = n \,|\, N_{\operatorname{Re}Tx} \le N_{\operatorname{Re}Tx}^{\max} - 1 \right] = \frac{g(n)}{\Pr(N_{\operatorname{Re}Tx} \le N_{\operatorname{Re}Tx}^{\max} - 1)} \tag{B3}$$

where

(i)
$$g(n) = f(n) \forall N_{\text{Re}Tx} \le N_{\text{Re}Tx}^{\text{max}} - 1$$
 and $g(n) = 0$ elsewhere; and

(ii)
$$\Pr(N_{\operatorname{Re}Tx} \le N_{\operatorname{Re}Tx}^{\max} - 1) \triangleq \sum_{k=0}^{N_{\operatorname{Re}Tx}^{\max} - 1} \Pr(N_{\operatorname{Re}Tx} = k)$$
 is the cumulative probability up to $N_{\operatorname{Re}Tx}^{\max} - 1$.

Now $Pr(N_{ReTx} \le N_{ReTx}^{max} - 1)$ can be expressed as

$$\Pr(N_{\text{Re}Tx} \le N_{\text{Re}Tx}^{\text{max}} - 1) = \sum_{k=0}^{N_{\text{Re}Tx}^{\text{max}} - 1} p_u^k p_s = p_s \frac{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}}{1 - p_u}, \ p_u \neq 1$$
(B4)

By property of geometric sequence, $\sum_{k=0}^{N_{\text{ReTx}}^{\text{max}}-1} p_u^k p_s = p_s \frac{1 - p_u^{N_{\text{ReTx}}^{\text{max}}}}{1 - p_u}$ is the sum of the first $N_{\text{ReTx}}^{\text{max}}$ terms

for $p_u \neq 1$. Substitute g(n) and equation (B4) into equation (B3) to obtain equation (B5) after rearranging

$$\Pr^{trunc} \left[N_{\text{Re}Tx} = n \mid N_{\text{Re}Tx} \le N_{\text{Re}Tx}^{\text{max}} - 1 \right] = \frac{p_u^n p_s}{p_s \frac{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}}{1 - p_u}}$$
$$= \frac{(1 - p_u) p_u^n}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}}, \ n \in \{0, 1, ..., N_{\text{Re}Tx}^{\text{max}} - 1\}$$
(B5)

Appendix C

Expressions for $\mathbf{E}[T_{MAC}(N_{\text{Re}Tx})]$ and $Var[T_{MAC}(N_{\text{Re}Tx})]$

For a single transmission attempt of a packet, the medium access time T_{MAC} is the duration of the back-off process and is known as back-off delay (*BOD*). If retransmissions occur, then the medium access time T_{MAC} is the sum of back-off delays of each transmission attempt and the sum of the transmission delays of each unsuccessful attempt. The number of back-off delays depends on the number of retransmissions. Therefore, T_{MAC} is a function of N_{ReTx} , denoted as $T_{MAC}(N_{ReTx})$, and is a random variable. From Appendix B, it is known that the number of retransmissions N_{ReTx} observed by a packet before successful transmission is a random variable and has a truncated geometric probability distribution (equation (B5)).

At any given time, since the number of retransmissions a WSN takes to transmit a packet successfully before the maximum number of retransmission is reached is unknown, the random variable T_{MAC} is defined as the weighted linear combination of each retransmission.

$$T_{MAC} = \frac{\left(1 - p_u\right)}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}^{\text{max}}-1} p_u^{N_{\text{Re}Tx}} T_{MAC}(N_{\text{Re}Tx}), \ N_{\text{Re}Tx} \in \{0, 1, 2, ..N_{ReTx}^{\text{max}} - 1\}$$
(C1)

where

$$T_{MAC}(N_{\text{Re}Tx}) = \sum_{n=0}^{N_{\text{Re}Tx}} BOD_{m=n+1} + \sum_{n=1:N_{\text{Re}Tx} \ge 1}^{N_{\text{Re}Tx}} \overline{T}_{unsucc}, \ N_{\text{Re}Tx} \in \{0, 1, 2, ..., N_{ReTx}^{\text{max}} - 1\}$$
(C2)

If a packet is successfully transmitted on the first attempt and no retransmission occur, i.e.,

 $N_{\text{Re}Tx} = 0$, then equations (C1) and (C2) reduce to $T_{MAC} = \frac{(1 - p_u)}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}} T_{MAC}(N_{\text{Re}Tx} = 0)$ and

 $T_{MAC}(N_{\text{Re}Tx} = 0) = BOD_{m=1}$, respectively. If a packet is transmitted on the second attempt and not on the first attempt, a retransmission is observed, i.e., $N_{\text{Re}Tx} = 1$, then equations (C1) and (C2) are

$$T_{MAC} = \frac{(1-p_u)}{1-p_u^{N_{ReTx}^{max}}} T_{MAC}(N_{ReTx} = 0) + \frac{(1-p_u)}{1-p_u^{N_{ReTx}^{max}}} p_u T_{MAC}(N_{ReTx} = 1) \text{ and}$$
$$T_{MAC}(N_{ReTx} = 1) = BOD_1 + BOD_2 + \overline{T}_{unsucc}, \text{ respectively and so on.}$$

Therefore, if each packet experiences $N_{\text{Re}Tx}$ retransmissions, then the random variable $T_{MAC}(N_{\text{Re}Tx})$ includes (i) each independent back-off delay BOD_m upto $N_{\text{Re}Tx} - 1$ retransmissions and (ii) $N_{\text{Re}Tx}$ unsuccessful transmissions each of average unsuccessful transmission delay of \overline{T}_{unsucc} . Mathematically, $T_{MAC}(N_{\text{Re}Tx})$ is expressed in equation (C2). Take the expectations on both sides of equation (4.95) to obtain

$$\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right] = \mathbf{E}\left[\sum_{n=0}^{N_{\text{Re}Tx}} BOD_{m}\right] + \mathbf{E}\left[\sum_{n=1}^{N_{\text{Re}Tx}} \overline{T}_{unsucc}\right]$$
(C3)

By assumption that each transmission attempt is independent and by additive law of expectation,^f $\mathbf{E}\left[\sum_{n=0}^{N_{\text{Re}Tx}} BOD_{m}\right] = \sum_{n=0}^{N_{\text{Re}Tx}} \mathbf{E}\left[BOD_{m}\right], \text{ equation (C4) is obtained}$

$$\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right] = \sum_{n=0}^{N_{\text{Re}Tx}} \mathbf{E}\left[BOD_{m}\right] + N_{\text{Re}Tx}\overline{T}_{unsucc}$$
(C4)

Similarly, to obtain the variance of $T_{MAC}(N_{ReTx})$ take variances on both sides of equation (C2) to obtain

$$Var\left[T_{MAC}(N_{\text{Re}Tx})\right] = Var\left[\sum_{n=0}^{N_{\text{Re}Tx}} BOD_{m}\right] + Var\left[\sum_{n=1}^{N_{\text{Re}Tx}} \overline{T}_{unsucc}\right]$$
(C5)

Again, by the assumption of independent transmission attempts and by variance sum law,^g

$$Var\left[\sum_{n=0}^{N_{\text{Re}Tx}} BOD_{m}\right] = \sum_{n=0}^{N_{\text{Re}Tx}} Var[BOD_{m}], \text{ equation (C6) is obtained}$$
$$Var[T_{MAC}(N_{\text{Re}Tx})] = \sum_{n=0}^{N_{\text{Re}Tx}} Var[BOD_{m}] + \sum_{n=1}^{N_{\text{Re}Tx}} Var(\overline{T}_{unsucc}) = \sum_{n=0}^{N_{\text{Re}Tx}} Var[BOD_{m}]$$
(C6)

The term $Var(\overline{T}_{unsucc})$ is zero as \overline{T}_{unsucc} is a fixed time value.

f. Additive law of expectation states that the expectation value of the sum of independent random variables is equal to the sum of expectation of random variables.

g. Variance sum law states that the variance of the sum or difference of independent random variables is equal to variance of the sum or difference of random variables.

Appendix D

Expressions for $E[T_{MAC}]$ and $Var(T_{MAC})$

The random variable T_{MAC} is expressed in terms of N_{ReTx}^{max} and N_{ReTx}^{max} as

$$T_{MAC} = \frac{\left(1 - p_u\right)}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}^{\text{max}} - 1} p_u^{N_{\text{Re}Tx}} T_{MAC}(N_{\text{Re}Tx})$$
(D1)

Take the expectation and variance on both sides of equation (D1). Again since each transmission attempt is independent, by additive laws of expectation and variance, equations (D2) and (D3) are obtained

$$\mathbf{E}\left[T_{MAC}\right] = \frac{\left(1 - p_u\right)}{1 - p_u^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}-1} p_u^{N_{\text{Re}Tx}} \mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right]$$
(D2)

and

$$Var\left[T_{MAC}\right] = \frac{\left(1 - p_{u}\right)}{1 - p_{u}^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}^{n}=0}^{N_{\text{Re}Tx}^{\text{max}}-1} p_{u}^{N_{\text{Re}Tx}} \mathbf{E}\left[\left(T_{MAC}(N_{\text{Re}Tx}) - \mathbf{E}\left[T_{MAC}\right]\right)^{2}\right]$$
(D3)

The value of $\mathbf{E}[T_{MAC}(N_{\text{Re}Tx})]$ in equation (D2) for each $N_{\text{Re}Tx}$ can be obtained from equation (C4). Expand the square term in equation (D3) and take expectation to obtain

$$\mathbf{E}\left[\left(T_{MAC}(N_{\text{Re}Tx}) - \mathbf{E}\left[T_{MAC}\right]\right)^{2}\right] = \mathbf{E}\left[T_{MAC}^{2}(N_{\text{Re}Tx})\right] + \mathbf{E}^{2}\left[T_{MAC}\right] - 2\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right]\mathbf{E}\left[T_{MAC}\right]$$
(D4)

Add and subtract $\mathbf{E}^{2}[T_{MAC}(N_{\text{Re}Tx})]$ from equation (D4) to obtain after simplification

$$\mathbf{E}\left[\left(T_{MAC}(N_{\text{Re}Tx}) - \mathbf{E}\left[T_{MAC}\right]\right)^{2}\right] = \left\{\mathbf{E}\left[T_{MAC}^{2}(N_{\text{Re}Tx})\right] - \mathbf{E}^{2}\left[T_{MAC}(N_{\text{Re}Tx})\right]\right\} + \left(\mathbf{E}^{2}\left[T_{MAC}(N_{\text{Re}Tx})\right] + \left(\mathbf{E}^{2}\left[T_{MAC}(N_{\text{Re}Tx})\right] - 2\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right]\mathbf{E}\left[T_{MAC}\right]\right)\right\}$$
$$= Var\left[T_{MAC}(N_{\text{Re}Tx})\right] + \left(\mathbf{E}\left[T_{MAC}(N_{\text{Re}Tx})\right] - \mathbf{E}\left[T_{MAC}\right]\right)^{2}$$
(D5)

Substitute equation (D5) into equation (D3) to obtain

$$Var[T_{MAC}] = \frac{\left(1 - p_{u}\right)}{1 - p_{u}^{N_{\text{Re}Tx}^{\text{max}}}} \sum_{N_{\text{Re}Tx}=0}^{N_{\text{Re}Tx}^{\text{max}}-1} p_{u}^{N_{\text{Re}Tx}} \left[Var[T_{MAC}(N_{\text{Re}Tx})] + \left(\mathbf{E}[T_{MAC}(N_{\text{Re}Tx})] - \mathbf{E}[T_{MAC}]\right)^{2}\right]$$
(D6)

Appendix E

Event Occurrence Probabilities

Let λ_{SE} and λ_{RE} denote the average sensing event arrival rate and average relay event arrival rate respectively. Suppose the arrival of events is modeled as a Poisson process. Then the probabilities of at least one sensing event (α) or at least one relay event (β) occurring during the maximum time period a WSN is active, denoted as T_A ,^h are given by following expressions

$$\alpha = 1 - e^{-\lambda_{\rm SE}T_A} \tag{E1}$$

$$\beta = 1 - e^{-\lambda_{\rm RE}T_A} \tag{E2}$$

In Appendix A, for a Poisson process, the probability that the number of events, say n, occurring in an interval of length t is given as

$$\Pr(N(t) = n) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$
(E3)

where λ is the arrival rate in case of a sensing event occurrence, $\lambda = \lambda_{SE}$ and in case of a relay event occurrence, $\lambda = \lambda_{RE}$. Hence, the probability that no sensing event occurs during T_A , i.e., n = 0, is $\Pr(N(T_A) = 0) = e^{-\lambda_{SE}T_A}$. Similarly, the probability that no relay event occurs during T_A is $\Pr(N(T_A) = 0) = e^{-\lambda_{RE}T_A}$. Therefore, the probability that at least one sensing event or one relay event occurs during T_A is $\Pr(N(T_A) = 0) = e^{-\lambda_{RE}T_A}$.

h. If any event takes place at T_A then of course the WSN remains active beyond the predefined length.

Appendix F

IEEE 802.15.4 MAC Parameter Values Used in Modeling and Simulation

Table 4. IEEE 802.15.4 default values of parameters used to compute the expected value of the energy consumed by a WSN in each state and in a cycle.

Attribute	Value
aUnitBackoffPeriod	320 µs
aBaseSlotDuration	960 μs
aBaseSuperFrameDuration	15.36 ms
aNumSuperframeSlots	16
aMaxPHYPacketSize	127 octets
aMinCAPLength	7.04 <i>m</i> s
macSuperFrameOrder	15
macBeaconOrder	15
macMinBE	3
macMaxBE	5
macMaxCSMABackoffs	4
macMaxFrameRetries	3
aTurnAroundTime	192 <i>µ</i> s
macAckWaitDuration	864 µs
phySHRDuration	160 µs

Appendix G

IEEE 802.11 MAC Parameter Values Used in Modeling and Simulation

Table 5. IEEE 802.11 default values of parameters used to compute the expected value of the energy consumed by a WSN in each state and in a cycle.

Parameters	Description	Values
T _{empty}	Time length of a empty medium of a WSN	20 µs
\bar{T}_{succ}	Average time length of a busy medium of a WSN because of successful transmissions by neighboring WSNs	944 μs
\overline{T}_{unsucc}	Average time length of a busy medium of a WSN because of unsuccessful transmissions by neighboring WSNs	944 μs
T _{SIFS}	Short Inter-Frame Slot	10 µs
T _{ACK}	Acknowledgement	304 µs
<i>CW</i> _{min}	Minimum Contention Window	32

Appendix H

Model and Simulation Parameter Values

Parameters	Description	Values (mW)
P_0	Power cost associated with the sleep state S_0	0.4620
P_{1i}	Power cost associated with the idle state S_{1i}	10.51
P_{1e}	Power cost associated with the event state S_{1e}	26.77
P_{1r}	Power cost associated with the relaying (receiving) state S_{1r}	45.00
<i>P</i> ₂	Power cost associated with the processing state S_2	24.15
<i>P</i> ₃	Power cost associated with the Transmission state S_3	49.51

Table 6. Power cost associated with each state for Mica2 WSN (Shnayder 2004)

Table 7. Transition Energy Cost

Parameters	Description	Values
E _{tr1}	Transition energy cost from S_0 to S_{1i}	0.0180 mJ
E _{tr2}	Transition energy cost from S_2 to S_3 or from S_{1r} to S_3	0.0384 mJ

Table 8. Model and Simulation Parameter Values

Parameters	Description	Values
T_{S_0}	Time length a WSN stays in S_0	2.5 sec
T_A	Maximum time duration a WSN stays in S_{1i} when no event occurs	2.5 sec
К	Average event processing rate	5 sec
R	Transmission rate in Kbps	250 kbps
L	Packet length	1024 bits
$N_{\text{Re}TX}^{\text{max}}$	Maximum number of allowed retransmissions	3
p_t	Probability that a medium is busy	0.5
p_s	Probability that a transmission attempt by a neighboring WSN is successful	0.5
<i>P</i> _u	Probability that a transmission attempt by a neighboring WSN is unsuccessful	0.5
T_e^{\max}	Maximum time duration for which a sensing event occurs	0.005
T_e^{\min}	Minimum time duration for which a sensing event occurs	0.001