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MFANS 2024 - Extensions of SysML Using Category Theory for Structured Analysis

April 2024

Changing the World's Energy Future

Lance Gregory Joneckis, William Lawrence Harrison

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Extensions of SysML

Using Category Theory for Structured Analysis

April 29, 2024

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Motivation

- Models provide understanding of the scientific and technical world
  - In Physics, Newton's Law of Universal Gravitation
  - In Technology, Model of a Rocket
- Models …
  - Developed to answer specific questions
  - Support specific reasoning
  - Have *Accuracy*, *Precision*, and *Fidelity*
  - Are Verifiable
  - Are Falsifiable
- If you don't have a model for something, you don't understand it

Elevate Physical Models
to be as important as the Cyber Model for a CPS

Modeling Spectrum

Requirements | Systems | Physical

Modeling Spectrum

Requirements

- Properties
  - Weight
  - Electrical Power
- Behaviors

Autonomous Emergency Braking System

Functions

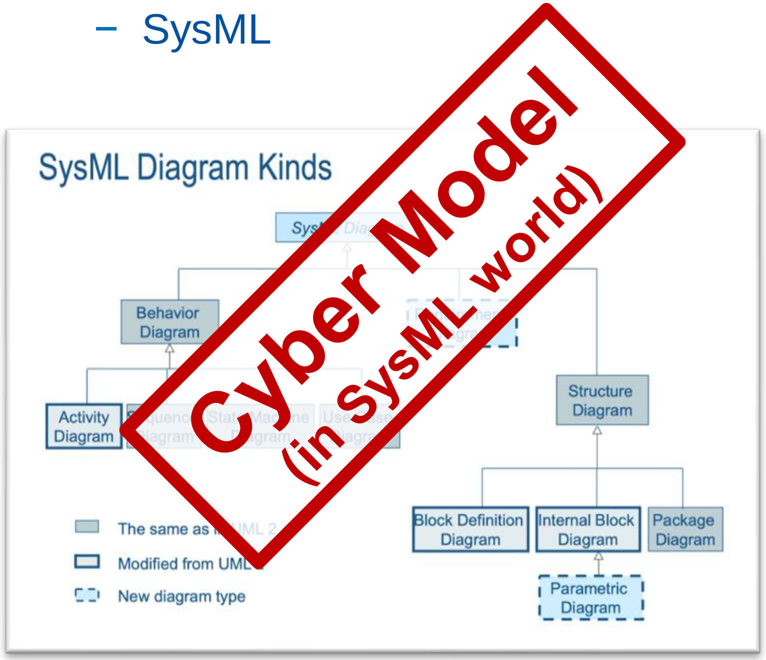
1. The AEB system shall provide the alert to driver when the vehicle is close to the obstacle.
2. The AEB system shall take automatic brake action after a certain time period if the driver ignores the alert and the vehicle is closer to the obstacle.

Constraints

1. The AEB system shall not take corrective action if the safety indicator of alert is not actuated.
2. The AEB system shall not take corrective action if the safety indicator of brake is not actuated.
3. The AEB system shall verify if corrective action has been conducted by system input or feedback.

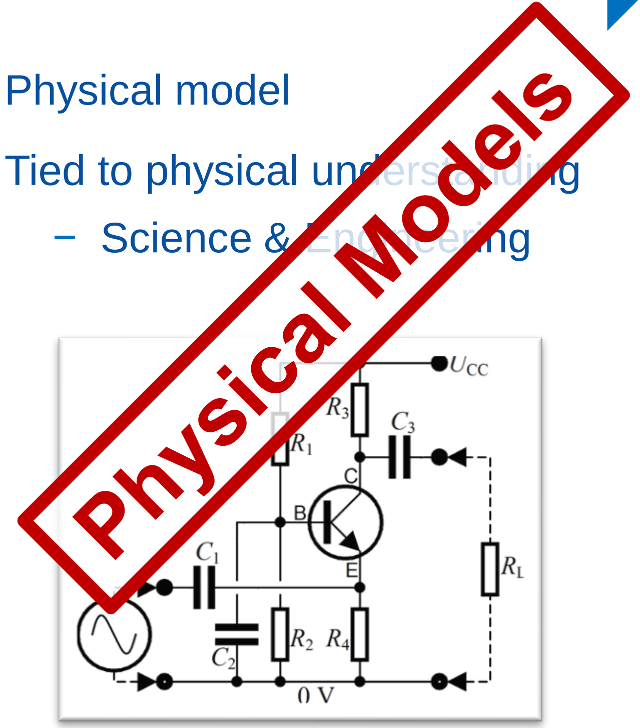
Systems

- Model-based systems engineering
  - SysML



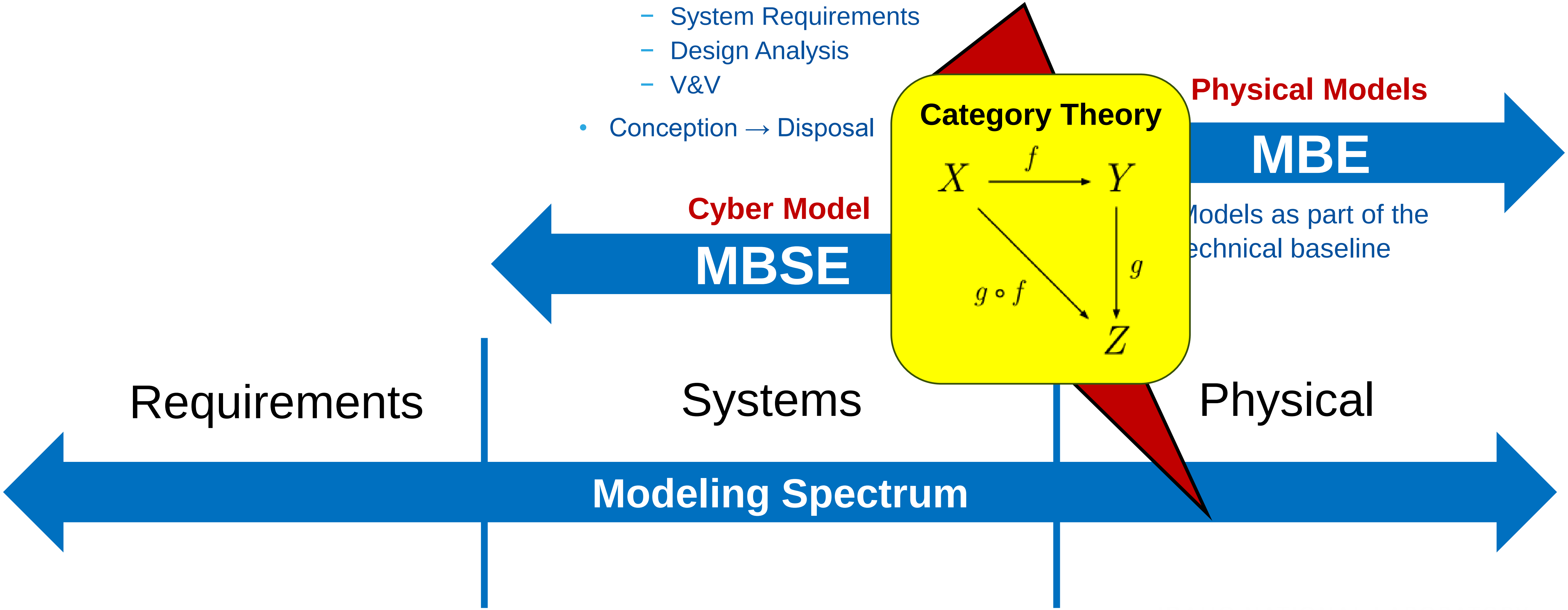
Physical

- Physical model
- Tied to physical understanding
  - Science & Engineering



Modeling Driven Development

- System Structure and Behavior
- Activities
  - System Requirements
  - Design Analysis
  - V&V
- Conception → Disposal

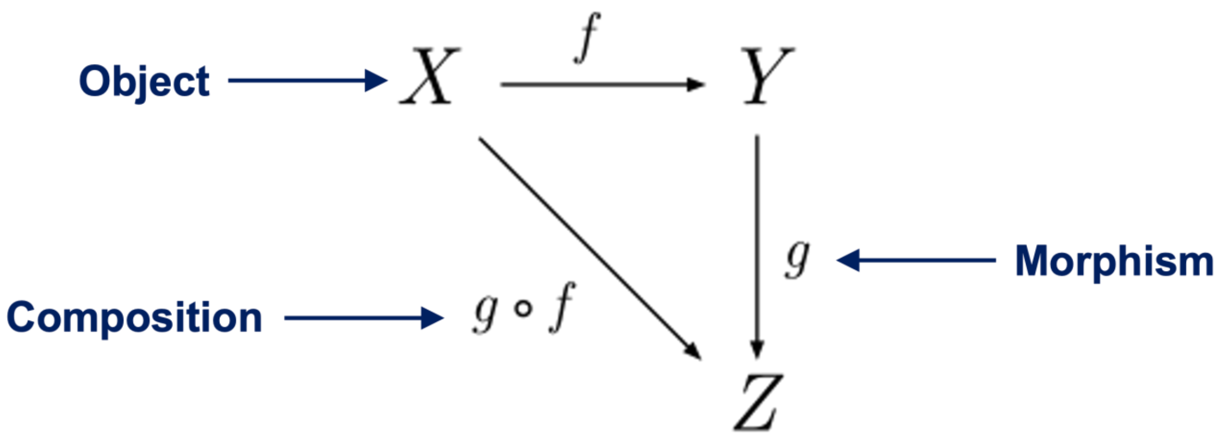


Outline

- Category Theory
- Epidemic Modeling
- SysML
- Physical Modeling of a Cyber Physical System
- Summary

Category Theory

- Category Theory
  - Objects: X, Y, Z
  - Morphisms: $\mathbb{I}$, f, g
  - Composition: $g \circ f$
- That's It
- Foundation for Mathematics
- Why ???
  - The power of category theory lies in its artful application to a topic
    - Definition of categories
    - Relationship that arise between the categories

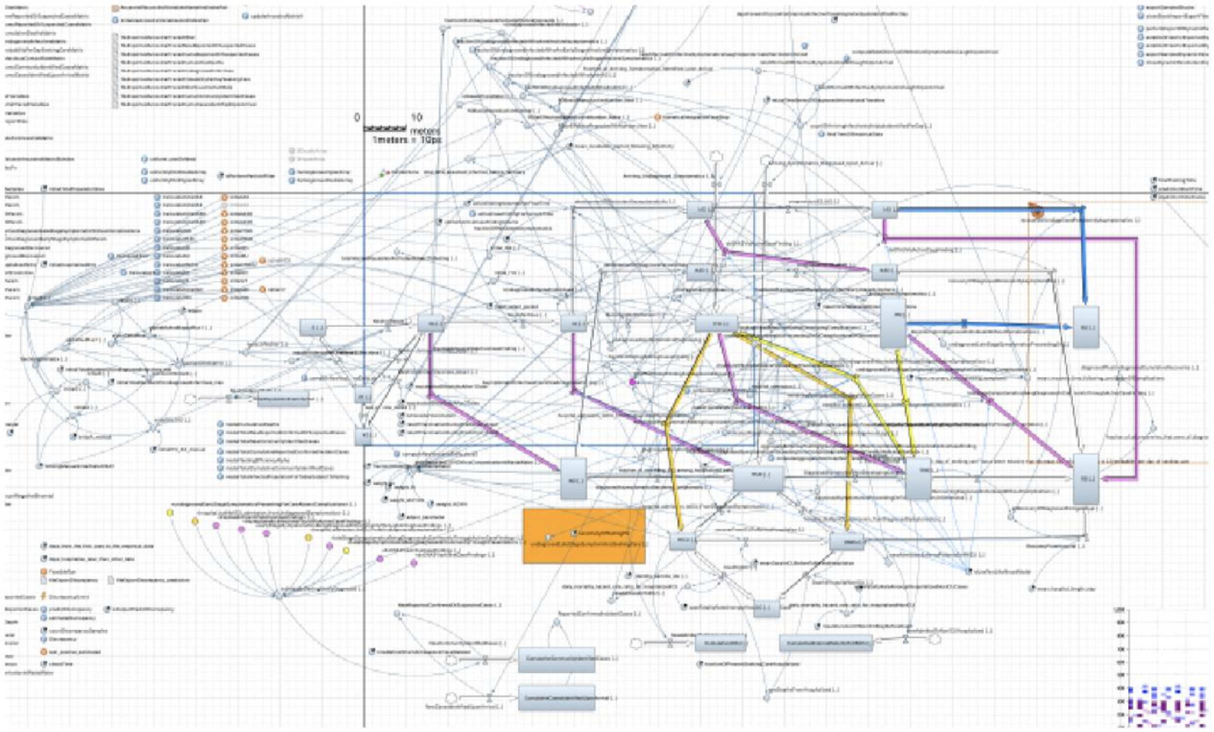


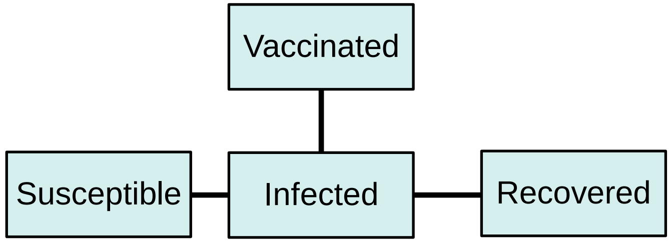
Epidemic Modeling

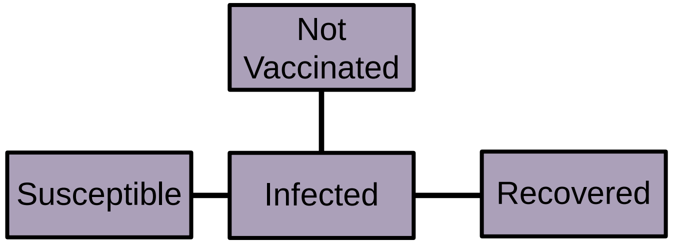
COVID-19 Epidemic Model for Canadian Government

John Baez, et. al.

The ability to *compose* models is crucial because realistic models are complicated and built out of many smaller parts. Here is Osgood and Li's COVID model used by the government of Canada:



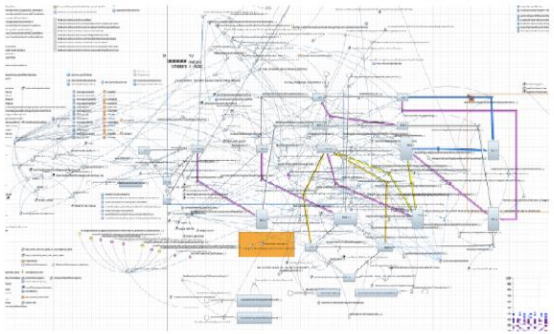




Stock Flow Model of COVID for Canadian Government

John Baez, et. al.

The ability to *compose* models is crucial because realistic models are complicated and built out of many smaller parts. Here is Osgood and Li's COVID model used by the government of Canada:



Most stock-flow modeling is done using software called AnyLogic. It's powerful, but it has several big problems:

- It has no support for *composing* models: that is, taking several smaller models and putting them together to form a larger model.
- It doesn't separate *syntax* from *semantics*.
- It has no support for *"stratifying"* models: that is, taking a model and splitting one stock into several stocks (e.g. age groups).
- It has no support for *collaboratively* building models.
- It is not *free* and not *open-source*!

Our new work aims to fix all these problems.

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An algebraic framework for structured epidemic modeling

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Research

Check for updates

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One contribution of 18 to a theme issue 'Technical challenges of modelling real-life epidemics and examples of overcoming these'.

Subject Areas:
computer modelling and simulation, algebra, applied mathematics, category theory, differential equations

Keywords:
computational epidemiology, applied category theory, modelling and simulation, operads, Petri nets, stratification

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An algebraic framework for structured epidemic modelling

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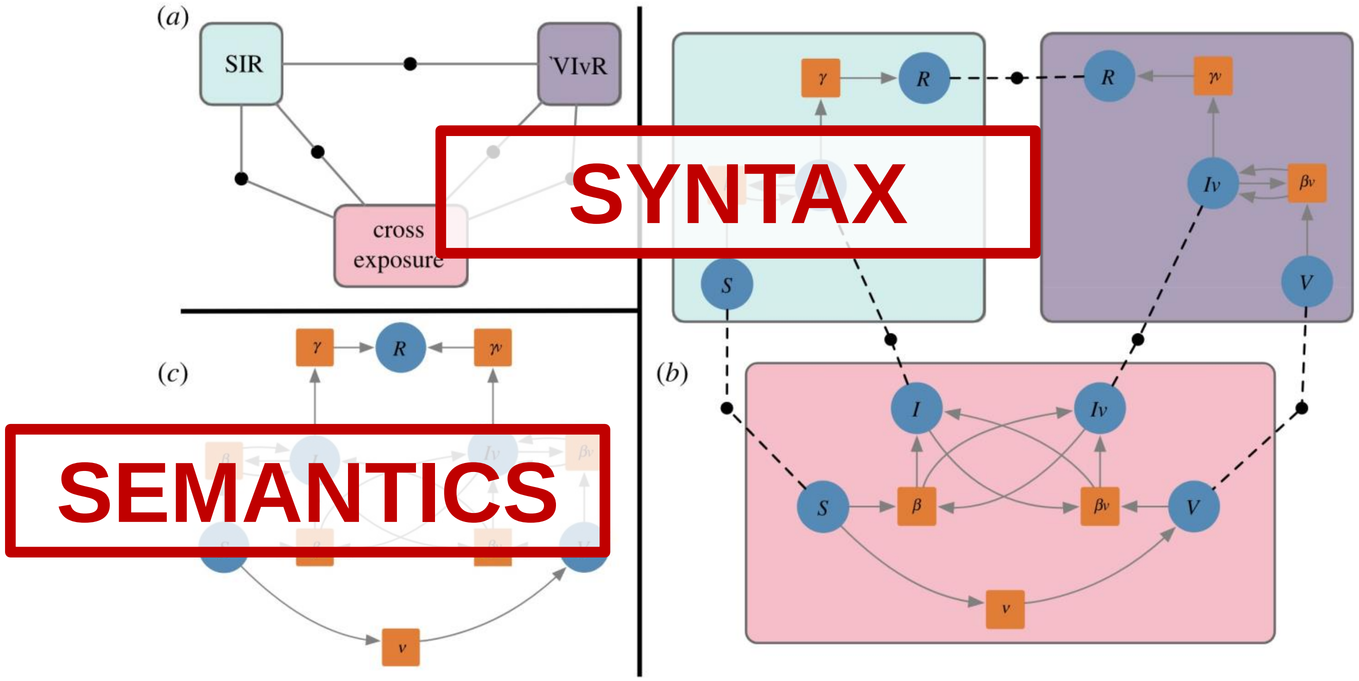
Pandemic management requires that scientists rapidly formulate and analyse epidemiological models in order to forecast the spread of disease and the effects of mitigation strategies. Scientists must modify existing models and create novel ones in light of new biological data and policy changes such as social distancing and vaccination. Traditional scientific modelling workflows detach the structure of a model—its submodels and their interactions—from its implementation in software. Consequently, incorporating local changes to model components may require global edits to the code base through a manual, time-intensive and error-prone process. We propose a compositional modelling framework that uses high-level algebraic structures to capture domain-specific scientific knowledge and bridge the gap between how scientists think about models and the code that implements them. These algebraic structures, grounded in applied category theory, simplify and expedite modelling tasks such as model specification, stratification, analysis and calibration. With their structure made explicit, models also become easier to communicate, criticize and refine in light of stakeholder feedback.

This article is part of the theme issue 'Technical challenges of modelling real-life epidemics and examples of overcoming these'.

THE ROYAL SOCIETY
PUBLISHING

"We propose a compositional modelling framework that uses high-level algebraic structures to capture domain-specific scientific knowledge and **bridge the gap between how scientists think about models and the code that implements them**."

"These algebraic structures, grounded in **applied category theory, simplify and expedite modelling tasks such as model specification, stratification, analysis and calibration.** "



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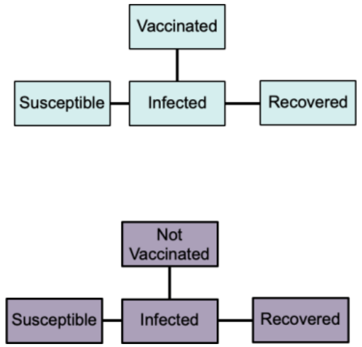
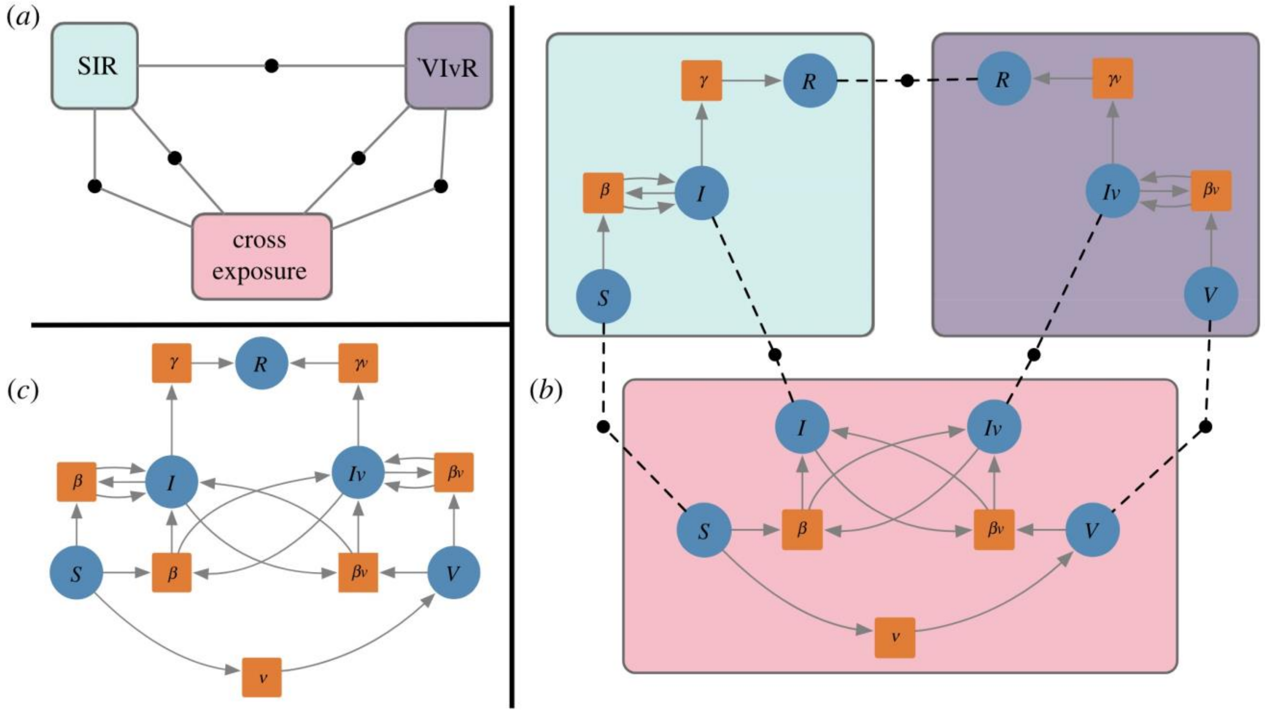
Syntax and Semantics

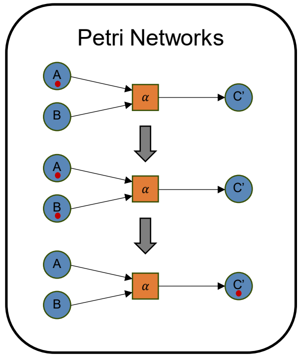
- SIR – Population
  - S = Susceptible
  - I = Infected
  - R = Recovered
- Syntax
  - Open Petri Nets
  - Composable, Nestable
- Semantics
  - Differential equation
  - Computer code for evaluation
    - Julia



SYNTAX



SEMANTICS



SYNTAX → Functor → **SEMANTICS**

Semantics is auto generated from the syntax

Syntax and Semantics

Semantics is auto generated from the syntax

- Modeler defines model using the **syntax** category
  - Resulting model has properties induced on it by the syntax category
- Simulation of the model is in the **semantic** category
  - Simulation computer code is generated by a mathematical transformation of model
  - In category theory, this mathematical transformation is called a functor
- Benefits
  - Easy to explore model variants
  - Model is automatically V&Ved, by construction

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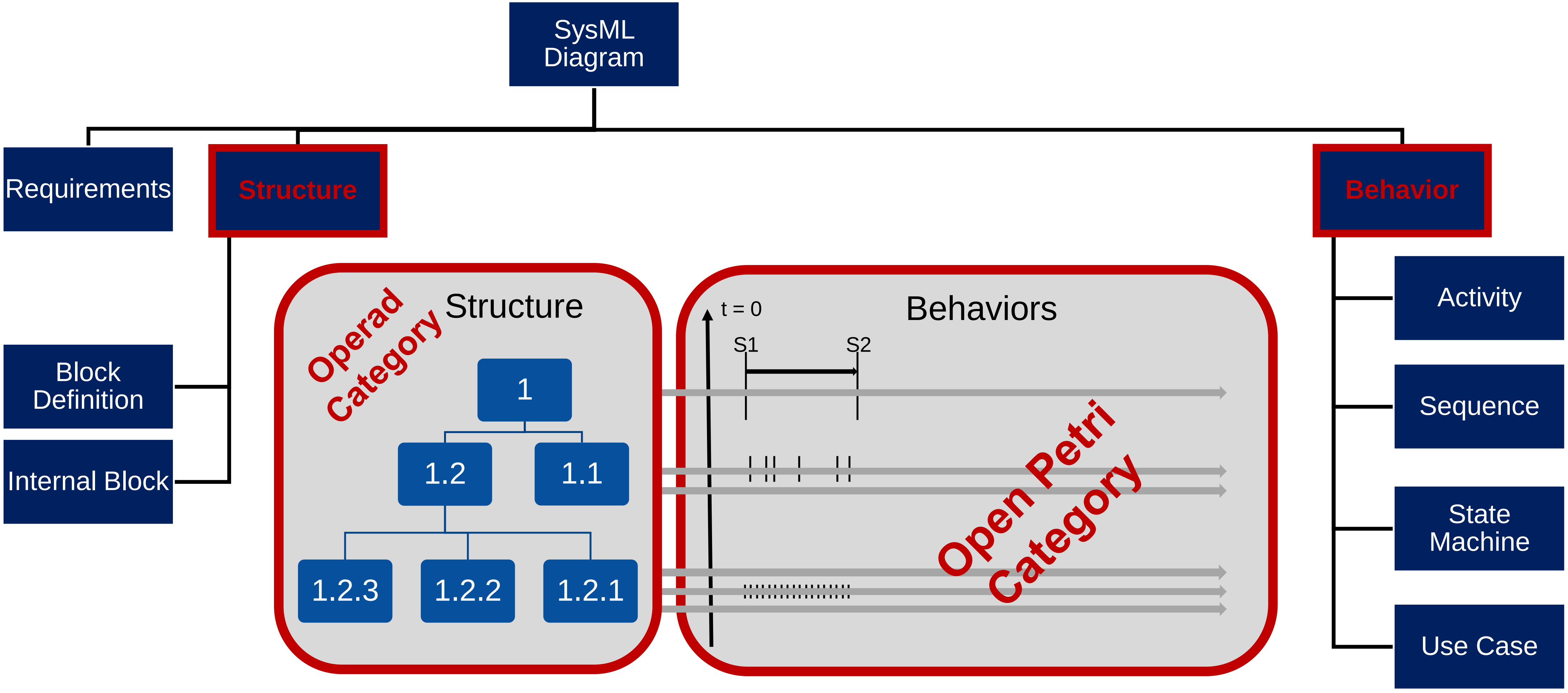
SysML

– and –

Physical Modeling of a CPS

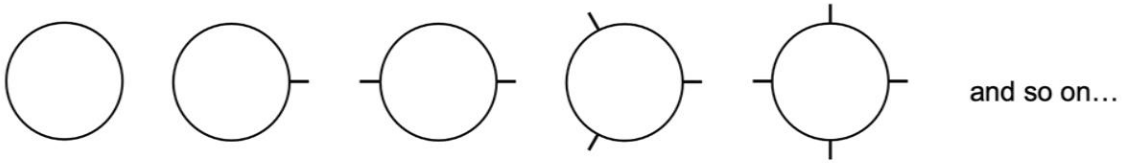
SysML and Category Theory for Syntax

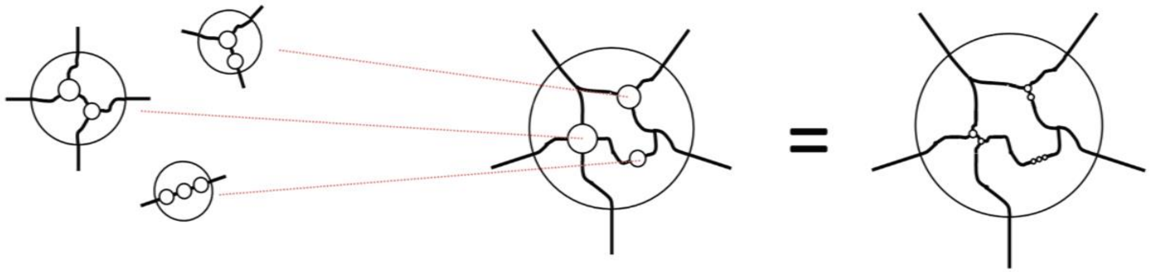


Operads

3.2.1 First example of operad and algebra

There is an operad $\mathcal{O}$ whose objects can be drawn as circles with a finite number of ports, like this:





Operads were invented for Algebraic Topology in 1968

HOMOTOPY-EVERYTHING *H*-SPACES

BY J. M. BOARDMAN AND R. M. VOGT

Communicated by F. P. Peterson, May 24, 1968

An *H-space* is a topological space X with basepoint e and a *multiplication* map $m: X^2 = X \times X \to X$ such that e is a homotopy identity element. (We take all maps and homotopies in the based sense. *We use k-topologies throughout* in order to avoid spurious topological difficulties. This gives function spaces a canonical topology.) We call X a *monoid* if m is associative and e is a strict identity.

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Operadic Analysis of Distributed Systems

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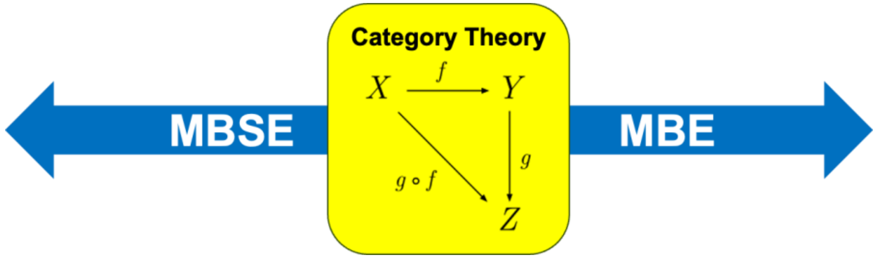
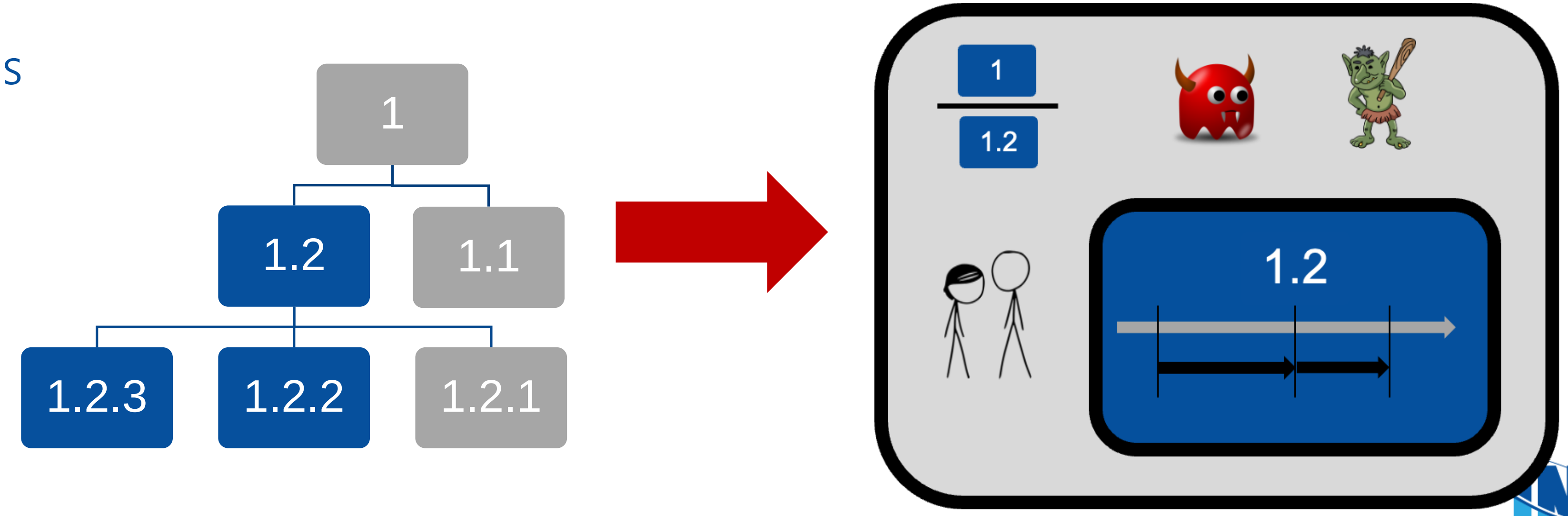
Syntax and Semantics for CPS

- Syntax
  - Categories: Operads, Petri, …

- Semantics
  1. Physical Simulation of System
     - Spatial and Logical
  2. Tailored Projections
  3. …





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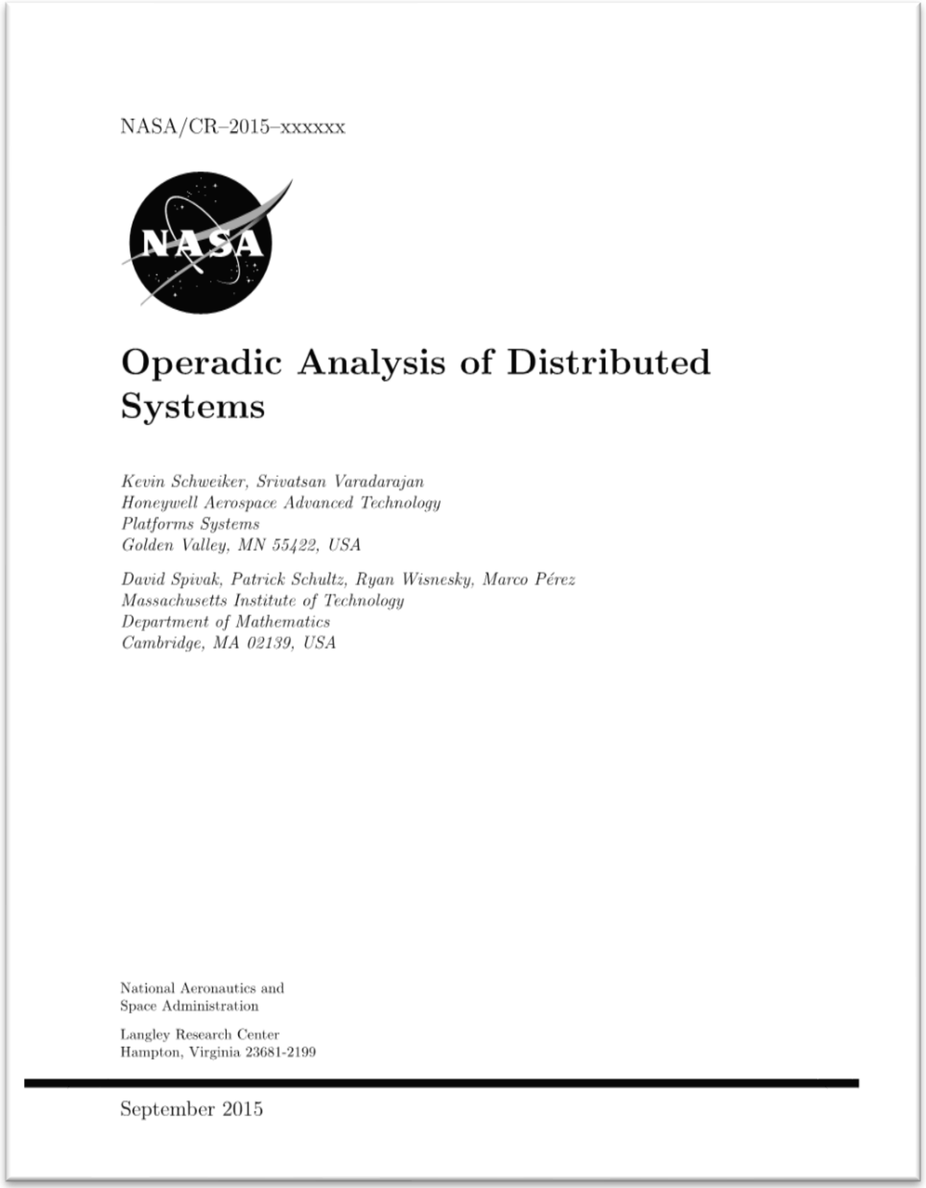
NextGen Airspace



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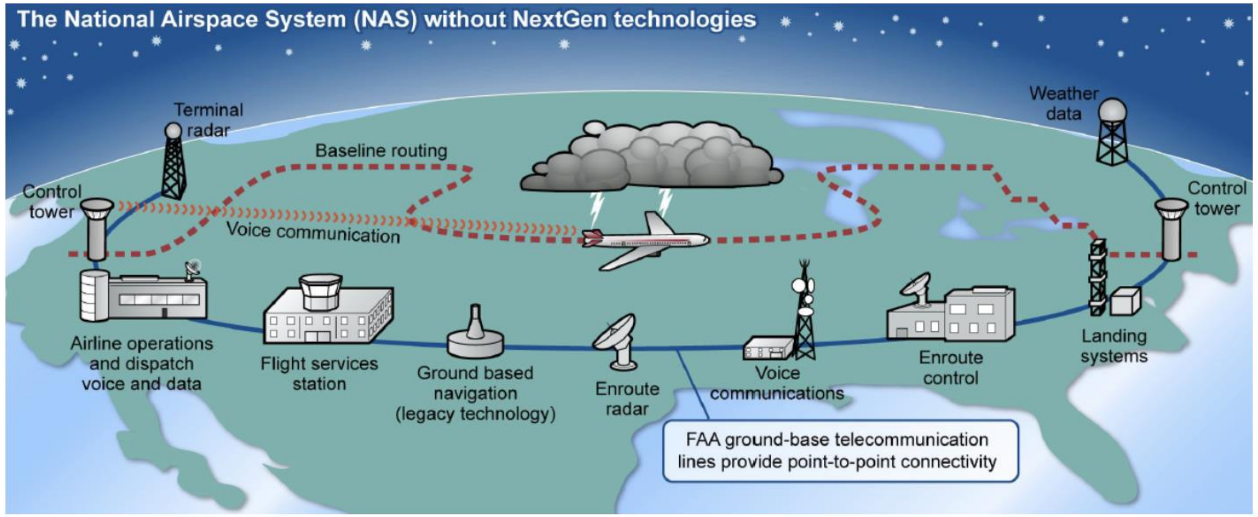
Current



Spatial and Logical Views of the National Airspace

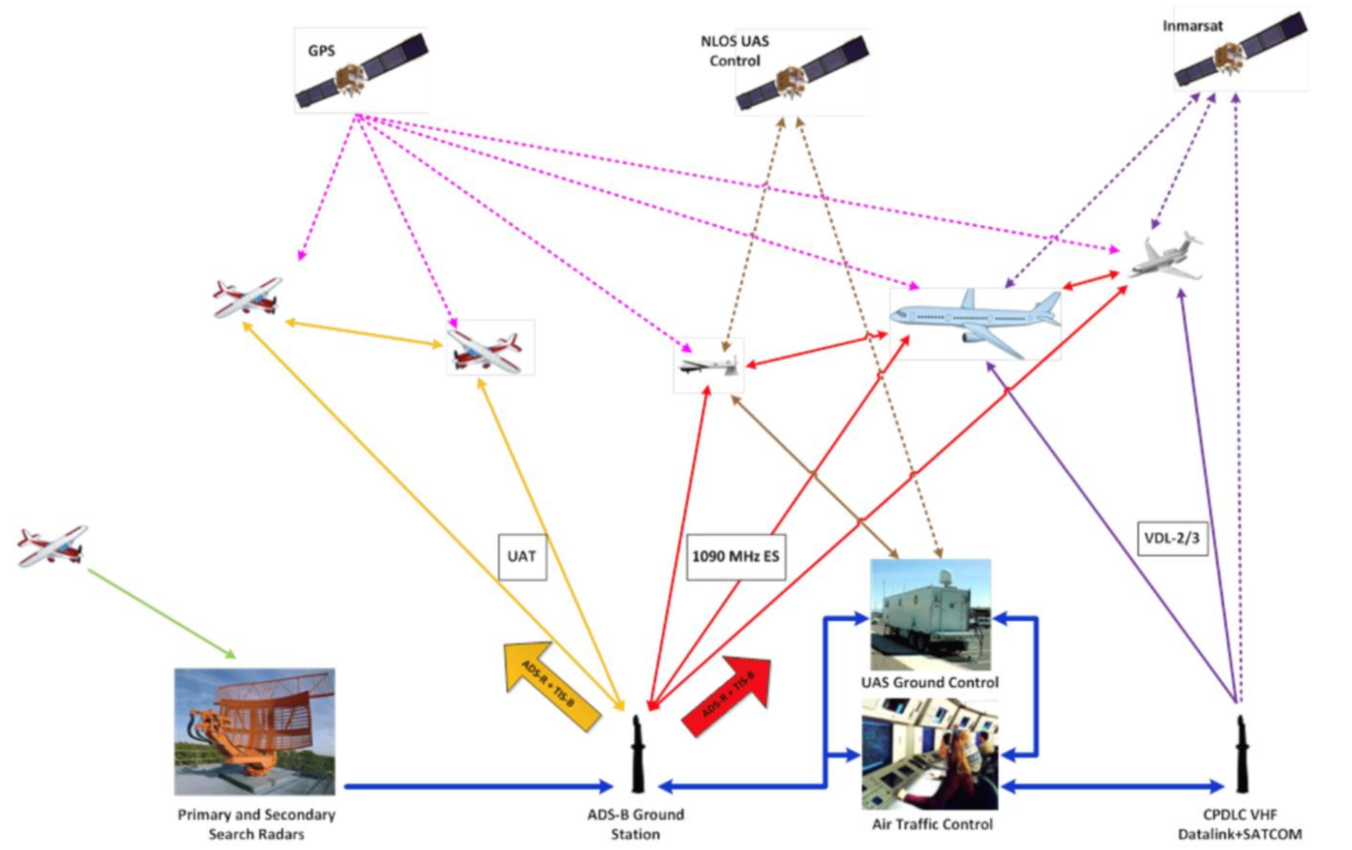


Figure 5: NextGen Communications used to aid Aircraft Separation

ID

Logical

Spatial

No.	ID	Description	Logical	Physical
1	ACT = (1,1)	Actuator	Command	Airplane
2	ACARS = (1,1)	Aircraft Communications Addressing and Reporting System	Command	Airplane
3	ADIRU = (1,1)	Air Data Inertial Reference System	Detect	Airplane
4	ASE = (1,1)	Aerodynamic Surfaces & Engines	Command	Airplane
5	CC = (1,1)	Controller Command	Command	ATC
6	CD = (2,1)	Computers & Displays	Estimate	ATC
7	CE = (2,1)	Controller Estimate	Estimate	ATC
8	CP =(1,1)	Controller Perception	Detect	ATC
9	MST = (1,2)	Mode S Transponder	Detect	Airplane
10	PC = (1,2)	Pilot Command	Command	Airplane
11	PE = (1,1)	Pilot Estimate	Estimate	Airplane
12	PP = (4,1)	Pilot Perception	Detect	Airplane
13	PSR = (1,1)	Primary Search Radar	Detect	Radars
14	Rx1 = (4,1)	Airplane Radio Receiver	Detect	Airplane
15	Rx2 = (1,1)	ATC Radio Receiver	Detect	ATC
16	SSR = (12)	Secondary Search Radar	Detect	Radars
17	T= (1,1)	TCAS	Detect	Airplane
18	Tx1 = (1,1)	Airplane Radio Transmitter	Command	Airplane
19	Tx2 = (1,1)	ATC Radio Transmitter	Command	ATC

Summary

- Models provide understanding
- Physical modeling is as important in a CPS as cyber modeling
  - Today, physical modeling is subordinate to cyber modeling in MBSE
  - Primarily based on interface standards for exchange of information
- Category Theory provides a foundation to unify modeling
  - Syntax for describing the system
  - Semantics for using the model

This work will be starting in Fall 2024
Your comments and participation would be appreciated