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Theoretical Four Pressure Model Development: A Real Characteristic Formulation for RELAP5-3D

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Theoretical Four Pressure Model Development

A Real Characteristic Formulation for RELAP5-3D

A “Well Posed” Equation System in 2-D

2-D One Sound Speed

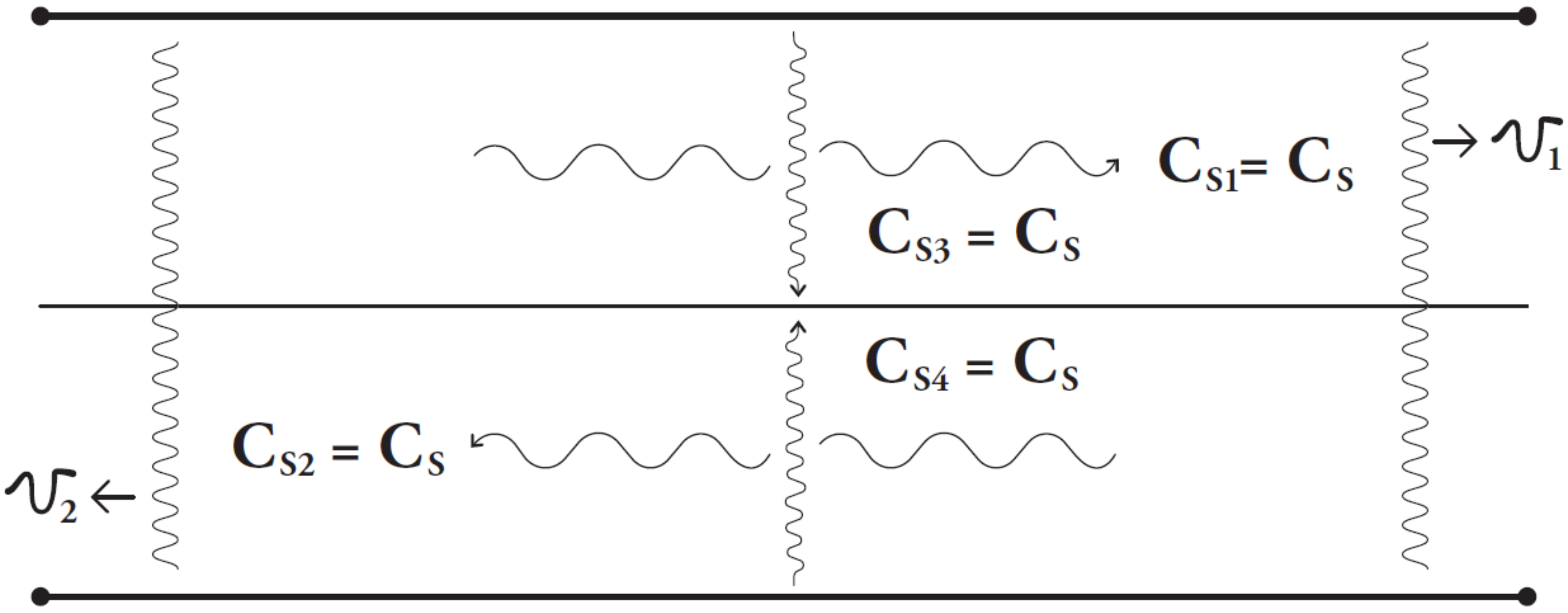

Figure 1.0 One Pressure Boundary Condition for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and Density

2-D Three Sound Speeds

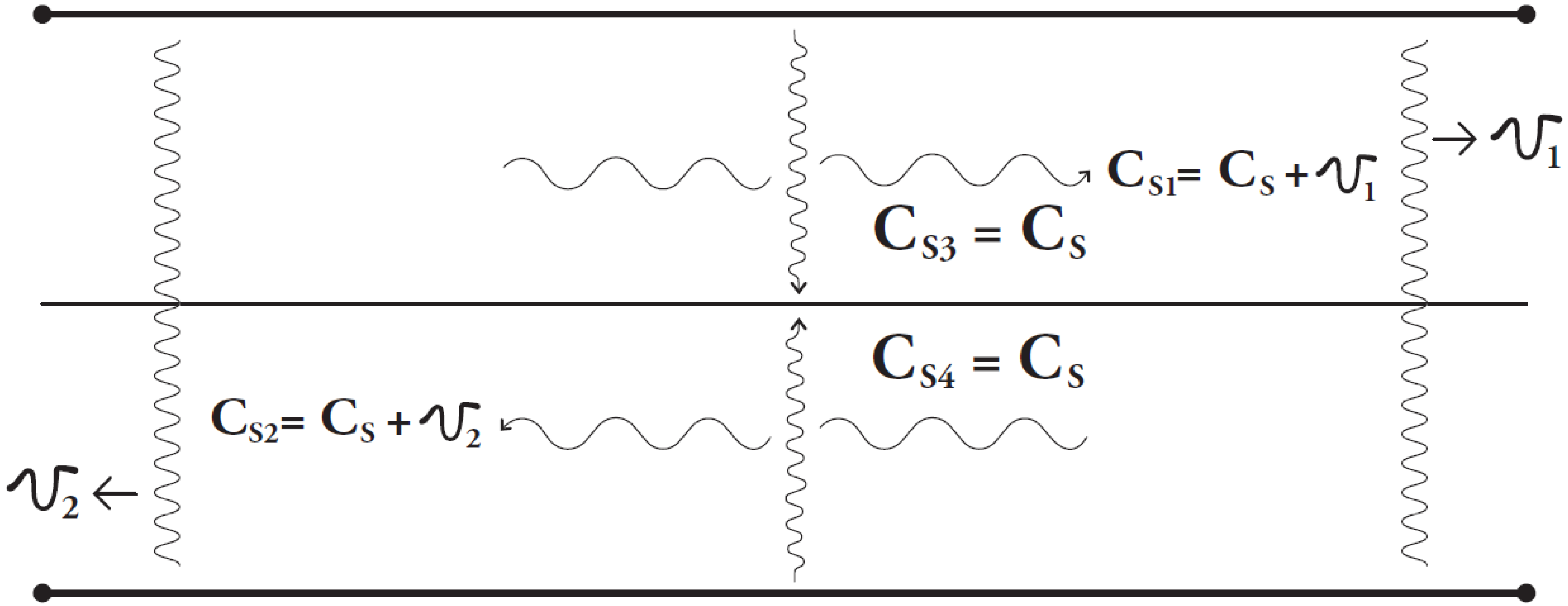

Figure 2.0 Three Pressure Boundary Conditions for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and Density

2-D Four Sound Speeds

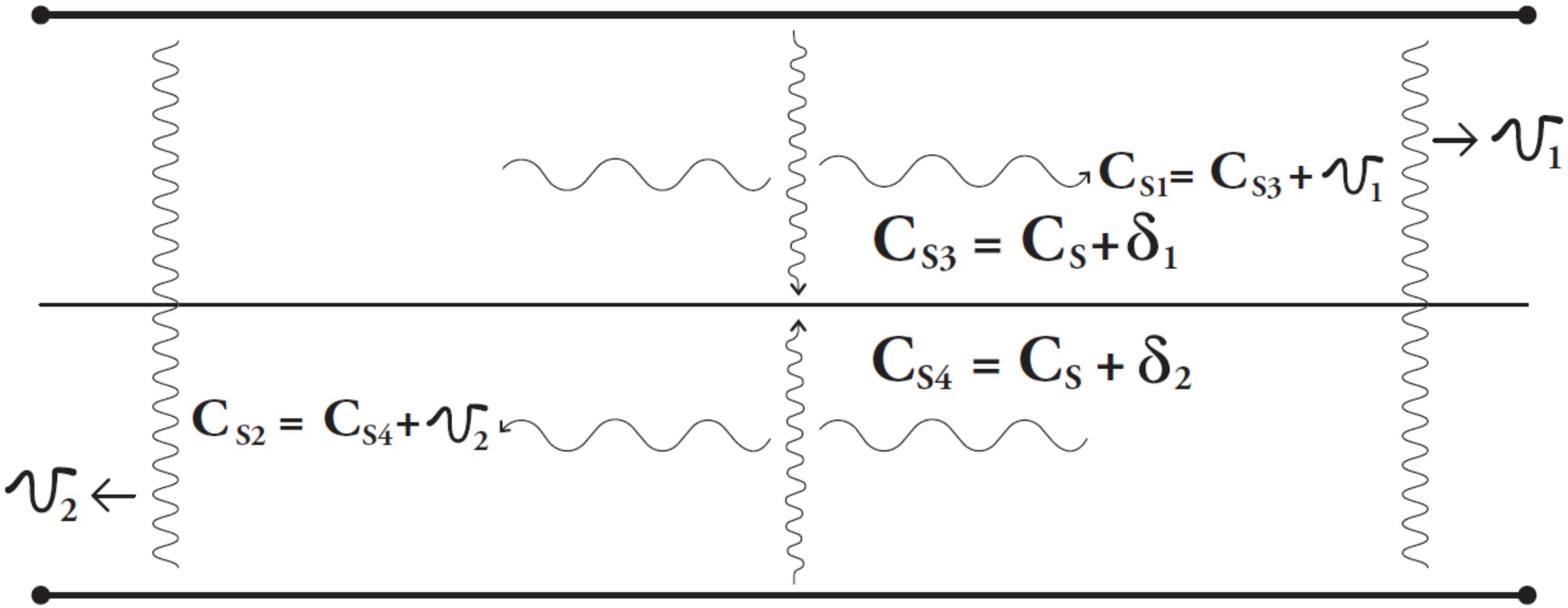

Figure 3.0 Four Pressure Boundary Conditions for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and unequal Density

Note : All three Figures illustrate flow at two velocities of two fluids between two flat plates with perfect slip at the fluid boundaries and plate boundaries.

$\mathbf{V}_{1}$ Velocity of Fluid One (can be constant or time dependant)

$\mathbf{V}_{2}$ Velocity of Fluid Two (can be constant or time dependant)

$\mathbf{C}_{S}$ Sound Speed at Rest ($V_1 = V_2 = 0$) (or given by One Pressure)

$\mathbf{C}_{S1}$ Sound Speed in Fluid One parallel to the motion of Fluid One

$\mathbf{C}_{S2}$ Sound Speed in Fluid Two parallel to the motion of Fluid Two

$\mathbf{C}_{S3}$ Sound Speed perpendicular to the motion of Fluid One

$\mathbf{C}_{S4}$ Sound Speed perpendicular to the motion of Fluid Two

δ_{1} Sound Speed Increment due to Density Difference of Fluid One

δ_{2} Sound Speed Increment due to Density Difference of Fluid Two

An Observation

- We have examined 2-D in the Figures

- For 1-D
-     - Consider ignoring the Transverse Doppler Shifts
-     - Consider only the two Axial Doppler Shifted Pressures

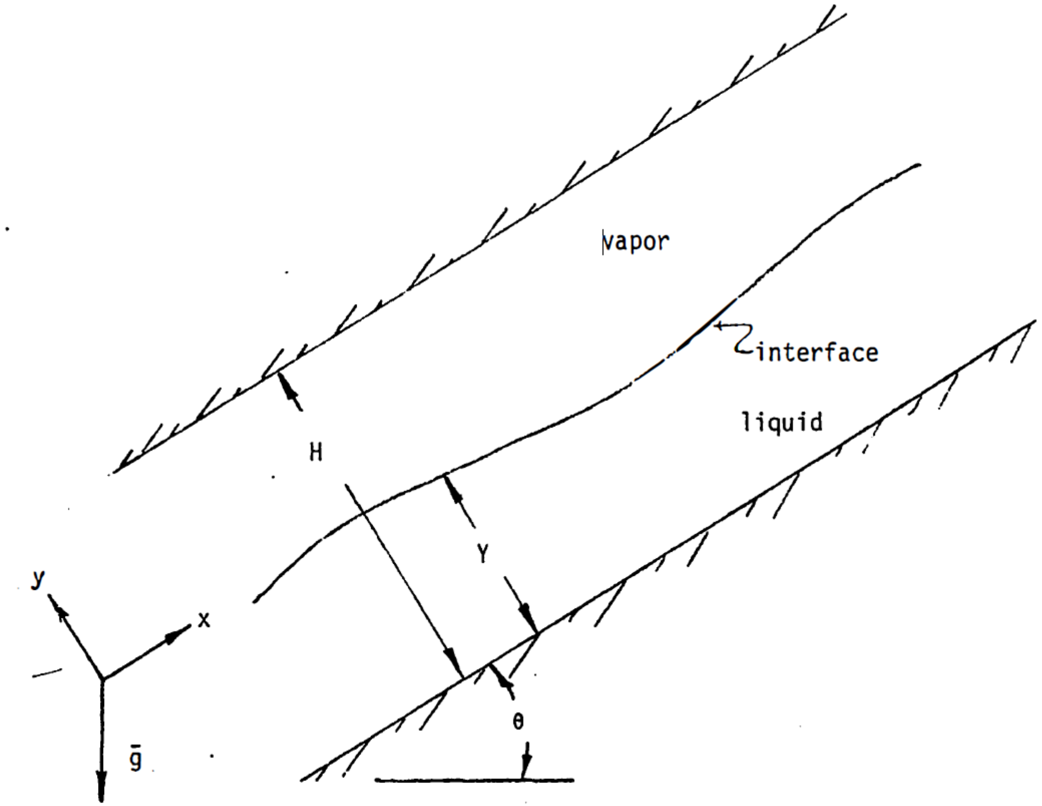

Figure 4. Two-Phase Plane Separated Flow Geometry

Glossary of Used Symbols

α Void Fraction for the Vapor Phase

$\bar{u_l}$ Averaged Liquid Velocity

$\bar{u_v}$ Averaged Vapor Velocity

$\bar{v_l}$ Averaged Transverse Liquid Velocity

$\bar{v_v}$ Averaged Transverse Vapor Velocity

v_m Transverse Velocity Parameter

$\bar{P}_l$ Averaged Liquid Axial Pressure

$\bar{P}_v$ Averaged Vapor Axial Pressure

$\bar{U}_l$ Averaged Liquid Axial Specific Internal Energy

$\bar{U}_v$ Averaged Vapor Axial Specific Internal Energy

$\bar{\rho_l}$ Averaged Liquid Density

$\bar{\rho_v}$ Averaged Vapor Density

P_l Transverse Liquid Pressure

P_v Transverse Vapor Pressure

c_l Transverse Liquid Sound Speed

c_v Transverse Vapor Sound Speed

$\bar{c_l}$ Averaged Axial Liquid Sound Speed

$\bar{c_v}$ Averaged Axial Vapor Sound Speed

$\dot{m}$ Mass Transfer across Liquid-Vapor Interface

Additional Definitions

(Eq. 1a) $\bar{\rho}_l = [\ \int_0^Y \rho_l \ \mathrm{dy}\ ] \ / \ \mathrm{Y}$

also

(Eq. 1b) $\bar{\rho}_v = [\ \int_Y^H \rho_v \ \mathrm{dy}\ ]/(\ \mathrm{H}\ -\ \mathrm{Y}\ )$

and

(Eq. 2a) $\overline{\rho_l u_l} = \bar{\rho}_l \bar{u}_l = [\ \int_0^Y \rho_l \ u_l \ \mathrm{dy}\ ] \ / \ \mathrm{Y}$

Additional Definitions (continued)

(Eq. 2b) $\overline{\rho_v u_v} = \bar{\rho_v}\bar{u_v} = [\ \int_Y^H \rho_l\ u_l\ \mathrm{dy}\ ]/(\ \mathrm{H}\ \text{-}\ \mathrm{Y}\ )$

We also define a transverse velocity variable v_m ;

(Eq. 3a) $\bar{v_l} = [\ \int_0^Y v_l\ \mathrm{dy}\ ]\ /\mathrm{Y} \equiv v_m\ /\ 2$

(Eq. 3b) $\bar{v_v} = [\ \int_Y^H v_v\ \mathrm{dy}\ ]/(\ \mathrm{H}\ \text{-}\ \mathrm{Y}\ ) \equiv v_m\ /\ 2$

Discussion

- We begin with a Two Dimensional Two Fluid compressible Euler Equation Model discussed by Ransom and Scofield (Feb 1976) containing an averaged low order Two Fluid formulation of an averaged equation set in 2-D.
- We utilize the Leibniz rule and their averaging formulation to interchange integration and differentiation, and assume as they do that the average of the product is the product the average. This is also assumed in the RELAP5-3D formulation.
- Note that this averaging assumption is not mathematically rigorous but represents the "state of the art" theoretically.
- If one does not make this assumption, closure issues arise in the formulation of the underlying two fluid model of two phase flow. We will investigate this issue further in future work. We retain the higher order terms and hence arrive with a Four Pressure Model summarized in this talk.
- We ignore all sources sinks in our current discussion and as well as $\dot{m}$ and set them all to zero. This is because we are only interested in the Characteristic Analysis of our Equation System for the "Basic Ill Posed" Model, as discussed in the literature. (We discuss this issue further in this talk)

ILL POSED ?

- ILL POSED (Elliptic) Equations Never Lead to a Correctly Set Cauchy Problem per Hadamard
    - Not a well defined Initial Value Problem
- Will not Converge
- Artificial Viscosity Will Eliminate the ILL POSEDNES
    - However NOT “Consistent” Mathematically With
        - Compressible Navier Stokes Equations
        - Compressible Euler Equations
        - Compressible Averaged Two Fluid Model Equations
- Don’t Know What Eqns. we are really solving
- But Good Fit With Data (Assessment Results)

The Model Equation System we are studying

(Eq. 4a) $\partial[(1-\alpha)\bar{\rho}_l]/\ \partial\mathrm{t}\ +$
$\partial[(1\text{ - }\alpha)\bar{\rho}_l\bar{u}_l]/\ \partial\mathrm{x} = 0$

(Eq. 4b) $\partial[\alpha\bar{\rho}_v]/\ \partial\mathrm{t} + \partial[\alpha\bar{\rho}_v\bar{u}_v]/\ \partial\mathrm{x} = 0$

(Eq. 5a) $(1\text{ - }\alpha)\ \bar{\rho}_l\partial\bar{u}_l/\ \partial\mathrm{t}\ +\ (1\text{ - }\alpha)\ \bar{\rho}_l\bar{u}_l\partial\bar{u}_l/\ \partial\mathrm{x}$
$+\ (1\text{ - }\alpha)\ \partial\bar{P}_l/\partial\ x + [\ \bar{P}_l - P_l\ ]\ \partial\ (1\text{ - }\alpha)\ /\partial\ x = 0$

(Eq. 5b) $\alpha\ \bar{\rho}_v\partial\bar{u}_v/\ \partial\mathrm{t}\ +\ \alpha\ \bar{\rho}_v\bar{u}_v\partial\bar{u}_v/\ \partial\mathrm{x}\ +\alpha\ \partial\bar{P}_v/\partial\ x$
$+\ [\ \bar{P}_v - P_v\ ]\ \partial\alpha\ /\partial\ x = 0$

(Eq. 6) $[\ (1\text{ - }\alpha\ )\ \bar{\rho}_l + \alpha\ \bar{\rho}_v\ ]\ \partial v_m\ /\ \partial\mathrm{t}$
$+\ [\ (1\text{ - }\alpha\ )\ \bar{\rho}_l\bar{u}_l + \alpha\ \bar{\rho}_v\bar{u}_v\ ]\ \partial v_m\ /\ \partial\mathrm{x}$
$+\ 2\ [\ P_v - P_l\ ] = 0$

(Eq. 7) $v_m/H + \partial\alpha\ /\ \partial t\ +$
$[(\ \bar{u}_l + \bar{u}_v\ )/2]\ \partial\alpha\ /\ \partial x = 0$

Closure Analysis

Our analysis results with an 8 x 8 Equation System of Eight Equations solving for Eight Unknowns.
(Only 6 x 6 for the Characteristic Analysis)

- The equations are
  - (a) Two averaged Continuity Equations
  - (b) Two averaged Axial Momentum Equations
  - (c) One averaged Transverse Momentum Equation
  - (d) One averaged Interface Equation
  - (e) Two averaged Phasic Thermal Energy Equations

- We chose the following solution unknowns

$$\alpha,\ \bar{u}_l\ ,\ \bar{u}_v\ ,\ v_m\ ,\ \bar{P}_l\ ,\ \bar{P}_v,\ \bar{U}_l,\ \bar{U}_v$$

- The remaining unknowns are

$$\bar{\rho}_l\ ,\ \bar{\rho}_v\ ,\ P_l\ ,\ P_v\ ,\ c_l\ ,\ c_v\ ,\ \bar{c}_l,\ \bar{c}_v$$

Closure Analysis (continued)

- We can utilize first order Taylor Expansions to obtain the desired closure relationships (no time in this talk for the details) or Tables ;

$$\bar{\rho_l} = f_l(\ \bar{P_l},\ \bar{U_l}\ )$$

$$\bar{\rho_v} = f_v(\ \bar{P_v},\ \bar{U_v}\ )$$

$$\bar{c_l} = f_{cl}\ (\ \bar{P_l},\ \bar{U_l},\ \bar{\rho_l},\ \bar{u_l}\ )$$

$$\bar{c_v} = f_{cv}\ (\ \bar{P_v},\ \bar{U_v},\ \bar{\rho_v},\ \bar{u_v}\ )$$

$$c_l = \bar{c_l}\ \text{-}\ \bar{u_l}$$

$$c_v = \bar{c_v}\ \text{-}\ \bar{u_v}$$

$$P_l = f_{Pl}\ (\ \bar{\rho_l}\ ,\ c_l^2\ )$$

$$P_v = f_{Pv}\ (\ \bar{\rho_v}\ ,\ c_v^2\ )$$

Closure Analysis (continued)

Note that in general ;

And (Eq. 8) $\left(\partial \bar{P}_l \,/\, \partial \bar{\rho}_l \right) = \bar{c}_l^2$

(Eq. 9) $\left(\partial \bar{P}_v \,/\, \partial \bar{\rho}_v \right) = \bar{c}_v^2$

at con … cteristic Analysis, assume that the two Axial F … arotropic.”

Characteristic Analysis

- We follow the below approach to the Characteristic Analysis. We assume that the two Axial Pressures for the two phases are "Barotropic." In general our equation system is of the form given below

$$\text{(Eq. 10)} \quad \underline{\vec{A}} \cdot \partial\, \vec{w} \,/\, \partial\, \mathrm{t} \;+ \vec{B} \cdot \partial\, \vec{w} \,/\, \partial\, \mathrm{x} \;+ \vec{C} = 0$$

$$\text{(Eq. 11)} \quad \vec{w} = [\bar{P}_l \;,\; \bar{P}_v \; \bar{u}_l \;, \bar{u}_v \; \alpha \;,\; v_m \;]^T$$

$$\text{(Eq. 12)} \quad \mathrm{Det}\;[\; \vec{A} \text{ - } \mu \cdot \; \vec{B} \;] = 0$$

where the μ ’s are the Characteristic Roots that should be Real for a "Well Posed" System. The superscript T denotes a matrix transpose, $\vec{A}$ and $\vec{B}$ are matrices.

Characteristic Analysis (continued)

- Note that $\vec{C}$ does not take part of the Characteristic analysis so that setting all sources and sinks to zero is justifiable as long as there are not any differential terms present. Additionally, the assumption that ṁ is zero does not effect the fundamental Kelvin Helmholtz or Rayleigh Taylor instabilities formulations and since it is believed that these are the underlying elements that result in an Ill Posed formulation per this assumption is justifiable.
- We obtain, utilizing MATLAB Six Real Roots for our 6 x 6 determinant and hence a "Well Posed“ Equation System. We list our Characteristic Roots below

$$2 / (\bar{u}_l + \bar{u}_v)$$

$$((1 - \alpha) \bar{\rho}_l) + \alpha \bar{\rho}_v) / ((1 - \alpha) \bar{\rho}_l \bar{u}_l + \alpha \bar{\rho}_v \bar{u}_v)$$

$$- 1 / (\bar{c}_l - \bar{u}_l)$$

$$+ 1 / (\bar{c}_l + \bar{u}_l)$$

$$- 1 / (\bar{c}_v - \bar{u}_v)$$

$$+ 1 / (\bar{c}_v + \bar{u}_v)$$

Characteristic Analysis (continued)

- Hence, we have obtained a "Well Posed" Equation System for a Two Phase Flow Model without any dissipation, surface tension or real or artificial viscosity without sources or sinks. Although we believe there are other Four Pressure Models that can be explored we suggest that the currently discovered model should be thoroughly investigated with sources and sinks as well as the Energy Equations included in the analysis. We leave this to future exploration of the Four Pressure systems. However, our results are very promising for the "Basic Ill Posed" model.

RELAP5-3D Implementation of New Model

- We suggest that the RELAP5-3D Software Two Phase Flow Computer Code be used as a test bed for our Four Pressure Models. It is a mature and well tested System Code for the study of Nuclear Reactor Dynamics under normal and transient conditions (and well verified) and also utilized for Nuclear Reactor Design and Analysis.
- Our current model, as discussed in this talk requires that the Artificial Viscosity Terms be zeroed out as an input parameter by the user and replaced by a suitable "patch".

RELAP5-3D Implementation of New Model (continued)

- Two Axial Pressures need to be solved for, and additional terms need to be added to the two momentum equations requiring modifications to the Solution Scheme. The Phasic Thermal Energy Equations as well as the Continuity Equations remain the same as long as the pressure terms in the Energy Equations are assumed to be the Averaged Axial Pressures for the Vapor and the Liquid. All other variables can be obtained from the Closure Relationships and the addition of the transverse Momentum and Interface Equations required by our Four Pressure model do not effect the RELAP5-3D code software implementation.
- It should be noted that some of the "Component" models in RELAP5-3D do not utilize the principal solution scheme of the code precisely (such as choked flow models and relief valves that utilize the Bernoulli Equations). We will avoid studying such model related issues in our initial Four Pressure solution scheme research and only look at "Simple" problems so that the new Four Pressure model problems can be understood simply and accurately.

Test Cases

- We suggest the following two test cases. They are the Water over Steam Problem and the Water Faucet Problem. Both Problems require the additional inclusion of Gravity as a parameter in our Four Pressure Formulation. The Water over Steam Problem requires the addition of the Energy Equations to our formulation per our previous discussion of the RELAP5-3D System Code and our current state of theoretical development for our Four Pressure Model.
- Note that the Water over Steam Problem is the Classical Rayleigh-Taylor Instability and exists in the Compressible Navier-Stokes and Compressible Euler Equation Systems. Similar behavior occurs for the in-compressible case

Summary

- A simple Four Pressure Two Fluid Model has been formulated and analyzed. Real Characteristic Roots have been obtained with a MATLAB analysis of a 6 x 6 Characteristic Matrix from the resulting PDE system for Continuity and Momentum. Hence, this system results in a "Well Posed" Model. We compared our Four Pressure Model MATLAB results with surface tension SIGMA Model MATLAB results and verified the SIGMA Model roots with the published literature.
- We have shown that Closure can be obtained for the resulting Eight Equation System of Continuity, Momentum and Energy for our Four Pressure Model and discuss the Closure methodology.
- We suggest two test cases for the Four Pressure Model and discuss implementation of the model in RELAP5-3D.

Conclusions

- We have shown that a Four Pressure Two Fluid Model Formulation for the Momentum and Continuity Equations results in real and physical Characteristics with a 6 x 6 MATLAB analysis.
- No new physics were required to obtain these results, and space and time transformation laws are preserved in this model. We have shown that closure can be obtained for an eight equation Two Fluid Model consisting of two Continuity, two axial Momentum, one Transverse Momentum, one Interface Balance and Two Energy Equation System. This Equation System is solved for Eight variables and results with Four Pressures, Two Axial Velocities, Two Densities, Four Sound Speeds, Two Energies, a Transverse Velocity and Void Fraction when closure relationships are applied.
- We believe that this model can be adapted to and implemented in the RELAP5-3D Two Fluid Computer Code for a limited implementation and tested with several simple model problems.
- We believe that the "Ill Posed Problem" issue associated with the Two Fluid One Pressure Model of Two Phase Flow is strictly a consequence of distorting space and time associated with a single sound speed in two frames of reference and is resolved by including Galilean Relativity and associated Doppler Shifts.

FUTURE WORK

- Extend Current Analysis to Full 2-D (Currently one and a half Dimensional)
- Extend Current Analysis to Full 3-D
- Formulate and Analyze a 1-D Model
  - RELAP5-3D is mostly a 1-D “PIPE” Code
  - Our Current Analysis suggests the following Model
    - 1 Continuity Eqn for each Phase
    - 1 Momentum Eqn for each Phase
      - Resulting in Two Pressures each with a Doppler Shift
    - 1 Energy Eqn for each Phase with an individual Pressure
    - Resulting in a 6 x 6 Eqn System –But Closure issue Exists

Available Information

- A complete set of write ups is available by request
- INL REPORT in review
- NS&E PAPER in the submittal stage

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