INL/CON-20-57351-Revision-0

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August 2020

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Prepared for the
U.S. Department of Energy
Under DOE Idaho Operations Office
Contract DE-AC07-05ID14517

CALCULATION OF CRITICAL HEAT FLUX USING AN INVERSE HEAT TRANSFER METHOD TO SUPPORT TREAT EXPERIMENT ANALYSIS

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ABSTRACT

Heat transfer between cladding and coolant during transient scenarios remains a critical area of uncertainty in understanding nuclear reactor safety. To advance the understanding of transient and accident scenarios involving critical heat flux (CHF), an in-pile experiment for the Transient Reactor Test facility (TREAT) at Idaho National Laboratory (INL) was developed. The experiment, named CHF-Static Environment Rodlet Transient Test Apparatus (CHF-SERTTA), consists of a hollow borated stainless-steel heater rod submerged in a static water pool heated via the (n, α) reaction in boron-10. This paper presents a novel inverse heat transfer method to determine CHF by using the optimization and uncertainty software Dakota to calibrate a RELAP5-3D model of CHF-SERTTA to temperature measurements obtained from a thermocouple welded to the surface of the rod.

Keywords: Transient boiling, CHF, heat transfer, TREAT

NOMENCLATURE

BNH	Borated Nuclear Heated
CHF	Critical Heat Flux
HGR	Heat Generation Rate
HTC	Heat Transfer Coefficient
INL	Idaho National Laboratory
MCNP	Monte Carlo N-Particle
n	Neutron
SERTTA	Static Environment Rodlet Transient Test Apparatus
TREAT	Transient Reactor Test Facility
α	alpha

1. INTRODUCTION

Heat transfer between cladding and coolant during transient scenarios remains a critical area of uncertainty in understanding nuclear reactor safety. Reactivity Initiated Accident experiments performed in France and Japan have shown that steady-state boiling correlations greatly overpredict the measured cladding temperature during these fast transients [1, 2]. Much of this overprediction can be attributed to the difference between the steady-state critical heat flux (CHF) correlations and the CHF values calculated for these transients. Transient CHF values tend to be greater than what the steady-state correlations predict; however, a lack of understanding results in the use of extremely conservative models that widely bound the onset and duration of transient boiling. The steady-state correlations are conservative because a lower CHF value causes the cladding to reach the CHF earlier in the transient resulting in a longer period under an insulating vapor film blanket. Improved understanding, along with predictive models to describe transient boiling, could meaningfully increase safety and design margins in both the current commercial fleet and advanced light water reactors.

To advance the understanding of transient and accident scenarios involving CHF, an in-pile experiment for the Transient Reactor Test facility (TREAT) at Idaho National Laboratory (INL) was developed. The experiment, named CHF-Static Environment Rodlet Transient Test Apparatus (CHF-SERTTA), consists of a hollow borated stainless-steel heater rod—termed the borated nuclear heated (BNH) rod—submerged in a static water pool. The BNH rod converts the neutron flux in TREAT into thermal energy via the (n, α) reaction in boron-10.

This paper presents a novel inverse heat transfer method to determine the CHF of a given transient. For this method, a RELAP5-3D model of CHF-SERTTA was created. A neutronic Monte Carlo N-Particle (MCNP) model of CHF-SERTTA was also created to translate TREAT reactor power into heat generation rates (HGRs) of the BNH rod. Using the optimization and uncertainty quantification software Dakota, the CHF of the transient is determined by calibrating the RELAP5-3D model to temperature data obtained by a thermocouple attached to the outside of the BNH rod.

2. EXPERIMENT DESCRIPTION

The TREAT facility provides the unique capability to test nuclear fuels under extreme conditions. Details on the facility and its capabilities have been reported elsewhere [3]. The design of TREAT allows for the insertion of experiment vehicles that are enclosed from the reactor. This allows for the flexibility to design experiments with different thermal-hydraulic boundary

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conditions and instrumentation packages that match the needs of the experiment. This flexibility enables TREAT to quickly transition between experiments of different fuel forms or thermal-hydraulic conditions.

The CHF-SERTTA experiment vehicle leverages a previous design, the Minimal Activation Retrievable Capsule Holder-SERTTA [4], but with the nuclear fuel rod replaced by a borated stainless-steel rod. The BNH rod is submerged in a static water pool, and a heater in the bottom of the capsule allows for experiments with elevated temperatures and pressures. An instrumentation package for capturing key heat transfer measurements was included. The instrumentation included a variety of temperature sensors: integral junction, bare-wire, type K thermocouples were welded to the outside of the BNH rod; thermocouples were placed in the capsule water; and optical fibers coupled to infrared pyrometer systems measured the temperature on the inner and outer surfaces of the BNH rod. A custom developed boiling detector to measure phase change in the system [5] and an accelerometer to measure acoustic signals from nucleate boiling and the departure from nucleate boiling (DNB) were also included. An overview of the CHF-SERTTA capsule and the BNH rod is shown in Figure 1.

The BNH rod is approximately 10 cm in length and made of stainless-steel 304 containing 2.05 wt.% natural boron. The rod is hollow and hourglass-shaped to make it thicker on the ends and thinner in the middle. This was done to reduce the localized heat flux in order to avoid end peaking effects.

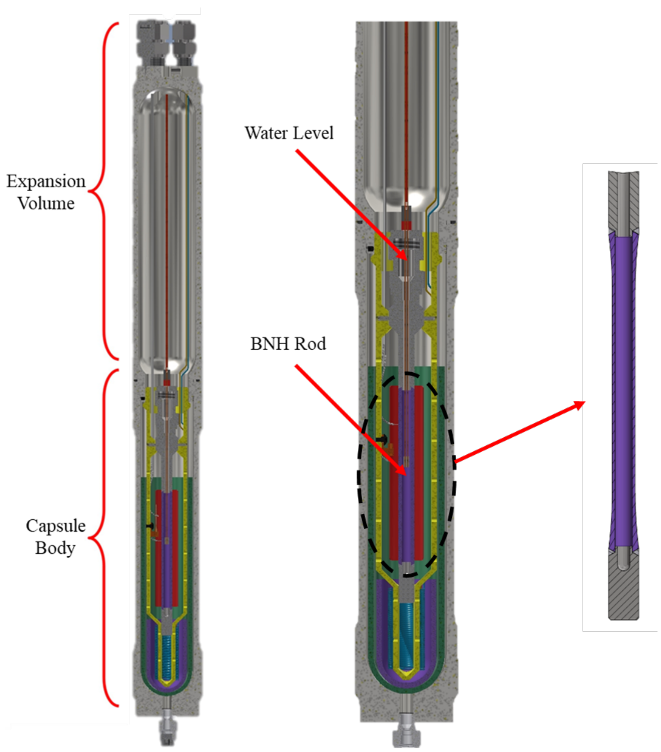

FIGURE 1: OVERVIEW OF THE CHF-SERTTA EXPERIMENT DESIGN

3. MODEL DESCRIPTIONS

3.1 Neutronics Model

An estimate of HGRs within the BNH rod and surrounding water is an essential starting point for thermal analysis and the determination of CHF. To estimate the HGRs for this experiment, MCNP version 6.1 [6] was used with ENDF/B-VII.1 nuclear data cross sections [7]. Since the nuclear heating of the BNH rod is not expected to be uniform, a cylindrical mesh was used to determine the axially and radially varying HGRs.

To accommodate the RELAP5-3D heat structure input requirements, the geometry of the BNH rod in MCNP was created in such a way that the HGRs could be directly used in the RELAP5-3D heat structure input. To do this, the rod was divided into three main sections: the upper curved region, the central straight region, and the lower curved region. The thin central region was divided into four concentric cylindrical tubes extending through the upper and lower curved regions. The remainder of the extended curved regions were divided into six additional, slightly narrower tubes. The length of the tube segments in the curved region were determined by using the MCNP plotter to estimate the location where the outermost, lower thickness tubes intersected with the smooth curved surface, and by assigning tube cut-off at the midpoint between intersections. This process is illustrated in Figure 2.

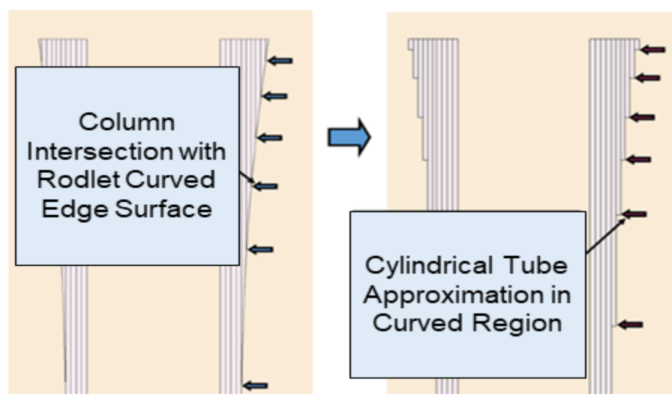

FIGURE 2: MCNP PLOT ILLUSTRATION SHOWING GEOMETRY MODIFICATIONS TO ACCOMMODATE RELAP5-3D HEAT STRUCTURE INPUT

The final mesh configuration of the BNH rod, along with the water directly surrounding the rod, is shown in Figure 3. The lower half of the rod is a mirror image of the upper half. From this mesh, HGRs can be estimated in terms of watts per gram of material per megawatt of TREAT core power. This allows for a translation between the TREAT power profile as a function of time, and the nuclear heating in the experiment.

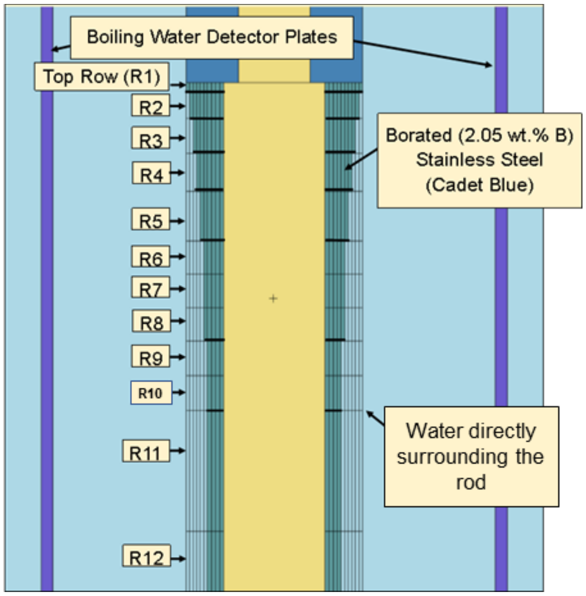

FIGURE 3: MCNP PLOT ILLUSTRATION OF THE MESH STRUCTURE USED FOR LOCAL NUCLEAR HGR ESTIMATES TO SERVE AS INPUT FOR THE RELAP5-3D MODEL (NOTE: HEATER ROD IS DARK BLUE, AND WATER IS LIGHT BLUE)

3.2 RELAP5-3D Model

To model the thermal-hydraulic conditions of the experiment, a RELAP5-3D model of the CHF-SERTTA capsule was created. The model consists of a pipe component representing the water in the capsule, eighteen heat structure geometries representing the BNH rod, and a heat structure to account for heat conducted through the capsule wall. Figure 4 shows the model nodalization, but due to the fine meshing of the BNH rod heat structures, they are not shown in full detail.

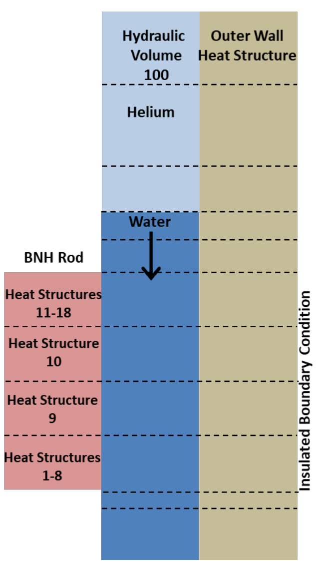

FIGURE 4: NODALIZATION OF THE CHF-SERTTA RELAP5-3D MODEL

The same nodalization used for the MCNP model for the BNH rod (Figure 3) was used to create the heat structures for the RELAP5-3D model. HGR estimates determined from the MCNP simulation were implemented in the heat structures to convert TREAT reactor power—provided in the model as a power table—to the internal heat generation of the BNH rod.

4. METHODOLOGY TO DETERMINE CHF

4.1 Sensitivity Analysis

Due to modeling uncertainties in heat transfer correlations, neutronic modeling, and material properties, initial RELAP5-3D model predictions do not accurately represent the experimentally measured data. To determine which uncertainties have the largest impact on the temperature of the BNH rod throughout the simulation, a sensitivity analysis using the RELAP5-3D model was performed.

The sensitivity analysis investigated a variety of input parameters. To assess the influence of the uncertainty in the MCNP estimate of the HGRs, these values were varied in the model using a multiplicative factor on the power table. Uncertainty in the volumetric heat capacity and thermal conductivity of the BNH rod were also considered. RELAP5-3D uses heat transfer correlations to determine what the heat transfer coefficient (HTC) should be. In the case of a static capsule, the heat transfer correlations of interest are the those in the natural convection, nucleate boiling, transition boiling, and film boiling regimes, as well as the correlation for CHF. RELAP5-3D allows the user to place multiplicative factors on the heat transfer coefficients predicted by the correlations, as well as on the predicted value of CHF [8]. The influence on the uncertainty of the heat transfer correlations was investigated by varying these multiplicative factors. Table I shows the range used in the sensitivity study for each parameter. The ranges used for the heat transfer multipliers were based on uncertainties reported in the RELAP5-3D manual [8]. The uncertainty in the heat generation rate of past TREAT experiments has been estimated to be approximately ± 14% via gamma spectroscopy. This value comes from a test which consisted of a UO2 fuel rod in a dry capsule [9]. To account for the water environment and the use of the BNH rod, a range of ± 30% was used. The nominal value for the parameters were based on the steady-state heat transfer values predicted by RELAP5-3D, the heat generation rate predicted by the MCNP model, and the thermal properties from measurements of the BNH rod material.

TABLE I: SENSITIVITY ANALYSIS PARAMETERS

Parameter	Deviation from Nominal
Heat Generation Rates	±30%
Natural Convection HTC	±25%
Nucleate Boiling HTC	±30%
Critical Heat Flux	50%-500%

Transition Boiling HTC	±20%
Film Boiling HTC	±20%
Thermal Conductivity	±5%
Volumetric Heat Capacity	±5%

The sensitivity analysis was performed using the optimization and uncertainty quantification software Dakota [10]. Latin hypercube sampling was used to sample the input parameters and a total of 50,000 simulations were ran. Results indicate that the multiplicative factors placed on the CHF and HGRs have the largest impact on the maximum temperature of the outer surface of the BNH rod. Figure 5 shows the Sobol indices of the sensitivity analysis. Sobol indices decompose the variance of the output into fractions that are associated with the importance of the input parameters, and are a way to determine which input parameters have the largest effect on the output [11]. The main indices show the first-order effects of each input parameter, or the effect of only varying one input parameter. The total indices show the first-order and higher-order effects, or the effects of changing multiple input parameters.

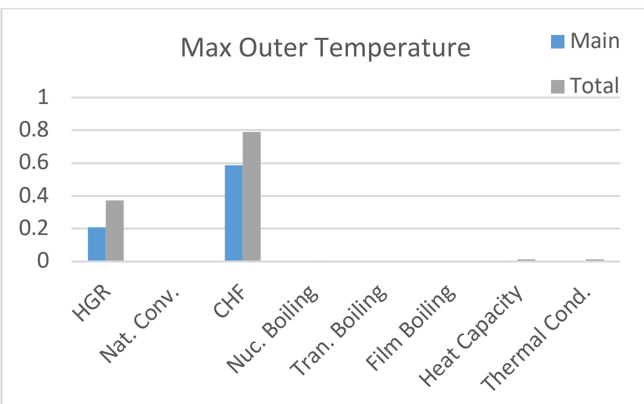

FIGURE 5: SENSITIVITY ANALYSIS SOBOL INDICES FOR THE MAX OUTER TEMPERATURE OF THE BNH ROD

The sensitivity analysis performed with the parameter ranges described in Table I resulted in simulations in which the BNH rod hit CHF and simulations in which it did not. To determine which parameters result in the most sensitivity in only cases where CHF occurred, the parameter range for the HGR and CHF factors were changed to 0%-30% and 50%-250%, respectively; all other ranges in Table I remained the same. With these updated ranges, results show that, along with the HGR and CHF factors, uncertainty in the volumetric heat capacity is also a sensitive parameter on the maximum outer surface temperature of the BNH rod. Sensitivity on the outer surface temperature at the time of CHF showed the CHF factor and the factor on the nucleate boiling heat transfer coefficient as being most important. These results are shown in Figures 6 and 7.

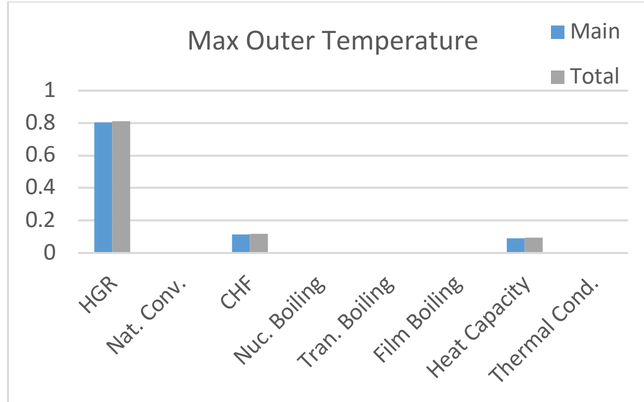

FIGURE 6: SENSITIVITY ANALYSIS SOBOL INDICES FOR THE MAX OUTER TEMPERATURE OF THE BNH ROD WHERE ALL SIMULATIONS HIT CHF

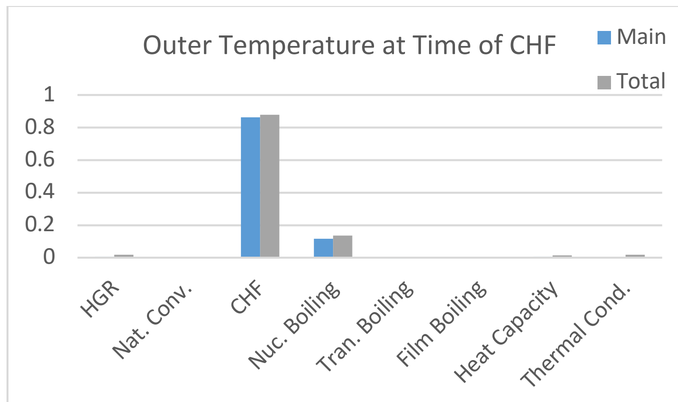

FIGURE 7: SENSITIVITY ANALYSIS SOBOL INDICES FOR THE OUTER TEMPERATURE OF THE BNH ROD AT THE TIME OF CHF

4.2 Model Calibration Process

By using experiment data and knowing what parameters influence the temperature of the BNH rod, the RELAP5-3D model can be calibrated to determine CHF. Sensitivity analysis showed that the most sensitive parameters are the multiplicative factors associated with the HGR, CHF, nucleate boiling heat transfer coefficient, and volumetric heat capacity. Although volumetric heat capacity is important when determining the maximum temperature of the rod, it is not important when determining the temperature of the rod at the time of CHF. Since it is not needed, it is not included in the model calibration to determine CHF.

To calibrate the RELAP5-3D model, experiment temperature data from an integral junction, bare-wire, type K thermocouple welded to the outer surface of the BNH rod was used. The thermocouple was located near the center of the rod. Based on this data, Dakota was used to find the combination of factors on the HGR, nucleate boiling heat transfer coefficient, and CHF that best matched the experiment data (i.e., the set of factors that minimized the residual between the experiment data and the model predictions).

Many different algorithms included in Dakota can be used for model calibration. All the algorithms have the same goal: to minimize the residual between the experiment and the simulation by determining the best set of input parameters. The algorithms simply differ in how they go about achieving this goal.

The different calibration algorithms can be divided into two categories: global calibration and local calibration. As their names suggest, local calibration algorithms aim to find the local optimum of a function, whereas global calibration algorithms aim to find the global optimum of a function. While the local optimum can also be the global optimum, this is not always the case. It was found that using both a global algorithm and a local algorithm to calibrate the model to the experiments yielded the best results. This is because global algorithms have difficulty converging on the optimum. So, a global algorithm was first used to find the region in which the global optimum exists, then a local algorithm was used to find the local optimum in that region—which is also the global optimum.

For the global algorithm, it was found that an evolutionary algorithm worked the best; in particular, the Single-objective Genetic Algorithm method that is part of the JEGA third-party library included in Dakota [12]. Evolutionary algorithms are based on Darwin's theory of "survival of the fittest," and they simulate certain evolutionary processes such as natural selection, mutation, and breeding. By starting with an initial population of input parameter sets and allowing those that best match the experiment data to survive, mutate (i.e., change a value in the parameter set), and mate with other surviving input parameter sets (i.e., take values from both input parameter sets to form a new one), the algorithm works to find the set of model input parameters that produces results closest to the experiment data.

The values produced by the evolutionary algorithm are then further refined through a local calibration method, NL2SOL, also included in Dakota [13]. This nonlinear least squares method works well on highly nonlinear problems and is described in more detail in the Dakota User Manual [10].

5. RESULTS

5.1 Experiment Results

The first series of CHF-SERTTA experiments were recently performed. Over the course of one week, a single CHF-SERTTA capsule underwent four identical transients. The prescribed transient was a 3.0% $\Delta k/k$ step insertion that resulted in a total reactor energy of 1050 MJ during a 160 ms full-width at half-maximum power pulse. Transients 2 and 4 were performed at slightly elevated temperatures due to TREAT not having cooled to ambient temperature before the transient occurred. Figure 8 shows the temperature measurements for each transient (as recorded by the thermocouple attached nearest the center of the BNH rod) as well as the reactor power pulse.

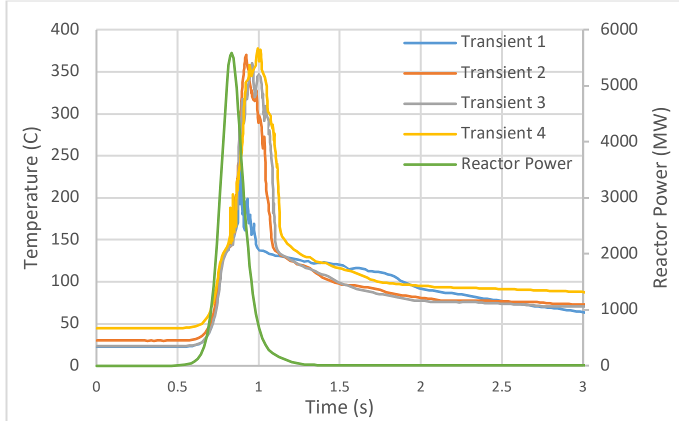

FIGURE 8: TEMPERATURE MEASUREMENTS FROM THE FIRST SERIES OF CHF-SERTTA EXPERIMENTS

Transients 2-4 show very similar temperature behavior, all undergoing DNB at approximately 850 ms and transitioning into film boiling. Transient 1 appears to reach DNB; however, the surface rewets without fully transitioning into film boiling. At this time, it is unclear why Transient 1 shows this different behavior and for this reason it is not included in the calibration to determine CHF. It is important to note that the temperature data does not include any corrections for thermocouple response time or fin effects.

5.2 Determination of CHF

The calibration process described in the previous section was used to determine the CHF value for Transients 2-4. The thermocouple data from the beginning of the transient up to the point where CHF (~850 ms) occurs, as well as the time that CHF occurs in each transient, was used as the calibration data. As shown in Table II, the HGRs predicted by the MCNP model are about 20% high; in other words, the energy deposited in the BNH rod is about 80% of what MCNP predicted. The multiplicative factors on the nucleate boiling heat transfer coefficient were found to be at least 50% greater than the predicted steady-state correlations, and the CHF value for each transient was approximately double the steady-state prediction.

Since the TREAT reactor pulse was identical for all the transients, the calibrated CHF values were expected to be in agreement with each other. This was proven out when they all fell within 10% of each other, with an average value of 5.4 MW/m^2.

TABLE II: RELAP5-3D CALIBRATED INPUT PARAMETERS

Factor	Transient 2	Transient 3	Transient 4
HGR	0.81	0.80	0.80
CHF	1.95 (5.6 MW/m^2)	1.73 (5.2 MW/m^2)	2.10 (5.4 MW/m^2)
Nucleate Boiling HTC	1.70	1.71	1.53

Figure 9 shows the calibrated and uncalibrated RELAP5-3D models' temperature predictions for the outer surface of the BNH rod per the three transients. This demonstrates that the calibration process successfully matched the experiment data and model predictions up until CHF. To match the experiment data after CHF, additional parameters need to be included in the calibration, due to the different heat transfer regimes that occur after DNB, as well as proper understanding of the response of thermocouples during this period of rapid temperature changes.

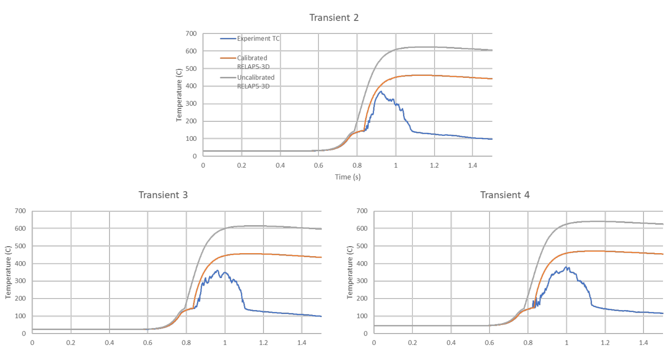

FIGURE 9: CALIBRATED AND UNCALIBRATED RELAP5-3D MODELS COMPARED TO EXPERIMENT THERMOCOUPLE DATA

Figure 10 zooms in on Transient 2 around the point where CHF occurs. This shows how well the calibrated RELAP5-3D model matches the experiment data. One can also see that changes in the slope of the temperature correspond to a change in heat transfer regime. This is shown in the figure by overlaying the heat transfer regime predicted by the RELAP5-3D model with the temperature. There is a second sharp increase in temperature that occurs at approximately 880 ms, this is assumed to be due to interaction between the thermocouple and the vapor film surrounding the BNH rod.

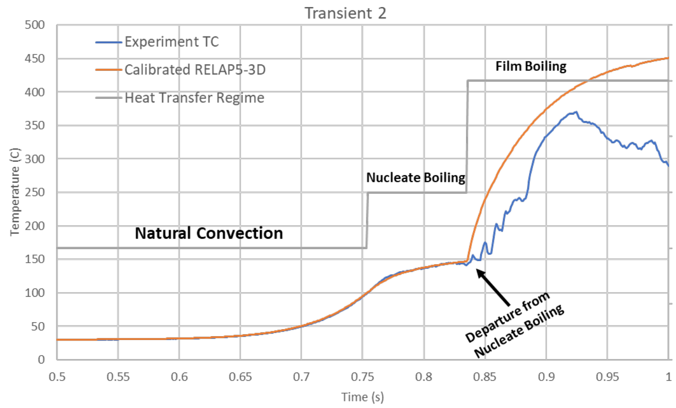

FIGURE 10: TRANSIENT 2 ZOOMED IN AROUND THE POINT WHERE CHF OCCURS, OVERLAYED WITH THE RELAP5-3D HEAT TRANSFER REGIME PREDICTIONS

6. FUTURE WORK AND CONCLUSIONS

This paper introduces a novel inverse heat transfer method to determine the CHF value for CHF-SERTTA transients. For this method, a RELAP5-3D model of CHF-SERTTA was created. An MCNP model of CHF-SERTTA was also created to translate TREAT reactor power into HGRs of the BNH rod. Using the optimization and uncertainty quantification software Dakota, the CHF of the transient is determined by calibrating the RELAP5-3D model to data obtained from a thermocouple attached to the outside of the BNH rod.

The presented method was then used to determine the CHF for Transients 2-4 from the first CHF-SERTTA test series. The results show that the calibrated CHF values are all within 10% of each other, with an average value of 5.4 MW/m^2. They also show that MCNP predictions overestimate the energy deposited into the BNH rod by about 20%, and that the nucleate boiling heat transfer coefficient is at least 50% greater than the predicted steady-state correlations.

Future work will focus on quantifying the uncertainties associated with this method. This includes work focused on determining the uncertainties in thermocouple measurements due to response time and fin effects. Once available, data from the pyrometer, which measures temperature on the inside of the BNH rod, will also be used as part of the calibration. This will help reduce uncertainty in the calibration of the HGR, since it was found that the inner temperature of the BNH rod largely depends on the energy deposited into the rod. Work will also be done to extend this method throughout the entire duration of the reactor pulse. This method will also be applied to future test series which will consist of different energy depositions and step insertions. The discrepancy between the results from Transient 1 and Transients 2-4 will also be investigated.

ACKNOWLEDGEMENTS

This work was supported by the Office of Nuclear Energy of the U.S. Department of Energy under DOE Idaho Operations Office Contract DE-AC07-05ID14517. Accordingly, the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for U.S. Government purposes.

This research made use of resources at the High Performance Computing Center at Idaho National Laboratory, which is supported by the Office of Nuclear Energy of the U.S. Department of Energy and the Nuclear Science User Facilities under Contract No. DE-AC07-05ID14517.

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