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Idaho National Laboratory
Idaho Falls, Idaho 83415

http://www.inl.gov

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A Non-cooperative Game-based Approach to Distributed Beam Scheduling in Millimeter-Wave Networks

Xiang Zhang*, Shamik Sarkar†, Sneha Kumar Kasera†*, Arupjyoti Bhuyan‡, and Mingyue Ji*†
Department of Electrical and Computer Engineering, University of Utah*
School of Computing, University of Utah †
Idaho National Laboratory‡
Email: *{xiang.zhang, mingyue.ji}@utah.edu,†shamik.sarkar@utah.edu, †kasera@cs.utah.edu, ‡arupjyoti.bhuyan@inl.gov

***Abstract*—This paper considers the problem of distributed beam scheduling in millimeter-Wave (mmWave) cellular networks with non-cooperating service operators sharing the spectrum. The base stations (BSs) may belong to different operators so there is no coordination or centralized control among them. Our goal is to design efficient distributed beam scheduling schemes to optimize the network-level utility defined as the sum of the logarithm of the average throughput of the users. Utilizing the Lyapunov optimization theory, the network utility optimization problem can be cast into two sub-problems to be solved iteratively in each frame. The second sub-problem is non-convex and challenging to solve in general. To this end, we propose a non-cooperative game-based beam scheduling and power allocation scheme in which the BSs are modeled as players that aim to optimize their individual payoff. The Nash Equilibrium (NE) then provides a distributed (approximate) solution to the second sub-problem. We prove the existence of NE and identify sufficient conditions guaranteeing the uniqueness of NE by establishing an equivalence of the formulated game to a variational inequality problem. A parallel power update algorithm is proposed with assured convergence. Numerical evaluation demonstrates the superiority of the proposed approach over several baselines.**

I. Introduction

The proliferation of mmWave frequencies in 5G cellular networks makes additional shared and unlicensed spectrum available, which improves the spectrum efficiency and contributes to orders of magnitude increase in data rate. Spectrum sharing enables the secondary utilization of the shared or unlicensed spectrum that is available by allowing concurrent beam-based transmission and has the potential to largely enhance the the system-level throughput performance [1], [2]. However, highly directional transmission also has drawbacks – it presents a severe interference condition for the users in ultra-dense small BS 5G networks if there is no proper coordination of beams. For efficient beam management and interference mitigation, two paradigms, centralized and distributed approaches are considered. Centralized approaches [3]–[5] can be effective in general but usually incur high complexity due to the global control, resulting in limited scalability. Systems with a central control entity is also vulnerable to attack.

Another line of work [6]–[23] considered distributed approaches, which are usually scalable and flexible, and can improve system security and robustness by removing any central point of attack. Nekovee *et al.* [15] considered a multi-RAT system where 5G BSs co-exist with existing networks like WiGig on a shared band. A co-existence mechanism was proposed where the 5G and WiGig BSs aim to optimize their own utilities. Wei *et al.* [16] proposed a two-stage schedule-and-align scheme that facilitates efficient communication in a scenario where a BS communicates with multiple UEs through a number of distributed remote mmWave radio units. Non-cooperative game-based formulation [6]–[8], [18], [19], [22], [23] is a natural way to model the distributed scheduling problem for cellular networks. Alpcan *et al.* [6] considered the CDMA uplink power control problem and formulated it as a non-cooperative game where the UEs are modeled as players each aiming to maximize its own payoff without information exchange among them. A fixed point iteration-based power update algorithm was proposed in order to find the NE. Pang *et al.* [7] considered a cognitive radio system consisting of primary and secondary users sharing the same spectrum. Each secondary user aims to maximize its own throughput but the aggregate interference caused by these users to the primary users should be controlled. A game-based formulation was presented and the existence and uniqueness of the NE were analyzed. Candogan *et al.* [19] studied the distributed power allocation problem in a multi-cell CDMA network. A *potential game*-based approach was proposed to provide an approximate solution to the formulated power allocation game. It was shown that by properly selecting the pricing factors of the potential game, it can converge to the unique equilibrium which is a globally optimal power allocation. This provides a good solution to the original game in the high SINR regime.

In this paper, we study the downlink beam scheduling and power allocation problem for non-cooperative mmWave cellular networks with BSs belonging to different operators and therefore there is no centralized control or explicit coordination among them. We formulate a network utility maximization problem that ensures fairness among the BSs. By utilizing the Lyapunov stochastic optimization framework, the network utility maximization problem is decomposed into two sub-problems that need to be solved distributedly in each time frame. The first sub-problem is convex and easy to cope with while the second sub-problem is non-convex and challenging to solve in general. To this end, we propose a non-cooperative game-theoretic beam scheduling and power allocation scheme by modeling each BS as a player which aims to maximize its own payoff defined as a weighted sum of the throughput and the power penalization. The pricing weights in the payoff are automatically and optimally determined by the virtual queues derived from the Lyapunov optimization. As a result, the Nash Equilibrium of the formulated power allocation game provides an approximate solution to the second sub-problem. We also prove the existence of the equilibrium and provide sufficient conditions which guarantee the uniqueness of the equilibrium via an equivalence of the formulated game to a specific variational inequality problem. A parallel power update algorithm is proposed where the BSs adapts their transmit powers simultaneously slot by slot based on the best response function. Simulation results show that the proposed approach outperforms several baselines and achieves near-optimal performance in the high SINR regime.

II. Problem Formulation

A. System Description

Consider a mmWave cellular network with M base stations (BSs) labeled by $\mathcal{M} = \{1, 2, \cdots, M\}$ and K user equipments (UEs) labeled by $\mathcal{K} = \{1, 2, \cdots, K\}$. Each BS i belongs to a different service operator and is responsible for serving a subset $\mathcal{K}_i \subseteq \mathcal{K}$ of the UEs. We assume that each BS can transmit to at most one UE at any given time and each UE is subscribed to exactly one BS. Therefore, we have $\mathcal{K}_i \neq \varnothing, \forall i$, $\mathcal{K}_i \cap \mathcal{K}_{i'} = \varnothing, \forall i \neq i'$ and $\cup_{i \in \mathcal{M}} \mathcal{K}_i = \mathcal{K}$, i.e., the M subsets $\{\mathcal{K}_i\}_{i \in \mathcal{M}}$ specifies the BS-UE association, which is assumed to be determined by some exogenous mechanism and stays unchanged during the scheduling process considered in this paper. The system operates synchronously over a shared and unlicensed frequency band of W Hz. We use a frame structure as follows: Each *frame* consists of N *blocks* and each block contains T_b time *slots*. Therefore, each frame contains $T_f = NT_b$ slots. Each block is a UE scheduling unit which means that the set of scheduled UEs stays fixed within any block but can change from block to block. Since the problem of UE scheduling will not be studied in this paper, we simply assume that the scheduled UEs are selected randomly from the UE pool of each BS and stay fixed among all blocks of each frame.

BS/UEs are equipped with directional antennas to facilitate beam-based data transmission and reception. A commonly used antenna model is the keyhole-like sectorized model (e.g., [24], [25]) which has a constant main-lobe power radiation gain $G^{\max}$ and a constant side-lobe gain $G^{\min}$. More specifically, the antenna gain $G(\theta)$ in the direction of $\theta \in [-\pi, \pi)$ is

$$G(\theta) = \begin{cases} G^{\max}, & |\theta| \leq \mathbf{\Theta}/2 \\ G^{\min}, & |\theta| > \mathbf{\Theta}/2 \end{cases} \tag{1}$$

where $\mathbf{\Theta}$ is the beamwidth. The antenna has a total power radiation gain of $E = \mathbf{\Theta} G^{\max} + (2\pi - \mathbf{\Theta}) G^{\min}$. We use $G_{j,i}^{\mathsf{BS}}$ and $G_{j,i}^{\mathsf{UE}}$ to respectively denote the antenna gain of BS i and UE j along the direction connecting them. The *main to side-lobe gain ratio* (MSR) is defined as $\text{MSR} \triangleq 10 \lg \left(G^{\max} / G^{\min} \right) \text{dB}$. A large MSR means that the antenna has strong radiation in the main-lobe while a small MSR implies energy leakage in the side-lobe. For any i, j, we let UE $j_i (j_i \in \mathcal{K}_i)$ denote the UE scheduled by BS i, and let BS i_j denote the BS that UE j subscribes to $(j \in \mathcal{K}_{i_j})$. The SINR at UE j can be written as

$$\mathsf{SINR}_{j,i_j} = \frac{p_{j,i_j} G_{j,i_j}^{\mathsf{UE}} G_{j,i_j}^{\mathsf{BS}} |h_{j,i_j}|^2 d_{j,i_j}^{-\eta}}{\sum_{\ell \in \mathcal{M}, \ell \neq i} p_{j_\ell,\ell} G_{j,\ell}^{\mathsf{UE}} G_{j,\ell}^{\mathsf{BS}} |h_{j,\ell}|^2 d_{j,\ell}^{-\eta} + \sigma^2}, \tag{2}$$

where $p_{j,i}$ denotes the transmit power of BS i to UE j if UE j is served by BS i; η is the path-loss exponent; $\sigma^2 = N_0 W$ is the Gaussian noise power with N_0 being the noise power spectrum density; $h_{j,i}$ is the small-scale fading between BS i and UE j which is assumed to follow the Nakagami-m distribution [26] with probability density

$$f(h|\mu, \Omega) = \frac{2\mu^\mu}{\Gamma(\mu)\Omega^\mu} h^{2\mu-1} \exp\left\{ -\frac{\mu}{\Omega} h^2 \right\}, \ h \geq 0, \tag{3}$$

where $\mu \triangleq \mathbb{E}[h^2]^2 / \text{Var}(h^2)$, $\Omega \triangleq \mathbb{E}[h^2]$ and Γ is the Gamma function. We assume a block fading channel for which the fading coefficients stay unchanged during each frame and are i.i.d. over different frames[1]. We further define the *equivalent channel gain* g_{j,i_j} between UE j and BS i_j as $g_{j,i_j} \triangleq \mathsf{SINR}_{j,i_j} / p_{j,i_j}$ if UE_j is scheduled and $p_{j,i_j} \neq 0$. Moreover, each BS has a instantaneous peak power constraint $p_i \leq P_i^{\max}$ and a long-term average power consumption constraint $\overline{p}_i \leq P_i^{\mathsf{avg}}, \forall i$ where $\overline{p}_i$ denotes the average transmit power of BS i.

B. Network Utility Maximization

The time-averaged expected throughput of UE j from the corresponding serving BS i_j is given by

$$\overline{X}_{j,i_j} = \lim_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} \mathbb{E}[X_{j,i_j}(k)], \tag{4}$$

where the expectation is taken over the system randomness (e.g., fading channel, scheduling *etc*). $X_{j,i_j}(k)$ is the achieved throughput of UE j in frame k and is calculated as

$$X_{j,i_j}(k) = \sum_{n=1}^{N} T_{j,i_j}^{\mathsf{d}}(k,n) W \log\left(1 + \mathsf{SINR}_{j,i_j}(k,n)\right), \tag{5}$$

where $T_{j,i_j}^{\mathsf{d}}(k,n)$ denotes the data reception time (unit: slot) of UE j in block n of frame k. The utility of BS i is defined as $U_i \triangleq \sum_{j \in \mathcal{K}_i} \log\left(\overline{X}_{j,i}\right)$, i.e., the sum of the logarithm of the average throughput of its associated UEs. The purpose of using the logarithm function is to ensure fairness among UEs.

[1] We do not consider UE mobility in this paper. However, the proposed approach applies to the case when UEs may move slowly such that the channel gains do not change violently from slot to slot.

The *network utility* is then defined as the sum utility of the BSs, i.e., $U_{\mathsf{sum}} \triangleq \sum_{i\in\mathcal{M}} U_i$. We aim to solve the following network utility maximization problem

$$\begin{aligned} \max \quad & U_{\mathsf{sum}} && \text{(6a)}\\ \text{s.t.} \quad & \sum_{j\in\mathcal{K}_i} \overline{p}_{j,i} \leq T_f P_i^{\mathsf{avg}}, \quad \forall i, && \text{(6b)}\\ & p_{j_i,i}(k,n) \leq P_i^{\mathsf{max}}, \quad \forall i,k,n, && \text{(6c)} \end{aligned}$$

where $\overline{p}_{j,i} = \lim_{T\to\infty} \frac{1}{T}\sum_{k=1}^{T}\sum_{n=1}^{N}\mathbb{E}[T^{\mathsf{d}}_{j,i}(k,n)p_{j,i}(k,n)]$ denotes the average power consumption of BS i to UE j. In Section III, we decompose the optimization problem (6) into two sub-problems and propose a non-cooperative game-theoretic beam scheduling and power allocation scheme which solves (6) in a distributed fashion.

III. Proposed Approach

A. Lyapunov Decomposition

The fact that U_{sum} is a non-linear function of the time average (4) makes (6) difficult to solve. To this end, we can transform problem (6) into an equivalent form with a new objective which is a time average of some non-linear function and is easier to solve. In particular, by introducing a set of auxiliary variables $\{\gamma_{j,i}(k)\}_{i\in\mathcal{M},j\in\mathcal{K}_i}$ for each frame k, (6) can be rewritten as [27]:

$$\begin{aligned} \max \quad & \lim_{T\to\infty}\frac{1}{T}\sum_{k\in[T]}\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{K}_i}\mathbb{E}\left[\log(\gamma_{j,i}(k))\right] && \text{(7a)}\\ \text{s.t.} \quad & \sum_{j\in\mathcal{K}_i}\overline{p}_{j,i} \leq T_f P_i^{\mathsf{avg}}, \quad \forall i && \text{(7b)}\\ & \overline{\gamma}_{j,i} \leq \overline{X}_{j,i}, \quad \forall i\in\mathcal{M}, \forall j\in\mathcal{K}_i && \text{(7c)}\\ & \sum_{j\in\mathcal{K}_i} p_{j,i}(k,n) \leq P_i^{\mathsf{max}}, \quad \forall i,k,n && \text{(7d)}\\ & 0 \leq \gamma_{j,i}(k) \leq T_f W \log\left(1+g_{j,i}^{\mathsf{max}}P_i^{\mathsf{max}}\right), \forall i,j,k && \text{(7e)} \end{aligned}$$

where $[T] \triangleq \{1,\cdots,T\}$, $g_{j,i}^{\mathsf{max}} \triangleq \max_{k,n} g_{j,i}(k,n)$ and $\overline{\gamma}_{j,i} \triangleq \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}\gamma_{j,i}(k)$ denotes the average value of $\gamma_{j,i}(k)$. Utilizing the Lyapunov drift-plus-penalty framework, (7) can be further decomposed into two sub-problems that need to be solved in each frame, together with two virtual queues to enforce the constraints (7b) and (7c). In particular, the transmit power queue $\{Z_i(k)\}_{k=1}^{\infty}$ is used to enforce (7b) and is updated in each frame by $\forall i\in\mathcal{M}$:

$$Z_i(k+1) = \max\left\{Z_i(k) + \sum_{j\in\mathcal{K}_i}\sum_{n\in[N]} T^{\mathsf{d}}_{j,i}(k,n)p_{j,i}(k,n) - T_f P_i^{\mathsf{avg}}, 0\right\}. \quad (8)$$

The throughput queue $\{H_{j,i}(k)\}_{k=1}^{\infty}$ is used to enforce (7c) and is updated by $\forall i\in\mathcal{M}, \forall j\in\mathcal{K}_i$:

$$H_{j,i}(k+1) = \max\left\{H_{j,i}(k) + \gamma_{j,i}(k) - X_{j,i}(k), 0\right\}. \quad (9)$$

The first sub-problem aims to solve the auxiliary variables $\gamma_{j,i}(k)$ in each frame k as

$$\begin{aligned} \max_{\gamma_{j,i}(k)} \quad & \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{K}_i}\left(V\log(\gamma_{j,i}(k)) - H_{j,i}(k)\gamma_{j,i}(k)\right) && \text{(10a)}\\ \text{s.t.} \quad & 0 \leq \gamma_{j,i}(k) \leq T_f W\log\left(1+g_{j,i}^{\mathsf{max}}(k)P_i^{\mathsf{max}}\right),\\ & \qquad \forall i\in\mathcal{M}, \forall j\in\mathcal{K}_i && \text{(10b)} \end{aligned}$$

where $g_{j,i}^{\max}(k) \triangleq \max_n g_{j,i}(k,n)$ and V is a constant that establishes a trade-off between the convergence (to the optimal solution of (7) by solving the sub-problems) speed and the optimality gap. This sub-problem is convex and thus easy to solve. More specifically, (10) can be optimally solved in a distributed manner by letting each BS i perform an independent convex optimization over the variables $\{\gamma_{j,i}(k)\}_{j\in\mathcal{K}_i}$ as follows:

$$\begin{aligned} \max_{\gamma_{j,i}(k)} \quad & \sum_{j\in\mathcal{K}_i}\left(V\log(\gamma_{j,i}(k)) - H_{j,i}(k)\gamma_{j,i}(k)\right) && \text{(11a)}\\ \text{s.t.} \quad & 0 \leq \gamma_{j,i}(k) \leq T_f W\log\left(1+g_{j,i}^{\mathsf{max}}(k)P_i^{\mathsf{max}}\right), \forall j\in\mathcal{K}_i. && \text{(11b)} \end{aligned}$$

Note that the solution is affected by the throughput queues $\{H_{j,i}(k), \forall k\}_{j\in\mathcal{K}_i}$ corresponding to the associated UEs of BS i which can be tracked by BS i. The second sub-problem aims to solve the transmit powers $p_{j,i}(k,n)$ in each block n:

$$\begin{aligned} \min \quad & \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{K}_i}\left(\sum_{n\in[N]}\mathbb{E}\left[T^{\mathsf{d}}_{j,i}(k,n)p_{j,i}(k,n)\right] - T_f P_i^{\mathsf{avg}}\right)\\ & \qquad \times Z_i(k) - H_{j,i}(k)\widehat{X}_{j,i}(k) && \text{(12a)}\\ \text{s.t.} \quad & \sum_{j\in\mathcal{K}_i} p_{j,i}(k,n) \leq P_i^{\mathsf{max}}, \quad \forall i,k,n && \text{(12b)} \end{aligned}$$

where $\widehat{X}_{j,i}(k) \triangleq \sum_{n=1}^{N}\mathbb{E}\left[T^{\mathsf{d}}_{j,i}(k,n)W\log(1+\mathsf{SINR}_{j,i}(k,n))\right]$ denotes the expected throughput achieved by UE j in frame k. The inclusion of the SINR in objective function renders this sub-problem challenging to solve in general. Therefore, we propose a game-theoretic distributed power allocation scheme and use the NE as an approximate solution to (12). The connection between the optimization (7) and (10), (12) is that if the two sub-problems can be solved with accuracy β in all frames, then (7) can be solved with accuracy $\mathcal{O}(\beta/V)$. This implies that we can make the optimality gap arbitrarily small by choosing arbitrarily large V. Let $U_{\mathsf{sum}}^{\mathsf{opt}}$ denote the optimal value of (6). Let $\overline{X}_{j,i}^{\mathsf{game}}$ denote the average throughput achieved by solving the sub-problems (10) and (12) iteratively in each frame using the proposed game-based approach, also let $U_{\mathsf{sub}}^{\mathsf{opt}}(k)$ and $U^{\mathsf{game}}(k)$ denote respectively the optimal value of (12) and the value achieved by the proposed approach in frame k. The above connection is then formally stated in the following lemma.

Lemma 1: (**Optimality Gap**) *Suppose that the sub-problem (12) can be solved with accuracy β (given that (10) is optimally solved) in each frame using the proposed game-based scheduling approach, i.e., $U_{\mathsf{sub}}^{\mathsf{opt}}(k) - U^{\mathsf{game}}(k) \leq \beta, \forall k$. Then*

$$\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{K}_i}\log\left(\overline{X}_{j,i}^{\mathsf{game}}\right) \geq U_{\mathsf{sum}}^{\mathsf{opt}} - \frac{\beta + C}{V}, \quad (13)$$

where C is a constant.

B. Proposed Beam Scheduling & Power Allocation

*1) **Non-cooperative Game-based Formulation**:* A non-cooperative game can represented by a triple $\mathcal{G} = \langle \mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{\phi_i\}_{i\in\mathcal{N}}\rangle$ where $\mathcal{N}$ is the set of players, $\mathcal{A}_i$ is the action space of play i, and ϕ_i is the payoff function of player i. Let $\boldsymbol{a} \triangleq \{a_i\}_{i\in\mathcal{N}}$ denote the action profile of the players where $a_i \in \mathcal{A}_i$ is the action chosen by player i. Let $\boldsymbol{a}_{-i} \triangleq \{a_{i'}\}_{i'\in\mathcal{N}\setminus\{i\}}$ denote the action profile excluding player i. For player i, the *best response* (BR) a_i^* is defined as an action such that ϕ_i is maximized given the action profile $\boldsymbol{a}_{-i}$ of other players, i.e., $\phi_i(a_i^*, \boldsymbol{a}_{-i}) \geq \phi_i(a_i, \boldsymbol{a}_{-i}), \forall a_i \in \mathcal{A}_i$. The Nash Equilibrium (NE) is an action profile $\boldsymbol{a}^* = \{a_i^*\}_{i\in\mathcal{N}}$ for which the players' actions are each other's BR, i.e., $\phi_i(a_i^*, \boldsymbol{a}_{-i}^*) \geq \phi_i(a_i, \boldsymbol{a}_{-i}^*), \forall a_i \in \mathcal{A}_i, \forall i \in \mathcal{N}$.

We model the distributed beam scheduling and power allocation problem as a non-cooperative game $\mathcal{G} = \langle \boldsymbol{\mathcal{M}}, \{\mathcal{P}_i\}_{i\in\boldsymbol{\mathcal{M}}}, \{\phi_i\}_{i\in\boldsymbol{\mathcal{M}}}\rangle$ where the M BSs are defined as the players which do not cooperate with each, i.e., there is no information exchange at all. The action of BS i is the transmit powers $\boldsymbol{p}_i \triangleq (p_{j,i})_{j\in\boldsymbol{\mathcal{K}}_i} \in \mathbb{R}^{|\boldsymbol{\mathcal{K}}_i|}$ to its associated UEs. Since only one UE can be scheduled in each block, we have $p_{j,i} = 0, \forall j \in \boldsymbol{\mathcal{K}}_i\setminus\{j_i\}$ where UE j_i is scheduled by BS i. WLOG, we assume that each BS is associated with the same number of UEs, i.e., $|\boldsymbol{\mathcal{K}}_i| = K/M, \forall i \in \boldsymbol{\mathcal{M}}$. The action space is correspondingly defined as $\mathcal{P}_i \triangleq \prod_{j=1}^{\frac{K}{M}}[0, P_i^{\mathsf{max}}]$. Let $\boldsymbol{p} = (\boldsymbol{p}_i)_{i\in\boldsymbol{\mathcal{M}}} \in \mathbb{R}^{\frac{K}{M}\times M}$ and $\mathcal{P} = \prod_{i=1}^{M}\mathcal{P}_i$ denote the joint action profile and the joint action space of the BSs respectively. The payoff of BS i is defined as

$$\phi_i(\boldsymbol{p}_i, \boldsymbol{p}_{-i}) \triangleq \alpha_i W \log\left(1 + \mathsf{SINR}_{j_i,i}\right) - \lambda_i p_{j_i,i},. \quad (14)$$

where $\alpha_i, \lambda_i \geq 0$ are the *pricing weights* that can be tuned (manually or determined by some other mechanisms) to find a desired trade-off between throughput maximization and power consumption. For the power allocation game $\mathcal{G}$, the BR of each player can be calculated by setting the first-order derivative of ϕ_i w.r.t. $p_{j_i,i}$ to be zero as stated in Lemma 2.

Lemma 2: (**Best Response**) *For the power allocation game $\mathcal{G} = \langle \boldsymbol{\mathcal{M}}, \{\mathcal{P}_i\}_{i\in\boldsymbol{\mathcal{M}}}, \{\phi_i\}_{i\in\boldsymbol{\mathcal{M}}}\rangle$, the BR of BS i is given by*

$$p_{j_i,i}^* = \min\left\{P_i^{\mathsf{max}}, \max\left\{\frac{\alpha_i W}{\lambda_i} - \frac{1}{g_{j_i,i}},\ 0\right\}\right\},\ \forall i, \quad (15)$$

where $g_{j_i,i} \triangleq \frac{G_{j_i,i}^{\mathsf{UE}} G_{j_i,i}^{\mathsf{BS}} |h_{j_i,i}|^2 d_{j_i,i}^{-\eta}}{\sum_{\ell\in\boldsymbol{\mathcal{M}},\ell\neq i} p_{j_\ell,\ell} G_{j_i,\ell}^{\mathsf{UE}} G_{j_i,\ell}^{\mathsf{BS}} |h_{j_i,\ell}|^2 d_{j_i,\ell}^{-\eta} + \sigma^2}$ is the equivalent channel gain between BS i and UE j_i.

*2) **Proposed scheme**:* Under the proposed game-based power allocation, the scheduled UE will be served throughout each block. Therefore, the data reception time of the scheduled UEs is equal to the block duration and is equal to zero for all other UEs. That is, for each i,

$$T_{j,i}^{\mathsf{d}}(k,n) = \begin{cases} T_b, & \text{if } j = j_i \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

As a result, after omitting the constant term $T_f P_i^{\mathsf{avg}}$, the objective of (12) can be rewritten as to maximize

$$\sum_{i\in\boldsymbol{\mathcal{M}}} \sum_{n\in[N]} H_{j_i,i}(k) T_b W \log\left(1 + \mathsf{SINR}_{j_i,i}(k,n)\right) - Z_i(k) T_b p_{j_i,i}(k,n). \quad (17a)$$

We propose to solve (17) in a distributed way by letting each BS i perform the following optimization problem in each block to solve $p_{j_i,i}(k,n)$:

$$\max \quad \alpha_i W \log\left(1 + \mathsf{SINR}_{j_i,i}(k,n)\right) - \lambda_i p_{j_i,i}(k,n) \quad (18a)$$

$$\text{s.t.} \quad p_{j_i,i}(k,n) \leq P_i^{\mathsf{avg}} \quad (18b)$$

where the pricing weights are determined by the virtual queues as

$$\alpha_i = H_{j_i,i}(k) T_b, \quad \lambda_i = Z_i(k) T_b, \quad \forall i. \quad (19)$$

Since (18) takes the same form as the payoff function (14) defined for $\mathcal{G}$, we can accordingly define a power allocation game $\mathcal{G}(k,n)$ for each block n with payoff functions defined in (18a). This game can be played from block to block and the NE can be used as an approximate solution to (12).

We next present a dynamic *parallel power update algorithm* based on the BR derived in Lemma 2 where the BSs updates their transmit powers simultaneously in each slot. In particular, let $p_{j_i,i}^{(t)}$ be the transmit power of BS i in slot t. Also let $I_{j_i}^{(t)}$ denote the measured interference (plus noise) at UE j_i in slot t which can be obtained by BS i via feedback. The transmit powers in the next slot can be computed according to the following update rule:

$$p_{j_i,i}^{(t+1)} = \min\left\{P_i^{\mathsf{max}}, \max\left\{\frac{H_{j_i,i}(k) W}{Z_i(k)} - \frac{I_{j_i}^{(t)}}{|\bar{h}_{j_i,i}|^2},\ 0\right\}\right\}, \quad (20)$$

where $|\bar{h}_{j_i,i}|^2 = G_{j_i,i}^{\mathsf{UE}} G_{j_i,i}^{\mathsf{BS}} |h_{j_i,i}|^2 d_{j_i,i}^{-\eta}$ denotes the direct channel gain from BS i to UE j_i which can be estimated by BS i via pilot training. The BSs perform the power update at the beginning of each slot. If the transmit power is not zero, the BSs then generate beams towards their scheduled UEs and start data transmission until the next power update. It should be noted that (20) is fully distributed as the power update of each BS does not require the knowledge of the cross channels or the virtual queues associated with other BSs. This update algorithm can be proved to converge as shown by Proposition 1.

Proposition 1: (**Convergence**, [7]) *The sequence $\left\{p_{j_i,i}^{(t)}, \forall i\right\}_{t=0}^{\infty}$ generated by the power update rule (20) always converges. Furthermore, if the matrix $\mathbf{Q}$ defined in (23) is a P-matrix, then the sequence $\left\{p_{j_i,i}^{(t)}, \forall i\right\}_{t=0}^{\infty}$ converges to the unique NE of $\mathcal{G}(k,n)$.*

C. Existence & Uniqueness of NE

Several properties of the formulated power allocation game in the previous section will be presented. In particular, we prove the existence of the NE and derive sufficient conditions which guarantee the uniqueness of NE by establishing an equivalence to a corresponding variational inequality (VI) problem [28] – if the VI problem has a unique solution, then the formulated game has a unique equilibrium. Due to space limit, the proof of the lemmas are omitted and can be found in [9]. The following lemma shows that the NE always exists.

Lemma 3: (**Existence of NE**) *The power allocation game $\mathcal{G} = \langle \boldsymbol{\mathcal{M}}, \{\mathcal{P}_i\}_{i\in\boldsymbol{\mathcal{M}}}, \{\phi_i\}_{i\in\boldsymbol{\mathcal{M}}}\rangle$ formulated in Section III-B always admits at least one pure strategy NE for any $\alpha_i, \lambda_i \geq 0, \forall i \in \boldsymbol{\mathcal{M}}$ and any set of channel realizations.*

To prove the uniqueness of NE, we first introduce two necessary concepts. A matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ is called a *P-matrix* if every principal minor of $\mathbf{A}$ is positive. In addition, a mapping $\overrightarrow{f} \triangleq \left[\overrightarrow{f_1}, \overrightarrow{f_2}, \cdots, \overrightarrow{f_n}\right] : \mathbb{R}^{m\times n} \mapsto \mathbb{R}^{m\times r}$ is called a *uniformly P-function* on a convex subset $\mathcal{C}$ of $\mathbb{R}^{m\times n}$ if there exists a constant $\epsilon > 0$ such that for any $\boldsymbol{x} = (\boldsymbol{x}_i)_{i=1}^n, \boldsymbol{y} = (\boldsymbol{y}_i)_{i=1}^n \in \mathbb{R}^{m\times n}$, it holds that

$$\max_{1\leq i\leq n} (\boldsymbol{x}_i - \boldsymbol{y}_i)^\mathsf{T} \left(\overrightarrow{f_i}(\boldsymbol{x}) - \overrightarrow{f_i}(\boldsymbol{x})\right) \geq \epsilon \|\boldsymbol{x} - \boldsymbol{y}\|^2. \tag{21}$$

where $\|\cdot\|$ is the Frobenius norm. In addition, for a closed and convex subset $\mathcal{S} \subseteq \mathbb{R}^n$, and a mapping $\overrightarrow{f} : \mathcal{S} \mapsto \mathbb{R}^n$, the VI problem, denoted by $\mathrm{VI}(\mathcal{S}, \overrightarrow{f})$ seeks to find a solution $\boldsymbol{x}^* \in \mathcal{S}$ such that $(\boldsymbol{x} - \boldsymbol{x}^*)^\mathsf{T} \overrightarrow{f}(\boldsymbol{x}^*) \geq 0, \forall \boldsymbol{x} \in \mathcal{S}$. It is known that if $\overrightarrow{f}$ is a uniformly P-function, then $\mathrm{VI}(\mathcal{S}, \overrightarrow{f})$ will have a unique solution $\boldsymbol{x}^*$ in $\mathcal{S}$.

We further define a vector function $\overrightarrow{F}(\boldsymbol{p}) \triangleq \left[\overrightarrow{F_1}(\boldsymbol{p}), \cdots, \overrightarrow{F_n}(\boldsymbol{p})\right] \in \mathbb{R}^{\frac{K}{M}\times M}$ where $\overrightarrow{F_i}$ is defined as the gradient of the payoff function ϕ_i w.r.t. $\boldsymbol{p}_i$, that is,

$$\overrightarrow{F_i}(\boldsymbol{p}) \triangleq -\nabla_{\boldsymbol{p}_i}\phi_i(\boldsymbol{p}_i, \boldsymbol{p}_{-i}) = \left[0, \cdots, -\frac{\partial \phi_i(\boldsymbol{p}_i, \boldsymbol{p}_{-i})}{\partial p_{j_i,i}}, \cdots, 0\right]^\mathsf{T}, \tag{22}$$

i.e., the only non-zero element appears at the scheduled UE j_i by BS i. It can be shown that the power allocation game $\mathcal{G}$ is equivalent to $\mathrm{VI}(\mathcal{P}, \overrightarrow{F})$ in the sense that if $\mathrm{VI}(\mathcal{P}, \overrightarrow{F})$ has a unique solution, then $\mathcal{G}$ admits a unique NE and vice versa. We also define a matrix $\mathbf{Q} \triangleq [Q_{p,q}]_{M\times M}$ as

$$Q_{p,q} = \begin{cases} \alpha_p W, & \text{if } p = q \\ -\alpha_p W \left|\frac{\overline{h}_{j_p,q}}{\overline{h}_{j_q,q}}\right|^2 \left(1 + \frac{\sum_{i\in\boldsymbol{\mathcal{M}}} |\overline{h}_{j_q,i}|^2 P_i^{\mathsf{max}}}{\sigma^2}\right), & \text{if } p \neq q \end{cases} \tag{23}$$

where $\overline{h}_{j,i} \triangleq \sqrt{G_{j,i}^{\mathsf{UE}} G_{j,i}^{\mathsf{BS}} |h_{j,i}|^2 d_{j,i}^{-\eta}}$. Recall that α_i is the pricing weights. We are now ready to present a sufficient condition which can guarantee the uniqueness of NE as stated in Lemma 4.

Lemma 4: (**Uniqueness of NE**) *If the matrix $\mathbf{Q}$ defined in (23) is a P-matrix, then $\overrightarrow{F}$ defined by (22) is a uniformly P-function on $\mathcal{P}$. Consequently, $\mathrm{VI}(\mathcal{P}, \overrightarrow{F})$ has a unique solution which implies that $\mathcal{G}$ admits a unique NE.*

IV. Simulation

A. Simulation Setup

Consider 10 BSs located on a 800×800 meter squared grid as shown in Fig. 1. Each BS is associated with 5 UEs and has a (possibly overlapping) disk coverage area with radius 150 meters. The total shared bandwidth is W = 400 MHz; path-loss exponent is $\eta = 4$; The power constraints are $P_i^{\mathsf{avg}} = 38.13$ dBm (6.5 Watt), $P_i^{\mathsf{max}} = 39$ dBm (7.9 Watt). The total noise power over the 400 MHz bandwidth is $\sigma^2 = -86.46$ dBm. Each frame has $N = 5$ blocks, each block has $T_b = 80$ slots, implying $T_f = NT_b = 400$ slots. Small-scale fading parameters are chosen as $\mu = 1, \Omega = 10^{-3}$. We consider two baseline

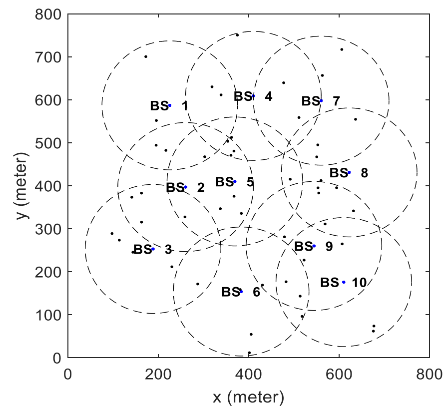


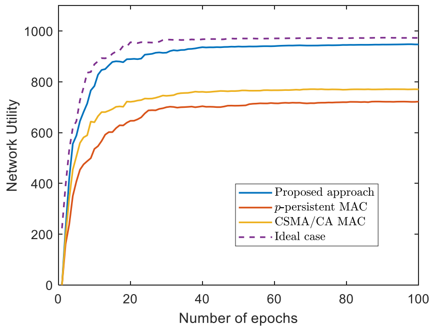
Fig. 1: Simulation network with 10 BSs and 50 UEs.

schemes which are p-persistent and CSMA/CA media access strategies widely used in real-world wireless networks. For p-persistent, the optimal contention probability is chosen as 0.1. For CSMA/CA, the minimum and maximum contention windows are chosen as 20 and 200 slots respectively and each data transmission duration contains two slots. For the baselines, based on an estimate of the data reception time of the UEs in each frame, the corresponding one-time transmit powers can be obtained by solving (12) similar to the proposed approach. In addition, we consider an 'ideal case' where we assume that there is no interference among BSs and thus the maximum transmit power will be chosen by every BS. This ideal case is not achievable but serves as an performance upper bound on any scheduling approach including centralized ones.

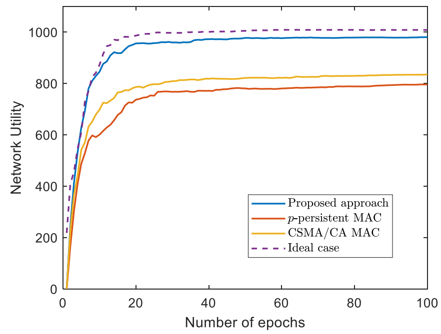
B. Simulation Result

We verify the effect of beamwidth and antenna gain on the proposed approach and compare it with two baselines. Simulation results show that the proposed approach has superior performance than the baselines and can achieve near-optimal performance in the high SINR regime as it approaches the ideal case when the BSs have sharp beams.

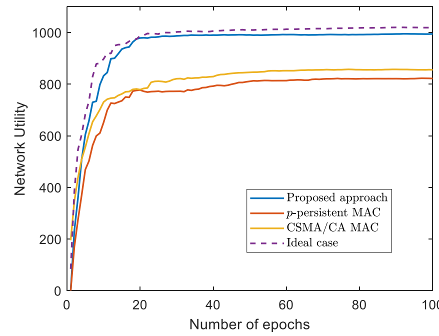
1) Effect of beamwidth: We assume all BSs have identical antenna configurations as well as the UEs. Because varying the UE beamwidth and antenna gain has a similar effect to varying that of the BSs, we fix the UE antenna configuration to be $(20 \text{ dB}, \pi/18)$ throughout the simulation. We fix the antenna gain as MSR = 20 dB and then change the beamwidth as $\boldsymbol{\Theta}^{\mathsf{BS}} \in \{\frac{\pi}{9}, \frac{\pi}{36}, \frac{\pi}{72}\}$ in order to observe the effect of beamwidth. The achieved network utility is shown in Fig. 2. It can be seen that the proposed approach outperforms the baselines in all three cases with both faster convergence and higher achieved utility. The achieved utility also increases as the beamwidth decreases. This is because narrow beams increases the power



(a) $\mathbf{\Theta}^{\text{BS}} = \pi/9$

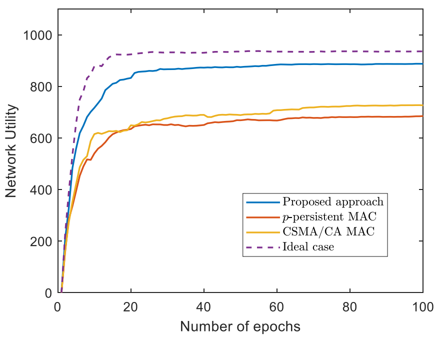


(b) $\mathbf{\Theta}^{\text{BS}} = \pi/36$

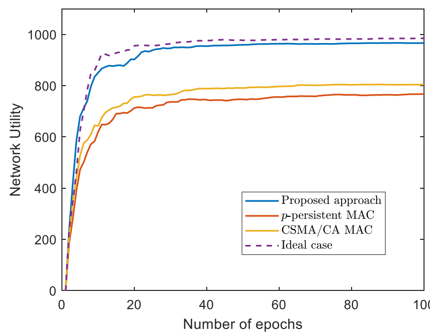


(c) $\mathbf{\Theta}^{\text{BS}} = \pi/72$

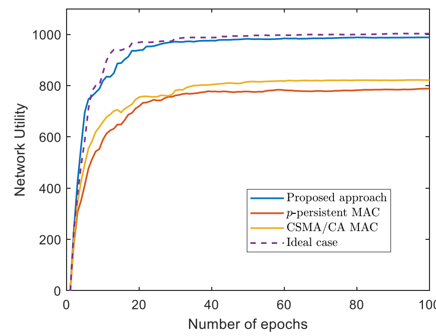
Fig. 2: Effect of beamwidth. BS antenna MSR is set to be 20 dB.



(a) $\text{MSR}^{\text{BS}} = 10$ dB



(b) $\text{MSR}^{\text{BS}} = 20$ dB



(c) $\text{MSR}^{\text{BS}} = 30$ dB

Fig. 3: Effect of antenna MSR. BS beamwidth is set to be $\pi/18$.

radiation towards the target UE and avoids covering non-target UEs and thus causing less interference to them. Moreover, the proposed approach achieves more than 90% of the utility of the ideal case.

2) Effect of antenna MSR: We fix the BS beamwidth to be $\pi/18$ and change the antenna gain as $\text{MSR}^{\text{BS}} \in \{10, 20, 30\}$ dB in order to verify the effect of antenna MSR. The results is shown in Fig. 3. It can be seen that the proposed approach has faster convergence speed and achieves higher utility than the baselines. The achieved utility also increases with the antenna MSR as a higher MSR increases the desired signal component power as well as reducing the power leakage in the side-lobe. Again, the proposed approach can achieve more than 90% utility of the upper bound.

3) Optimality: A comparison of the proposed approach with the ideal case under various BS antenna beamwidth-MSR configurations $(\pi/3, 6\text{ dB}), (\pi/20, 30\text{ dB})$ and $(\pi/30, 40\text{ dB})$ is shown in Fig. 4. It can be seen that when the beams become sharper, the performance gap decreases, which is because the interference from unintended BSs is effectively suppressed. In the extreme case of $(\pi/30, 40\text{ dB})$, the proposed approach achieves almost identical performance to the ideal case. This demonstrates the near-optimality of the proposed approach in the high SINR regime.

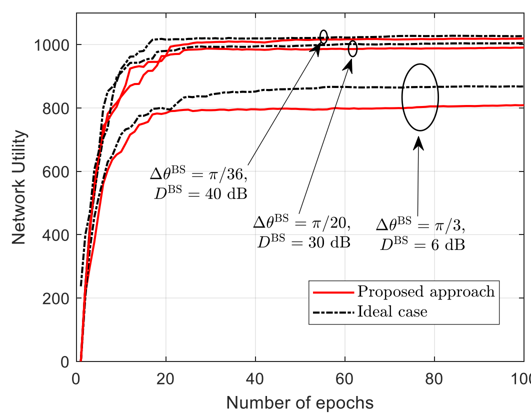


Fig. 4: Comparison with the ideal case.

V. Conclusion

In this work, we studied the distributed beam scheduling problem in mmWave cellular networks for the purpose of network utility maximization. We proposed a non-cooperative game-based beam scheduling and power allocation scheme by formulating the power allocation task as a game. The Nash Equilibrium of the game then serves as a distributed and approximate solution to the second sub-problem extracted from the original network utility maximization problem. The existence of the equilibrium was proved and sufficient conditions guaranteeing the uniqueness of the equilibrium were derived utilizing the connection to the variational inequality problem. A parallel power adaptation algorithm was proposed and shown to converge to the equilibrium. Simulation results demonstrated the efficiency of the proposed approach.

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