



Understanding Melting and Freezing/Solidification and the Associated Effects on Heat Transfer

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1. INTRODUCTION

Phase change (solid/liquid or liquid/vapor) is a process that enables a large heat transfer capacity for a broad range of engineering applications, such as steam generators, condensers, thermal energy storage, thermosyphons, and heat pipes. Water has been the most usable phase-change cooling medium. Advanced applications are proposing alkali metals (e.g., sodium, potassium, lithium) to increase the total heat transfer rate for moderate and high-temperature applications. Sodium and potassium have the advantage of relatively low melting temperatures (98 and 64°C, respectively) make them suitable for high-temperature applications (700–1100 K). At too high of temperatures (above 1200 K), lithium is an excellent choice as a working fluid because of its high latent heat vaporization and high-surface tension. The low vapor pressure of alkali metals makes them candidates for closed spaces applications, such as thermosyphons and heat pipes. Lithium has the lowest vapor pressure than potassium and sodium, which can work at a much higher temperature without over-pressurizing the container.

Modeling and simulation of the phase-change process is mathematically a complicated problem. The presence of multiple phases behaving differently and tracing the heat and mass transfer through the interface between different phases is challenging. It is also essential to consider the solid surface's characteristics intact to the fluid during the phase change. Among engineering applications of interest associated with the phase-change process on distinctive surfaces are heat pipes and thermosyphons. In these applications, fluids change phases between solid, liquid, and vapor on porous media surfaces. The process is complicated by the presence of the solid-liquid-vapor phase simultaneously in a porous medium. The porous media geometrical and thermophysical properties, including shape (spherical or cylindrical), porosity, and thermal conductivity of the solid matrix, are playing crucial roles in the heat and mass transfer of these applications.

This report summarizes mathematical models available in the literature that describes the fluid flow and heat transfer process in porous media. The models discussed are for the single-phase and phase-change (solid/liquid or liquid/vapor) heat transfer process. These models are tools to validate any ongoing experimental or numerical modeling of heat and mass transfer in porous media or packed bed applications, including thermosyphons and heat pipes.

2. LITERATURE REVIEW OF POROUS MEDIA MODELING

Efforts have been reported in the literature on the development of heat and mass transfer models of different liquids in porous media. Most of these models include a broad range of assumptions to simplify the governing equation for one-dimensional (1-D), laminar, and turbulent steady-state analysis. These models were introduced within the focus of heat and mass transfer in heat pipe and thermosyphon applications. The next section briefly summarizes these literature efforts.

Tilton et al. experimentally and numerically studied the transient mass and heat transfer of sodium in porous media for Inconel heat pipe [1,2]. The heat capacity of the porous media was assumed to be uniform. The global energy balance was used to calculate the total energy stored per-unit length of the porous media, assuming the porous media conduit's isothermal condition. The vapor pressure and temperature profiles were calculated using steady-state 1-D momentum balance, assuming laminar incompressible flow friction and dynamic coefficients. The liquid-vapor interface temperature was assumed uniform and equal to the axially distributed temperature along with the porous media container. The radiant heat flux at the interface was equated to the latent rate heat of vaporization multiplied by the phase-change rate occurring at the interface. This model is limited to high-temperature, low-heat input, and slow-transient application under which axial temperature variation can be neglected.

Faghri and Chen introduced a two-dimensional (2-D) steady-state model assuming thermodynamic equilibrium at the liquid-vapor interface [3]. The phase-change rate at the porous media matrix interface

was calculated using the interface's energy balance. The model predicted the axial conduction, vapor compressibility, and viscous dissipation on the water and sodium heat pipes' operation. The model was then extended to perform a transient analysis of a sodium heat pipe. The model neglected the liquid axial flow through the porous media [4–6].

Jang et al. developed a transient heat conduction model for porous media walled rectangular cross-sectional conduits, such as heat pipes and thermosyphons [7,8]. The porous media was assumed fully saturated with liquid, and axial capillary flow-through porous media was neglected. The phase change at the interface was evaluated using the kinetic theory. The model was improved by solving a 1-D transient compressible flow equation suitable for cylindrical shape conduit and counting for the startup from the frozen conditions [9].

Peery and Best used a 2-D transient conduction model to heat transfer through heat pipe walls [10]. The liquid and vapor flow through the porous media and intermediate space are analyzed as 1-D compressible flow regimes. The phase change at the interface was evaluated using the modified kinetic theory relationships. Once the phase change and temperature of the interface were obtained, the axial vapor and liquid velocity were calculated by solving the kinetic and energy theory equations. The axial pressure distribution was obtained from the 1-D momentum equation.

Hall et al. developed a transient heat transfer model for liquid metal heat pipes [11–13]. The model count for melting from frozen and evaporation of lithium. The axial flow of liquid in a porous medium was considered in the model. The model successfully predicted the void fraction of the stable, liquid, and vapor phases. The model used the kinetic theory to predict the thermal-hydraulic response of the solidification, condensation, and evaporation process along the porous media using a 1-D axial model. The model counts for four phases: frozen state (all solid), startup or melting, regular operation, and shutdown or refreezing.

Seo and El-Genk developed a transient model to predict a fully thawed liquid metal heat pipe's heat transfer performance [14–16]. The model assumed 2-D, transient, laminar incompressible flow in the porous medium. The vapor region was treated separately using a quasi-steady-state model to discretize laminar 2-D Navier Stokes equations. The specific heat capacities were assumed constant, solving the transient conduction equation to calculate the conduit temperature distribution. The phase-change rates (evaporation and condensation) at the liquid-vapor interface were obtained from the energy balance. The model accurately predicted the temperature distribution of the working fluid before the fluid reached its freezing conditions.

Turnier performed an extensive review for existing models to simulate the mass and heat transfer in porous media for heat pipe applications to develop a more efficient model that counts for the startup from a frozen state of liquid metals [17]. A detailed summary of his developing an efficient full transient model to predict the heat and mass transfer in porous media for different related applications is included in this report.

3. FLUID FLOW AND HEAT TRANSFER IN POROUS MEDIA

3.1 Conservation of Mass

The continuity equation of a single-phase (liquid or gas) flow in a porous medium has the following forms:

(1)

(2)

V and U are the velocity vector of the liquid and vapor phases in the porous medium, respectively.

3.2 Conservation of Momentum

Initial efforts of describing fluid flow in porous media were introduced by Darcy's equation [18]. The equation was developed for a single-phase steady flow through a fully saturated homogenous porous

medium.

Or

(3)

where:

= fluid velocity within the porous medium (m/s),

= Permeability of the porous medium (m^2),

= External acceleration such as gravity (m/s^2),

= Pressure (Pa), and

= Density (kg/m^3) and dynamic viscosity of the working fluid ($kg/m \cdot s$)

The Darcy equation is applicable for a low range of fluid velocity in the porous medium. At higher velocity, the inertia effect must be considered by adding the term to the Darcy model. Multiple models modified the Darcy equation to consider higher velocity for laminar and turbulent flow in a porous medium. The initial model for unidirectional laminar and turbulent flow in the porous medium was developed by Forchheimer [19].

The Forchheimer equation presents the superposition of viscous and inertia terms of the flowing fluid in the porous medium.

(4)

where:

= Constants for a given porous medium.

An analysis by Ward, J. C. [20] showed that Forchheimer constants could be presented as . Moreover, C is the inertia coefficient. These values are valid for many porous media. Forchheimer equation (neglecting gravity forces) expressed as:

(5)

The Fanning friction factor () driven from the pressure gradient and flow Reynolds number () in porous media using characteristic dimensions which is a function of the permeability () expressed as:

(6)

(7)

(8)

The latest equation clearly shows that the friction coefficient is inversely proportional to Reynolds's number in the laminar flow regime (Darcy flow) and increased by the inertia coefficient as the flow becomes turbulent with no transition flow in the porous media.

Efforts were reported to drive formulas for the porous media permeability and inertia coefficient Sabri-Ergun [21] experimentally developed an expression for permeability and inertia coefficient for flowing gases in various solid particle porous beds.

(9)

(10)

The Kozeny-Carman introduced a much-modified permeability formula [22] equation to calculate spherical particles' permeability porous media.

(11)

Where:

and μ_p = Particles, mean diameter, and the bed porosity, respectively.

Other models have been proposed; however, the Kozeny-Carman equation is the most widely used. The equation is applicable for isotropic and anisotropic structured porous medium.

The Darcy equation only counts for inertia and gravity forces but neglect the viscous force. The equation is then valid for a low-permeability (porosity) porous medium. There is a need for an equation valid for a large permeability medium (Navier Stokes equation). Brinkman [23-24] modified the Darcy equation in a similarity to Navier Stokes equation to count for the shear force into the following form:

(12)

where: μ_{eff} μ_p

= the effective dynamic viscosity (different than fluid viscosity).

Lundgren [25] proposed $1/\mu_p$ for <0.7 .

Introducing Forchheimer and Brinkman's extensions to Darcy's law, a generalized equation for a flow-through porous media can be written as:

(13)

Darcy Forchheimer Brinkman
Term Term
 μ_p μ_p μ_p

Irmay [19] work resulted in adding the transient term to the general equation of fluid flow in a porous medium. Another investigation by Stark [26] showed that the difference between the pressure drop between low and high permeability could be attributed to a new convection inertia term in the Navier Stokes equations. Therefore, the general flow equation for porous media flow is obtained:

(14)

Gary and O'Neill [27] performed an analysis to develop the inertia term. The results of complicated long mathematical series of integration of Navier Stokes equation, a general equation to describe the flow of Newtonian fluid through an isotropic porous medium as:

(14)

The above equations approach the Darcy-Forchheimer model as permeability, decreases, and reduced Navier Stokes equations as porosity, goes to 1 and permeability approach infinity. The equation presents the conservation of radial and axial momentum of liquid in a porous medium. The differential forms are:

Radial Momentum

(15)

Axial Momentum

(16)

where:

(17)

(18)

3.3 Conservation of Energy

3.3.1 Heat Transfer for Liquid-Vapor Phases

The continuity and momentum equations deal only with a single-phase flow in a porous medium, while heat transfer is more complicated as it must count for phase change, solid/liquid or liquid/vapor in all directions. The temperature-based model and volume-average enthalpy methods are two models to address the heat diffusion between different phases. In the temperature method developed by Wolff and Viskanta [28], each phase's energy equation conservation is presented for solid and liquid phases. Additional temperature, velocity, and heat transfer through the two phases' interfacial boundary must be introduced. The three equations are connected through the sole dependent variable—temperature. In the enthalpy method (Voller and Parakash [29]), the enthalpy is used as the dependent variable and temperature to satisfy the interfacial phase-change boundary. The volume-average homogenous enthalpy method counts for the complicated interfacial structure for the various constituents, liquid, solid matrix of the porous medium, and the working fluid's frozen phase.

Whitaker [30–31] developed a model to predict the heat transfer using the volume-average homogenous enthalpy in a porous medium. The resulting model assumes thermal and hydrodynamic equilibrium between phases. The resulting model has the form:

(19)

where:

and = the porosity, effective thermal conductivity of the porous medium, and the viscous dissipation function.

Subscripts L and m = liquid and solid matrix of the porous medium, respectively.

The above equation can be rewritten in a more representative format for the radial and axial heat transport in the porous medium as:

(20)

(21)

For slow mass transport in a porous medium (gravity or capillary forces), the dissipation and compressible effects can be neglected for the liquid phase.

3.3.2 Heat Transfer for Solid-Liquid Phases

The heat transfer model discussed above the only count for the porous medium's liquid and solid matrix. It is worth to mention that in porous media applications, initial energy transfer is required to change phase from solid to liquid and vice versa "Freeze and Thaw." The heat transfer model count for this process can be developed from the volume-average enthalpy equation as:

(22)

The subscripts L, S, and m are for the liquid, solid matrix of the porous medium, frozen phase, and solid matrix of the porous medium, respectively.

(23)

The effective thermal conductivity () is a crucial parameter for the heat transfer model in a porous medium. The next section summarizes the reported model in the literature on calculating .

3.3.3 Interfacial Conditions

The interface between the liquid and vapor phases is a discontinuity surface at which conditions jump related to the flow conditions on both sides of the interface must be simulated. Delhayé determined the local governing equations and jump conditions at the liquid-vapor interface, including the instantaneous local mass, momentum, and energy equations [32–33]:

(24)

The radial momentum condition at the interface can be related to the pressure drop across the interface:

(25)

R is the radius of curvature of the liquid meniscus in the wick, which is a function of the pore's vapor volume.

The transverse momentum condition at the interface:

(26)

The simplified energy equation at the liquid-vapor interface:

(27)

Another relationship to close out the governing equations at the liquid-vapor interface is assuming continuous temperature at the interface:

(28)

3.4 Effective Thermal Conductivity of a Porous Medium

Reported in the literature efforts to determine the effective thermal conductivity of porous media saturated with liquid. The depends on the porous media structure, thermophysical properties, and volume fraction of the constituents. The parallel and series theoretical models are used to determine the limits of structured porous media's effective thermal conductivity like a set of strata and the solid surface parallel to heat flux [34].

(29)

The series model determines the lower limit of the porous media effective thermal conductivity as follows:

(30)

Where:

, and = the porosity, porous media thermal conductivity, and liquid thermal conductivity, respectively.

Maxwell developed another model to determine the effective thermal conductivity of randomly packed and sized cylinder porous media [35].

(31)

A model to determine the effective thermal conductivity for identical cylinders porous media using the potential theory developed and reported in the literature by Rayleigh [36].

(32)

An expression for continuous liquid containing randomly distributed solid sphere [37]:

(33)

Veinberg [38] developed a model for a wide range of permeability of packed spherical spheres dispersed in a fluid.

(34)

Krupiczka [39] solved the Laplace equation analytically for two different types of porous media off specific porosity and extrapolated the data using experimental data from the literature to cover a broader range of porosity. The corrected formula for defined as:

(35)

(36)

All models listed above to predict the effective thermal conductivity of the porous medium are capable of predicting the with an acceptable accuracy if the matrix thermal conductivity is larger the fluid thermal conductivity (. Also, the uncertainty of predicting for the gas-filled porous medium are higher [37]. For both liquid or frozen phases of liquid metals such as sodium, potassium, and lithium and their compatible matrix materials, including stainless steel, nickel, zirconium, and tungsten, the matrix to fluid conductivity ratios ranges 0.1 to 3.5, and the parallel, Maxwell, and Rayleigh models are suitable. These models are valid for porous media of porosity ranges between 0.2 and 0.5. In the case of water-filled porous media (e.g., water in stainless steel or copper,). Maxwell's model predicts the solid matrix's effective thermal conductivity to fluid thermal conductivity ratio <1.

3.5 Porous Media Description Case Study

It is crucial to describe the porous medium mathematically with a model that counts for all the geometrical parameters of the porous media and the matrix's thermophysical properties. One of the widely used porous medium in heat pipe and thermosyphon application is the wire screen. The wire screens are commercially available and have a broad range of mesh sizes and materials made. This report summarizes the method to calculate and relate the wire screen parameters, including porosity, permeability, and effective pore diameter. Figure 1 shows a typical standard square wire screen.

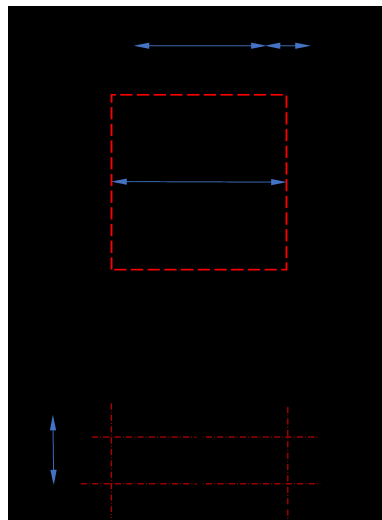


Figure 1. Schematic of wire screen porous media.

A standard square mesh wire screens of thickness t has several openings per unit length , also known as the mesh number:

(37)

Where:

d = Wire diameter

W = Wire spacing

L = Mesh length

e = Effective interlayer spacing (10–75% of $2d$).

Practical recommendations indicated that the screen with $N < 40$ is not capable of developing enough capillary force to drive the fluid through the enormous value of $N > 300$, the screen is not sufficiently rigid to be used for practical applications. As shown in Figure 1, the formula to calculate the void fraction or porosity are:

(38)

The parameters N and d are sufficient to define the wire screen's geometry, including porosity completely.

During operation, the wires are not straight, resulting in a non-uniform distribution of the cavities or the pores. A shrinking coefficient, S , is the wire length per unit mesh size and calculated as:

(39)

The geometrical parameters listed above are for the plan wire screen and based on 2-D analysis of a single layer of wire screen. The porous medium would consist of multiple layers of wire screens, and the geometrical parameters must be reflected for the porous media volume. The volume porosity of porous media composed of multiple layers of wire screen with neglected intermeshing between the layer is expressed as:

(40)

The calculated porosity of wire screen layers can then be used to calculate the permeability using the simplified formula developed by Ivanovskii et al. [38] for the apparent permeability, K , and inertia coefficient, C given by Erguns' equation and Kozeny-Carman equation.

(41)

(42)

It is essential to relate the mesh number to the wire diameter. Ikeda [39] and Dunn and Reay [40] collected data from the literature and developed a simple formula to calculate the required wire diameter to construct a porous medium of wire screen to achieve a specific mesh number:

(43)

Data from a set of wires commercially available were collected, and the correlation showed a good agreement between the measured and predicted values [17].

3.6 Summary

The summarized set of equations in this report forms a strong basis to model mass and heat transfer in porous media considering all fluid phases. These governing equations are applicable for modeling a wide range of liquids, including water and alkali metals, including sodium and potassium. A summary of these equations is below:

Conservation of Mass:

(liquid), (vapor)

Conservation of Momentum:

(radial)

(axial)

Conservation of Energy:

(liquid - vapor)

(solid - vapor)

Interfacial Condition:

Additional geometry description and calculation may be needed for other porous media different than the wire mesh described in this report as a selected case study.

3.7 Next Step

As presented by this report, the process of heat and mass transfer in porous media is complicated. Multiple phases must be considered simultaneously in the analysis. The following step is to build up a simple 1-D steady-state and transient simulation code using an appropriate language. The activity will focus initially on water and sodium in the liquid phase in wire mesh porous media. The initial step is crucial to validate the model with existing models and experimental data and build enough fidelity in the results. Additional steps include freezing condition and upgrading the code to a 2-D model to become usable for engineering applications, including heat pipe and thermosyphons.

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