INL/MIS-23-71357-Revision-0

Scaling for Nuclear Reactor System: Overview

January 2023

Changing the World's Energy Future

Palash Kumar Bhowmik

INL/MIS-23-71357-Revision-0

Scaling for Nuclear Reactor System: Overview

Palash Kumar Bhowmik

January 2023

Idaho National Laboratory
Idaho Falls, Idaho 83415

http://www.inl.gov

Prepared for the
U.S. Department of Energy
Under DOE Idaho Operations Office
Contract DE-AC07-05ID14517

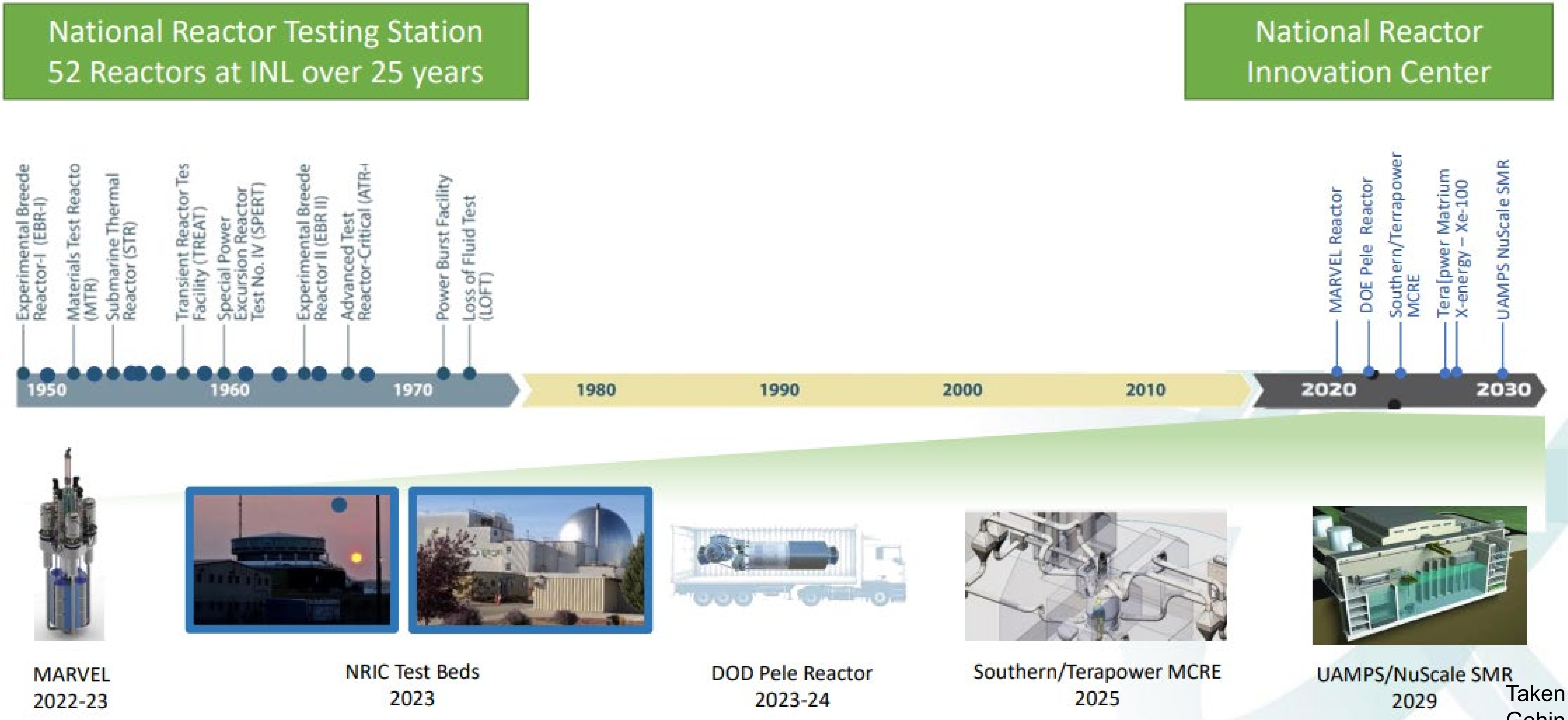
Jan 18, 2023

Palash Bhowmik
C140

Scaling for Nuclear Reactor System: Overview

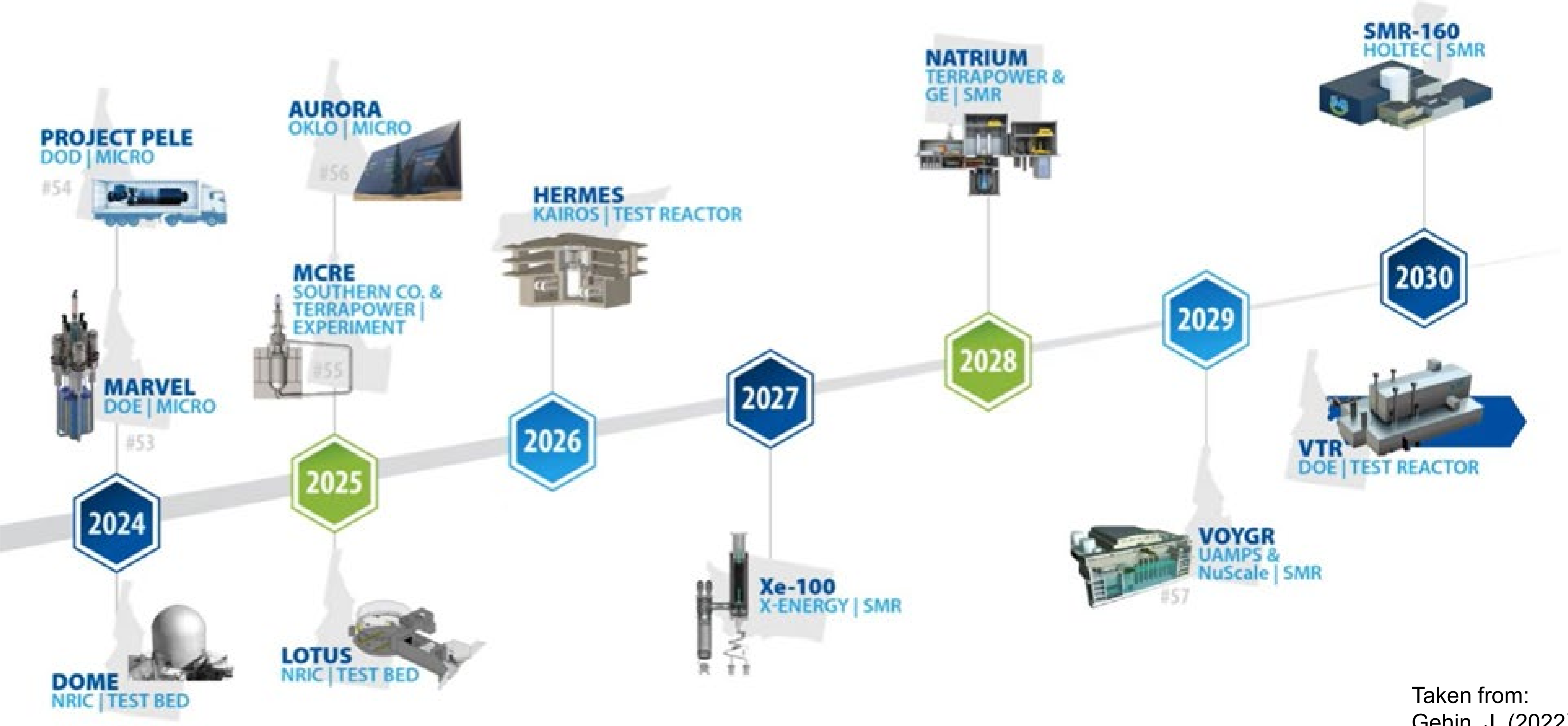
Scaling Analysis to Support Integral Effect Test Facility Development

Motivation: Advanced Reactors Testing at INL



Taken from:
Gehin, J. (2022) [1]

Motivation: Advanced Reactors Demonstration and Deployment



Taken from:
Gehin, J. (2022) [1]

Content Note

The content of this presentation is taken from the lecture notes, publish articles and INL's internal scaling reports.

This presentation focused on the general overview of the scaling for developing experimental facilities for reactor systems.

Presentation outline

- **Part-I: Objectives, Scope and Importance**
- **Part-II Scaling approaches**
- **Part-III (specific application)**
  - Scaling of reactor system primary loop

Objectives, Scope and Importance

- **Reactor systems are complex**
  - When designing experiments, it is best to vary *each parameter* to observe how the result changes
  - Practicality
  - How can we simplify these problems?

Prior studies:

- Phenomena Identification and Ranking Tables (PIRT)
  - Identify the phenomena of interest (POI), state of knowledge (SOK) and figure of merits (FOM)

Objectives, Scope and Importance (cont'd)

- **For LWR analysis**
  - Single phase and two-phase flow
  - When considering facility size, many factors to consider
    - Available space, scale relations of existing facilities
    - Need to compensate for shortcomings in existing facilities
    - Justifiable rationale, impact on total cost

- **For ideally scaled facility**
  - Full pressure, prototypic fluid
  - All materials to be same as model and prototype
  - Ratios (scale facility and prototype)
    - Length ratio—to match available space
      - Determine velocity ratio
    - Area ratio—to match commercially available pipe
    - Volume ratio—from length and area ratios
      - Determine power ratio

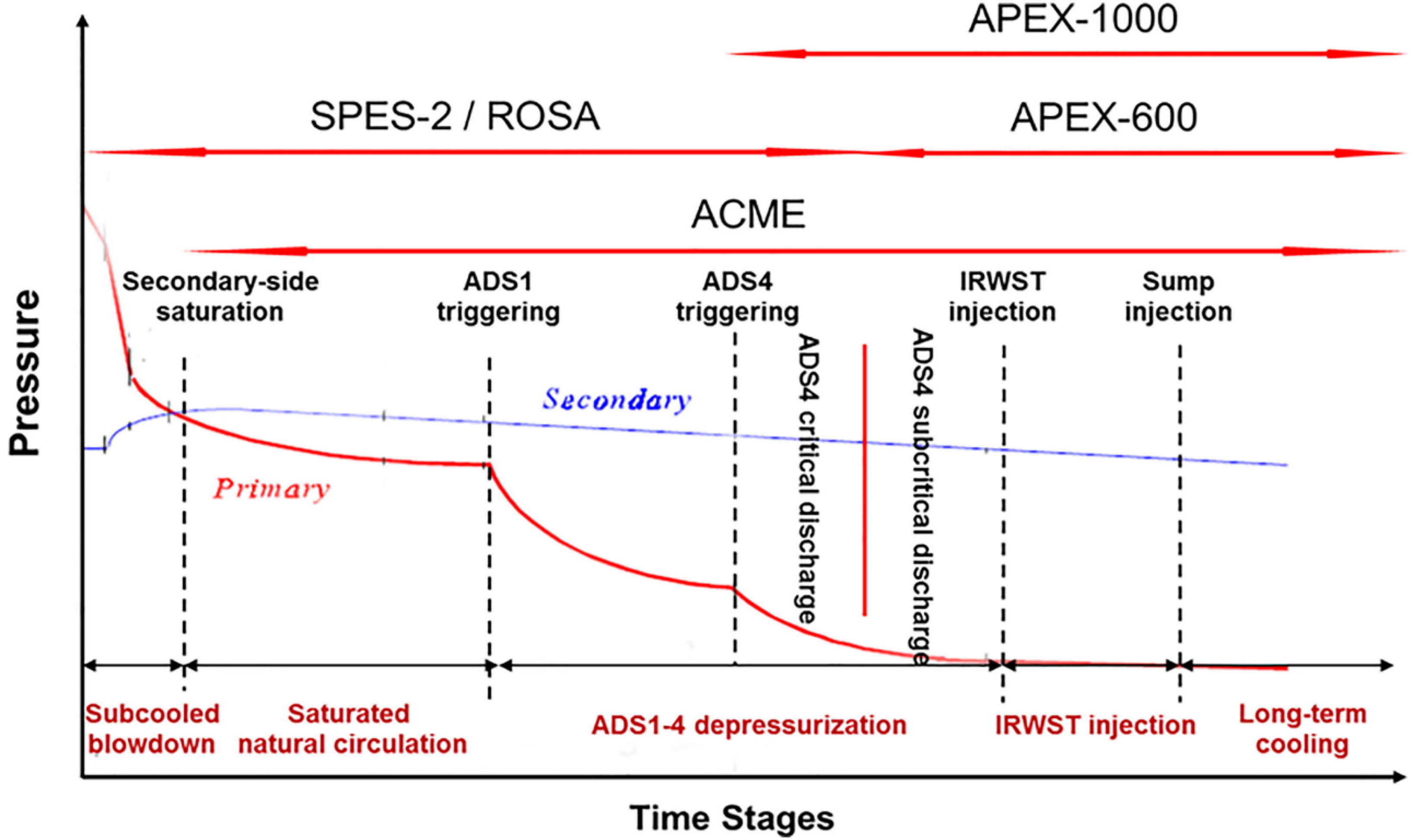
Objectives, Scope and Importance (cont'd)

- **Evaluation model development and assessment process (EMDAP)**
  - Ideal scaled model evaluated with system code (e.g., RELAP5)
    - Full power operation
    - Results are scaled
    - Transient (startup and after shutdown)
      - Break (i.e., LOCA) analysis

- **Modifying ideally scale to engineering scale facility**
  - After ideal scaling verification, need engineering decisions for realistic facility
    - Use component that can actually be obtained or built
- Scaling ratios should be same as IET
- Some distortions unavoidable, examples
    - Reactor core, steam separator assembly,
    - Commercially available pipe diameters and thicknesses

Objectives, Scope and Importance (cont'd)

- Test facilities development to mimic reactor transient scenarios



SBLOCA accident temporal progression [2]

Scaling Approaches: Buckingham Pi Theorem

- **Buckingham Pi Theorem [3-4]**

(# of π terms) = (#of variables) – (# of reference dimensions)

- Dimensionally homogeneous equations:
  - Dimensionless products
  - Reference dimensions
- **Method of Repeating Variables**
  - ***List the variables*** involved in the problem
  - Express each variable in terms of ***basic dimensions*** (MLtT)
  - ***Determine*** the number of Π terms
  - Select ***repeating variables***
  - Form the ***dimensionless Pi terms***
  - ***Verify*** that all Pi terms are dimensionless

Method of Repeating Variables

- This method can be used in all parts of engineering (i.e., reactor physics!), not just fluid mechanics
- Instead of having to run several different sets of experiments, we really need to run one set of experiments
  - Vary Π1 and measure Π2
  - Reduces complexity, cost, and effort of experiments
- Tells us what terms are important in an equation
  - DOES NOT tell us what the relationship between Π groups is
  - Π groups are not necessarily unique, they depend on your choice of repeating variables

Governing Equations

- Start with an incompressible fluid with constant viscosity in a gravity field
  - Continuity

$$\nabla \cdot \vec{v} = 0$$

  - Momentum

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

  - Thermal Energy

$$\rho c\left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T\right) = k \nabla^2 T + \nabla \cdot \left(\mu \nabla^2 \vec{v} \cdot \vec{v}\right)$$

- Define dimensionless variables:

$$\begin{aligned}
\vec{x}^* &= \frac{\vec{x}}{L} & & & \vec{x} &= \vec{x}^* L \\
\vec{v}^* &= \frac{\vec{v}}{V} & & & \vec{v} &= \vec{v}^* V \\
t^* &= \frac{t}{\tau} & &\Rightarrow & t &= t^* \tau \\
p^* &= \frac{p}{p_0} & & & p &= p^* p_0 \\
T^* &= \frac{T}{T_0} & & & T &= T^* T_0
\end{aligned}$$

Governing Equations

- Substitute and simplify:
  - Continuity $$\nabla^* \cdot \vec{v}^* = 0$$
  - Momentum $$\left(\frac{L}{V\tau}\right)\frac{\partial \vec{v}^*}{\partial t^*} + \vec{v}^* \cdot \nabla^* \vec{v}^* = -\left(\frac{p_0}{\rho V^2}\right)\nabla^* p^* + \left(\frac{|\vec{g}|L}{V^2}\right)\frac{\vec{g}}{|\vec{g}|} + \left(\frac{\mu}{\rho V L}\right)\nabla^{*2}\vec{v}^*$$
  - Energy $$\left(\frac{L}{V\tau}\right)\frac{\partial T^*}{\partial t^*} + \vec{v}^* \cdot \nabla^* T^* = \left(\frac{k}{\mu c}\right)\left(\frac{\mu}{\rho V L}\right)\nabla^{*2} T^* + \left(\frac{V^2}{c T_0}\right)\left(\frac{\mu}{\rho V L}\right)\nabla^* \cdot \left(\nabla^{*2}\vec{v}^* \cdot \vec{v}^*\right)$$

Dimensionless Groups

- ***Strouhal Number*** $$St = \frac{L}{V\tau}$$
- ***Euler Number*** $$Eu = \frac{p_0}{\rho V^2}$$
- ***Reynolds Number*** $$\mathrm{Re} = \frac{\rho V L}{\mu}$$
- ***Froude Number*** $$Fr = \frac{V}{\sqrt{gL}}$$
- ***Prandtl Number*** $$\mathrm{Pr} = \frac{k}{\mu c} = \frac{k}{\rho c}\frac{\rho}{\mu} = \frac{\alpha}{\nu}$$
- ***Eckert Number*** $$Ec = \frac{V^2}{cT_0}$$
- ***Weber Number*** $$We = \frac{\rho V^2 L}{\sigma}$$
- ***Mach Number*** $$Ma = \frac{V}{c}$$

Two-Fluid Model

- Continuity

$$\rho_k^* = \frac{\overline{\overline{\rho}}_k}{\rho_{k0}}, v_k^* = \frac{\hat{\vec{v}}_k}{v_{k0}}, t^* = \frac{t}{\tau_0}, \nabla^* = L_0 \nabla, \Gamma_k^* = \frac{\overline{\overline{\Gamma}}_k}{\Gamma_{k0}}$$

$$\Downarrow$$

$$\frac{1}{Sl_k}\frac{\partial}{\partial t^*}\alpha_k \rho_k^* + \nabla^* \cdot \alpha_k \rho_k^* v_k^* = Zu_k \Gamma_k^*$$

$$Sl_k = \frac{\tau_0}{L_0 / v_{k0}} = \frac{\text{mixture time constant}}{\text{phase k time constant}}$$

$$Zu_k = \frac{\Gamma_{k0} L_0^2}{\rho_{ko} v_{k0} L_0} = \frac{\text{mass tranfer rate}}{\text{inertia}}$$

Taken from:
Schlegel, J. (2022) [5]

Two-Fluid Model

- Momentum:

$$P_k^* = \frac{\overline{\overline{P}}_k - P_{k0}}{\Delta P_0}, \tau_k^* = \frac{\overline{\overline{\tau}}_k}{\mu_{k0} v_{k0} / L_0}, M_{ik}^* = \frac{\vec{M}_{ik}}{a_{i0}(\rho_{d0} + \rho_{c0})(v_{d0} - v_{c0})^2}, g^* = \frac{\vec{g}}{|\vec{g}|}$$

$$\Downarrow$$

$$\frac{1}{Sl_k} \frac{\partial}{\partial t^*} \alpha_k \rho_k^* v_k^* + \nabla^* \cdot \alpha_k \rho_k^* v_k^* v_k^* = -Eu_k \alpha_k \nabla^* P_k^* + \frac{1}{\mathrm{Re}_k} \nabla^* \cdot \alpha_k \left(\tau_k^* + \tau_{k,T}^*\right)$$

$$+ \frac{1}{Fr_k} \alpha_k \rho_k^* g^* + N_{D,k} M_{ik}^* - \frac{1}{\mathrm{Re}_k} \nabla^* \alpha_k \cdot \tau_{ki}^* + Zu_k \Gamma_k^* \left(v_{ki}^* - v_k^*\right)$$

$$+ Eu_k \left(P_{ki}^* - P_k^*\right) \nabla^* \alpha_k$$

Reynolds number
Euler number
Froude number
Drag number

Taken from:
Schlegel, J. (2022) [5]

$$\mathrm{Re}_k = \frac{\rho_{ko} v_{k0} L_0}{\mu_{k0}} = \frac{\text{inertia}}{\text{viscous forces}} \qquad Eu_k = \frac{\Delta P_0}{\rho_{k0} v_{k0}^2} = \frac{\text{pressure losses}}{\text{dynamic pressure}}$$

$$Fr_k = \frac{v_{k0}^2}{g L_0} = \frac{\text{inertia}}{\text{body forces}} \qquad N_D = \frac{a_{i0} L_0 (\rho_{d0} + \rho_{c0})(v_{d0} - v_{c0})^2}{\rho_{k0} v_{k0}^2} = \frac{\text{surface area} \cdot \text{drag losses}}{\text{dynamic pressure}}$$

Two-Fluid Model

- Enthalpy:

$$i_k^* = \frac{\hat{i}_k - i_{k0}}{\Delta i_0}, (q_k'')^* = \frac{\bar{\bar{\vec{q}}}_k'' L_0^2}{k_{k0}\Delta T_0 L_0}, (q_{ki}'')^* = \frac{\bar{\bar{\vec{q}}}_{ki}''}{a_{i0}k_{k0}(T_{i0} - T_{k0})},$$

$$\Downarrow$$

$$\frac{1}{Sl_k}\frac{\partial}{\partial t^*}\alpha_k\rho_k^* i_k^* + \nabla^* \cdot \alpha_k \rho_k^* h_k^* i_k^* = -\frac{1}{Pe_k}\nabla^* \cdot \alpha_k (q_k'' + q_{k,T}'')^*$$

$$+ Eu_k Ec_k \alpha_k \left\{ \frac{1}{Sl_k}\frac{\partial P_k^*}{\partial t^*} + v_k^* \cdot \nabla^* P_k^* \right\} + \frac{Ec_k}{\mathrm{Re}_k}\alpha_k \tau_k^* : \nabla^* v_k^*$$

$$+ \frac{Ec_k}{\mathrm{Re}_k}\nabla^* \alpha_k \cdot \tau_{ki}^* \cdot (v_{ki}^* - v_k^*) + N_{D,k} Ec_k M_{ik}^* (v_{ki}^* - v_k^*) + Zu_k \Gamma_k^* (i_{ki}^* - i_k^*) + N_{q,k} a_i^* (q_{ki}'')^*$$

Peclet number
Eckert number
Interface Heating number

$$Pe_k = \frac{\rho_{k0} v_{k0} \Delta i_{k0} L_0}{k_{k0}\Delta T_{k0}} = \frac{\text{convection}}{\text{conduction}}$$

$$Ec_k = \frac{v_{k0}^2}{\Delta i_{k0}} = \frac{\text{mechanical energy}}{\text{thermal energy}}$$

$$N_q = \frac{a_{i0} L_0 k_{k0}(T_{i0} - T_{k0})}{\rho_{k0} v_{k0} \Delta i_{k0}} = \frac{\text{interfacial heat transfer}}{\text{convection}}$$

Taken from:
Schlegel, J. (2022) [5]

Two-Fluid Model

- Interfacial Jump Conditions:

$$a_i^* = \frac{a_i}{a_{i0}}, H_{dc}^* = \frac{\overline{\overline{H}}_{dc}}{H_{dc0}},$$

$$\Downarrow$$

$$\sum_k \Gamma_k^* = 0$$

$$\sum_k M_{ik}^* = M_m^*$$

$$P_c^* - P_d^* = -2N_\sigma H_{dc}^* \sigma^*$$

$$P_c^* - P_{sat}^* = 2N_\sigma \left(\frac{\rho_d^*}{\rho_d^* - \rho_c^* / N_\rho} \right) H_{dc}^* \sigma^*$$

$$\left(N_{i,c} i_{ic}^* - N_{i,d} i_{id}^* - 1 \right) \Gamma_c^* + \sum_k \frac{N_{q,k} N_{i,k}}{Zu_k} a_i^* q_{ki}^* = 0$$

Converted enthalpy ratio and density ratio. Surface tension number is often replaced by Weber number—requires weber numbers for each phase.

$$N_i = \frac{\Delta i_{k0}}{i_{d0} - i_{c0}} = \frac{\text{enthalpy change}}{\text{latent heat}}$$

$$N_\rho = \frac{\rho_{d0}}{\rho_{c0}} = \text{density ratio}$$

$$N_\sigma = \frac{H_{dc0}\sigma_0}{\Delta P_0} = \frac{\text{surface tension force}}{\text{pressure losses}}$$

$$\Rightarrow \frac{Eu}{N_\sigma} = We = \frac{\rho_{k0} v_{k0}^2}{H_{dc0}\sigma_0} = \frac{\text{dynamic pressure}}{\text{surface tension force}}$$

Taken from:
Schlegel, J. (2022) [5]

Modeling and Similarity

- To be useful, experiments must provide data that can be applied to full-scale prototypes
  - How to assure ***similarity***?

- ***Geometric Similarity***
  - Height/length ratio, etc.
    - i.e., a 1/100 scale model airplane
  - Often easiest type of similarity
- ***Kinematic similarity***
  - Geometric similarity is a prerequisite
  - Arises from continuity equation
    - Strouhal number
    - Zuber number
- ***Dynamic similarity***
  - Most restrictive requirements for most scale models
  - Arises from momentum equation
    - Reynolds, Froude, Euler numbers
- ***Thermal similarity***
  - Also, can be very restrictive for heat transfer problems
  - Arises from energy equation
    - Peclet, Prandtl, Eckert numbers

Scaling vs. Similarity

- Scaling is much different than simple similarity
  - **How do we design a system so that it is similar?**
    - Task is finding key design parameters that must be addressed!
    - Often we look at system design, rather than just single-component
      - **How to preserve friction losses and other system-integral parameters?**

Scaling Methods

- **Linear scaling**
- **Power-to-volume scaling**
- **Three-level scaling**
- **Hierarchical two-tiered scaling (H2TS)**
- **Fractional scaling analysis (FSA)**

- **Others scaling methods**, such as power-to-mass scaling, dynamic system scaling (DSS) and sequential-parallel interdependent complement scaling (SPIC)

Taken from:
Wang et al. (2021) [6]

Scaling Methods (cont'd)

- Traditionally scaling parameters are expressed using scaling ratios
- The ratio of the nondimensional numbers in the scaled-down model and prototype should be close to unity to maintain and preserve the phenomenology such as flow and heat transfer dynamics, as shown in equation:

$$\psi_R = \frac{\psi_{model}}{\psi_{prototype}} = \frac{\psi_m}{\psi_p} = 1$$

Scaling Methods (cont'd)

Scaling parameters used in various scaling method [2]

Name	Parameter Symbol	Linear scaling	Power-to-volume scaling	Three-level (Ishii) scaling	H2TS scaling
Length ratio	l_R	l_R	1	l_R	l_R
Diameter ratio	d_R	l_R	d_R	d_R	$l_R\ f_R$
Area ratio	a_R	l_R^2	d_R^2	d_R^2	d_R^2
Volume ratio	V_R	l_R^3	d_R^2	$l_R\ d_R^2$	$l_R\ d_R^2$
Velocity ratio	u_R	1	1	$l_R^{1/2}$	$l_R^{1/2}$
Time ratio	t_R	l_R	1	$l_R^{1/2}$	$l_R^{1/2}$
Power-volume ratio	q_R'''	l_R^{-1}	1	$l_R^{-1/2}$	$l_R^{-1/2}$
Power ratio	P_R	l_R^2	d_R^2	$d_R^2\ l_R^{1/2}$	$d_R^2\ l_R^{1/2}$
Acceleration ratio	g_R	l_R^{-1}	1	1	1

Scaling Methods (cont'd)

Scaling method	Advantages	Disadvantages	Facilities
Linear scaling (1960s)	• Miniature replica of the prototype • Interpretation of the component's interactions	• Distortion in acceleration and energy transfer • Acceptability of the scaling factor's low limit	LOFT, SEMISCALE, LSTF, PKL, ROSA, BETHSY, LOBI
Power-to-volume scaling (1970s)	• Time preserving • Height preserving	Distorting in • pressure drop • heat loss • multi-D phenomena	PANDA, INKA, PKL, PACTEL, BETHSY, SMART, SPES, ROSA IV
Three-level scaling, Ishii (1980s-1990s)	• Reduction in construction costs • Scaling in important local phenomena	• Time scale is shifted • Distortions in multi-D phenomena	PUMA, ATLAS
H2TS (1990s)	• Scaling hierarchy for complex system • Scaling in important phenomena	• Contradiction of scaling criteria • Distortions in multi-D phenomena • Definition of distortion	APEX, ACME
FSA method (2000s)	• Analysis of state variable and scaling distortion	• Static evaluation of scaling distortion	

Taken from:
Wang et al. (2021) [6]

Linear Scaling

- Developed by Carbiener and Cudnik (1969) [7] and Nahavandi et al. (1979) [8] independently
  - obtained identical similarity laws for IET
- Dimensions are proportionally reduced by an identical factor
  - traditionally the characteristic length ratio
  - miniature replica of the prototype
- Both the transportation time of fluid and sound is proportionally scaled

Linear Scaling (cont'd)

Adv.:

- Better interpretation of the interactions between components
  - When the effect of gravity is relatively smaller than the system pressure drop
- Acceptability of the scaling factor's low limit

Disadv.:

- Sensitive to length scale (e.g., entrance effects, and boundary layer) phenomena
- Some energy transfer processes are difficult to be simulated
  - difficult to reduce diameter or thickness, e.g., fuel rods and SG tubes

Modified method was suggested by Yun et al. (2004 [9], which requires the same geometry similarity criterion **but preserve the gravity effect**

Power-to-volume Scaling

- Suggested by Nahavandi et al. (1979) [8]
- It can be seen as the first proposed solution to the scaling problem (Deng et al., 2019) [10]
- It is a simple, straightforward method for the design of test facilities with **full-pressure, full-height, and time-preserving**
- The conserving of **time and heat flux in the prototype is helpful to reproduce the phenomena with strong gravity effect**
- A good choice for **LBLOCA tests because it guarantees an equal time ratio**

Power-to-volume Scaling(cont'd)

Adv.:

- Same working fluid, height, pressure and similar structural materials, etc.

Disadv.:

- An **increase in hydraulic resistance** in the model due to the reduction diameter
  - which is important for SBLOCAL dominated by natural circulation
  - the full height require high material costs

- An **increase in heat loss in heater and cooler**
  - heat losses and core power ratio are higher in model than prototype
  - **distort the system natural circulation** and long-term cooling transient
    - especially in low power experiments.
- Distortion in multi-D flow phenomenon
  - e.g., effects of surface tension and transitions in the flow regime observed

Three-Level Scaling

- Suggested by Ishii and Kataoka (1983, 1984) [11-12]
  - assumed each component is a 1-D system
  - use 1-phase and the drift-flux two-phase flow formulation
- Scaling criteria used for 1-D conservation equations with boundary conditions

These three levels of scaling mainly include:

(1) **Scaling of integral response function:** system conservation equations are solved with a linear small-perturbation method
   - this level gives the dynamic scaling of a whole component

(2) **Scaling of control volume and boundary flow:** secondly, the mass and energy inventory and boundary flow are scaled
   - which preserve inter-connected components TH interaction

(3) **Scaling of local phenomena:** compromising with conflicting similarity criteria.
   - maintain and conserve the physical phenomena

Three Level Scaling

- For two-phase flow scaling we use small perturbation analysis
  - Start with the 1D drift-flux model
  - Determine characteristic response function
  - Highlights key dimensionless numbers

Zuber Number $Zu = \left(\frac{4q_0'''\delta L_0}{DV_0\rho_f i_{fg}}\right)\left(\frac{\Delta\rho}{\rho_g}\right)$

Subcooling Number $N_{sub} = \left(\frac{i_{sub}}{i_{fg}}\right)\left(\frac{\Delta\rho}{\rho_g}\right)$

Froude Number $Fr = \left(\frac{V_0^2}{gL_0\alpha_0}\right)\left(\frac{\Delta\rho}{\rho_g}\right)$

Drift Number $N_d = \left(\frac{\overline{V}_{gj,i}}{V_0}\right)_i$

Time Ratio Number $T_i^* = \left(\frac{L_0/V_0}{\delta^2/\alpha_s}\right)_i$

Thermal Inertia Ratio $N_{th,i} = \left(\frac{\rho_s c_{ps}\delta}{\rho_f c_{pf} D}\right)_i$

Friction Number $N_{f,i} = \left(\frac{fL}{D}\right)_i\left(\frac{1+x(\Delta\rho/\rho_g)}{\left(1+x(\Delta\mu/\mu_g)\right)^{0.25}}\right)\left(\frac{A_0}{A_i}\right)^2$

Orifice Number $N_{o,i} = K_i\left(1+x^{3/2}\left(\frac{\Delta\rho}{\rho_g}\right)\right)\left(\frac{A_0}{A_i}\right)^2$

Void Ratio $\alpha_0 = \left(\frac{\rho_f}{\Delta\rho}\right)\left[\frac{1}{1+(N_d+1)/(Zu-N_{sub})}\right]$

Taken from:
Schlegel, J. (2022) [5]

Three-Level Scaling (cont'd)

Adv.:

- No requirement to keep the same height, pressure, fluid and structural material
  - flexibility in the design of test facilities, especially for the reduced-height facilities
  - the flow regime and multi-D phenomena can be well preserved
  - reduction in scaling distortions, such as those originated by structural heat loss and pressure drops

Since the axial length is reduced, the time scale in the model is shifted in the two-phase flow natural circulation loops, reduction in time

Adv: simulating slow transients, such as SBLOCA

Disadv: such as vapor generation and condensation that may be distorted

Three-Level Scaling(cont'd)

Disadv.:

- Distortions due to
  - reduction in scaling distortions, such as those originated by **structural heat loss and pressure drops**
  - difficulty in matching the local scaling criteria
    - e.g., flow stagnation, reverse flow in individual tubes, critical heat flux (CHF) and reflooding
  - An optimal height scale but, no criteria on the minimum-height scales without significant scaling distortions

Top-down vs. Bottom-up Scaling

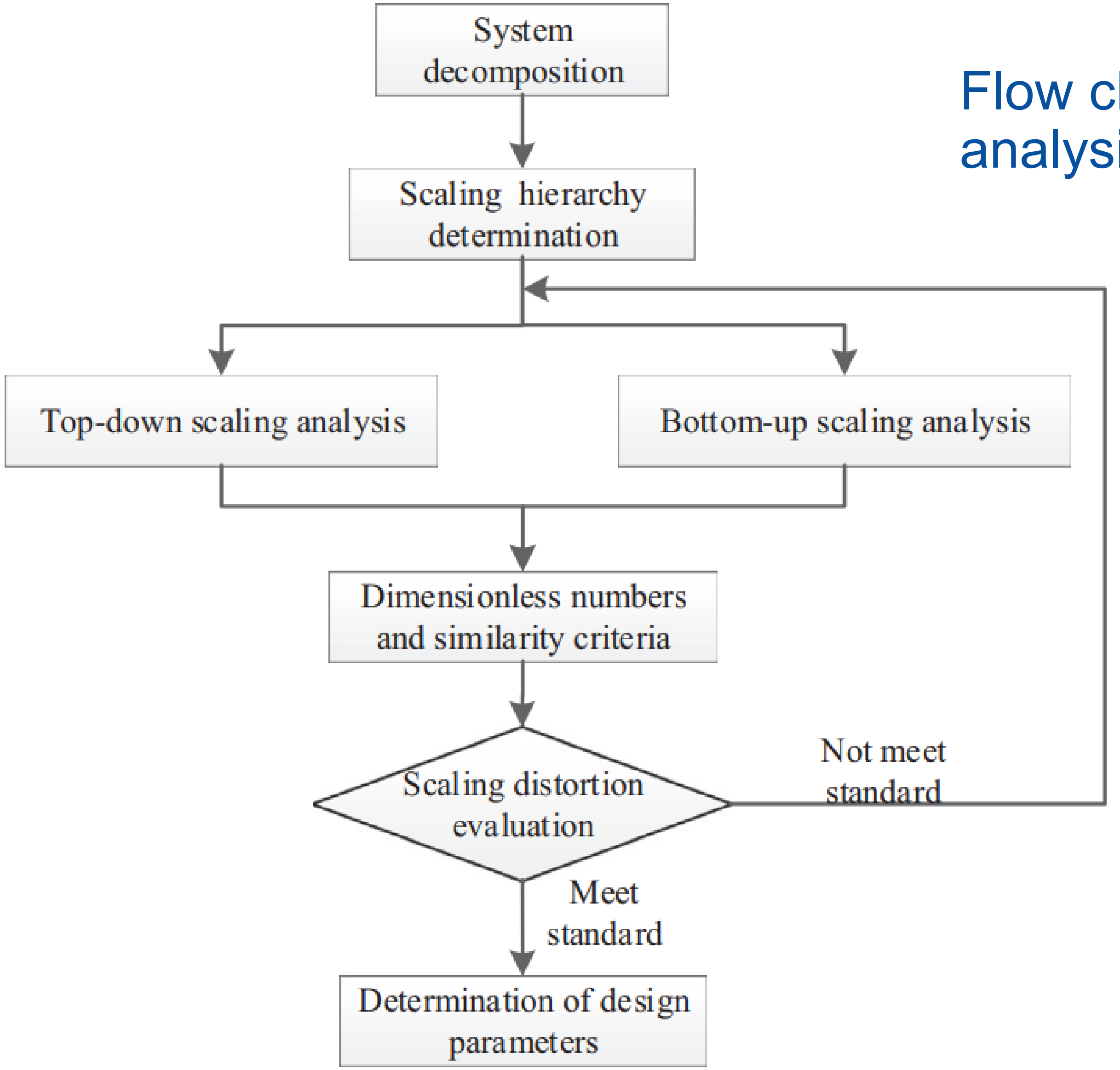
- **Top-down scaling**
  - Based on conservation equations for a control volume – mass, momentum and energy
  - Comprehensive groups of similarity parameters
  - Includes boundary and initial conditions
  - Time ratios for various processes
- **Bottom-up scaling**
  - Focus is on dominant mechanisms

H2TS Scaling

- Developed by Zuber's research team [13-15]
  - scaling forbids exact similitude between the prototype and the test facility
    - ability to rank processes according to their importance on the system
- A hierarchical architecture of the system is generally accompanied with a spatial, temporal, and energetic hierarchy

- A lower level in the hierarchy only transfers its average to the higher level since less detailed information is needed at higher levels.

H2TS Scaling (cont'd)



Flow chart of the H2TS scaling analysis method [6]

H2TS Scaling (cont'd)

- The procedures of H2TS method are mainly composed of four stages.

(1) System breakdown: Separating the system into subsystems, modules, constituents, phases, geometrical configurations, fields, and processes.

(2) Scale identification: Developing a hierarchy for characteristic volume fraction, spatial scale, and temporal scale.

(3) Top-down scaling: Establishing a scaling hierarchy with conservation equations. For each scale level, these equations are nondimensionalized to obtain characterized time ratios and similarity criteria.

(4) Bottom-up scaling: Scaling analysis of key processes and phenomena to achieve similar criteria for local phenomena.

Taken from:
Wang et al. (2021) [6]

H2TS scaling (cont'd)

Adv.:

- The top-down scaling is efficient, and the bottom-up scaling is sufficient
  - Before scaling, PIRT studies identifies the important and complex phenomena
  - Fluid properties and geometric parameters are the two main scaling factors
- It is an effective way scaling-down the model test facilities
  - H2TS scaling is used for the APEX facility, the most accurate geometric representation of AP600's steam supply system
    - developed by setting the characteristic time ratios
    - by sensibly selecting the geometry, fluid properties and operating conditions

Fractional Scaling

- **Based on concept of fractional rates of change:**

$$\frac{\partial\Phi/\partial t}{\Phi} \Rightarrow \omega = \frac{1}{v}\frac{\partial v}{\partial t} = \frac{1}{\tau} = \frac{F}{mv}$$

- Based on integral scaling
  - Turnover time and turnover length
  - Concept of ‘action’
    - **Scaling parameters expressed in terms of energy and time**
  - Similar to control volume analysis
- Spatial and temporal scales determined from geometry and energy considerations

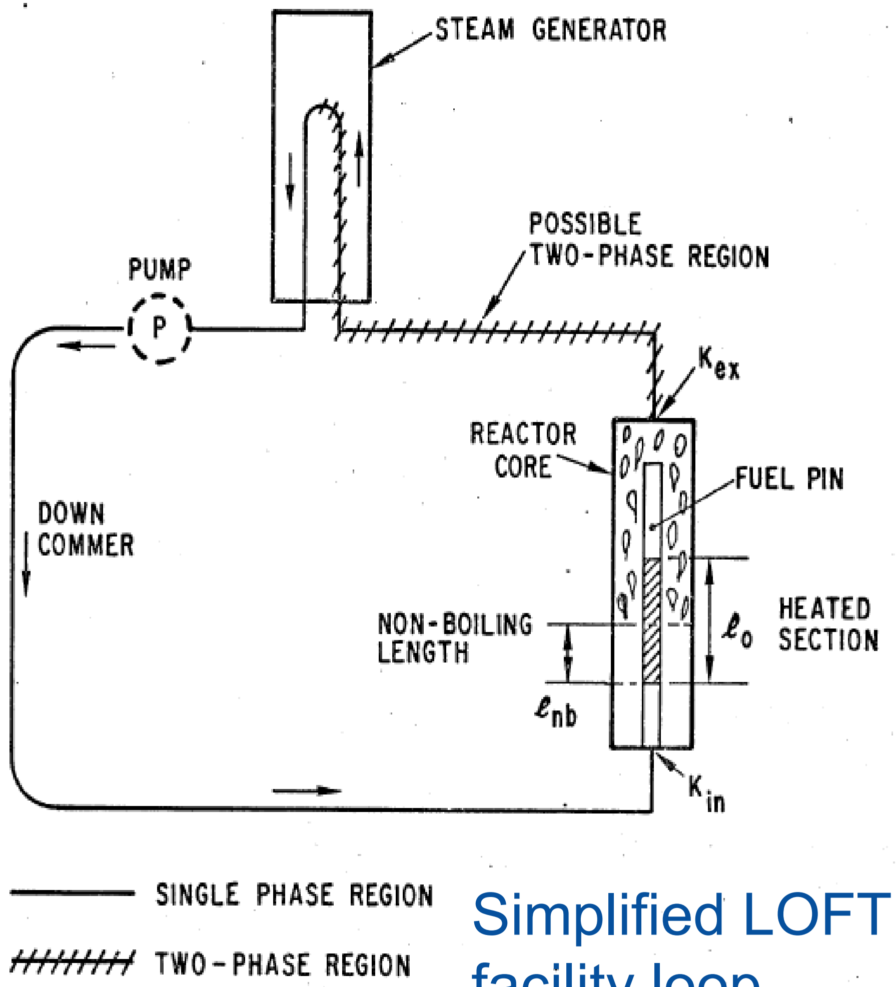
- For any signal transfer process:

$$\lambda = V/A$$

  - For energy, λ_t represents the length required to change/dissipate energy
- Two-time scales
  - Clock time
  - Process time constant,
- **Three transmission modes**
  - Diffusion
  - Convection
  - Wave Propagation

Scaling: Natural Circulation Loop

- **Scaling of natural circulation**
  - Single phase and two phase
  - Important for LWR accidents
  - Small perturbation method
    - Based on drift-flux model
    - Simplified balance equation (for single phase)
      - Continuity
      - Momentum
      - Energy
        - Solid
        - Fluid
      - Boundary conditions



Simplified LOFT facility loop

Primary Loop Single-Phase (1-Φ) Natural Circulation (NC)

	Equations and Variables	Remarks
A1. Governing Equations		
1-D loop momentum balance equation for 1-Φ natural circulation (Each term represents mass flux times velocity or momentum rate of change per unit area or force per unit area)	$$\sum_{i=1}^{N}\left(\frac{l_i}{a_i}\right)\cdot\frac{d\dot{m}}{dt}=\beta g\rho_l(T_H-T_c)L_{th}-\frac{\dot{m}^2}{\rho_l a_c^2}\sum_{i=1}^{N}\left[\frac{1}{2}\left(\frac{fl}{d_h}+K\right)_i\left(\frac{a_c}{a_i}\right)^2\right]$$ where the *i* subscripts refer to the *ith* component and L_{th} is the thermal center length. The core cross-sectional flow area, a_c, is used as the reference flow area	Primary mechanism of core heat removal during normal operation and certain accident scenarios Provide a basic for scaling analysis of the IET
1-D energy equation for 1-Φ natural circulation (Each term in this equation has dimensions of power)	$$C_{vl}M_{sys}\frac{d(T_M-T_C)}{dt}=\dot{m}C_{pl}(T_H-T_C)-q_{SG}-q_{loss}$$ Rate of change of thermal energy = heat generation and heat losses	Rate of change of thermal energy is equal to the sum of heat generation and heat losses
Loop time constant	$$\tau_{loop}=\sum_{i=1}^{N}\frac{l_i}{u_i}=\sum_{i=1}^{N}\tau_i=\frac{M_{sys}}{\dot{m}_o}=\frac{M_{sys}}{\rho_l u_{co}a_c}$$	Used to nondimensionalized loop momentum and energy equations

Taken from:
James E. O'Brien (2022) [16]

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
A2. Nondimensional Parameters [16]		
Loop reference length number	$$\Pi_L = \sum_{i=1}^{N} \frac{l_i}{l_{ref}} \frac{a_c}{a_i}, \qquad \text{where } l_{ref} = \frac{M_{sys}}{\rho_l a_c}$$	Primary mechanism of core heat removal during normal operation and certain accident scenarios Provide a basic for scaling analysis of the IET
Richardson number	$$Ri = \frac{\beta g (T_H - T_C)_o L_{th}}{u_{co}^2} = \frac{\beta g q_{co} L_{th}}{\rho_l a_c C_{pl} u_{co}^3}$$ These two forms of Ri comes from an energy balance across the core $$q_{co} = \rho_l a_c u_{co} C_{pl} (T_H - T_c) = \dot{m}_{co} C_{pl} (T_H - T_C)$$ $$Ri = \frac{\beta g \Delta T L_c}{u^2} = \frac{\frac{g \beta \Delta T L^3}{\upsilon^2}}{\left(\frac{uL}{\upsilon}\right)^2} = \frac{Gr}{Re^2} = \frac{u_o^2}{u^2} = \frac{u_o^2}{u_{co}^2}$$ For the reactor system natural circulation, $$u_o = \sqrt{\beta g (T_H - T_C) L_{th}}\ and\ u = u_{co}$$	Richardson number (Ri) represents the ratio of buoyancy forces to inertial forces Ri can also be expressed in terms of the Grashof numbers (*Gr*) and Reynolds number (*Re*) Ri can be simplified to the square ratio of characteristic velocity for natural convection (u_o) and local velocity (u)

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
A2. Nondimensional Parameters		
Loop Resistance number	$$\Pi_{Fl} = \sum_{i=1}^{N} \left\{ \frac{1}{2} \left(\frac{fl}{d_h} + K \right)_i \left(\frac{a_c}{a_i} \right)^2 \right\}_o$$ For steady state momentum case $$Ri = \Pi_{Fl} = \frac{\beta g q_{co} L_{th}}{\rho_l a_c C_{pl} u_{co}^3}$$ Therefore, core inlet velocity $$u_{co} = \left(\frac{\beta q_{co} L_{th} g}{\rho_l a_c C_{pl} \Pi_{Fl}} \right)^{1/3}$$	Provides a solution for the steady-state core inlet velocity If the loop resistance number is known, from calculation or simulation, the core inlet velocity can be calculated as well as the Richardson number and the loop time constant. Experimental validation could be achieved through direct measurement of the core inlet velocity in the IET.

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
A2. Nondimensional Parameters [16]		
Loop energy ratio	$$\Pi_T = \frac{(T_H - T_C)_o}{(T_M - T_C)_o}$$	
Steam generator transport number	$$\Pi_{SG} = \frac{q_{SGo}}{\rho_l u_{co} a_c C_{pl} (T_M - T_c)_o}$$	Represents the ratio of the heat transfer to the steam generator to the core power heat input
Loop heat loss number	$$\Pi_{Loss} = \frac{q_{loss,o}}{\rho_l u_{co} a_c C_{pl} (T_M - T_c)_o}$$	Represents the ratio of the other system heat losses to the core heating

Primary Loop 1-Φ NC (cont'd)

The constituent energy equations can be nondimensionalized using the core inlet velocity as the reference velocity, core height as the reference length, core temperature difference as the reference temperature difference, core flow area as the reference flow area, and the conduction thickness, δ, as the reference thermal depth for the solid

B1. Geometry-Level Governing Equations (for 1-Φ natural circulation)		
Conduction thickness	$$\delta_i = \frac{a_{s_i}}{P_w}$$	Defined as the ratio of the solid cross-sectional area of each section divided by the wetted perimeter
Hydraulic diameter	$$d_{h_i} = 4\delta_i \frac{a_i}{a_{s_i}}$$	Conduction thickness definition is similar to the definition of hydraulic diameter (flow cross sectional area divided by wetted perimeter).

Primary Loop 1-Φ NC (cont’d)

	Equations and Variables	Remarks
B1. Geometry-Level Governing Equations (for 1-Φ natural circulation)		
Fluid energy equation for 1-Φ natural circulation	$$\rho C_p \left\{ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right\} = \frac{4 h_{conv}}{d_h} (T_s - T)$$	Examines fluid-solid transient heat transfer processes and the associated scaling issues.
Solid energy equation for 1-Φ natural circulation	$$\rho_s C_{ps} \frac{\partial T_s}{dt} + k_s \nabla^2 T_s = \dot{q}_s$$	Consideration of this energy transfer mechanism necessitates the inclusion of an energy equation for both the fluid (1-Φ or 2-Φ) and the solid.
Boundary condition	$$-k_s \frac{\partial T_s}{\partial y} = h_{conv} (T_s - T)$$	Balancing between conductive heat transfer in solid and convective heat transfer in fluid

Part-III:

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
B2. Geometry-Level Nondimensional Parameters (for 1-Φ natural circulation)		
Modified Stanton number	$St_i = \left(\frac{4h_{conv}l_o}{\rho C_p u_{co} d_h}\right)_i$	Reference velocity is the core inlet velocity Reference length is the heated height of the core
Conduction time number	$T_i^* = \left(\frac{\alpha_s l_o}{\delta^2 u_{co}}\right)_i$	
Biot number	$Bi_i = \left(\frac{h_{conv}\delta}{k_s}\right)_i$	
Heat source number	$Q_{si} = \left(\frac{\dot{q}_s l_o}{\rho_s C_{p_s} u_{co} \Delta T_o}\right)_i$	
The reference temperature rise	$\Delta T_o = \left(\frac{\dot{q}_s l_o}{\rho C_p u_{co}}\right)\left(\frac{a_{s_o}}{a_o}\right)$	Obtained from an energy balance across the core

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
C1. Similarity Criteria and Scale Ratios (for 1-Φ natural circulation)		
Loop length scale ratio	$l_R = (L_{th})_R = 1/6$	Ratio, R (subscript) represents a model-to-prototype (i.e., IET-to-SMR) ratio
Time constant scale ratio	$\tau_{loop,R} = \frac{1}{2} = \left(\frac{M_{sys}}{\rho_l u_{co} a_c}\right)_R = \left(\frac{l_{ref}\rho_l a_c}{\rho_l u_{co} a_c}\right)_R = \left(\frac{l_{ref}}{u_{co}}\right)_R$	Selected a value less than 1.0 to allow for more reasonable velocity ratios for reduced-height test loops
Fluid velocity scale ratio	$u_{co,R} = \left(\frac{l_{ref}}{\tau_{loop}}\right)_R = \left(\frac{1/6}{1/2}\right)_R = 1/3$	
For steady state natural circulation, requirements		
To maintain kinematic similarity, require geometric similarity in terms of cross-sectional flow areas	$\left(\frac{a_i}{a_c}\right)_R = 1$	The ratio of each component flow area to the core area should be the same in the model as in the prototype
Full transient buoyancy and friction scaling adopted by	$Ri_R = \Pi_{Fl,R}$	Steady-state natural circulation scaling can be applied
Full pressure testing with fluid property similitude	$\left(\frac{L_{th}}{u_{co}^2}\right)_R = \left(\frac{l_{ref}}{d_h}\right)_R$	matched temperature rise across the core, and assuming that the minor pressure losses and area ratios can be met by geometric similarity

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
C2. Similarity Criteria and Scale Ratios (for 1-Φ steady-state natural circulation) [16]		
Diameter, area, and volume scale ratios	$(d_h)_R = (u_{co}^2)_R, (a_i)_R = (d_h^2)_R$ $(V_i)_R = (a_i l_i)_R$	Reference velocity is the core inlet velocity Reference length is the heated height of the core
Mass flow rate scale ratio	$\dot{m}_R = (u_{co} a_c)_R$	
Core power scale ratio	$Ri_R = \left(\frac{\beta g q_{co} L_{th}}{\rho_l a_c C_{pl} u_{co}^3}\right)_R = \Pi_{Fl,R} = \left(\frac{l_{ref}}{d_h}\right)_R$ $q_{co,R} = \left(\frac{l_{ref}}{L_{th}}\right)_R \left(\frac{a_c u_{co}^3}{d_h}\right)_R$	Simplified form of the friction number scale ratio Again, assuming fluid property similitude
Power-to-Volume ratio	$(q_{co}/V_i)_R$	

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
C2. Similarity Criteria and Scale Ratios (for 1-Φ steady-state natural circulation)		
Richardson number ratio	$$Ri_R = \left(\frac{L_{th}}{u_{co}^2}\right)_R$$ $$Ri_R = \left(\frac{\beta g q_{co} L_{th}}{\rho_l a_c C_{pl} u_{co}^3}\right)_R$$	Ri number ratio can be obtained from any of these two equations
Friction number ratio	$$\Pi_{Fl,R} = \left(\frac{l_{ref}}{d_h}\right)_R =$$	Same as the Ri ratio, as required The model should have a 50% higher friction number than the prototype Can be achieved by adding resistance elements (e.g., orifices)

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
C3. Additional Energy Scale Ratios (for 1-Φ steady-state natural circulation)		
Loop energy scale ratio	$E_R = \left[\frac{(T_H - T_C)_o}{(T_M - T_C)_o}\right]_R = \frac{(T_H - T_C)_{o,R}}{(T_M - T_C)_{o,R}} = 1$ Also require, $(T_M - T_C)_{o,R} = 1$	Indicates that the ratio of the temperature difference across the core to the difference between the mixed mean system temperature and the core inlet temperature remain fixed in the model
Steam generator power scale ratio	$\left(\frac{q_{SGo}}{q_{co}}\right)_R = 1$	indicates that the ratio of SG heat transfer to core power should be the same in the IET and the SMR
Heat loss scale ratio	$\left(\frac{q_{loss,o}}{q_{co}}\right)_R = 1$	So, the ratios of the SG heat transfer and heat losses to core power should be preserved in the model Due to its inherently higher surface area-to-volume ratio, relative heat losses will tend to be larger in the IET compared to the prototype, need thermal insulation and guard heating
Fluid-Solid heat transfer scale ratios	$St_{i_R} = T^*_{i_R} = Bi_{i_R} = Q_{s_{o_R}} = 1$	St and Bi ratios involve heat transfer coefficient Ti and Bi includes conduction thickness, which will not be automatically matched in the IET

Primary Loop 1-Φ NC (cont'd)

	Equations and Variables	Remarks
C4. Additional Energy Scale Ratios (for 1-Φ steady-state natural circulation) [16]		
Tube Reynolds number ratio	$Re_{tube,R} = \frac{\dot{m}_{tube,R}}{d_{tube,R}} = \frac{\dot{m}_R/N_{tube,R}}{d_{tube,R}} =$	
Primary side inside-tube Re during normal operation can be estimated for the SMR from	$Re_{tube,SMR} = \frac{4\dot{m}_{tube,SMR}}{\mu\pi d_{tube,SMR}} = \frac{4\dot{m}_{SMR}/N_{tubes,SMR}}{\mu\pi d_{tube,SMR}}$	
Nusselt number ratio for heat transfer inside the SG tubes	$Nu_{SGtubes,R} = \left(\frac{Re_{tube,IET}}{Re_{tube,SMR}}\right)^{0.8}$	
Tube inside-surface thermal resistance ratio and overall heat transfer coefficient	$R_{th,inside\ tubes,R} = 1/Nu_{SGtubes,R}$ Overall heat transfer coefficient ratio $U_{SGtubes,R} = \frac{1}{R_{th,Tot,R}}$	

Summary and Path-Forward

- General overview of scaling and similarity
- Approaches or types
  - Defining and overview
  - Remarks
- Scaling analysis for natural circulation loop for reactor system
  - Single-phase flow and two-phase flow
    - Governing equations and nondimensional parameters
    - Geometry-level governing equations and nondimensional parameters
  - Similarity criteria and scaling ratios
    - Specified scale ratios Finalizing the scaling analysis
- Scaling for nuclear IET and SET facility design

References

[1] Gehin, J. (2022), “Introduction to Today's Nuclear R&D,” Lecture notes, MeV Summer School, Idaho National Laboratory.

[2] Deng, C.; Zhang, X.; Yang, Y.; and Yang, J. (2019), “Research on scaling design and applicability evaluation of integral thermal-hydraulic test facilities: A review,” *Annals of Nuclear Energy*, vol. 131, pp. 273–290.

[3] Chapter5. Dimensional analysis and similarity, http://www.pmt.usp.br/ACADEMIC/martoran/NotasModelosGrad/Dimensional%20Analysis.pdf (Accessed on 18th January 2023).

[4] Dumka, P., Chauhan, R., Singh, A., Singh, G., & Mishra, D. (2022). Implementation of Buckingham's Pi theorem using Python. Advances in Engineering Software, 173, 103232.

[5] Schlegel, J. (2022). “Lecture notes: Advanced two-phase flow” Missouri S&T, USA.

[6] Wang, L. S., & Yan, B. H. (2021). The scaling technology in nuclear reactor thermal hydraulic. Annals of Nuclear Energy, 161, 108440.

[7] Carbiener, W. A., & Cudnik, R. A. (1969). Similitude Consideration for Modeling Nuclear Reactor Blowdowns. Battelle Columbus Labs., Ohio.

[8] Nahavandi, A. N., Castellana, F. S., & Moradkhanian, E. N. (1979). Scaling laws for modeling nuclear reactor systems. Nuclear science and engineering, 72(1), 75-83.

[9] Yun, B. J., Cho, H. K., Euh, D. J., Song, C. H., & Park, G. C. (2004). Scaling for the ECC bypass phenomena during the LBLOCA reflood phase. Nuclear Engineering and Design, 231(3), 315-325.

[10] Deng, C., Zhang, X., Yang, Y., & Yang, J. (2019). Research on scaling design and applicability evaluation of integral thermal-hydraulic test facilities: A review. Annals of Nuclear Energy, 131, 273-290.

References

[11] Ishii, M., & Kataoka, I. (1983). Similarity analysis and scaling criteria for LWRs under single-phase and two-phase natural circulation (No. NUREG/CR-3267; ANL-83-32). Argonne National Lab., IL (USA).

[12] Ishii, M., & Kataoka, I. (1984). Scaling laws for thermal-hydraulic system under single phase and two-phase natural circulation. Nuclear Engineering and Design, 81(3), 411-425.

[13] Zuber, N. (1991). Appendix D: hierarchical, two-tiered scaling analysis. An Integrated Structure and Scaling Methodology for Severe Accident Technical Issue Resolution, 20555.

[14] Zuber, N., Wilson, G. E., Ishii, M., Wulff, W., Boyack, B. E., Dukler, A. E., ... & Valente, J. (1998). An integrated structure and scaling methodology for severe accident technical issue resolution: development of methodology. Nuclear Engineering and Design, 186(1-2), 1-21.

[15] Zuber, N. (2001). The effects of complexity, of simplicity and of scaling in thermal-hydraulics. Nuclear Engineering and Design, 204(1-3), 1-27.

[16] James E. O'Brien (2022), "Preliminary Ideal Scaling Analysis for the Holtec SMR-160 Integral Effects Test Loop Based on Steady-State Natural Circulation Scaling at 1/6 Length Scale," SMR-160 Development Report, October 2022.

Idaho National Laboratory