INL/CON-23-71452-Revision-0

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January 2023

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Prepared for the
U.S. Department of Energy
Under DOE Idaho Operations Office
Contract DE-AC07-05ID14517

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Abstract—**We develop a methodology based on Bayesian inference over Probabilistic Graphical Models (PGMs) to understand and quantify risk in solar-centered grids using targeted measurements and learned system behavior. Being non-prescriptive but, rather, able to infer system behavior and, ultimately, address risk queries from data, our machine learning-type paradigm is tailored for diverse topologies and threat scenarios often associated with distributed energy generation and photovoltaic distributed energy resources (PV-DERs) in particular. We describe algorithmic processes for: (i) learning the structure of PGMs that result from attack-prone PV-DER-proliferated distribution systems, (ii) quantifying cause-effect relationships, and (iii) evaluating risk queries based on diverse evidence. The contributions are illustrated on a residential grid subject to output impairment attacks on its PV-DER infrastructure.**

Index Terms—**Photovoltaic systems, Power grids, Risk analysis, Probabilistic graphical models, Bayes methods, Machine learning.**

I. Introduction

Present and forthcoming energy landscapes, consisting of numerous and diverse distributed energy resources (DERs), necessitate increased situational awareness and understanding. We indicatively refer to the general problem statements for contemporary grid challenges in the recent National Academy of Engineering study [1], and to [2] for an example effort to building situational awareness capabilities from distributed measurements. Although DERs and related devices (e.g., inverters, storage) are inherently tied to green energy sources, and their ongoing proliferation is not only welcome by stakeholders but also a high-priority global objective, the presence of DERs introduces numerous opportunities for multi-modal faults and/or adversarial behavior to enter the system, affect its operation, and potentially impair critical infrastructure that depends on seamless and reliable power delivery.

Contributing to the trend of building situational awareness and understanding for nontrivial grid phenomena, as well as alongside other recent and contemporary efforts to leverage machine learning and related techniques for smart grid applications [3]–[5], we use contemporary methods for Bayesian learning and inference over probabilistic graphical models (PGMs) to evaluate the condition of solar-centered grids consisting of numerous DERs and loads, with particular focus here on inferring risks of losing power at *critical loads*, or with emphasis on another probabilistic metric of interest for some particular mission. As critical loads we define electricity customers providing essential services (e.g., hospitals, emergency response, water treatment plants), the operation of which can be jeopardized by the volatility of distributed and/or solar-centered power generation. A conceptual illustration of the problem's logic is illustrated by Fig. 1. Among other benefits summarized in Section V, our PGM-based methodology allows for efficient learning of cause-effect relationships with synthesized and/or operational data, as well as intuitively produces effective topologies and dispatchable analytics of great potential benefit in modern grids.

The learning part of the presented algorithm consists of using datasets, collected from simulations (e.g., from Digital Twins) and/or from real operations, that relate key events of interest, chosen via engineering judgement (i.e., health estimation for PV-DERs, voltage measurements across distribution systems, environmental and network conditions, status of critical loads). Cause-effect relationships between events are learned automatically from preprocessed data using Bayesian structure learning algorithms resulting in what we refer to as the effective topology, conceptually illustrated by Fig. 2. Subsequently, cause-effect relationships are further characterized by quantifying conditional probability tables from the same dataset. Finally, the resulting object, having absorbed knowledge on the effective topology and CPTs, is used operationally to perform inference queries on any combination of events of interest using any available evidence, including insightful omnidirectional queries as illustrated by Fig. 3.

Whereas in [6] PGM techniques are used to infer outage locations in distribution grids based on historically-recorded multisource information, but, explicitly, *not* real-time DER-specific events and potential faults and/or attacks, we follow an orthogonal approach motivated by accelerating PV-DER proliferation, trends in data analytics, and the need for greater visibility and fidelity. More aligned with the novel premise of digital twins, where perpetually-updated models are used to operationally mimic complex real assets (in our case, a solar-centered grid), we base our inference engine on grid-wide data driven by DER-specific physics and other phenomena in anticipation of greater penetration of DER-embedded health and state estimation (including Behind-the-Meter techniques), IoT-type networks, and analytics capabilities that will enable

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Work supported by the U.S. DOE Office of Energy Efficiency and Renewable Energy / Solar Energy Technologies Office, under contract number DE-AC07-05ID14517. The views expressed herein do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

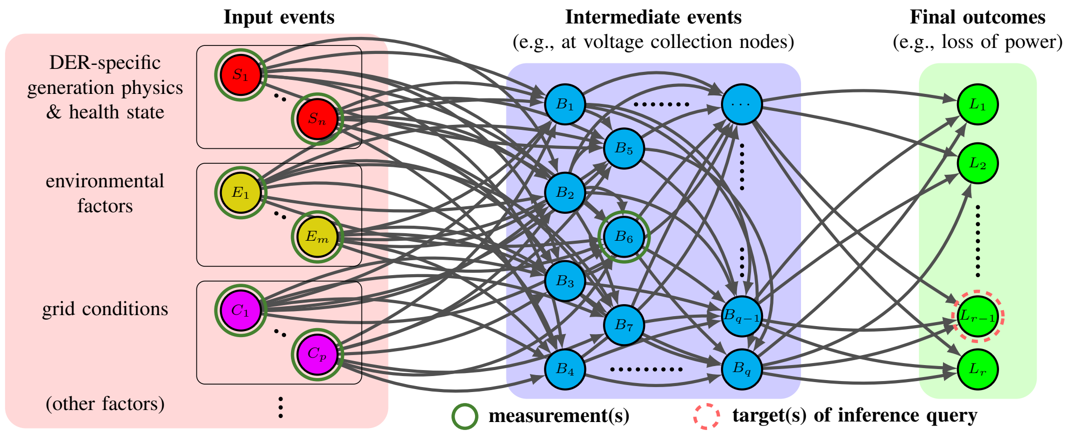


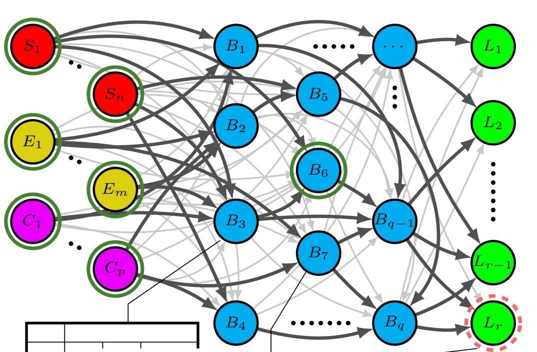
Fig. 1: Conceptual illustration of a *forward* type of analysis for a solar-centered distributed grid, where input events of interest drive processes that ultimately lead to assessments regarding mission-critical final outcomes.

DER-specific inputs, among other diverse evidence, to our learned inference models.

Notation: The *probability* of event A is denoted by $P(A)$; the probability of A *conditioned* on event B is denoted by $P(A|B)$. A *graph* with vertices V and edges $E \subseteq \{(i,j) : (i,j) \in V^2, i \neq j\}$ is denoted by $G = (V,E)$. A graph $G = (V,E)$ is *undirected* if, for all $i,j \in V$, the statement $(i,j) \in E$ (i.e., edge from i to j) implies $(j,i) \in E$; otherwise, G is *directed*. A graph $G = (V,E)$ is a *Directed Acyclic Graph (DAG)* if G is a directed graph and no path starting from any $i^* \in V$ forms a cycle by including i^* more than once.

II. PGMs and Bayesian methods

A Directed Acyclic Graph (DAG) $G = (X,E)$, defined on events $X = \{x_1, \dots, x_n\}$, accompanied with the global distribution Θ of X, and where each $x_i \in X$ is conditioned only on events x_j such that $(x_j, x_i) \in E$, except for x_i which have no incoming edges, constitutes a *Probabilistic Graphical Model* (PGM), which we hereby denote as $B = (G,\Theta)$. When X consists of discrete-valued events that follow a multinomial distribution (e.g., $X_i = \{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}$), Θ can be conveniently expressed by a set of *Conditional Probability Tables*, which relate local discrete-valued distributions for each x_i to values of parent nodes, except for events x_i with no incoming edges, which follow multinomial distributions that are not conditioned on any other event.



	conditions $\mathscr{C}$ at identified parent nodes		
	$(B_{q-1}, B_q) = (\cdot,\cdot)$	$(B_{q-1}, B_q) = (\cdot,\cdot)$	...
$P(L_r = 1\|\mathscr{C})$	0.641	0.327	
$P(L_r = 2\|\mathscr{C})$	0.125	0.550	
...	...	...	
	$\sum(\dots) = 1$	$\sum(\dots) = 1$	...

Fig. 2: Identification of effective topologies showing (strong) cause-effect relationships in a solar-centered distributed grid.

From the preceding description, it is evident that PGMs equip a problem with a rich structure reminiscent of diagnostic or prognostic workflows, the conceptual distinction between which is up to the interpretation assigned to events in X. In addition, the PGM-based structure allows for easy and systematic identification not only of direct, qualitative cause-effect relationships among *parent* and *children* events, but also indirect effects among *sibling* events sharing a common parent.

In a fully specified PGM $B = (G,\Theta)$, one can calculate the probability for any event to attain a particular value, using information from G and Θ. We call such information *prior*. Moving from all "input" events and along the edges E renders such calculations trivial (e.g., one just multiplies CPT elements). Increasingly more tedious calculations allow reasoning on any logical combination of events based on the prior. Fusing the prior with actual measurements or evidence (i.e., to narrow down, in a way, the probabilistic behavior of the system and condition it to events we already know have occurred) and allowing evidence and query targets across the PGM leads us to inference problems:

Problem 1. ***(Bayesian inference on PGMs)*** *Given, for subsets of events from X, evidence $\mathbf{Y} = \cup_{i\in\mathcal{Z}}\{X_i = y_i\}$ and a query*

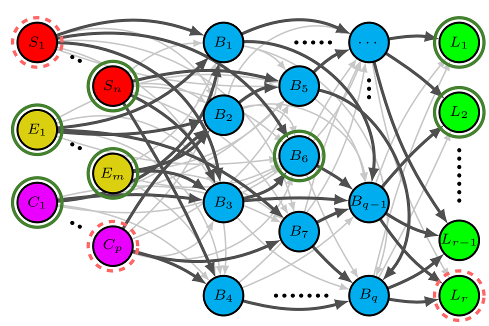


Fig. 3: Example of *omnidirectional* inference, where information from sibling (e.g., L_1, L_2) and other nodes is used to condition events of interest (here: S_1, C_p, L_r). Information about an adjacent load can help condition probabilities for loads for which no information is available. Also, observations throughout the network can help diagnose conditions at input events.

$\mathbf{Q} = \cup_{j \in \mathcal{W}} \{X_j = q_j\}$, *where* $\mathcal{Z}, \mathcal{W} \subseteq X$, *calculate:*

$$P(\mathbf{Q}|\mathbf{Y}) = \left(P(\mathbf{Y}|\mathbf{Q})P(\mathbf{Q})\right)/P(\mathbf{Y}). \tag{1}$$

In principle, Bayesian inference queries can be addressed by repeated use of Bayes formula (i.e., Eq. (1)) across the PGM. Nevertheless, this *exact* inference approach scales unfavorably, from a computational viewpoint. *Approximate* inference based on Monte Carlo methods is known to be more effective in practice for PGMs of realistically useful size and complexity.

Even if one can construct a PGM with expert knowledge and/or with first principles modelling, it is often the case that a problem's PGM-like structure is only suspected but not completely known. Bayesian inference principles, fused with conditional independence tests and various combinatorial algorithms, enable the construction of candidate structures (i.e., the DAG $G = (X, E)$) from data.

Problem 2. ***(Structure learning for PGMs)*** *For a given set of events* X*, let* $\mathcal{D}$ *be a dataset containing realizations of* X*, and* $\mathcal{E}_{\text{forced}}, \mathcal{E}_{\text{forbidden}} \subseteq \{(X_i, X_j) : (X_i, X_j) \in X^2, i \neq j\}$ *be sets of edges. Find the graph* $G = (X, E)$ *which solves:*

$$\max_{G=(X,E)} \quad P(G|\mathcal{D})$$
$$\text{s.t.} \quad G \text{ is DAG}, \;\; \mathcal{E}_{\text{forced}} \subseteq E, \;\; \mathcal{E}_{\text{forbidden}} \cap E = \varnothing,$$

where $\mathcal{E}_{\text{forced}}$ *and* $\mathcal{E}_{\text{forbidden}}$ *are edges that must and cannot, respectively, be in* E*.*

The DAG specification and constraints $\mathcal{E}_{\text{forced}} \subseteq E$, $\mathcal{E}_{\text{forbidden}} \cap E = \varnothing$ define a set of admissible structures for the sought-after graph $G = (X, E)$; all free parameters are then chosen to maximize the objective $P(G|\mathcal{D})$, that is, the probability that graph G resulted from dataset $\mathcal{D}$. Structure learning methods can be divided to constraint-based, score-based, and hybrid algorithms. The relative performance (incl. correctness, speed, scalability) of each class of algorithms depends on the particular problem setup [7].

Problem 3. ***(Parameter learning for PGMs)*** *For a given* X *and DAG* $G = (X, E)$*, let* $\mathcal{D}$ *be a dataset containing realizations of* X*. Find the distribution* Θ *which solves:*

$$\max_{\Theta} \quad P(\Theta|G, \mathcal{D}).$$

Admissible Θ can be constructed by considering the set of local CPTs for each node, according to parent nodes indicated by the DAG G resulting from the solution of Problem 2. Then, the sought-after global distribution is obtained as the maximizer of the objective $P(\Theta|G, \mathcal{D})$, that is, the probability the resulting distribution resulted from dataset $\mathcal{D}$ and DAG G.

Various libraries exist to perform Bayesian inference and learning tasks on PGMs. We indicatively refer to `bnlearn` [8]–[10] in R, and to the `PyMC3` Python package [11].

III. SOLAR-CENTERED DISTRIBUTION SYSTEM MODEL

We consider a distribution network topology consisting of 93 one- and 49 three-phase PV-DERs, with individual capacities taking values in $\{60, 180, 500, 1000, 10.26\}$ kW, 2520 collection nodes, 1138 loads, and one substation. The topology is illustrated by Fig. 4. The substation is assumed to be of finite capacity, so that disturbances in PV-DERs can propagate and cause a measurable effect on the network. Six particular loads distributed across the topology are designated as critical loads. Each critical load is assumed to be equipped with a protection system (e.g., a low-voltage protection scheme) that will automatically shut off its power depending on the voltage levels collected from nearby collection nodes. The –possibly stochastic– behavior of each particular critical load is unique to itself, to better promote the premise of heterogeneity in the network. PV-DERs are logically organized in 6 fleets: A, B, C, D, E, and F reflecting both locality and logical (e.g., ownership and/or management) aspects.

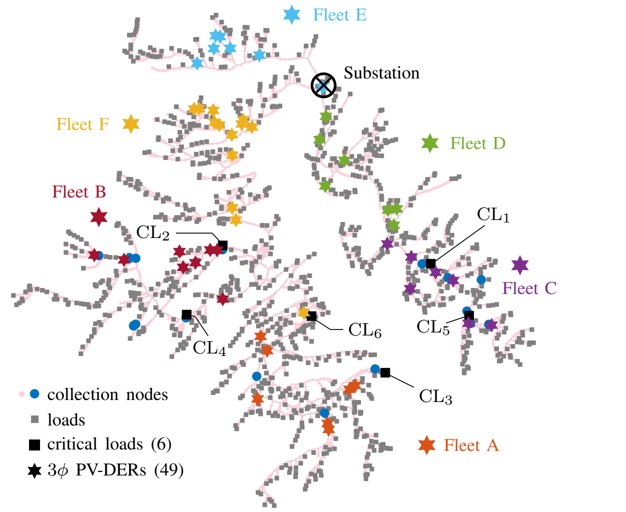


Fig. 4: Topology of the solar-centered distribution system considered in this work, consisting of 142 PV-DERs organized in 6 fleets, 2520 collection nodes, 1138 loads of which 6 are designated as *critical* loads, and 1 substation.

All PV-DERs are vulnerable to output-impairment attacks, which can be thought to be of malicious, accidental, and/or wear-and-tear nature. Such attacks are modeled to act on the output power of each PV-DER given its individual characteristics (e.g., capacity, geometry) for particular environmental conditions (e.g., irradiance, temperature) and setpoint information. Given the finite capacity of the substation and the presence of numerous other network elements, significant attacks can cause voltage violations throughout the distribution system. The propagation of voltage violations and the probabilistic localization of loss of power events given individual DER health are highly nontrivial phenomena that we address with the proposed algorithm. Each PV-DER fleet is equipped with the capability to assess its own health under output-impairment attacks in terms of the ratio of *actually generated* to *nominal* total (i.e., fleet-wide) power, given fleet characteristics, environmental conditions, and setpoint information.

To streamline the presentation, it is hereafter assumed that: (i) PV-DERs are in maximum setpoint tracking mode and (ii) solar irradiance varies between 80% and 100% of its maximum value for the particular locale. It is further assumed that (iii) grid conditions (e.g., power demand) correspond to a busy day. Assumptions (i), (ii), and (iii) enable us to focus on the propagation of attacks through the distribution system and on their effect on particular critical loads, as the environment and the grid correspond to stationary events.

IV. RICE: Risk-Informed Condition Evaluation

Steps of RICE are outlined next and illustrated by Fig. 5.

A. Data source

Similarly to typical supervised statistical machine learning workflows, RICE requires datasets from where it will learn not only the probabilistic behavior but also the influence structure of the underlying system. We define the dataset $\mathcal{D}_c$ as a collection of N instantiations of random variables which correspond to a (nonstrict) superset of the sought-after PGM's events. At this stage, the dataset can contain both continuous (e.g., real numbers) and discrete (e.g., `True`/`False`) values.

It is envisioned that the primary source of such datasets for the class of problems of interest will be simulations of adequate coverage and fidelity. For the system of Section III, such simulations were performed by appropriately augmenting OpenDSS models of the topology to capture effects of output-impairment attacks. Dataset variables cover aggregate DER fleet health (real values in the $[0, 1]$ interval), normalized voltage violation at each of the collection nodes (real values, referenced to nominal voltages under perfect PV-DER health), and loss of power information at each identified critical load (`True`/`False`). For each of the $N = 10,000$ runs, DER health was sampled from the uniform random distribution, whereas the response of downstream topology elements was obtained from the augmented, OpenDSS-based simulation. None of the intermediate probability density/mass functions were explicitly specified; rather, they were implicitly determined by the grid elements and grid at-large dynamics.

B. Preparation

1) Collection node grouping: As expected in any electrical network, adjacent collection nodes will have similar values during transients and steady-state conditions. This can jeopardize our goals to learn effective topologies and, ultimately, PGMs from observations, since cause-effect relationships become obscure (e.g., a particular critical load may appear to have tens or hundred of parent nodes). To retain the discriminating abilities of related algorithms, we group collection nodes by examining pairwise correlation coefficients of the respective dataset values[1]. In what follows, any two collection nodes with data yielding a correlation coefficient greater than or equal to 0.98 were grouped together, resulting in a new fictitious collection node, with numerical values for voltage violation picked arbitrarily from any of the grouped nodes.

2) Event discretization and labelling: Values for DER health events were discretized into 7 levels of uniform size. Labels were assigned incrementally, as shown in Table I. Values for each voltage violation event were discretized into $\ell = 8$ levels designed to contain an equal number of dataset cases; we denote the respective partition scheme by $\mathscr{P}(\mathcal{D}_c, \ell)$.

3) Topology assumptions: We populate $\mathcal{E}_{\text{forbidden}}$ with all edges that would render DER-related events neighbors, thus enforcing the assumption that PV-DERs are independent and do not affect each other directly. Similarly, we forbid direct connections between loads and collection nodes. In addition, we add edges to $\mathcal{E}_{\text{forbidden}}$ that render DER-related events sources (i.e., without incoming edges), and load-related events as roots, to dictate a prognostic PGM. $\mathcal{E}_{\text{forced}}$ is left empty.

C. PGM learning

1) Structure learning: Equipped with the discrete dataset $\mathcal{D}_d$, a collection of events X, and any assumptions on the topology, we proceed and solve Problem 2 from Section II. It was found that, for the scale of problems similar to that of Section III, score-based structure learning algorithms (e.g., Hill Climbing, Tabu Search) performed well and seemingly better than constraint-based and hybrid algorithms.

The learned structure $G = (X, E)$ for the system of Section III, as obtained by the Tabu Search implementation of `bnlearn` [8], is illustrated by Fig. 6.

2) Parameter learning: Given the learned structure $G = (X, E)$, we proceed and solve Problem 3 from Section II with a Bayesian parameter fit for the global distribution Θ over the dataset $\mathcal{D}_d$, resulting in the learned PGM $B = (G, \Theta)$.

D. Operational use

We consider different inference queries on the learned PGM $B = (G, \Theta)$, which we evaluate with Monte Carlo sampling-based approximate Bayesian inference.

■ **Baseline query - uniform health:** Numerical results for the probability to (independently) lose power at each of

[1] Given two N-dimensional observation vectors $A = \{A_i, \ldots, A_N\}$, $B = \{B_1, \ldots, B_N\}$ with means μ_A, μ_B, and standard deviations σ_A, σ_B, we use the following form of the *Pearson correlation coefficient*:

$$\rho(A, B) := \frac{1}{N-1} \sum_{i=1}^{N} \left(\frac{A_i - \mu_A}{\sigma_A}\right) \left(\frac{B_i - \mu_B}{\sigma_B}\right).$$

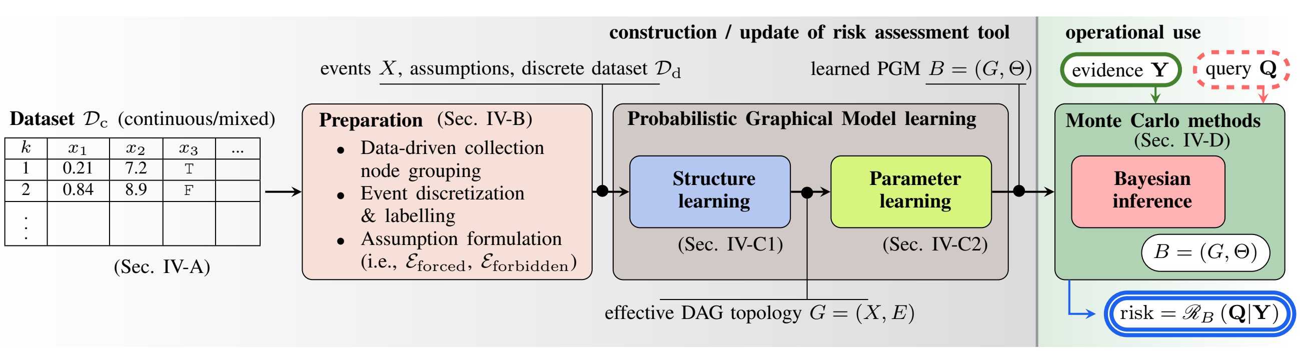


Fig. 5: Steps and main workflow of RICE (Risk-Informed Condition Evaluation).

Critical load	uniform PV-DER fleet physical health levels L1 ... L7						
	L1 (poor PV-DER health)	**L2**	**L3**	**L4**	**L5**	**L6**	**L7** (excellent PV-DER health)
1	0.319	0.224	0.149	0.021	0.005	0.001	0.000
2	0.466	0.322	0.227	0.093	0.069	0.013	0.001
3	0.496	0.363	0.276	0.157	0.052	0.010	0.000
4	0.492	0.387	0.237	0.163	0.082	0.013	0.001
5	0.762	0.585	0.429	0.338	0.197	0.103	0.011
6	0.389	0.272	0.144	0.084	0.016	0.002	0.000

TABLE I: Probability to lose power at each critical load $1, \ldots, 6$, for PV-DER fleet health $h_i \equiv H$ varying uniformly for $i \in \{\mathrm{A}, \ldots, \mathrm{F}\}$ across 7 discrete levels partitioning uniformly the $[0, 1]$ interval: $\{$L1: $H \in [0, 0.143)$, L2: $H \in [0.143, 0.286)$, $\ldots$, L6: $H \in [0.714, 0.857)$, L7: $H \in [0.857, 1]\}$.

the critical loads, as health of all PV-DER fleets is varying uniformly, are given in Table I.

■ **Case I** $(\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{I}})$**:** **[**$\mathbf{Q}_{\mathrm{I}}$**]** Lose power at critical load 5 when **[**$\mathbf{Y}_{\mathrm{I}}$**]** Fleet C is at health level 2 and other fleets (A, B, D, E, F) are at level 4. ► **response:** $P(\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{I}}) = 0.47$ (notice apparent consistency with regards to Table I).

■ **Case II** $(\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{II}})$**:** **[**$\mathbf{Q}_{\mathrm{I}}$**]** when **[**$\mathbf{Y}_{\mathrm{II}}$**]** Fleet C is at health level 1 and all other fleets are at health level 2. ► **response:** $P(\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{II}}) = 0.71$ (notice apparent consistency with regards to Table I and expected deterioration compared to Case I).

■ **Case III** $(\mathbf{Q}_{\mathrm{III}}|\mathbf{Y}_{\mathrm{II}})$**:** **[**$\mathbf{Q}_{\mathrm{III}}$**]** Lose power at critical load 1, when **[**$\mathbf{Y}_{\mathrm{II}}$**]**. ► **response:** $P(\mathbf{Q}_{\mathrm{III}}|\mathbf{Y}_{\mathrm{II}}) = 0.27$.

■ **Case IV** $(\mathbf{Q}_{\mathrm{I}}$ AND $\mathbf{Q}_{\mathrm{III}}|\mathbf{Y}_{\mathrm{II}})$**:** ► **response:** $P(\mathbf{Q}_{\mathrm{I}}$ AND $\mathbf{Q}_{\mathrm{III}}|\mathbf{Y}_{\mathrm{II}}) = 0.22$ (as expected from probability laws for non-mutually exclusive events).

■ **Case V** $(\mathbf{Q}_{\mathrm{III}}$ AND NOT $\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{II}})$**:** ► **response:** $P(\mathbf{Q}_{\mathrm{III}}$ AND NOT $\mathbf{Q}_{\mathrm{I}}|\mathbf{Y}_{\mathrm{II}}) = 0.07$ (i.e., it is an unlikely event to have power in one load and not in another when both depend on the same voltage node group).

Each individual query was completed in 1 second or less, on a system with a 2.3 GHz CPU and 64 GB RAM. However, it is strongly expected that the Monte Carlo methods for addressing inference queries do not need that computational horsepower and can easily be embedded on low-power hardware.

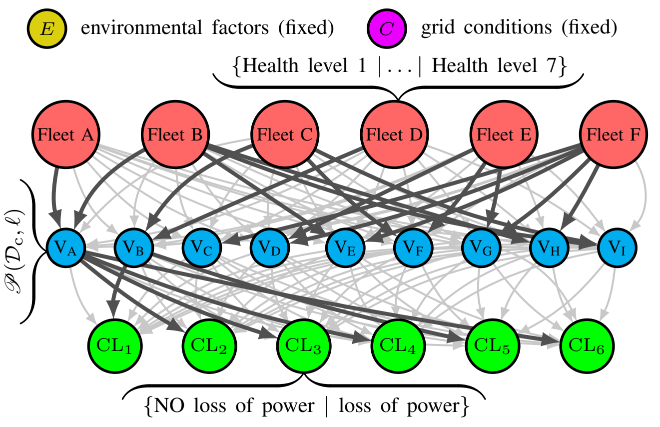


Fig. 6: Network structure or *effective topology* learned for the model of Section III. Comparing against Figure 4 shows that PV-DER fleets near the substation have no actual effect on voltage levels associated with critical loads. Also, given the particular connectivity among collection nodes and many of them moving closely together (according to their groups), PV-DER fleet effects on critical loads are lumped together, creating interesting structures that are influenced by the topology, capacities, load behavior, and other factors.

V. Conclusion

We presented RICE, an algorithm and associated methodology based on Bayesian learning on PGMs to assess risk in attack prone solar-centered distributed grids. It is worth noting that graphical model learning is fundamentally different than fitting neural networks or other function approximators in a typical statistical learning context, even if workflows appear to have some similarity. Whereas a neural network can be configured to approximate certain aspects of distributions, it cannot, typically, be equipped with the flexible reasoning power to address diverse and, as referred to above, omnidirectional inference queries, without a priori constructed separate approximators for each query of interest. It is equally worth noting that the proposed methodology and its envisioned extensions go well beyond targeted or large-scale sensitivity analyses, given the built-in capabilities to understand and quantify underlying structures and cause-effect relationships, as well as the *dispatchability* of the resulting analytics and the associated value to operators and grid stakeholders.

Future work is expected to focus on multi-modal attacks (e.g., physical and cyber), transient phenomena (as opposed to purely quasi-steady state case considered here), consideration of more variables (e.g., environmental, grid conditions, that were fixed herein), and scalability analyses.

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