Bayesian Inference for Time Trends in Parameter Values: Case Study for the Ageing PSA Network of the European Commission

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Abstract: There is a nearly ubiquitous assumption in PSA that parameter values are at least piecewise-constant in time. As a result, Bayesian inference tends to incorporate many years of plant operation, over which there have been significant changes in plant operational and maintenance practices, plant management, etc. These changes can cause significant changes in parameter values over time; however, failure to perform Bayesian inference in the proper time-dependent framework can mask these changes. Failure to question the assumption of constant parameter values, and failure to perform Bayesian inference in the proper time-dependent framework were noted as important issues in NUREG/CN-6813, performed for the U. S. Nuclear Regulatory Commission’s Advisory Committee on Reactor Safeguards in 2003. That report noted that “industry lacks tools to perform time-trend analysis with Bayesian updating.” This paper describes an application of time-dependent Bayesian inference methods developed for the European Commission Ageing PSA Network. These methods utilize open-source software, implementing Markov chain Monte Carlo sampling. The paper also illustrates the development of a generic prior distribution, which incorporates multiple sources of generic data via weighting factors that address differences in key influences, such as vendor, component boundaries, conditions of the operating environment, etc.

Keywords: PRA, aging, time-dependent reliability, Bayesian inference.

1. INTRODUCTION

There is a common assumption in almost all probabilistic safety assessment (PSA) models that parameters in reliability models are constant over time (failure rate, probability of failure, etc.). As a result, Bayesian inference tends to incorporate many years of plant operation, over which there have been significant changes in plant operational and maintenance practices, plant management, etc. These changes can cause significant changes in parameter values over time; however, failure to perform Bayesian inference in the proper time-dependent framework can mask these changes. Failure to question the assumption of constant parameter values, and failure to perform Bayesian inference in the proper time-dependent framework were noted as important issues in (1), performed for the U. S. Nuclear Regulatory Commission’s Advisory Committee on Reactor Safeguards. That report noted that “industry lacks tools to perform time-trend analysis with Bayesian updating.”

In different PSA models there are different time windows over which reliability parameters are often assumed to be constant. Some of these PSA time windows can span significant changes in plant operational and maintenance practices, plant management, etc. As an example, equipment failure criteria, as well as reporting practices could be changed and these changes could impact the data being collected for parameter estimation. Changes to maintenance method and procedures could also impact component/equipment reliability. Hence PSA models with reliability parameters assumed to be constant over long time periods could misrepresent the actual state of the plant, which may be quite dynamic.

Another important factor that could lead to variation in the values of reliability parameters is the aging of components. Because of aging, the value of reliability parameters at the current phase of the component lifetime may be significantly higher than a constant value estimated considering the

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lifetime as a whole. Different plant components have different lifetimes and the component reliability may be varying considerably over the component lifetime, due to burn-in and wear-out. Such variations in reliability parameters can lead to changes in the risk profile over plant life, and can thus affect risk-informed decision-making.

Particularly in the case of older nuclear power plants it may be more reasonable to use instantaneous values of reliability parameters rather than constant average values. To do this one needs to perform trend analysis of reliability parameters using plant-specific data. However, a statistical trend analysis is often not sufficient by itself, due to the sparseness of plant-specific data. Quantitative trend analysis of reliability parameters using plant-specific data can be supplemented by the qualitative analysis of operational experience, taking into account all major changes during the assessed time window. This also implies that the data sources employed for the analysis (both specific and generic) should be as specific and applicable to the component under consideration as possible.

2. TIME-DEPENDENT BAYESIAN INFERENCE

Component specific data required for trending reliability parameters is often sparse or unavailable, and thus generic data is needed as a supplement. There are multiple sources of reliability data for nuclear power plant equipment. Most of these sources provide reliability data in terms of reliability parameter estimates that are constant over time (2), (3). Such sources would be inapplicable for trend identification and would require at least representative generic trends. Therefore it would be more efficient to use year-to-year reliability data from similar plants. In other words there is an advantage to using operating experience of similar plants for specific PSA purposes.

A case study of VVER-440 component age-dependent reliability data was performed under the framework of the European Commission Joint Research Centre (EC JRC) Ageing PSA (APSA) Network. This case study employed time-dependent Bayesian inference methods described in the EC JRC guideline on aging data analysis (4). These methods utilize OpenBUGS, open-source Bayesian inference software, implementing Markov chain Monte Carlo (MCMC) sampling (5). Time dependent trend calculations were performed for parameters in Poisson and binomial aleatory failure models to check the reasonableness of the assumption that the aleatory model parameter ($\rho$ or $\lambda$) is a specific function of time (constant, linear, log-linear, power-law, etc.). The methods applied in the case study support calculations for plant-specific data, and also allow for plant-specific data to be supplemented with generic information.

In the case study mentioned above, plant-specific and generic data were binned together. However when factoring both plant-specific and generic data into the analysis, one must take into account possible differences in the equipment being assessed for a trend. In that case one may use the generic information to develop a prior distribution, which can then be combined with the plant-specific component data. Additionally, one may also have to account for variability among the generic sources, when more than one source is used. This variability corresponds mostly to differences in vendor, component boundaries, operational/environmental conditions, and technical parameters of the component (equipment, system). These are the key factors which could influence the applicability of information used in the analysis.

3. CONSIDERATION OF KEY INFLUENCING FACTORS

The following list of key factors influencing the applicability of generic data is not exhaustive, but it illustrates important factors to consider when utilizing generic data for time-dependent Bayesian inference, and provides suggestions for future detailed consideration.

Selected components for which generic information was collected were assumed to be similar from the standpoint of performed function, design, and operational parameters. However, there are other factors besides these that need to be considered. The following is a more complete list of key factors that may influence the applicability of generic data:
Qualitative scoring of key influencing factors could vary with the type of component, and separate scales may need to be developed for different components. An example scoring scheme is shown in Table 1. The purpose of this example is to demonstrate the notion of factor scoring and estimating the applicability level of generic data based on those factors. As an example of generic data applicability, the key factors were assessed for the main feedwater pump. Each factor is assessed as to whether its influence is high, medium, or low, and the resultant score from Table 1 is assigned for that factor.

### Table 1: Qualitative grading of key influencing factors for Main Feedwater Pump

<table>
<thead>
<tr>
<th>Qualitative parameters</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vendor/type</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Component boundaries</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Environment</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Critical/non-critical failures</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Operational conditions (e.g. medium, pressure, temperature)</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Operational load</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Operational parameters (e.g. flow rate)</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The scoring system in Table 1 was applied to the main feedwater pumps at the Armenian nuclear power plant. Armenian plant-specific data was considered, with data from Dukovany and Paks taken as sources of generic information. Table 2 presents a comparison of the scoring of these two sources for applicability to the Armenian plant, using the key factors and the scoring scale in Table 1. As this table indicates, the Dukovany pumps show more difference (accordingly showing higher scores on key factors) than the Paks pumps.

### 3.1. Use of Weighting Factors for Data Applicability in Bayesian Inference

Data for the Armenian main feedwater pumps are relatively sparse, so a model with constant demand failure probability \( p \) over the life of the plant is reasonable, considering only the plant-specific data. However, time-dependent assessment shows that there is some evidence of an increasing trend in \( p \) over time. Because of the sparseness of the data, it is desired to supplement the Armenian data with data from similar plants (Paks and Dukovany).

A complementary log-log link function was used to describe the time-dependence in \( p \) at Paks and Dukovany.\(^2\) For more details on generalized linear models for binomial data, see (6). This gives the following equation for \( p \) at each of these plants as a function of time, \( t \):

\[
p(t) = 1 - \exp[-\exp(a + bt)]
\]

\(^2\) For small values of \( p \), the choice of link function is not crucial as the value of the complementary log-log is approximately equal to the value of the logit which is the canonical link function.
The parameters of this model are $a$ and $b$. If $b < (>) 0$, there is a decreasing (increasing) trend in $p$ with time, $t$. A decreasing trend was found for both Paks and Dukovany, and the resulting models, were checked for reasonableness using the posterior predictive checks described in (7). The models, while not outstanding, were reasonable at replicating the observed data, and better than models with no time variation in $p$. The joint posterior distributions of $a$ and $b$ for Paks and Dukovany were used to construct a mixture prior distribution for the Armenian plant, using the scores in Table 2. The mixing coefficients were calculated from the scores in Table 2 according to the following equations.

$$W_i = \frac{1}{\sum_{k=1}^{2} \frac{1}{\sum_{j} F_{k,j}}} \quad W_2 = \frac{1}{\sum_{k=1}^{2} \frac{1}{\sum_{j} F_{k,j}}}$$

Using the scores in Table 2 we find

$$W_{Paks} = \frac{1}{\frac{6}{1} + \frac{1}{14}} = 0.7 \quad W_{Dukovany} = \frac{1}{\frac{14}{1} + \frac{1}{14}} = 0.3$$

\[3\] There is a strong negative correlation between $a$ and $b$ in the joint posterior distribution. The marginal posterior distributions for $a$ and $b$ are approximately normal, and the joint posterior distribution is approximately bivariate normal. OpenBUGS accounts for this correlation automatically.
<table>
<thead>
<tr>
<th>Qualitative parameters</th>
<th>Armenian plant</th>
<th>Paks</th>
<th>Dukovany Score</th>
<th>Dukovany Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vendor/type</td>
<td>PE-850-65 (Soviet)</td>
<td>PE850-65-2U4 (Soviet/Yugoslavia)</td>
<td>Sigma (Czech Republic) 250 KHX 415-27-4-UC-01</td>
<td>3</td>
</tr>
<tr>
<td>Component boundaries</td>
<td>Breaker within pump boundaries</td>
<td>Does not include voltage supply breakers</td>
<td>Does not include voltage supply breakers</td>
<td>3</td>
</tr>
<tr>
<td>Environment</td>
<td>In turbine hall</td>
<td>In turbine hall</td>
<td>In turbine hall</td>
<td>0</td>
</tr>
<tr>
<td>Operational conditions</td>
<td>Deaerated water, 1590°C, 7.01Mpa.</td>
<td>Deaerated water, 164°C, 7.14Mpa.</td>
<td>Deaerated water, 164°C, 6.6Mpa.</td>
<td>0</td>
</tr>
<tr>
<td>Critical/non-critical failures</td>
<td>12/23</td>
<td>N/A</td>
<td>Comparing to Armenian NPP data there is a slight difference</td>
<td>0</td>
</tr>
<tr>
<td>Operational load</td>
<td>4 of 5 pumps in operation (monthly switch to backup)</td>
<td>4 of 5 pumps in operation (monthly switch to backup)</td>
<td>4 of 5 pumps in operation (monthly switch to backup)</td>
<td>0</td>
</tr>
<tr>
<td>Operational parameters (e.g. flow rate)</td>
<td>850 m³/h</td>
<td>850 m³/h</td>
<td>680 m³/h</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6</strong></td>
<td><strong>14</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The OpenBUGS package (5) was used to perform Bayesian inference for this model. Independent diffuse prior distributions were used for $a$ and $b$ at Paks and Dukovany. Three MCMC chains were used, with initial values dispersed around the maximum likelihood estimates of $a$ and $b$. The cut() function was used to remove feedback from the Armenian nuclear plant data on the estimates of $a$ and $b$ for Paks and Dukovany. The script is shown in the appendix. The mean of $b$ for the Armenian plant was estimated to be slightly negative, -0.12/yr, with a 95% credible interval of (-0.35, 0.10). The posterior distribution is shown in Figure 1 below. Figure 2 shows a box plot of $p$ in each year, illustrating the slight decreasing trend that results from the analysis, along with the increasing uncertainty in $p$. Because of this increasing uncertainty, the posterior median of $p$ decreases monotonically, while the posterior mean actually increases somewhat over time.
4. CONCLUSIONS

This paper has illustrated time-dependent Bayesian inference using modern, open-source software that implements MCMC sampling. It has also illustrated the development of weighting factors based on consideration of key influencing factors, which allow PSA parameter estimation to be carried out in a more precise and informed manner with regard to use of generic data, which may be an important consideration in PSAs performed for new or newly designed plants, for which little or no plant-specific empirical data are available.

The data used in the example presented in this paper were taken from Armenian, Czech, and Hungarian nuclear power plants of a similar VVER design, and these data were also used in the above-mentioned case-study performed under the auspices of the EC JRC. The results obtained for this example were similar to those derived during the case study. However, this paper extends the case study by incorporating both quantitative and qualitative assessments into the Bayesian inference.

The paper has attempted to highlight important operational issues to be considered while performing data analysis in support of PSA. In comparison with standard data analysis methods utilized in PSA, this example has attempted to incorporate important operational factors into the parameter estimation process. Unfortunately, the sparse empirical data for developing and updating the prior distribution do not lead to a clear result in terms of a time-dependent trend for the Armenian plant. Nonetheless, the methods can be employed using freely available, open-source software. More work on methods to incorporate such operational factors into the parameter estimation process is suggested so that the PSA could include these factors. Such work could be very helpful for new designs, as it addresses the question of the applicability of data from current designs to new designs, for which plant-specific empirical data are lacking.
Figure 2  95% credible intervals for failure probability vs. time at the Armenian plant, illustrating slight decreasing trend over time and increasing uncertainty, which causes mean to increase slightly. Age increases from top to bottom.

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References

Appendix

This appendix lists the OpenBUGS script used in the analyses in the main body of the paper.

```openbugs
model {
  for(i in 1:N.Paks) {
    x.Paks[i] ~ dbin(p.Paks[i], n.Paks[i])
    cloglog(p.Paks[i]) <- a[1] + b[1]*i
  }
  for(j in 1:N.Duk) {
    x.Duk[j] ~ dbin(p.Duk[j], n.Duk[j])
  }
  for(m in 1:2) {
    a.cut[m] <- cut(a[m])
    b.cut[m] <- cut(b[m])
    a[m] ~ dflat()
    b[m] ~ dflat()
  }
  a.anpp <- a.cut[q]
  b.anpp <- b.cut[q]
  q ~ dcat(w[])
  for(k in 1:N.ANPP) {
    x.anpp[k] ~ dbin(p.anpp[k], n.anpp[k])
    cloglog(p.anpp[k]) <- a.anpp + b.anpp*age.anpp[k]
    x.rep.anpp[k] ~ dbin(p.anpp[k], n.anpp[k])
    diff.obs.anpp[k] <- pow(x.anpp[k] - n.anpp[k]*p.anpp[k], 2)/(n.anpp[k]*p.anpp[k]*(1-p.anpp[k]))
    diff.rep.anpp[k] <- pow(x.rep.anpp[k] - n.anpp[k]*p.anpp[k], 2)/(n.anpp[k]*p.anpp[k]*(1-p.anpp[k]))
  }
  chisq.obs.anpp <- sum(diff.obs.anpp[])
  chisq.rep.anpp <- sum(diff.rep.anpp[])
  p.value.anpp <- step(chisq.rep.anpp - chisq.obs.anpp)
}
```